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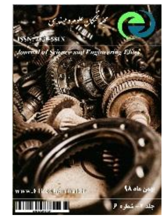

Fuzzy Based Approach for Monitoring P-Control Chart by Means of A-Level Fuzzy Midrange

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Abstract

Fuzzy set theory is an inevitable tool for fuzzy control charts as well as other applications subjected to uncertainty in any from. In this paper, we will used the α-cut p-control chart and the α-level fuzzy midrange of control limits fuzzy when the observations are trapezoidal fuzzy numbers. We are representing α-cut p-control limits of fuzzy decision, that shown in control or out of control of process $\tilde{\bar{P}}$ .Illustrates applications are presented for control limits fuzzy of fraction and numbers non-conforming.

Keyword: Control chart, Fuzzy control chart, α-cut p-control chart, α-level fuzzy midrange of trapezoidal number.

1- Introduction

Cheng [1] constructed fuzzy control charts for a process with fuzzy outcomes derived from the subjective quality ratings provided by a group of experts. The fuzzy quality ratings are then plotted on fuzzy control charts, whose construction and out-of control conditions are developed using possibility theory. Faraz et al. [2] constructed a fuzzy statistical control chart that explained existing fuzziness in data by considering variability between observations. Sorooshian [3] proposed monitoring attribute quality characteristic with consideration of uncertainty and ambiguous. Shu and Wu [4] applied resolution identity to construct the control limits fuzzy p-control chart using fuzzy data. Laviolette et al. [5] used a Probabilistic and statistical view of fuzzy methods. Gulbay and Kahraman [6,7] introduce a direct fuzzy approach (DFA), where fuzzy sample data and the imprecise number of nonconformities found in the manufacturing process are directly used to construct the fuzzy center line and fuzzy control limits for a fuzzy C-control chart. Hsu and Chen [8] described a new diagnosis system based on fuzzy reasoning to monitor the performance of a discrete manufacturing process and to justify the possible causes. Kanagawa et al. [9] adopted the fuzzy probabilistic approach where they have used the probability density function of a fuzzy random variable to form the center line and control limits of the fuzzy control chart.

The structure of the α-level fuzzy midrange for control chart have been proposed for triangular and trapezoidal membership functions by Gulbay et al. [11].]. Shewhart [12] proposed economic control of quality of manufactured product. Construction of control of charts by using fuzzy multinomial-FM and EWMA chart "comparative study " have introduced by kawa and haydar [23] . The fuzzy sets theory was first proposed by zadeh [13]. Senturk and Erginel [15] introduced the framework of fuzzy $\tilde{\bar{\bar{X}}}$- $\tilde{R}$ and $\tilde{\bar{\bar{X}}}$- $\tilde{S}$ control charts Sogandi et al. [14] considered the control limits and computed them by using the regular arithmetic calculations. In this paper, to compute the control limits and fuzzy decision for in control or out-of-control the process. we have also used the percentage sample area within the control limits criterion for more investigation. Several graphs are given to show better performances of using the trapezoidal fuzzy numbers with respect to the regular numbers. The objective of this paper is to develop traditional p-control charts which monitor the process without any transformation techniques and also maintain the basic structure of Shewhart control charts for trapezoidal fuzzy numbers. The rest of the paper is organized in the following order. Fuzzy transformation techniques are introduced in section 2. In section 3, firstly we present a short review on the classic P-control charts and fuzzy P-control charts, then α-cut fuzzy are developed for observation in these control charts. Finally, α-cut and α-level fuzzy midrange is presented for P-control charts. In section 4, introduced fuzzy C-control charts and α-cut C-control charts for each fuzzy sample are introduced. Finally, the proposed condition of control. Section 5 illustrates the methods using a numerical example. The conclusion is also presented in section 6.

2. Fuzzy converters

To construct standard of control charts and facilitate the plotting of observations on the chart, we need to convert the fuzzy sets associated with the linguistic or uncertain values into scalars. The four fuzzy measures of central tendency are: fuzzy mode, α-level fuzzy midrange, fuzzy median and fuzzy average, which are well known is descriptive statistics regarded representative values [3].

1. Fuzzy mode, f_{mod} : The fuzzy mode of a fuzzy set F is the value of the base variable where the membership function equals 1. This is stated.

$$f_{mod} =\{ x|\mu_F(x)=1\}, \quad \forall\, x \in F. \tag{1}$$

2. Fuzzy median, f_{med} **:** This is the point which partitions the curve under the membership function of a fuzzy set into equal regions satisfying the following equation :

$$\int_a^{f_{med}} \mu_F(x)\mathrm{d}\,x = \int_{f_{med}}^{c} \mu_F(x)\mathrm{dx} = \frac{1}{2}\int_a^c \mu_F(x)\mathrm{d}\,x\,, \tag{2}$$

where a and c are the end points in the base variable of the fuzzy set F such that $a< c$.

3. Fuzzy average, f_{avg}: based on zadeh [13], the fuzzy average is:

$$f_{avg}=\mathrm{Av}(x;\mathrm{F})=\frac{\int_0^1 x\mu_F(x)\mathrm{d}x}{\int_0^1 \mu_F(x)\mathrm{d}x} \quad (3)$$

4. α-level fuzzy midrange, $f_{mr}[\alpha]$**:** This is defined as the midpoint of the ends of the α-level cut. An α-level cut, denoted by A[α] is non fuzzy set which comprises all elements whose membership is greater than α or equal to α. The α-level fuzzy midrange is determined for a trapezoidal fuzzy set as the midpoint of the crisp interval that divides the set into two subsets One subset is related to all the values that have a membership larger than or equal to α in the original set. The other subset concludes all the membership less than α. If $\tilde{A}$ = (*a* , *b* ,*c* ,*d*) be trapezoidal fuzzy number , then α-level cut , given by $A[\alpha]$ is :

$$A[\alpha]=[A^L[\alpha],A^U[\alpha]]=[a+\alpha(b-a),d-\alpha(d-c)]. \quad (4)$$

Thus, α-level fuzzy midrange is calculated by:

$$f_{mr}(\alpha)=\frac{A^L[\alpha]+\ A^U[\alpha]}{2} \quad (5)$$

In Figure 1, α-cut on a sample by trapezoidal fuzzy number is plotted. The α-level fuzzy midrange of sample j, $S^{\alpha}_{mr,j}$ is determined by :

$$S_{mr,j}[\alpha]=\frac{\left(a_{j+d_j}\right)+\alpha[(b_j-a_j)-(d_j-c_j)]}{2} \quad (6)$$

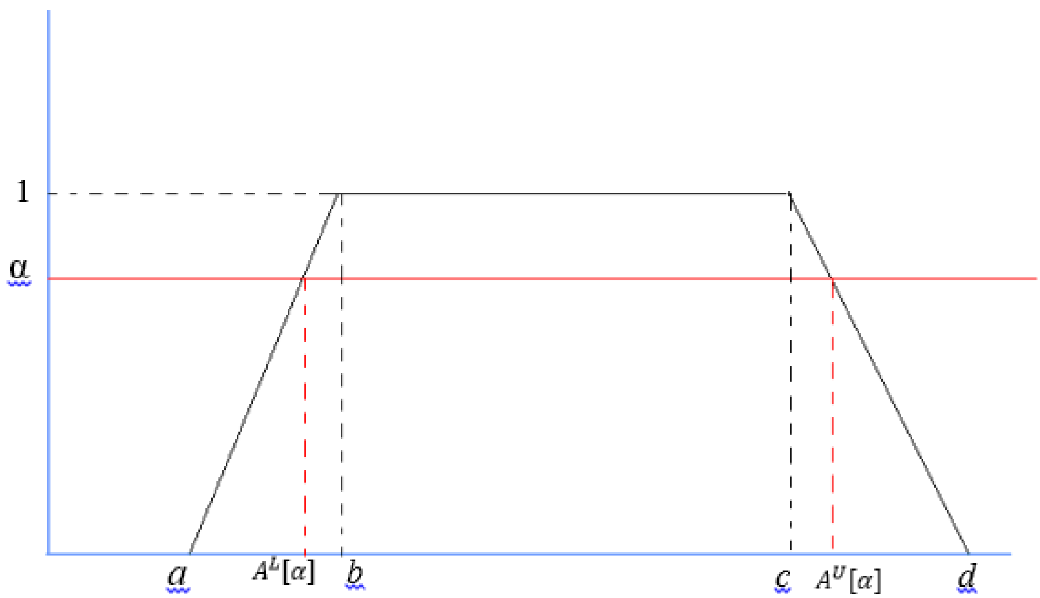

Fig.1- the plot of α-cut a sample of trapezoidal fuzzy number

3- P-control chart based on fuzzy logic

Traditionally, the P-control chart is used to monitor the fraction rejected units of products. It shows the number of non-conforming items, which is existed in entire process. The well-known upper and lower bounds fraction non-conforming chart are given by:

$$UCL_p=\bar{P}+3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}}, \quad (7)$$
$$\mathrm{CL}=\bar{P},$$

LCL_p=$\bar{P}$ -3$\sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$

regular arithmetic calculations. Here, to compute the LCL and UCL, we have used fuzzy rule-based methods.

Depending on the value of *p* and *n*, the values of LCL is sometimes less than zero. In these cases, we customarily set LCL=0 and assume that the control chart only has upper control limit. A fuzzy approach is suitable for attributes control charts (P, NP, C and U charts).When the data is linguistic ,categorical or uncertain, the human dependent subjective judgment is possible. In classical P-charts, products are distinctly classified as conformed or non-conformed when determining fraction rejected. In this study, we consider number of defects as trapezoidal fuzzy numbers, represented by (X_a ,X_b,X_c,X_d) for each fuzzy sample CL_P, and UCL_P , LCL_P represent the center line and control limits of fuzzy P-control charts, respectively, and they are trapezoidal fuzzy sets. They are determined by the following equations:

$$\tilde{P_i}=(\frac{X_{ai}}{n},\frac{X_{bi}}{n},\frac{X_{ci}}{n},\frac{X_{di}}{n})=(P_{ai},P_{bi},P_{ci},P_{di}) \quad ;i=1,2,\ldots,m$$
$$\widetilde{CL}_P=\tilde{\bar{P}}=(\frac{\sum_{i=1}^{m}P_{ai}}{m},\frac{\sum_{i=1}^{m}P_{bi}}{m},\frac{\sum_{i=1}^{m}P_{ci}}{m},\frac{\sum_{i=1}^{m}P_{di}}{m}) \tag{8}$$
$$=(\frac{\sum_{i=1}^{m}X_{ai}}{mn},\frac{\sum_{i=1}^{m}X_{bi}}{mn},\frac{\sum_{i=1}^{m}X_{ci}}{mn},\frac{\sum_{i=1}^{m}X_{di}}{mn})=(\bar{P}_a,\bar{P}_b,\bar{P}_c,\bar{P}_d)$$

$$\widetilde{LCL_P}=(\bar{P}_a-3\sqrt{\frac{\bar{P}_d(1-\bar{P}_d)}{n}},\bar{P}_b-\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}},\bar{P}_c-3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}},\bar{P}_d-3\sqrt{\frac{\bar{P}_a(1-\bar{P}_a)}{n}}),$$
$$\widetilde{UCL}_P=(\bar{P}_a+3\sqrt{\frac{\bar{P}_a(1-\bar{P}_a)}{n}},\bar{P}_b+3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}},\bar{P}_c+3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}},\bar{P}_d+3\sqrt{\frac{\bar{P}_d(1-\bar{P}_d)}{n}}),$$

Where n and m display the fuzzy sample size and number of subgroups, respectively.

We first construct fuzzy P-control charts by means of α-cut method. The interpretation of these charts is the same as mentioned in the previous section. By applying α-cuts fuzzy set $\tilde{\bar{P}}$ the values of center line (see, Fig.2) are determined by:

$$\bar{P}[\alpha]=[\bar{P}^L[\alpha],\bar{P}^U[\alpha]] \tag{9}$$

Where $\bar{P}^L[\alpha]$ and $\bar{P}^U[\alpha]$ are given by:

$$\bar{P}^L[\alpha]=\bar{P}_a+\alpha(\bar{P}_b-\bar{P}_a), \qquad \bar{P}^U[\alpha]=\bar{P}_d-\alpha(\bar{P}_d-\bar{P}_c).$$

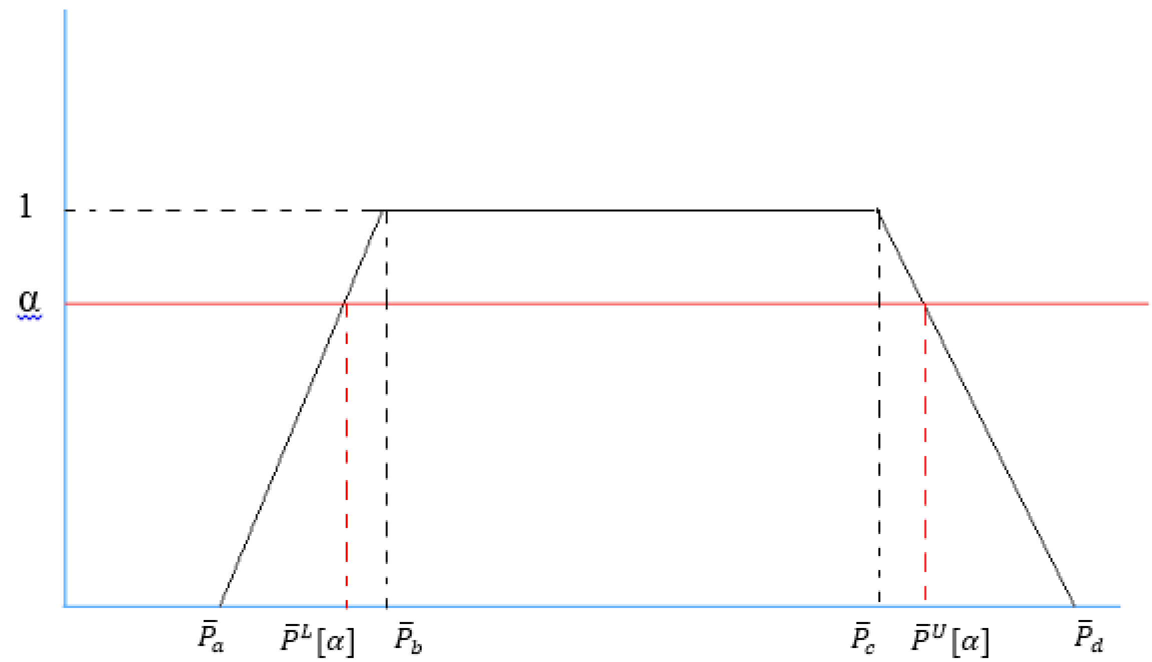

Fig.2- Representation of α-cut fuzzy set $\tilde{\bar{P}}$

Using these α-cut representations, the fuzzy control limits can be rewritten as:

$$\widetilde{CL}_P[\alpha] = (\bar{P}^L[\alpha], \overline{P_b}, \bar{P}_c, \bar{P}^U[\alpha]) = (CL_1, CL_2, CL_3, CL_4), \quad (10)$$

$$\widetilde{LCL}_P[\alpha] = \left(\bar{P}^L[\alpha] - 3\sqrt{\frac{\bar{P}^U[\alpha](1-\bar{P}^U[\alpha])}{n}}, \bar{P}_b - 3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}}, \bar{P}_c - 3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}}, \bar{P}^U[\alpha] - 3\sqrt{\frac{\bar{P}^L[\alpha](1-\bar{P}^L[\alpha])}{n}}\right)$$
$$= (LCL_1[\alpha], LCL_2, LCL_3, LCL_4[\alpha]),$$

$$\widetilde{UCL}_P[\alpha] = \left(\bar{P}^L[\alpha] + 3\sqrt{\frac{\bar{P}^L[\alpha](1-\bar{P}^L[\alpha])}{n}}, \bar{P}_b + 3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}}, \bar{P}_c + 3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}}, \bar{P}^U[\alpha] + 3\sqrt{\frac{\bar{P}^U[\alpha](1-\bar{P}^U[\alpha])}{n}}\right)$$
$$= (UCL_1[\alpha], UCL_2, UCL_3, UCL_4[\alpha]).$$

A graphical representation of these equations can be seen in Figure 3.

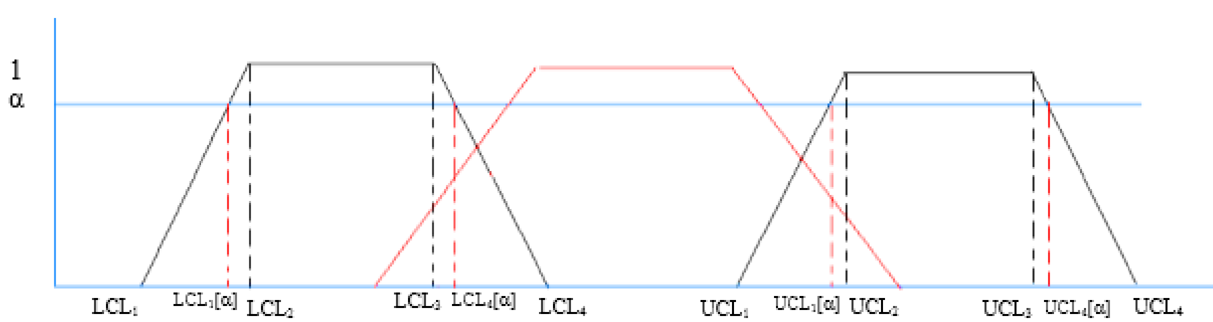

Fig.3- a graphical representation of α-cut control limits of $\tilde{\bar{P}}$

Fuzzy transformation techniques are used for deciding whether the process is "under control or out of control" after calculating the control limits. In the second step, we calculate the control limits for α-level fuzzy midrange for α-cut fuzzy P-control chart, by using the following midrange transformation:

$$CL_{mr-p}[\alpha] = \frac{\bar{P}^L[\alpha]+\bar{P}^U[\alpha]}{2}, LCL_{mr-p}[\alpha] = CL_{mr-p}[\alpha] - 3\sqrt{\frac{CL_{mr-p}[\alpha](1-CL_{mr-p}[\alpha])}{n}}, \quad (11)$$

$$UCL_{mr-p}[\alpha] = CL_{mr-p}[\alpha] + 3\sqrt{\frac{CL_{mr-p}[\alpha](1-CL_{mr-p}[\alpha])}{n}}.$$

The α-level fuzzy midrange of sample j , $S_{mr-j}[\alpha]$, is determined by :

$$S_{mr-p}[\alpha] = \frac{(P_a+P_d)+\alpha\{(P_b-P_a)-(P_d-P_c)\}}{2}. \quad (12)$$

Therefore, the control condition of process for each sample can be specified as:

$$\text{Process is} \begin{cases} \text{in control ;} & LCL_{mr-p}[\alpha] \le S_{mr-p,j}[\alpha[\le UCL_{mr-p}[\alpha] \\ \text{out of control ;} & \text{otherwise} \end{cases} \quad (13)$$

4- Fuzzy C-control chart

In the crisp case, the control limits for the number of non-conformities are calculated by:

$$\begin{aligned} &CL=\bar{C}, \\ &LCL= \bar{C}-3\sqrt{\bar{C}}, \quad (14)\\ &UCL= \bar{C}+3\sqrt{\bar{C}}, \end{aligned}$$

where $\overline{C}$ is the mean of the non-conformities .In the fuzzy case, where number of non-conformity includes human subjective or uncertainty values such as " between 10 or 14 " or " approximately 12 " can be used to defined number of non-conformities in a sample. Hence, the number of non-conformity in each sample, or subgroups, can be represented by a trapezoidal fuzzy number (X_a,$X_{b,}$ X_c,$X_{d,}$) . Here, we propose a direct fuzzy approach (DFA) to deal with the vague data for the control charts.
Transforming the vague data by representing them with other values may effect on the result in based decision for particular data especially when they are represented by asymmetrical fuzzy numbers. For fuzzy case, where the numbers of non-conformities are represented by trapezoidal fuzzy numbers, the fuzzy center line, $\widetilde{CL}$, can be determined by using the following arithmetic mean of the fuzzy numbers. In the other words,

$$\begin{aligned} &\widetilde{CL}_C = \tilde{\bar{C}}=(\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d)=(\frac{\sum_{i=1}^{n} X_{ai}}{n}, \frac{\sum_{i=1}^{n} X_{bi}}{n}, \frac{\sum_{i=1}^{n} X_{ci}}{n}, \frac{\sum_{i=1}^{n} X_{di}}{n}), \quad (15)\\ &\widetilde{LCL}_C=\widetilde{CL}_C - 3\sqrt{\widetilde{CL}_C}=(\bar{X}_a-3\sqrt{\bar{X}_d}, \bar{X}_b-3\sqrt{\bar{X}_c}, \bar{X}_c-3\sqrt{\bar{X}_b}, \bar{X}_d-3\sqrt{\bar{X}_a}), \\ &\widetilde{UCL}_C=\widetilde{CL}_C + 3\sqrt{\widetilde{CL}_C}= (\bar{X}_a+3\sqrt{\bar{X}_a}, \bar{X}_b+3\sqrt{\bar{X}_b}, \bar{X}_c+3\sqrt{\bar{X}_c}, \bar{X}_d+3\sqrt{\bar{X}_d}), \end{aligned}$$

where n is the number of fuzzy sample. An α-cut is a non fuzzy set which comprises all elements whose membership is greater than or equal to α. The α-cut of fuzzy sets for of $\widetilde{CL}$ of center line are determined by:

$$\bar{C}^L[\alpha] = \bar{X}_a + \alpha(\bar{X}_b - \bar{X}_a), \bar{C}^U[\alpha]= \bar{X}_d - \alpha(\bar{X}_d - \bar{X}_c), \quad (16)$$

where $\bar{C}^L[\alpha]$, $\bar{C}^U[\alpha]$ are start and point of the α-cut of $\tilde{\bar{C}}$, respectively. Using α-cut representations, the fuzzy control limits can be rewritten as:

$$\widetilde{CL}_C[\alpha]=(\bar{C}^L[\alpha], \bar{X}_b, \bar{X}_c, \bar{C}^U[\alpha])=(CL_1[\alpha], CL_2, CL_3, CL_4[\alpha]),$$
$$\widetilde{LCL}_C[\alpha]=\widetilde{CL}[\alpha]-3\sqrt{\widetilde{CL}[\alpha]} = (\bar{C}^L[\alpha]-3\sqrt{\bar{C}^U[\alpha]}, \bar{X}_b-3\sqrt{\bar{X}_c}, \bar{X}_c-3\sqrt{\bar{X}_b}, \bar{C}^U[\alpha]-3\sqrt{\bar{C}^L[\alpha]})$$
$$=(LCL_1[\alpha], LCL_2, LCL_3, LCL_4[\alpha]), \quad (17)$$
$$\widetilde{UCL}_C[\alpha]=\widetilde{CL}[\alpha]+3\sqrt{\widetilde{CL}[\alpha]} = (\bar{C}^L[\alpha]+3\sqrt{\bar{C}^L[\alpha]}, \bar{X}_b+3\sqrt{\bar{X}_b}, \bar{X}_c+3\sqrt{\bar{X}_c}, \bar{C}^U[\alpha]+$$
$$3\sqrt{\bar{C}^U[\alpha]})=(UCL_1[\alpha], UCL_2, UCL_3, UCL_4[\alpha]).$$

The decision about whether the process is in control can be made according to the percentage area of the sample which remains inside the $\widetilde{UCL}$ and or $\widetilde{LCL}$ defined as fuzzy sets. When the fuzzy sample is completely involved by the fuzzy control limits, the process is said to be " in control ". If a fuzzy sample is totally excluded the fuzzy control limits, the process is said to be " out control ". Otherwise, a sample is partially included by the fuzzy control limits. In this case, if the percentage area (β) which remains inside the fuzzy control limits is equal or greater than a predefined acceptable percentage (β), then the process can be accepted as " rather in control "; otherwise it can be stated as " rather out of control.

Hence, the total sample area outside the fuzzy control limits, A_{out}, is the sum of the areas below the fuzzy lower control limit and above the fuzzy upper control limits. The percentage sample area within the control limits is calculated by:

$$\beta_j{}^{\alpha}= \frac{S_j{}^{\alpha}-A_{out,j}{}^{\alpha}}{S_j{}^{\alpha}} \quad (18)$$

where $S_j{}^{\alpha}$ is the sample area at α-level cut.

5- Numerical results

A Sample of 200 units is taken every 4h to control number of non-conformities. Data from 30 subgroups given in Table 1 are linguistic such as "approximately 30" or " between 25 and 30 ". The linguistic expressions in Table 1 are represented by fuzzy trapezoidal numbers as shown in Table 2.Table 3 calculates the fraction non-conforming of each subgroup based on (8) ; the α-level fuzzy midrange based on (6) and (12). Table 3 also shows the overall percentage of area of each observation remains outside the fuzzy control limits. It shows that all sample are in control as all β is equal to one. Figure 5 shows that for α=0.6 fuzzy midrange is completely in control .The control limits, α-cut fuzzy and α-level fuzzy midrange for P-control chart are calculated, respectively.

Table 1- Number of non- conformities 30 subgroups

No	Approximately	Between	No	Approximately	Between
1	30		16	40	
2		20-30	17		32-50
3		5-12	18	39	
4	6		19		15-21
5	38		20	28	
6		20-24	21		32-35
7		4-8	22		10-25
8		36-44	23	30	
9		11-15	24	25	
10		10-13	25		31-41
11	6		26		10-25
12	32		27		5-14
13	13		28		28-35
14		50-52	29		20-25
15		38-41	30	8	

Table 2- Fuzzy number (X_a,X_b, X_c,X_d,) representation of 30 subgroups

No	X_a	X_b	X_c	X_d	No	X_a	X_b	X_c	X_d
1	25	30	30	35	16	33	40	40	44
2	15	20	20	35	17	28	32	50	60
3	4	5	12	15	18	33	39	39	43
4	3	6	6	8	19	12	15	21	38
5	32	38	38	45	20	23	28	28	36
6	16	20	24	28	21	28	32	35	42
7	3	4	8	12	22	14	18	28	33
8	27	36	44	50	23	24	30	25	25
9	9	11	15	20	24	20	25	31	41
10	7	10	13	15	25	25	31	41	46
11	3	6	6	10	26	7	10	25	28
12	27	32	32	37	27	3	5	14	20
13	11	13	13	15	28	23	28	35	38
14	39	50	52	55	29	17	20	25	29
15	28	38	41	45	30	25	8	8	15

Table 3-The fuzzy zones fraction non-conforming for the samples and control limits of midrange

No	P_a	P_b	P_c	P_d	$S_{mr-p}(0.6)$	$UCL_{mr-p}[0.6]$	$LCL_{mr-p}[0.6]$	β
1	0.125	0.15	0.15	0.175	0.15	0.2257	0	1
2	0.075	0.1	0.1	0.175	0.11	0.1764	0	1
3	0.02	0.025	0.06	0.075	0.0445	0.08824	0	1
4	0.015	0.03	0.1	0.04	0.05	0.09623	0	1
5	0.16	0.19	0.19	0.225	0.191	0.27438	0	1
6	0.08	0.1	0.12	0.14	0.11	0.17637	0	1
7	0.015	0.02	0.04	0.06	0.033	0.07089	0	1
8	0.135	0.18	0.22	0.25	0.197	0.28137	0	1
9	0.045	0.055	0.075	0.1	0.068	0.1214	0	1
10	0.035	0.05	0.065	0.075	0.0565	0.105478	0	1
11	0.015	0.03	0.03	0.05	0.031	0.067766	0	1
12	0.135	0.16	0.16	0.185	0.16	0.237769	0	1
13	0.055	0.065	0.065	0.075	0.065	0.1173	0	1
14	0.195	0.25	0.26	0.275	0.247	0.33848	0	1
15	0.14	0.19	0.205	0.225	0.1915	0.27497	0	1
16	0.14	0.16	0.25	0.3	0.211	0.29755	0	1
17	0.165	0.195	0.195	0.215	0.193	0.2767	0	1
18	0.06	0.075	0.105	0.19	0.104	0.16875	0	1
19	0.115	0.14	0.14	0.18	0.143	0.21726	0	1
20	0.14	0.16	0.175	0.21	0.1705	0.250277	0	1
21	0.07	0.09	0.14	0.165	0.116	0.18393	0	1
22	0.12	0.15	0.125	0.125	0.1315	0.20319	0	1
23	0.1	0.125	0.155	0.205	0.145	0.21969	0	1
24	0.125	0.155	0.205	0.23	0.179	0.2603	0	1
25	0.035	0.05	0.125	0.14	0.0875	0.14744	0	1
26	0.015	0.025	0.07	0.1	0.0515	0.09838	0	1
27	0.115	0.14	0.175	0.19	0.1555	0.23237	0	1
28	0.085	0.1	0.125	0.145	0.1135	0.18079	0	1
29	0.025	0.04	0.04	0.075	0.044	0.08751	0	1
30	0.165	0.2	0.2	0.22	0.197	0.28137	0	1

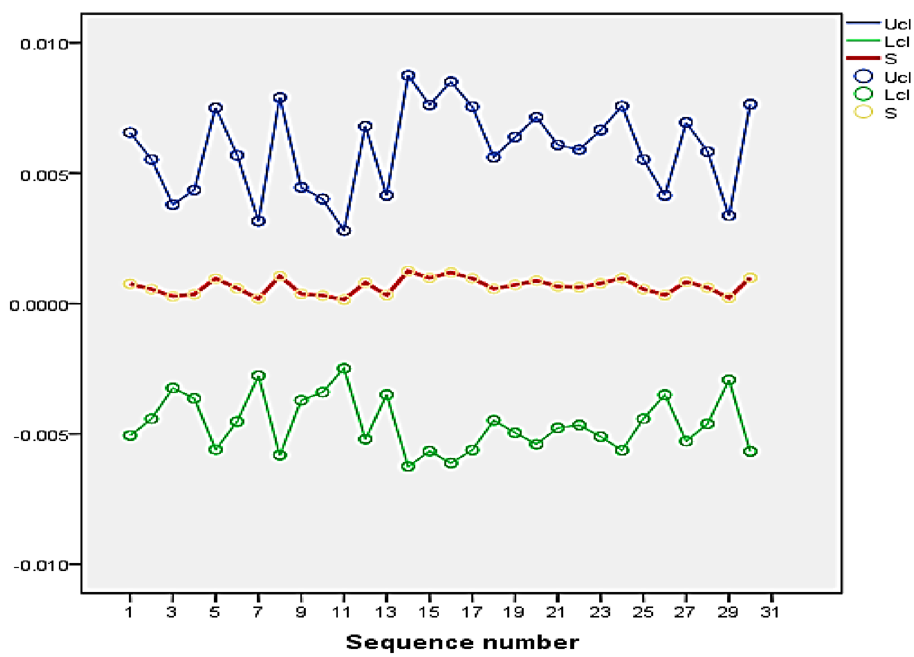

Fig.5-The α-level (α=0.6) fuzzy midrange of the fuzzy fraction.

Note that control limits fuzzy mean of fraction non-conforming each of subgroup are calculated based on (8). During the value components of $\widetilde{LCL}_P$ be less than zero ,in this case ,we have let zero:

$\widetilde{CL}_P=(\overline{P_a},\overline{P_b},\overline{P_c},\overline{P_d})$=(0.0907,0.1133,0.1355,0.1605) ,

$\widetilde{LCL}_P=(\bar{P}_a-3\sqrt{\frac{\bar{P}_d(1-\bar{P}_d)}{n}},\bar{P}_b-3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}},\bar{P}_c-3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}},\bar{P}_d-3\sqrt{\frac{\bar{P}_a(1-\bar{P}_a)}{n}})$,
=(-0.1103,-0.0742,-0.0381,0.0032)=(0,0,0,0.0032),

$\widetilde{UCL}_P=(\bar{P}_a+3\sqrt{\frac{\bar{P}_a(1-\bar{P}_a)}{n}},\bar{P}_b+3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}},\bar{P}_c+3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}},\bar{P}_d+3\sqrt{\frac{\bar{P}_d(1-\bar{P}_d)}{n}})$,
=(0.2480,0.2869,0.3230,0.3615) .

The α-cut fuzzy mean of fraction non-conforming P-control charts each of subgroup obtained based on (9) and (10). Note that, we set the values less than zero ,replace whit zero:

$\bar{P}^L[0.6]=\bar{P}_a+0.6(\bar{P}_b-\bar{P}_a)$=0.1043,
$\bar{P}^U[0.6]=\bar{P}_d-0.6(\bar{P}_d-\bar{P}_c)=0.1455$,
$\widetilde{CL}_P[0.6]=(\bar{P}^L[0.6],\overline{P_b},\overline{P_c},\bar{P}^U[0.6])$ =(0.1043,0.1133,0.1355,0.1455),
$\widetilde{LCL}_P[0.6]=(\bar{P}^L[0.6]-3\sqrt{\frac{\bar{P}^U[0.6](1-\bar{P}^U[0.6])}{n}},\bar{P}_b-3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}},\bar{P}_c-3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}},\bar{P}^U[0.6]-3\sqrt{\frac{\bar{P}^L[0.6](1-\bar{P}^L[0.6])}{n}})$=(0,0,0,0),

$\widetilde{UCL}_P[0.6]=(\bar{P}^L[0.6]+3\sqrt{\frac{\bar{P}^L[0.6](1-\bar{P}^L[0.6])}{n}},\bar{P}_b+3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}},\bar{P}_c+3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}},\bar{P}^U[0.6]+3\sqrt{\frac{\bar{P}^U[0.6](1-\bar{P}^U[0.6])}{n}})=(0.2717,0.2869,0.3230,0.3386)$.

The α-level fuzzy midrange for P-control chart is obtained from (11). Because LCL_{mr-p} is less than zero , it is replaced whit number of zero:

$CL_{mr-p}[0.6]=\frac{\bar{P}^L[0.6]+\bar{P}^U[0.6]}{2}=0.1249$,
$LCL_{mr-p}[0.6]=CL_{mr-p}[0.6]-3\sqrt{\frac{CL_{mr-p}[0.6](1-CL_{mr-p}[0.6])}{n}}=0$,
$UCL_{mr-p}[0.6]=CL_{mr-p}[0.6]+3\sqrt{\frac{CL_{mr-p}[0.6](1-CL_{mr-p}[0.6])}{n}}=0.6373$.

Hence , the three above approaches state that the total sample of subgroups are in control.
Similarly, the α-cut of C-control chart limits based on number non-conforming are calculated as given follow. The control limits fuzzy mean of number non-conforming each of subgroup calculates based on (15) are given by:

$\widetilde{CL}_c=(\overline{X_a},\overline{X_a},\overline{X_a},\overline{X_a})$ =(18.13 , 22.67 , 26.93 , 32.07),
$\widetilde{LCL}_C=\widetilde{CL}_C-3\sqrt{\widetilde{CL}_C}$=(1.1408,7.1018,12.6461,192962),
$\widetilde{UCL}_c=\widetilde{CL}_C+3\sqrt{\widetilde{CL}_C}$=(30.9038,36.9381,42.4982,49.0591).

The α-cut fuzzy mean of number non-conforming P-control charts each of subgroup obtained based on ((16) and (17) are:

$\bar{C}^L[\alpha] = \bar{X}_a + \alpha(\bar{X}_b - \bar{X}_a)$=20.854 , $\bar{C}^U[\alpha] = \bar{X}_d - \alpha(\bar{X}_d - \bar{X}_c) = 28.989$.

$\widetilde{CL}_C[0.6]$=($\bar{C}^L[0.6]$, $\bar{X}_b, \bar{X}_c$, $\bar{C}^U[0.6]$)=(4.7016,7.1018,12.6461,15.2891),

$L\widetilde{CL}_c$ [0.6]=($\widetilde{CL}$ [0.6]-3$\sqrt{\widetilde{CL}[0.6]}$ = ($\bar{C}^L[0.6] - 3\sqrt{\bar{C}^U[0.6]}, \bar{X}_b-3\sqrt{\bar{X}_c}, \bar{X}_c-3\sqrt{\bar{X}_b}, \bar{C}^U[0.6] -$ 3$\sqrt{\bar{C}^L[0.6]}$)= (4.7016,7.1018,12.6461,15.2891),

$\widetilde{UCL}_C[0.6] = (\widetilde{CL}\ [0.6] + 3\sqrt{\widetilde{CL}[0.6]} =\ (\bar{C}^L[0.6] + 3\sqrt{\bar{C}^L[\alpha]}, \bar{X}_b + 3\sqrt{\bar{X}_b}, \bar{X}_c + 3\sqrt{\bar{X}_c}, \bar{C}^U[0.6] + 3\sqrt{\bar{C}^U[0.6\alpha]})$ =(34.5538,36.9381,42.82,45.1414).

Finally, the α-level fuzzy midrange for C-control chart is obtained by (11). So,

$CL_{mr-C}[0.6] = \frac{\bar{C}^L[0.6]+\bar{C}^U[0.6]}{2}$ = 24.9215 ,

$LCL_{mr-C}[0.6] = CL_{mr-C}[0.6] - 3\sqrt{\frac{CL_{mr-C}[0.6](1-CL_{mr-C}[0.6])}{n}}$ = 9.9451,

$UCL_{mr-C}[0.6] = CL_{mr-C}[0.6] + 3\sqrt{\frac{CL_{mr-C}[0.6](1-CL_{mr-C}[0.6])}{n}}$ = 39.8979.

Hence, we conclude that based on the two above approaches; P-control chart and c-control charts, the total sample of subgroups are in control.

6- Conclusions

In literature, numerous zone tests or run rules have been developed to assist quality practitioners in the detection of unnatural patterns for the crisp control chart. The control charts have extensive applications in detecting of unnatural patterns for the crisp control charts. In this paper, we have calculated a direct fuzzy approach to fuzzy control charts without any defuzzification, The proposed P-control chart , the α-cut fuzzy mean of fraction non-conforming and the α-level fuzzy midrange when the observations are trapezoidal fuzzy numbers. This article also proposed a fuzzy decision for in control or out of control we used illustrate applications to fraction non-conforming of fuzzy number. In the result of this study, it is possible to say that building fuzzy control charts have more flexible and more appropriate statistical description frame than control chart approach and give more meaning results than traditional quality control. For further research, new fuzzy unnatural pattern rules can be develop and tested using fuzzy random variables and the methodology can be extended for the other three measures and intuitionistic fuzzy numbers.

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