



Symbolic Generation of Adomian Polynomials for Different Nonlinearities by Python

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ARTICLE INFO	ABSTRACT
<p>Article History: Received: 09 November 2023 Revised: 11 January 2024 Accepted: 13 January 2024</p> <p>Article type: Research</p> <p>Keywords: Adomian Decomposition Method, Adomian Polynomials, Differential Equations, Nonlinear Equations, Python</p>	<p>The Adomian decomposition method (ADM) is a powerful mathematical technique to find closed-form solutions to nonlinear functional equations including ODEs, PDEs, differential-difference, integral, integro-differential, algebraic, and transcendental equations or systems of such equations. It features a particular infinite series for the representation of nonlinear terms of the equation under study, referred to as the Adomian polynomials. Nevertheless, the computation of such polynomials manually, devoid of any assistance from computational resources, can often be a laborious and protracted endeavor. In this paper, an innovative Python code is proposed, which exploits the SymPy library to perform the involved symbolic calculus operations to generate the Adomian polynomials of any given nonlinear expressions. The use of the code would substantially facilitate the implementation of the ADM to the equations arising in various branches of science and engineering. A number of nonlinear expressions are decomposed to their relevant Adomian polynomials for the sake of demonstration.</p>

Introduction

Years of studying nature and making efforts to quantify real world problems by means of mathematics has proven that physical and engineering models usually lead to equations that incorporate nonlinearities. The more accuracy is demanded, the more complicated equations are derived. As a result, there is always a need for more efficient solution methods. These methods are exhaustively categorized into two groups: analytical methods and numerical methods. Numerical methods will not provide an exact answer for all the domains of the problem nor for a specific point of the problem domain. In other words, there will be no closed-form solution using the numerical methods. On the other hand, analytical solutions will provide a closed-form solution to the problem. However, they are usually achievable for specific cases, like linear equations or some specific nonlinearities which are rather simple. Therefore, in 1983, George Adomian introduced a potent analytical technique which can be implemented to find the solution to all kinds of nonlinear functional equations, called the Adomian Decomposition

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Method (ADM) [1]. Needless to say, this method can also be used to find the solutions to linear equations. The ADM does not impose any linearization, perturbation or discretization and leads to convergent solutions quickly. Many problems from various branches of science and engineering have been solved by the ADM. Areas that the ADM have been applied in include heat transfer [2-9], population dynamics [10-13], fluid dynamics [14-17], thermodynamics [18-22], applied chemistry [23-27], optics [28, 29], etc. Particularly, the ADM has been applied to problems arising from chemical engineering; for instance, in catalysis and reactor engineering [30], gas absorption [31], bioreactors [32], and electrochemical systems [33]. Particularly, The Hammerstein integral equation manifests in chemical reactor engineering subsequent to employing the Green's function technique and has been proficiently solved using the ADM [34].

As it will be discussed, the ADM requires a particular series representation for the nonlinearities involved in the equation, namely the Adomian Polynomials. Several efforts have been made to derive procedures for computing these polynomials. However, some of them are restricted to only special cases of nonlinearity and many of them involve complexity. A number of algorithms to calculate the Adomian polynomials have been developed so far [35-40]. They consist intricate structures and all are coded in Maple or MATLAB.

The present paper, proposes a code to efficiently compute the Adomian polynomials in Python, an open source, high level, widely used programming language, getting its basic concept from a straightforward technique. Illustrative examples are provided to show the reliability of the program.

Methodology

Consider, without loss of generality, the following functional equation:

$$u - N(u) = f \quad (1)$$

where N is a nonlinear operator on a Banach space E , f is a specified element of E and we are seeking $u \in E$, which satisfies Eq. 1. Assuming that Eq. 1 has a unique solution for every $f \in E$, then the ADM decomposes the solution u as an infinite series $u = \sum_{i=0}^{\infty} u_i$ and the nonlinearity as $N(u) = \sum_{i=0}^{\infty} A_i$, where the A_i are called the Adomian polynomials and are defined as:

$$A_i = \frac{1}{i!} \frac{d^i}{d\lambda^i} N \left(\sum_{k=0}^{\infty} u_k \lambda^k \right) \Big|_{\lambda=0} \quad (2)$$

By selecting the initial solution component as $u_0 = f$, the ADM uses the following expression to generate components of the solution as:

$$\begin{cases} u_0 = f, \\ u_{i+1} = A_i, i \geq 0 \end{cases} \quad (3)$$

The convergence and reliability of the ADM have been ascertained in prior research [41, 42].

Among all the different formulas for the Adomian polynomials, we are going to use the mentioned definitional one, i.e. Eq. 2 due to its simplicity and succinctness. The code will be generated in the Python programming language, which is nowadays really popular among researchers and is implemented globally, since its syntax is very close to human's speech and

is an object-oriented open-source programming language with many practical libraries. (See Appendix)

Illustrative Examples

To illustrate the program results, several Adomian polynomials related to the most frequent, in real-world applications, nonlinearities are presented here. Note that in our code, the variable ‘s’ has been used to express the nonlinearity term $N(s) = s^2$ as an example. Hence, one can replace it with any desired nonlinearity operating on variable ‘s’.

Table 1. First ten Adomian polynomials for $N(u) = u^2$ and $N(u) = u^3$

$N(u) = u^2$	$N(u) = u^3$
$A_0 = u_0^2$	$A_0 = u_0^3$
$A_1 = 2u_0u_1$	$A_1 = 3u_0^2u_1$
$A_2 = 2u_0u_2 + u_1^2$	$A_2 = 3u_0(u_0u_2 + u_1^2)$
$A_3 = 2u_0u_3 + 2u_1u_2$	$A_3 = 3u_0^2u_3 + 6u_0u_1u_2 + u_1^3$
$A_4 = 2u_0u_4 + 2u_1u_3 + u_2^2$	$A_4 = 3u_0^2u_4 + 6u_0u_1u_3 + 3u_0u_2^2 + 3u_1^2u_2$
$A_5 = 2u_0u_5 + 2u_1u_4 + 2u_2u_3$	$A_5 = 3u_0^2u_5 + 6u_0u_1u_4 + 6u_0u_2u_3 + 3u_1^2u_3 + 3u_1u_2^2$
$A_6 = 2u_0u_6 + 2u_1u_5 + 2u_2u_4 + u_3^2$	$A_6 = 3u_0^2u_6 + 6u_0u_1u_5 + 6u_0u_2u_4 + 3u_0u_3^2 + 3u_1^2u_4 + 6u_1u_2u_3 + u_2^3$
$A_7 = 2u_0u_7 + 2u_1u_6 + 2u_2u_5 + 2u_3u_4$	$A_7 = 3u_0^2u_7 + 6u_0u_1u_6 + 6u_0u_2u_5 + 6u_0u_3u_4 + 3u_1^2u_5 + 6u_1u_2u_4 + 3u_1u_3^2 + 3u_2^2u_3$
$A_8 = 2u_0u_8 + 2u_1u_7 + 2u_2u_6 + 2u_3u_5 + u_4^2$	$A_8 = 3u_0^2u_8 + 6u_0u_1u_7 + 6u_0u_2u_6 + 6u_0u_3u_5 + 3u_0u_4^2 + 3u_1^2u_6 + 6u_1u_2u_5 + 6u_1u_3u_4 + 3u_2^2u_4 + 3u_2u_3^2$
$A_9 = 2u_0u_9 + 2u_1u_8 + 2u_2u_7 + 2u_3u_6 + 2u_4u_5$	$A_9 = 3u_0^2u_9 + 6u_0u_1u_8 + 6u_0u_2u_7 + 6u_0u_3u_6 + 6u_0u_4u_5 + 3u_1^2u_7 + 6u_1u_2u_6 + 6u_1u_3u_5 + 3u_1u_4^2 + 3u_2^2u_5 + 6u_2u_3u_4 + u_3^3$

Table 2. First ten Adomian polynomials for $N(u) = u^4$

$N(u) = u^4$
$A_0 = u_0^4$
$A_1 = 4u_0^3u_1$
$A_2 = 2u_0^2(2u_0u_2 + 3u_1^2)$
$A_3 = 4u_0(u_0^2u_3 + 3u_0u_1u_2 + u_1^3)$
$A_4 = 4u_0^3u_4 + 12u_0^2u_1u_3 + 6u_0^2u_2^2 + 12u_0u_1^2u_2 + u_1^4$
$A_5 = 4u_0^3u_5 + 12u_0^2u_1u_4 + 12u_0^2u_2u_3 + 12u_0u_1^2u_3 + 12u_0u_1u_2^2 + 4u_1^3u_2$
$A_6 = 4u_0^3u_6 + 12u_0^2u_1u_5 + 12u_0^2u_2u_4 + 6u_0^2u_3^2 + 12u_0u_1^2u_4 + 24u_0u_1u_2u_3 + 4u_0u_2^3 + 4u_1^3u_3 + 6u_1^2u_2^2$
$A_7 = 4u_0^3u_7 + 12u_0^2u_1u_6 + 12u_0^2u_2u_5 + 12u_0^2u_3u_4 + 12u_0u_1^2u_5 + 24u_0u_1u_2u_4 + 12u_0u_1u_3^2 + 12u_0u_2^2u_3 + 4u_1^3u_4 + 12u_1^2u_2u_3 + 4u_1u_2^3$
$A_8 = 4u_0^3u_8 + 12u_0^2u_1u_7 + 12u_0^2u_2u_6 + 12u_0^2u_3u_5 + 6u_0^2u_4^2 + 12u_0u_1^2u_6 + 24u_0u_1u_2u_5 + 24u_0u_1u_3u_4 + 12u_0u_2^2u_4 + 12u_0u_2u_3^2 + 4u_1^3u_5 + 12u_1^2u_2u_4 + 6u_1^2u_3^2 + 12u_1u_2^2u_3 + u_2^4$



$$A_9 = 4u_0^3u_9 + 12u_0^2u_1u_8 + 12u_0^2u_2u_7 + 12u_0^2u_3u_6 + 12u_0^2u_4u_5 + 12u_0u_1^2u_7 + 24u_0u_1u_2u_6 + 24u_0u_1u_3u_5 + 12u_0u_1u_4^2 + 12u_0u_2^2u_5 + 24u_0u_2u_3u_4 + 4u_0u_3^3 + 4u_1^3u_6 + 12u_1^2u_2u_5 + 12u_1^2u_3u_4 + 12u_1u_2^2u_4 + 12u_1u_2u_3^2 + 4u_2^3u_3$$

Table 3. First ten Adomian polynomials for $N(u) = e^u$,

$N(u) = e^u$
$A_0 = e^{u_0}$
$A_1 = u_1 e^{u_0}$
$A_2 = \frac{(u_1^2 + 2u_2) e^{u_0}}{2}$
$A_3 = \frac{(u_1^3 + 6u_1u_2 + 6u_3) e^{u_0}}{6}$
$A_4 = \frac{(u_1^4 + 12u_1^2u_2 + 24u_1u_3 + 12u_2^2 + 24u_4) e^{u_0}}{24}$
$A_5 = \frac{(u_1^5 + 20u_1^3u_2 + 60u_1^2u_3 + 60u_1u_2^2 + 120u_1u_4 + 120u_2u_3 + 120u_5) e^{u_0}}{120}$
$A_6 = \frac{(u_1^6 + 30u_1^4u_2 + 120u_1^3u_3 + 180u_1^2u_2^2 + 360u_1^2u_4 + 720u_1u_2u_3 + 720u_1u_5 + 120u_2^3 + 720u_2u_4 + 360u_3^2 + 720u_6) e^{u_0}}{720}$
$A_7 = \frac{\left(u_1^7 + 42u_1^5u_2 + 210u_1^4u_3 + 420u_1^3u_2^2 + 840u_1^3u_4 + 2520u_1^2u_2u_3 + 2520u_1^2u_5 + 840u_1u_2^3 + 5040u_1u_2u_4 + 2520u_1u_3^2 + 5040u_1u_6 + 2520u_2^2u_3 + 5040u_2u_5 + 5040u_3u_4 + 5040u_7 \right) e^{u_0}}{5040}$
$A_8 = \frac{\left(u_1^8 + 56u_1^6u_2 + 336u_1^5u_3 + 840u_1^4u_2^2 + 1680u_1^4u_4 + 6720u_1^3u_2u_3 + 6720u_1^3u_5 + 3360u_1^2u_2^3 + 20160u_1^2u_2u_4 + 10080u_1^2u_3^2 + 20160u_1^2u_6 + 20160u_1u_2^2u_3 + 40320u_1u_2u_5 + 40320u_1u_3u_4 + 40320u_1u_7 + 1680u_2^4 + 20160u_2^2u_4 + 20160u_2u_3^2 + 40320u_2u_6 + 40320u_3u_5 + 20160u_4^2 + 40320u_8 \right) e^{u_0}}{40320}$
$A_9 = \frac{\left(u_1^9 + 72u_1^7u_2 + 504u_1^6u_3 + 1512u_1^5u_2^2 + 3024u_1^5u_4 + 15120u_1^4u_2u_3 + 15120u_1^4u_5 + 10080u_1^3u_2^3 + 60480u_1^3u_2u_4 + 30240u_1^3u_3^2 + 60480u_1^3u_6 + 90720u_1^2u_2^2u_3 + 181440u_1^2u_2u_5 + 181440u_1^2u_3u_4 + 181440u_1^2u_7 + 15120u_1u_2^4 + 181440u_1u_2^2u_4 + 181440u_1u_2u_3^2 + 362880u_1u_2u_6 + 362880u_1u_3u_5 + 181440u_1u_4^2 + 362880u_1u_8 + 60480u_2^3u_3 + 181440u_2^2u_5 + 362880u_2u_3u_4 + 362880u_2u_7 + 60480u_3^3 + 362880u_3u_6 + 362880u_4u_5 + 362880u_9 \right) e^{u_0}}{362880}$

Table 4. First ten Adomian polynomials for $N(u) = \frac{au}{b+u}$

$N(u) = \frac{au}{b+u}$	
$A_0 = \frac{au_0}{b+u_0}$	
$A_1 = \frac{abu_1}{(b+u_0)^2}$	
$A_2 = \frac{ab(bu_2 + u_0u_2 - u_1^2)}{(b+u_0)^3}$	
$A_3 = \frac{ab}{(b+u_0)^4} (b^2u_3 + 2bu_0u_3 - 2bu_1u_2 + u_0^2u_3 - 2u_0u_1u_2 + u_1^3)$	
$A_4 = \frac{ab}{(b+u_0)^5} \left(b^3u_4 + 3b^2u_0u_4 - 2b^2u_1u_3 - b^2u_2^2 + 3bu_0^2u_4 - 4bu_0u_1u_3 - 2bu_0u_2^2 \right. \\ \left. + 3bu_1^2u_2 + u_0^3u_4 - 2u_0^2u_1u_3 - u_0^2u_2^2 + 3u_0u_1^2u_2 - u_1^4 \right)$	
$A_5 = \frac{ab}{(b+u_0)^6} \left(b^4u_5 + 4b^3u_0u_5 - 2b^3u_1u_4 - 2b^3u_2u_3 + 6b^2u_0^2u_5 - 6b^2u_0u_1u_4 \right. \\ \left. - 6b^2u_0u_2u_3 + 3b^2u_1^2u_3 + 3b^2u_1u_2^2 + 4bu_0^3u_5 - 6bu_0^2u_1u_4 \right. \\ \left. - 6bu_0^2u_2u_3 + 6bu_0u_1^2u_3 + 6bu_0u_1u_2^2 - 4bu_1^3u_2 + u_0^4u_5 - 2u_0^3u_1u_4 \right. \\ \left. - 2u_0^3u_2u_3 + 3u_0^2u_1^2u_3 + 3u_0^2u_1u_2^2 - 4u_0u_1^3u_2 + u_1^5 \right)$	
$A_6 = \frac{ab}{(b+u_0)^7} \left(b^5u_6 + 5b^4u_0u_6 - 2b^4u_1u_5 - 2b^4u_2u_4 - b^4u_3^2 + 10b^3u_0^2u_6 - 8b^3u_0u_1u_5 \right. \\ \left. - 8b^3u_0u_2u_4 - 4b^3u_0u_3^2 + 3b^3u_1^2u_4 + 6b^3u_1u_2u_3 + b^3u_2^3 + 10b^2u_0^3u_6 \right. \\ \left. - 12b^2u_0^2u_1u_5 - 12b^2u_0^2u_2u_4 - 6b^2u_0^2u_3^2 + 9b^2u_0u_1^2u_4 + 18b^2u_0u_1u_2u_3 \right. \\ \left. + 3b^2u_0u_2^3 - 4b^2u_1^3u_3 - 6b^2u_1^2u_2^2 + 5bu_0^4u_6 - 8bu_0^3u_1u_5 - 8bu_0^3u_2u_4 \right. \\ \left. - 4bu_0^3u_3^2 + 9bu_0^2u_1^2u_4 + 18bu_0^2u_1u_2u_3 + 3bu_0^2u_2^3 - 8bu_0u_1^3u_3 - 12bu_0u_1^2u_2^2 \right. \\ \left. + 5bu_1^4u_2 + u_0^5u_6 - 2u_0^4u_1u_5 - 2u_0^4u_2u_4 - u_0^4u_3^2 + 3u_0^3u_1^2u_4 + 6u_0^3u_1u_2u_3 \right. \\ \left. + u_0^3u_2^3 - 4u_0^2u_1^3u_3 - 6u_0^2u_1^2u_2^2 + 5u_0u_1^4u_2 - u_1^6 \right)$	
$A_7 = \frac{ab}{(b+u_0)^8} \left(b^6u_7 + 6b^5u_0u_7 - 2b^5u_1u_6 - 2b^5u_2u_5 - 2b^5u_3u_4 + 15b^4u_0^2u_7 - 10b^4u_0u_1u_6 \right. \\ \left. - 10b^4u_0u_2u_5 - 10b^4u_0u_3u_4 + 3b^4u_1^2u_5 + 6b^4u_1u_2u_4 + 3b^4u_1u_3^2 + 3b^4u_2^2u_3 \right. \\ \left. + 20b^3u_0^3u_7 - 20b^3u_0^2u_1u_6 - 20b^3u_0^2u_2u_5 - 20b^3u_0^2u_3u_4 + 12b^3u_0u_1^2u_5 \right. \\ \left. + 24b^3u_0u_1u_2u_4 + 12b^3u_0u_1u_3^2 + 12b^3u_0u_2^2u_3 - 4b^3u_1^3u_4 - 12b^3u_1^2u_2u_3 \right. \\ \left. - 4b^3u_1u_2^3 + 15b^2u_0^4u_7 - 20b^2u_0^3u_1u_6 - 20b^2u_0^3u_2u_5 - 20b^2u_0^3u_3u_4 + 18b^2u_0^2u_1^2u_5 \right. \\ \left. + 36b^2u_0^2u_1u_2u_4 + 18b^2u_0^2u_1u_3^2 + 18b^2u_0^2u_2^2u_3 - 12b^2u_0u_1^3u_4 - 36b^2u_0u_1^2u_2u_3 \right. \\ \left. - 12b^2u_0u_1u_2^3 + 5b^2u_1^4u_3 + 10b^2u_1^3u_2^2 + 6bu_0^5u_7 - 10bu_0^4u_1u_6 - 10bu_0^4u_2u_5 \right. \\ \left. - 10bu_0^4u_3u_4 + 12bu_0^3u_1^2u_5 + 24bu_0^3u_1u_2u_4 + 12bu_0^3u_1u_3^2 + 12bu_0^3u_2^2u_3 \right. \\ \left. - 12bu_0^2u_1^3u_4 - 36bu_0^2u_1^2u_2u_3 - 12bu_0^2u_1u_2^3 + 10bu_0u_1^4u_3 + 20bu_0u_1^3u_2^2 \right. \\ \left. - 6bu_1^5u_2 + u_0^6u_7 - 2u_0^5u_1u_6 - 2u_0^5u_2u_5 - 2u_0^5u_3u_4 + 3u_0^4u_1^2u_5 + 6u_0^4u_1u_2u_4 \right. \\ \left. + 3u_0^4u_1u_3^2 + 3u_0^4u_2^2u_3 - 4u_0^3u_1^3u_4 - 12u_0^3u_1^2u_2u_3 - 4u_0^3u_1u_2^3 + 5u_0^2u_1^4u_3 \right. \\ \left. + 10u_0^2u_1^3u_2^2 - 6u_0u_1^5u_2 + u_1^7 \right)$	



$$A_8 = \frac{ab}{(b+u_0)^9} \left(\begin{aligned} & b^7u_8 + 7b^6u_0u_8 - 2b^6u_1u_7 - 2b^6u_2u_6 - 2b^6u_3u_5 - b^6u_4^2 + 21b^5u_0^2u_8 - 12b^5u_0u_1u_7 \\ & - 12b^5u_0u_2u_6 - 12b^5u_0u_3u_5 - 6b^5u_0u_4^2 + 3b^5u_1^2u_6 + 6b^5u_1u_2u_5 + 6b^5u_1u_3u_4 \\ & + 3b^5u_2^2u_4 + 3b^5u_2u_3^2 + 35b^4u_0^3u_8 - 30b^4u_0^2u_1u_7 - 30b^4u_0^2u_2u_6 - 30b^4u_0^2u_3u_5 \\ & - 15b^4u_0^2u_4^2 + 15b^4u_0u_1^2u_6 + 30b^4u_0u_1u_2u_5 + 30b^4u_0u_1u_3u_4 + 15b^4u_0u_2^2u_4 \\ & + 15b^4u_0u_2u_3^2 - 4b^4u_1^3u_5 - 12b^4u_1^2u_2u_4 - 6b^4u_1^2u_3^2 - 12b^4u_1u_2^2u_3 - b^4u_2^4 \\ & + 35b^3u_0^4u_8 - 40b^3u_0^3u_1u_7 - 40b^3u_0^3u_2u_6 - 40b^3u_0^3u_3u_5 - 20b^3u_0^3u_4^2 + 30b^3u_0^2u_1^2u_6 \\ & + 60b^3u_0^2u_1u_2u_5 + 60b^3u_0^2u_1u_3u_4 + 30b^3u_0^2u_2^2u_4 + 30b^3u_0^2u_2u_3^2 - 16b^3u_0u_1^3u_5 \\ & - 48b^3u_0u_1^2u_2u_4 - 24b^3u_0u_1^2u_3^2 - 48b^3u_0u_1u_2^2u_3 - 4b^3u_0u_2^4 + 5b^3u_1^4u_4 \\ & + 20b^3u_1^3u_2u_3 + 10b^3u_1^2u_2^3 + 21b^2u_0^5u_8 - 30b^2u_0^4u_1u_7 - 30b^2u_0^4u_2u_6 - 30b^2u_0^4u_3u_5 \\ & - 15b^2u_0^4u_4^2 + 30b^2u_0^3u_1^2u_6 + 60b^2u_0^3u_1u_2u_5 + 60b^2u_0^3u_1u_3u_4 + 30b^2u_0^3u_2^2u_4 \\ & + 30b^2u_0^3u_2u_3^2 - 24b^2u_0^2u_1^3u_5 - 72b^2u_0^2u_1^2u_2u_4 - 36b^2u_0^2u_1^2u_3^2 - 72b^2u_0^2u_1u_2^2u_3 \\ & - 6b^2u_0^2u_2^4 + 15b^2u_0u_1^4u_4 + 60b^2u_0u_1^3u_2u_3 + 30b^2u_0u_1^2u_2^3 - 6b^2u_1^5u_3 - 15b^2u_1^4u_2^2 \\ & + 7bu_0^6u_8 - 12bu_0^5u_1u_7 - 12bu_0^5u_2u_6 - 12bu_0^5u_3u_5 - 6bu_0^5u_4^2 + 15bu_0^4u_1^2u_6 \\ & + 30bu_0^4u_1u_2u_5 + 30bu_0^4u_1u_3u_4 + 15bu_0^4u_2^2u_4 + 15bu_0^4u_2u_3^2 - 16bu_0^3u_1^3u_5 \\ & - 48bu_0^3u_1^2u_2u_4 - 24bu_0^3u_1^2u_3^2 - 48bu_0^3u_1u_2^2u_3 - 4bu_0^3u_2^4 + 15bu_0^2u_1^4u_4 \\ & + 60bu_0^2u_1^3u_2u_3 + 30bu_0^2u_1^2u_2^3 - 12bu_0u_1^5u_3 - 30bu_0u_1^4u_2^2 + 7bu_1^6u_2 + u_0^7u_8 \\ & - 2u_0^6u_1u_7 - 2u_0^6u_2u_6 - 2u_0^6u_3u_5 - u_0^6u_4^2 + 3u_0^5u_1^2u_6 + 6u_0^5u_1u_2u_5 + 6u_0^5u_1u_3u_4 \\ & + 3u_0^5u_2^2u_4 + 3u_0^5u_2u_3^2 - 4u_0^4u_1^3u_5 - 12u_0^4u_1^2u_2u_4 - 6u_0^4u_1^2u_3^2 - 12u_0^4u_1u_2^2u_3 - u_0^4u_2^4 \\ & + 5u_0^3u_1^4u_4 + 20u_0^3u_1^3u_2u_3 + 10u_0^3u_1^2u_2^3 - 6u_0^2u_1^5u_3 - 15u_0^2u_1^4u_2^2 + 7u_0u_1^6u_2 - u_1^8 \end{aligned} \right)$$

$$A_9 = \frac{ab}{(b + u_0)^{10}}$$

$$\begin{aligned}
 & b^8u_9 + 8b^7u_0u_9 - 2b^7u_1u_8 - 2b^7u_2u_7 - 2b^7u_3u_6 - 2b^7u_4u_5 + 28b^6u_0^2u_9 \\
 & - 14b^6u_0u_1u_8 - 14b^6u_0u_2u_7 - 14b^6u_0u_3u_6 - 14b^6u_0u_4u_5 + 3b^6u_1^2u_7 \\
 & + 6b^6u_1u_2u_6 + 6b^6u_1u_3u_5 + 3b^6u_1u_4^2 + 3b^6u_2^2u_5 + 6b^6u_2u_3u_4 + b^6u_3^3 + 56b^5u_0^3u_9 \\
 & - 42b^5u_0^2u_1u_8 - 42b^5u_0^2u_2u_7 - 42b^5u_0^2u_3u_6 - 42b^5u_0^2u_4u_5 + 18b^5u_0u_1^2u_7 \\
 & + 36b^5u_0u_1u_2u_6 + 36b^5u_0u_1u_3u_5 + 18b^5u_0u_1u_4^2 + 18b^5u_0u_2^2u_5 + 36b^5u_0u_2u_3u_4 \\
 & + 6b^5u_0u_3^3 - 4b^5u_1^3u_6 - 12b^5u_1^2u_2u_5 - 12b^5u_1^2u_3u_4 - 12b^5u_1u_2^2u_4 - 12b^5u_1u_2u_3^2 \\
 & - 4b^5u_2^3u_3 + 70b^4u_0^4u_9 - 70b^4u_0^3u_1u_8 - 70b^4u_0^3u_2u_7 - 70b^4u_0^3u_3u_6 - 70b^4u_0^3u_4u_5 \\
 & + 45b^4u_0^2u_1^2u_7 + 90b^4u_0^2u_1u_2u_6 + 90b^4u_0^2u_1u_3u_5 + 45b^4u_0^2u_1u_4^2 + 45b^4u_0^2u_2^2u_5 \\
 & + 90b^4u_0^2u_2u_3u_4 + 15b^4u_0^2u_3^3 - 20b^4u_0u_1^3u_6 - 60b^4u_0u_1^2u_2u_5 - 60b^4u_0u_1^2u_3u_4 \\
 & - 60b^4u_0u_1u_2^2u_4 - 60b^4u_0u_1u_2u_3^2 - 20b^4u_0u_2^3u_3 + 5b^4u_1^4u_5 + 20b^4u_1^3u_2u_4 \\
 & + 10b^4u_1^3u_3^2 + 30b^4u_1^2u_2^2u_3 + 5b^4u_1u_4^2 + 56b^3u_0^5u_9 - 70b^3u_0^4u_1u_8 - 70b^3u_0^4u_2u_7 \\
 & - 70b^3u_0^4u_3u_6 - 70b^3u_0^4u_4u_5 + 60b^3u_0^3u_1^2u_7 + 120b^3u_0^3u_1u_2u_6 + 120b^3u_0^3u_1u_3u_5 \\
 & + 60b^3u_0^3u_1u_4^2 + 60b^3u_0^3u_2^2u_5 + 120b^3u_0^3u_2u_3u_4 + 20b^3u_0^3u_3^3 - 40b^3u_0^2u_1^3u_6 \\
 & - 120b^3u_0^2u_1^2u_2u_5 - 120b^3u_0^2u_1^2u_3u_4 - 120b^3u_0^2u_1u_2^2u_4 - 120b^3u_0^2u_1u_2u_3^2 \\
 & - 40b^3u_0^2u_2^3u_3 + 20b^3u_0u_1^4u_5 + 80b^3u_0u_1^3u_2u_4 + 40b^3u_0u_1^3u_3^2 + 120b^3u_0u_1^2u_2^2u_3 \\
 & + 20b^3u_0u_1u_2^4 - 6b^3u_1^5u_4 - 30b^3u_1^4u_2u_3 - 20b^3u_1^3u_3^2 + 28b^2u_0^6u_9 - 42b^2u_0^5u_1u_8 \\
 & - 42b^2u_0^5u_2u_7 - 42b^2u_0^5u_3u_6 - 42b^2u_0^5u_4u_5 + 45b^2u_0^4u_1^2u_7 + 90b^2u_0^4u_1u_2u_6 \\
 & + 90b^2u_0^4u_1u_3u_5 + 45b^2u_0^4u_1u_4^2 + 45b^2u_0^4u_2^2u_5 + 90b^2u_0^4u_2u_3u_4 + 15b^2u_0^4u_3^3 \\
 & - 40b^2u_0^3u_1^3u_6 - 120b^2u_0^3u_1^2u_2u_5 - 120b^2u_0^3u_1^2u_3u_4 - 120b^2u_0^3u_1u_2^2u_4 \\
 & - 120b^2u_0^3u_1u_2u_3^2 - 40b^2u_0^3u_2^3u_3 + 30b^2u_0^2u_1^4u_5 + 120b^2u_0^2u_1^3u_2u_4 + 60b^2u_0^2u_1^3u_3^2 \\
 & + 180b^2u_0^2u_1^2u_2^2u_3 + 30b^2u_0^2u_1u_2^4 - 18b^2u_0u_1^5u_4 - 90b^2u_0u_1^4u_2u_3 - 60b^2u_0u_1^3u_3^2 \\
 & + 7b^2u_0^6u_3 + 21b^2u_1^5u_2^2 + 8bu_0^7u_9 - 14bu_0^6u_1u_8 - 14bu_0^6u_2u_7 - 14bu_0^6u_3u_6 \\
 & - 14bu_0^6u_4u_5 + 18bu_0^5u_1^2u_7 + 36bu_0^5u_1u_2u_6 + 36bu_0^5u_1u_3u_5 + 18bu_0^5u_1u_4^2 \\
 & + 18bu_0^5u_2^2u_5 + 36bu_0^5u_2u_3u_4 + 6bu_0^5u_3^3 - 20bu_0^4u_1^3u_6 - 60bu_0^4u_1^2u_2u_5 \\
 & - 60bu_0^4u_1u_2^2u_4 - 60bu_0^4u_1u_2u_3^2 - 20bu_0^4u_2^3u_3 + 20bu_0^3u_1^4u_5 \\
 & + 80bu_0^3u_1^3u_2u_4 + 40bu_0^3u_1^3u_3^2 + 120bu_0^3u_1^2u_2^2u_3 + 20bu_0^3u_1u_2^4 - 18bu_0^2u_1^5u_4 \\
 & - 90bu_0^2u_1^4u_2u_3 - 60bu_0^2u_1^3u_3^2 + 14bu_0u_1^6u_3 + 42bu_0u_1^5u_2^2 - 8bu_1^7u_2 + u_0^8u_9 \\
 & - 2u_0^7u_1u_8 - 2u_0^7u_2u_7 - 2u_0^7u_3u_6 - 2u_0^7u_4u_5 + 3u_0^6u_1^2u_7 + 6u_0^6u_1u_2u_6 \\
 & + 6u_0^6u_1u_3u_5 + 3u_0^6u_1u_4^2 + 3u_0^6u_2^2u_5 + 6u_0^6u_2u_3u_4 + u_0^6u_3^3 - 4u_0^5u_1^3u_6 \\
 & - 12u_0^5u_1^2u_2u_5 - 12u_0^5u_1^2u_3u_4 - 12u_0^5u_1u_2^2u_4 - 12u_0^5u_1u_2u_3^2 - 4u_0^5u_2^3u_3 \\
 & + 5u_0^4u_1^4u_5 + 20u_0^4u_1^3u_2u_4 + 10u_0^4u_1^3u_3^2 + 30u_0^4u_1^2u_2^2u_3 + 5u_0^4u_1u_2^4 - 6u_0^3u_1^5u_4 \\
 & - 30u_0^3u_1^4u_2u_3 - 20u_0^3u_1^3u_3^2 + 7u_0^2u_1^6u_3 + 21u_0^2u_1^5u_2^2 - 8u_0u_1^7u_2 + u_1^9
 \end{aligned}$$



Conclusion

The proposed algorithm intended to calculate the Adomian polynomials enjoys a great deal of simplicity, succinctness and efficiency. As it is converted to a function file in Python, it can easily be integrated into any code dealing with solutions of nonlinear functional equations and be called anywhere in the program to compute the desired Adomian polynomial component of an interested nonlinearity. By taking advantage of the symbolic infrastructure of the *SymPy* library in Python, we have managed to shorten the code considerably. The authors are working on the application of this computer code to tackle nonlinear equations in chemical engineering, especially in the field of equations of state in thermodynamics.

Nomenclature

A_i	i -th Adomian polynomial component
d	differentiation operator
E	Banach space
f	known function in E
u_k	k -th solution component of functional equation (1)
u	analytical solution of functional equation (1)
$N(u)$	nonlinear operator acting on u
λ	auxiliary variable

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Appendix

The Python Code

```

#Copyright (c) [2023] [A. Houshmand, M. Noorimohammad, and H.
Fatoorehchi]
#The code presented in this paper is protected by copyright law.
#Permission is granted for personal or educational use, subject to
#the following conditions: (1) Include the copyright notice in all
#copies, (2) Prohibit commercial use and distribution without
#written permission, and (3) Disclaimer of warranties.
#For other usage or questions, contact [hfatoorehchi@ut.ac.ir].
# Import the necessary mathematical libraries
import math
import sympy as sym

# Define the order of the Adomian series expansion
n = 10

# Create variables u0, u1, ..., un-1 for the coefficients in the
series
u = {}

for i in range(n):
    key = 'u{}'.format(i)
    u[key] = sym.symbols(key)
u_list = list(u.values())
Lambda = sym.symbols('Lambda')

# Calculate the series representation S(Lambda)
s = 0
for i in range(n):
    s = s + pow(Lambda,i)*u_list[i]

# Compute the nonlinearity term N(s)
N_s = s**2

# Calculate the Adomian polynomials A0, A1, ..., An-1
kk = range(n)
A_K = [None] * len(kk)
A_KK = [None] * len(kk) # A_KK stores the Adomian result
for i in kk:
    A_K [i] = (1/(sym.factorial(i)))*(sym.diff(N_s, Lambda,i))
    A_KK[i] = A_K[i].subs(Lambda,0)

# Print the Adomian polynomials
print (A_KK)

```

The *Python* code can be downloaded from this address:

<https://hfatoorehchi.com/Admpolypython.ipynb>