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Comparison of surface wave tomography results using the first order Tikhonov regularization and Yanovskaya-Ditmar (1990) method

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Extended Abstract

Summary

Two-dimensional (2-D) surface wave tomography is one of the most practical approach to reveal the lateral variation of phase/group velocity. Hence, various methods have been introduced to attain the more accurate images. In this paper, we compare two different tomography methods (Tikhonov regularization method and Yanovskaya-Ditmar method) and try to show their advantages and drawbacks resulted from their basic assumptions. We first investigate the lateral resolution capability of the methods using the synthetic checkerboard test in cases of noise free and noise level of 3%. Synthetic models are considered with different grid spacings of 0.5˚×0.5˚, 1˚×1˚ and 2˚×2˚. Then, they were applied on real data containing phase velocity obtained from the teleseismic Rayleigh waves recorded at broad band stations located in Iran at the period of 30 s. A comparison of the results obtained from the Tikhonov regularization and Yanoskaya-Ditmar methods shows that the former method has a better lateral resolution than the latter method.

Introduction

Surface wave tomography has been used as a very popular method to determine lateral variations of phase or group velocity. Due to special conditions of data, surface wave travel time tomography is an ill-conditioned inverse problem and has non-unique solution. The conditions arise from the fact that the observed data are contaminated by noise, and also, distributed non-uniformly. Uneven data coverage in tomography imposes two different systems on the inverse problem. The over-determined system is defined for portion of the study area with high density ray, and the under-determined system is used where there is no enough data. In such case, regularization is exploited to choose a suitable model. In this paper, we use the method of Yanovskaya and Ditamr (1990), based on Backus-Gilbert approach, and compare its results with those estimated from Tikhonov regularization method (Tikhonov et al., 1977) in two cases of synthetic and real data.

Methodology and Approaches

Under ray theory, in surface wave travel time tomography problem, we incorporate the measured frequency-dependent travel times of surface wave phase/group velocity along their rays with formulation of $t_i(\omega) = \int_{l_i} S_i(l.\omega)dl$, where $t_i(\omega)$ is the travel time of *i*th ray path with phase/group slowness S_i along the ray path l_i at angular frequency ω. After discretization, matrix form of the problem is d=Gm+e, where $d \in \mathbb{R}^m$ is data, $G \in \mathbb{R}^{m\times n}$ is the forward operator, $m \in \mathbb{R}^n$ is model and $e \in \mathbb{R}^m$ is noise. The general form of Tikhonov regularization is expressed as: $\underset{m \in \mathbb{R}^n}{\operatorname{argmin}}\{\|d - Gm\|_2^2 + \lambda\|Lm\|_2^2\}$, where $\lambda \in \mathbb{R}^+$ as a regularization parameter controls trade-off between two terms and L denotes the regularization operator, which is the first-order derivative in this study. The problem is quadratic, and thus, its minimizer *m* is defined explicitly by the associated normal equations $(G^TG + \lambda L^TL)m = G^Td$, which can be solved directly.

In the Yanoskaya-Ditmar method, the main goal is the optimization of $\sum\left[\delta t_i - \iint_\Omega G_i(r)m(r)d(r)\right]^2 + \alpha\iint_\Omega |\nabla m(r)|^2 dr$, where δt_i is the travel time residual along *i*th ray path, $G_i(r)$ is kernel that is different from zero in vicinity of the ray, $m(r)$ is the relative slowness defined as $\frac{[V(r)^{-1} - V_0^{-1}]}{V_0^{-1}} = -\frac{\delta V(r)}{V_0}$, where V_0 is the velocity in the initial approximation model, α is a regularization parameter and $\nabla m(r)$ is the gradient of model.

Results and Conclusions

In this study, we performed two methods of tomography on synthetic and real data. The results obtained from synthetic test show that in two cases of noise free and noise level of 3%, the Tikhonov regularization method is more capable in the recovery of all three synthetic models in comparison with the Yanovskaya-Ditmar method. It should be mentioned that the synthetic checker board test model with grid spacing of $0.5^{\circ}\times0.5^{\circ}$ is poorly recovered by both methods. It refers that apart from the method of inversion, the ray coverage of the study area does not suit this grid spacing. Tomography maps obtained using real data are compared with the previous studies. In comparison with the results of Yanovskaya-Ditmar method, the results of Tikhonov inversion show more similarities with other studies, especially for small features. The lower lateral resolution of the results obtained from the Yanovskaya-Ditmar method can be explained by the imposed error resulted from its assumptions and consideration of Fresnel zone in a calculation of slowness along the ray path.