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A New Approach for Solving Grey Assignment Problems

S. H. Nasseri [1,*], D. Darvishi Salokolaei[2], A. Yazdani Charati[3]

[1,3] Department of Mathematical Sciences, University of Mazandaran, Babolsar, Iran
[2] Department of Mathematics, Payame Noor University, P.O. Box. 19395-3697, Tehran, Iran.

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Abstract. Linear assignment problem is one of the most important practical models in the literature of linear programming problems. Input data in the cost matrix of the linear assignment problem are not always crisp and sometimes in the practical situations is formulated by the grey systems theory approach. In this way, some researchers have used a whitening technique to solve the grey assignment problem. Since the whitening technique only provides a crisp equivalent model and does not reflect the evolutionary characteristics of a grey set, it cannot keep the uncertainty properties in an interval involving the optimal solution. Based on these shortcomings, in this paper a new direct approach is introduced to solve linear assignment problem in grey environments. For preparing the mentioned method, some theoretical results are given to support the methodology. Finally, a numerical example will be solved to test the validity of the proposed method. Based on the suggested methodology, we emphasize that the same approach can be used whenever any linear programming model is formulated in grey environments.

Keywords. Assignment problem, Grey system theory, Grey number, Uncertainty, Whitening technique.

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∗ Corresponding author
nasseri@umz.ac.ir, d_darvishi@pnu.ac.ir, yazdani@umz.ac.ir
http://mathco.journals.pnu.ac.ir

1 Introduction

The assignment problem refers to a special class of linear programming problem where the objective is to assign a number of resources to an equal number of activities on a one-to-one basis to minimize total costs of performing the tasks or maximize total profit of allocation [24]. Assignment problem is used worldwide in solving the real world problems. An assignment problem plays an important role in industry and other applications. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. In this problem, c_{ij} denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one person can be assigned exactly one job; also, each person can do at most one job. The problem is to find an optimal assignment so that the minimum is the total cost of performing all jobs or the maximum is total profit [21]. The assignment problem is to resolve the problem of assigning a number of origins to the equal number of destinations at a minimum cost or maximum profit. It can assign persons to jobs, classes to rooms, operators to machines, drivers to trucks, trucks to delivery routes, or problems to research teams, etc. [26]. There are many other applications of the linear assignment problem, e.g., in locating and tracing objects in space, scheduling parallel machines, inventory planning, vehicle and crew scheduling, wiring of typewriters, etc. To find solutions to assignment problems, various algorithms such as linear programming [20], Hungarian Algorithm [13], Neural Network [12] and Genetic Algorithm [4], have been developed. Authors have proposed different methods to handle different types of assignment problems. In this context, Albrecher [1] introduced an asymptotic behavior of bottleneck problems and Aldous [2] studied asymptotes in the random assignment problem. Many authors in different approaches [3] have analyzed quadratic assignment. Pardalos and Pitsoulis [23] developed some works on nonlinear assignment problems.

Recently, the fuzzy generalized assignment problem has become very popular because data may not be known with certainty in the real life. Therefore, to consider uncertainty in the real life situations, fuzzy data are more advantageous [25] than crisp data. As a different model for uncertainty representation, professor Julong Deng [10] proposed grey systems theory in 1982 [11]. The grey systems theory focuses on the study of such uncertain systems with partially known and partially unknown information whose matters of the characteristics of poor information are seen as its research subjects [14]. By combining the grey systems theory with the principle and method of linear assignment problem, the linear assignment model is established based on the grey systems theory. Bernardo and Blin [8] developed linear assignment method as a compensatory model of consumer choice among multi-attribute brands. This method is further extended and applied in some multi criteria decision-making problems [6]. Bai [5] showed some methods for solving the grey assignment problems. There are some special methods for solving a grey assignment problem. So, we can obtain the whitened value of the grey optimal value of the grey assignment problem and the semi-optimal value, such as the cost of assignment worker i to job j is given by the time sequence or determining the whitened value of the grey cost which is difficult. Hence, for overcoming the obvious shortcoming discussed above, here we propose a direct approach for solving the grey assignment problem based on grey arithmetic.

We emphasize that the proposed method is easy to understand and apply it for finding an optimal solution of assignment problem occurring in the real-life situation.

The rest of the paper is organized as follows. Some necessary backgrounds and definitions to the grey systems theory, which are needed in the next sections, are presented in Section 2. A definition of the grey linear assignment problem is given in Section 3 and then the proposed approach and its related issues are explained. An algorithm to solve the grey assignment problem based on grey arithmetic by using Hungarian method is proposed in Section 4. In Section 5, a numerical example is solved by the proposed procedure. Finally, Section 6 consists of conclusions.

2 Notations and Preliminaries

In this section, some definitions, notions and results are introduced which are useful to our further consideration of grey systems theory and to arithmetic on the grey numbers calculus as a tool for solving the grey assignment problem [14, 15, 16, 17, 18, 28, 29, 30].

2.1 Grey system theory and grey numbers

Grey system theory is one of the convenient tools as well as fuzzy set theory, which is used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information.

Definition 1. A grey system is defined as a system containing uncertain information presented by a grey number and grey variables.

Definition 2. Let X be the universal set $X = \mathbb{R}$, the set of all real numbers. Then grey system G of X is defined by two maps $\overline{\mu}_G(x)$ and $\underline{\mu}_G(x)$, where $\overline{\mu}_G(x) : X \longrightarrow [0, 1]$ and $\underline{\mu}_G(x) : X \longrightarrow [0, 1]$, $\underline{\mu}_G(x) \leq \overline{\mu}_G(x)$, where $\overline{\mu}_G(x)$ and $\underline{\mu}_G(x)$ are the upper and lower membership functions in G, respectively.

Remark 1. When $\overline{\mu}_G(x) = \underline{\mu}_G(x)$, grey set G becomes a fuzzy set. Thus, the grey theory considers the condition of the fuzziness and can deal with the fuzziness situation.

Grey systems theory introduces the concept of interval grey numbers. Grey numbers are regarded as the basic unit of grey systems to participate in the construction of grey model. It plays an important role in grey system theory [27]. Let X denote a closed and bounded set of real numbers. Grey number is the basic unit of the grey system and the operations of grey numbers are different from regular interval numbers. Interval number refers to a special one in grey number conception terms [29]. It is easy to prove the following results.

Definition 3. A grey number is a number with clear upper and lower boundaries, yet with an unknown position within the boundaries. A grey number in the system is expressed mathematically as $\otimes x \in [\underline{x}, \overline{x}] = \{\underline{x} \leq t \leq \overline{x}\}$ where $\otimes x$ is a grey number, t is information, $\underline{x}$ and $\overline{x}$ are the lower and upper limits of the information [9].

There are several types of grey numbers (See [17]), but the interval grey numbers are considered here as the convenient kind in the literature among them. In fact, the grey number is a number whose exact value is unknown, but a range within which the value lies in is known [16].

Definition 4. Interval grey number is the number with both lower limit $\underline{x}$ and upper limit $\overline{x}$: $\otimes x \in [\underline{x}, \overline{x}]$.

Definition 5. (Black and white numbers)When $\otimes x \in (-\infty, +\infty)$, i.e., when $\otimes x$ has neither an upper limit nor lower limit or the upper and the lower limits are all grey numbers, $\otimes x$ is called a black number. When $\otimes x \in [\underline{x}, \overline{x}]$ and $\underline{x} = \overline{x}$, $\otimes x$ is called a white number.

In the rest of the paper, we consider interval grey numbers; for the sake of simplicity, we hasve shortly called it grey number.

Remark 2. We denote the set of all grey numbers by $R(\otimes)$. We also show an element of $R(\otimes)$ that is $\otimes x \in [\underline{x}, \overline{x}]$ by $[\underline{x}, \overline{x}]_G$.

Remark 3. The transformation of an interval grey number to the appropriate crisp value can be done by using the whitening function, which can be shown as $\tilde{\otimes} G = \lambda \overline{x} + (1-\lambda)\underline{x}$ with as whitening coefficient and $\lambda \in [0, 1]$ (See [17]).

2.2 Grey degree and kernel

In this subsection, the concepts and definitions are taken from [22, 28, 30].

Definition 6. Suppose that the grey number $\otimes x \in [\underline{x}, \overline{x}]$, where $\underline{x} \leq \overline{x}$, in the case of the lack of the distributing information about the values of grey number $\otimes x$, if a grey number $\otimes x$ is continuous, then $\tilde{\otimes} x = \frac{1}{2}(\underline{x} + \overline{x})$ is called kernel of grey number $\otimes x$.

Definition 7. Suppose that the background, which makes grey number $\otimes x$ come into being is Ω and $\mu(\otimes x)$ is the measure of Ω, then $g^\circ(\otimes x) = \mu(\otimes x) / \mu(\Omega)$ is called the degree of greyness of grey number $\otimes x$ (denoted as g°).

Definition 8. Let $\tilde{\otimes} x$ and $g^\circ(\otimes x)$ be respectively the kernel and the degree of greyness of a grey number $\otimes x$. Then, $\otimes x = \hat{\otimes} x_{(g^\circ)}$ is seen as a simple form of grey number $\otimes x$.

The main arithmetic operations can be defined in a simple form of the grey numbers. Let $\hat{\otimes}_1 x_{(g^\circ)}$ and $\hat{\otimes}_2 x_{(g^\circ)}$ be two simple forms of grey numbers. The following operations can be defined:

$$\hat{\otimes}_1 x_{(g_1^\circ)} + \hat{\otimes}_2 x_{(g_2^\circ)} = (\hat{\otimes}_1 x + \hat{\otimes}_2 x)_{(g_1^\circ \vee g_2^\circ)}$$

$$\hat{\otimes}_1 x_{(g_1^\circ)} - \hat{\otimes}_2 x_{(g_2^\circ)} = (\hat{\otimes}_1 x - \hat{\otimes}_2 x)_{(g_1^\circ \vee g_2^\circ)}$$

Proposition 1. For grey numbers, there is a one-to-one correspondence between the simplified forms $\otimes x = \hat{\otimes} x_{(g^\circ)}$ and grey numbers $\otimes x \in [\underline{x}, \overline{x}]$ where $\underline{x} \leq \overline{x}$.

Based on the simplified form $\otimes x = \hat{\otimes} x_{(g^\circ)}$ of grey numbers, we can obtain the following properties of the arithmetic operations of grey numbers:

$$\hat{\otimes_1} x_{(g_1^\circ)} = \hat{\otimes_2} x_{(g_2^\circ)} \iff \hat{\otimes_1} x = \hat{\otimes_2} x \text{ and } g^\circ(\otimes_1 x) = g^\circ(\otimes_2 x)$$

With the increasing development of grey system theory in various scientific fields and the need to compare grey numbers in different areas, ranking of grey numbers plays a very important role in decision-making and some other grey system applications. Several strategies have been proposed to rank grey numbers. In this paper, we reviewed a recent ranking method, which will be useful for the researchers who are interested in this area.

Definition 9. For any grey number $\otimes x$ we have $\otimes x - \otimes x = \otimes 0$.

Definition 10. Suppose that $\otimes x$ and $\otimes y$, are two grey numbers and $\otimes \hat{x}$ and $\otimes \hat{y}$ are the kernel of $\otimes x$ and $\otimes y$, respectively, $g^\circ(\otimes x)$ and $g^\circ(\otimes y)$ are the degrees of greyness of $\otimes x$ and $\otimes y$, respectively. so
if $\otimes \hat{x} < \otimes \hat{y}$ then $\otimes x <_G \otimes y$;
if $\otimes \hat{x} = \otimes \hat{y}$ then

$$\begin{aligned}
&\text{if } g^\circ(\otimes x) = g^\circ(\otimes y) \text{ then } \otimes x =_G \otimes y;\\
&\text{if } g^\circ(\otimes x) < g^\circ(\otimes y) \text{ then } \otimes x >_G \otimes y;\\
&\text{if } g^\circ(\otimes x) > g^\circ(\otimes y) \text{ then } \otimes x <_G \otimes y.
\end{aligned}$$

Now, we are established a direct approach to solved a grey assignment problem.

3 Grey Assignment Problem

In this section, we give some essential concepts and definitions, which are taken from [5, 6, 7, 19, 26]. Let there be n jobs and n persons available with different skills. Assume that each person can do each work at a time, though with an unreliable grade of efficiency. If the grey cost of doing the j^{th} work by the i^{th} person is $\otimes c_{ij}, i, j = 1, 2, ..., n$. Now the problem is which work is to be assigned to whom so that the grey cost of completion of work will be minimized. The grey assignment problem with grey cost is represented in Table 1.

It is assumed that the cost of assigning worker i to job j is a grey number $\otimes c_{ij}, i, j = 1, 2, ..., n$, and the grey assignment problem is

Table 1: Cost matrix of the grey assignment problem

works/persons	1	2	$\dots$	j	$\dots$	n
1	$\otimes c_{11}$	$\otimes c_{12}$	$\dots$	$\otimes c_{1j}$	$\dots$	$\otimes c_{1n}$
2	$\otimes c_{21}$	$\otimes c_{22}$	$\dots$	$\otimes c_{2j}$	$\dots$	$\otimes c_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$\otimes c_{i1}$	$\otimes c_{i2}$	$\dots$	$\otimes c_{ij}$	$\dots$	$\otimes c_{in}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$\otimes c_{n1}$	$\otimes c_{n2}$	$\dots$	$\otimes c_{nj}$	$\dots$	$\otimes c_{nn}$

$$\begin{aligned} &min \quad \otimes Z =_G \textstyle\sum_{i=1}^n \sum_{j=1}^n \otimes c_{ij} x_{ij} \\ &subject\ to: \quad \textstyle\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n, \\ &\textstyle\sum_{i=1}^n x_{ij} = 1, \ j = 1, 2, \dots, n, \\ &x_{ij} = 0 \ or \ 1 \quad , i, j = 1, 2, \dots, n, \end{aligned} \tag{1}$$

where $x_{ij} = 1$ if the i^{th} person is assigned the j^{th} work and $x_{ij} = 0$ if the i^{th} person is not assigned the j^{th} work with the restrictions.

If grey number $\otimes c_{ij} \in [\underline{c}_{ij}, \overline{c}_{ij}], \underline{c}_{ij} \leq \overline{c}_{ij}$, then the following assignment problem (2) is said to be a lower limit assignment problem of the grey assignment problem (1):

$$\begin{aligned} &min \quad Z = \textstyle\sum_{i=1}^n \sum_{j=1}^n \underline{c}_{ij} x_{ij} \\ &subject\ to: \quad \textstyle\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n, \\ &\textstyle\sum_{i=1}^n x_{ij} = 1, \ j = 1, 2, \dots, n, \\ &x_{ij} = 0 \quad or \quad 1 \quad , i, j = 1, 2, \dots, n. \end{aligned} \tag{2}$$

The following assignment problem (3) is said to be an upper limit assignment problem of the grey assignment problem (2).

$$\begin{aligned} &min \quad Z = \textstyle\sum_{i=1}^n \sum_{j=1}^n \overline{c}_{ij} x_{ij} \\ &subject\ to: \quad \textstyle\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n, \\ &\textstyle\sum_{i=1}^n x_{ij} = 1, \ j = 1, 2, \dots, n, \\ &x_{ij} = 0 \quad or \quad 1 \quad , i, j = 1, 2, \dots, n. \end{aligned} \tag{3}$$

Theorem 1. In the grey assignment problem (1), if the optimal values of (2) and (3) are α and β respectively, then $\alpha \leq \beta$.

Proof. It is straightforward as given in [6]. □

Definition 11. Suppose that in a grey assignment problem, the number of rows (persons) is equal to the number of columns (works); then this problem is the balanced assignment problem.

Theorem 2. Let α and β be the optimal values of (2) and (3) respectively, if $x_{ij}^0 = (x_{11}^0, \ldots, x_{1n}^0, x_{21}^0, \ldots, x_{nn}^0)$ is a synchronal optimal solution to the grey assignment problem (1), then for every t, $0 \leq t \leq 1$, x_{ij}^0 is also an optimal solution to the following assignment problem.

$$
\begin{aligned}
&Min \quad Z = \textstyle\sum_{i=1}^{n} \sum_{j=1}^{n} \underline{c}_{ij} + t(\bar{c}_{ij} - \underline{c}_{ij}) x_{ij} \\
&subject\ to: \quad \textstyle\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n, \\
&\textstyle\sum_{i=1}^{n} x_{ij} = 1, \ j = 1, 2, \ldots, n, \\
&x_{ij} = 0 \ or \ 1 \ \ , i, j = 1, 2, \ldots, n.
\end{aligned} \tag{4}
$$

The optimal value of assignment problem (4) is equal to $\alpha + t(\beta - \alpha)$.

Proof. It is straightforward as given in [6]. □

Definition 12. Let grey number $\otimes c_{ij} \in [\alpha, \beta]$ be a grey optimal value of the grey assignment problem (1). If for some $n \times n$ real numbers γ_{ij}, $\underline{c}_{ij} \leq \gamma_{ij} \leq \bar{c}_{ij}, i, j = 1, 2, \ldots, n$, there exists $x_{ij}^0 = (x_{11}^0, \ldots, x_{1n}^0, x_{21}^0, \ldots, x_{nn}^0)$, $x_{ij}^0 \geq 0$, $\sum_{j=1}^{n} x_{ij}^0 = 1$, $\sum_{i=1}^{n} x_{ij}^0 = 1$, $i, j = 1, \ldots, n$ such that $\alpha \leq \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} x_{ij}^0 \leq \beta$ then x_{ij}^0 is said to be a semi-optimal solution to the grey assignment problem (1) and $\sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} x_{ij}^0$ is said to be the semi-optimal value corresponding to the semi-optimal solution.

Theorem 3. A balanced grey assignment problem always has semi-optimal solutions and grey optimal value.

Proof. See [6]. □

It is easy to see that the grey assignment problem is not only suitable to study static, but also suitable to study dynamic assignment problems. The solution to the grey assignment problem is fundamentally based on the following two theorems.

Theorem 4. The grey assignment minimizes the total grey cost of the new grey cost matrix; it also minimizes the total grey cost of the original grey cost matrix. If the addition (subtraction) of a constant grey number to an every grey element of a row (or column) of the grey cost matrix $\otimes c_{ij}$,then the results of the new assignment problem are the same as the previous optimal solution.

Proof. Let $x_{ij} = X_{ij}$ minimizes the total grey cost:

$$\otimes Z = \sum_{i=1}^{n}\sum_{j=1}^{n} \otimes c_{ij} x_{ij} \tag{5}$$

over all $x_{ij} \geq 0$ and

$$\sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij} = 1 \tag{6}$$

It is shown that assignment $x_{ij} = X_{ij}$ also minimizes the new total grey cost

$$\otimes \acute{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n}(\otimes c_{ij} - \otimes u_i - \otimes v_j) x_{ij}$$

For all $i, j = 1, 2, , n$, where $\otimes Z$ and $\otimes \acute{Z}$ are grey constants subtracted from the i^{th} row and the j^{th} column of the cost matrix $\otimes C_{ij}$. To prove this, it may be written as

$$\otimes \acute{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \otimes c_{ij} x_{ij} - \sum_{i=1}^{n} \otimes u_i \sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} \otimes v_j \sum_{i=1}^{n} x_{ij}$$

using equations 5 and 6, we get

$$\otimes \acute{Z} = \otimes Z - \sum_{i=1}^{n} \otimes u_i - \sum_{j=1}^{n} \otimes v_j \tag{7}$$

The terms that are subtracted from $\otimes Z$ to present $\otimes \acute{Z}$ are independent of x_{ij} and it follows that $\otimes \acute{Z}$ is minimized whenever $\otimes Z$ is minimized, and conversely. □

Theorem 5. If $x_{ij}, i, j = 1, 2, ..., n$ is an optimal solution to an assignment problem with cost $\otimes c_{ij}$, then it is also optimal for the problem with cost $\otimes \acute{c}_{ij}$ when
1)$\otimes c_{ij} = \otimes \acute{c}_{ij}$ for $i, j = 1, 2, ..., n; j \neq k$
2) $\otimes c_{ij} = \otimes \acute{c}_{ij} - \otimes a$ where $\otimes a$ is a grey constant.

Proof. We have

$$\otimes \acute{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \otimes \acute{c}_{ij} x_{ij}$$

$$= \sum_{i=1}^{n}(\sum_{j\neq k}^{n} \otimes \acute{c}_{ij} + \otimes \acute{c}_{ik}) x_{ij}$$

$$= \sum_{i=1}^{n}(\sum_{j\neq k}^{n} \otimes \acute{c}_{ij} + \otimes \acute{c}_{ik} - \otimes a) x_{ij}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \otimes c_{ij} x_{ij} - \otimes a \sum_{i=1}^{n} x_{ij}$$

$$= \otimes Z - \otimes a.$$

Thus, if x_{ij} minimizes $\otimes Z$, $\otimes \acute{Z}$ is also minimized. □

Theorem 6. If all $\otimes c_{ij} \geq \otimes 0$ in an assignment problem with the cost $\otimes c_{ij}$, then a feasible solution x_{ij} which satisfies $\sum_{i=1}^{n}\sum_{j=1}^{n} \otimes c_{ij} x_{ij} = \otimes 0$ is optimal for the problem.

Proof. Since all $\otimes c_{ij}$ and $x_{ij} \geq 0$, objective function $\sum_{i=1}^{n} \sum_{j=1}^{n} \otimes c_{ij} x_{ij}$ cannot be negative. The minimum possible value that $\otimes Z$ attained is zero. Thus, any feasible solution x_{ij} that satisfies $\sum_{i=1}^{n} \sum_{j=1}^{n} \otimes c_{ij} x_{ij} = \otimes 0$, will be optimal. □

Now based on the above theoretical discussion, we are going to present the solution method in the next section.

4 Solving Process

To solve the assignment problems there are different methods. Among them, Hungarian method [13] is simple and easy to understand. The Hungarian method is an algorithm which finds an optimal assignment for a given cost matrix. In this section, an algorithm to solve grey assignment problem with grey arithmetic by using Hungarian method is proposed. For obtaining the minimum total grey cost, we use Algorithm 4.1 which is established as follows.

4.1 Algorithm: The main steps of algorithm

Assumption. Suppose that an assignment problem, which is formulated in grey environment, is given to solve.

Step1: First test whether the given grey cost matrix of a grey assignment problem is a balanced one or not. If balanced then go to step 3.

Step2: Introduce dummy rows or columns with zero grey costs so, as to form a balanced one.

Step3: Find out the kernel and the degree of greenness of a grey number for the simplest form of each jury number in the cost matrix as mentioned in section 3 and obtain the minimum grey number of each row according to Definition 10.

Step4: Subtract the minimum grey number in each row from all the entries of its row by using the grey arithmetic to get the reduced grey cost matrix.

Step5: Obtain the minimum grey number of each column according to Definition 10 and subtract these grey numbers of the reduced grey cost matrix from those columns which have no grey numbers containing zero from all the entries of its column to get the first modified grey cost matrix.

Step6: Draw the minimum number of horizontal and vertical lines through appropriate rows and columns so that all the grey numbers containing zero of the cost matrix are covered and the minimum number of such lines is used.

Step7: Test for optimality: If the minimum number of covering lines is equal to the order of the cost matrix then optimality is reached and then stop.

Step8: Determine the smallest kernel value of the grey number, which is not covered by any lines. Subtract this entry from all uncrossed elements and add it to the crossing having a grey number containing zero and then go to step 6.

5 Numerical Example

In this section, for an illustration of the above approach, a numerical example of the grey assignment problem will be solved based on the proposed method.

Example 1. Let us consider a grey assignment problem with 3 rows representing 3 jobs, J_1, J_2 and J_3 and 3 columns representing 3 persons A, B and C. Cost matrix $\otimes c_{ij}$ is given which is grey numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum. Cost matrix $\otimes c_{ij}$ is shown in Table 2.

Table 2: Cost matrix with grey entries

works/persons	1	2	3
A	$[2,6]_G$	$[7,12]_G$	$[4,8]_G$
B	$[8,13]_G$	$[10,15]_G$	$[5,10]_G$
C	$[2,6]_G$	$[6,10]_G$	$[9,12]_G$

Find out a simple form of each grey number in the cost matrix based on the kernel and the degree of greenness. Determine minimum grey number of each row in the cost matrix.

Table 3: Simple form of grey number in cost matrix

works/persons	1	2	3	minimum of row
A	$4_{0.08}$	$9.5_{0.1}$	$6_{0.08}$	$4_{0.08}$
B	$10.5_{0.1}$	$12.5_{0.1}$	$7.5_{0.1}$	$7.5_{0.1}$
C	$4_{0.08}$	$8_{0.08}$	$10.5_{0.06}$	$4_{0.08}$

Subtract the minimum grey number of each row from all the entries of its row to get the reduced grey cost matrix.

Table 4: Reduced grey cost matrix

works/persons	1	2	3
A	$\otimes 0$	$[1,10]_G$	$[-2,6]_G$
B	$[-2,8]_G$	$[0,10]_G$	$\otimes 0$
C	$\otimes 0$	$[0,8]_G$	$[3,10]_G$

Subtract the minimum grey number of each column of the reduced grey cost matrix from all the entries of its column to get the first modified grey cost matrix.

Table 5: First modified grey cost matrix

works/persons	1	2	3
A	$\otimes 0$	$5.5_{0.18}$	$2_{0.16}$
B	$3_{0.1}$	$5_{0.1}$	$\otimes 0$
C	$\otimes 0$	$4_{0.16}$	$6.5_{0.14}$
minimum of column	$\otimes 0$	$4_{0.16}$	$\otimes 0$

Draw three vertical lines to cover all the grey numbers containing zero of the first modified grey matrix.

Table 6: Optimal assignment matrix

works/persons	1	2	3
A	$\otimes 0$	$[-7, 10]_G$	$[-2, 6]_G$
B	$[-2, 8]_G$	$[-8, 10]_G$	$\otimes 0$
C	$\otimes 0$	$\otimes 0$	$[3, 10]_G$

Since the minimum number of covering lines is equal to the order of the cost matrix, then solve it by Hungarian method to get the optimal solution as follows: optimal assignment as A, B, C persons are assigned to 1, 3, 2 jobs, respectively. Also, the optimum assignment cost as follows:

$$\otimes z =_G [2, 6] + [5, 10] + [6, 10] =_G [13, 26].$$

According to Table 7, an optimum assignment cost of the lower limit assignment problem is 13 and optimal assignments as A, B, C persons are assigned to 1, 3, 2 jobs, respectively.

According to Table 8, we get an optimum assignment cost of the upper limit assignment problem as 26 and optimal assignments as A, B and C persons are assigned to 1, 3 and 2 jobs, respectively.

Table 7: Lower limit assignment problem

works/persons	1	2	3
A	2	7	4
B	8	10	5
C	2	6	9

Table 8: Upper limit assignment problem

works/persons	1	2	3
A	6	12	8
B	13	15	10
C	6	10	12

6 Conclusion

In this paper, we investigated a practical problem, which was formulated in grey environments. We observed that the grey kind of ambiguity of parameters is concentrated in a convenient linear programming model such as assignment problem. Unlike the existing method for solving these problems, in this study, we suggested a new approach that can solve the grey linear assignment problem directly without converting it to a classical equivalent crisp model. In particular, the mentioned approach keeps greyness of all parameters in the solving process. We finally emphasize that the given approach can be also used for other convenient models, such as Transportation, Network Flows and assignment problem in the grey environment, which is discussed in the paper.

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یک رویکرد جدید برای حل مسایل تخصیص خاکستری

ناصری، س، ه.
دانشیار ریاضی کاربردی- نویسنده مسئول
ایران، بابلسر، دانشکده علوم ریاضی، دانشگاه مازندران.
nasseri@umz.ac.ir

درویشی سلوکلایی، د.
استادیار ریاضی کاربردی
ایران، تهران، دانشگاه پیام‌نور، گروه ریاضی، صندوق پستی ۳۶۹۷-۱۹۳۹۵
d_darvishi@pnu.ac.ir

یزدانی چراتی، ا.
استادیار ریاضی کاربردی
ایران، بابلسر، دانشکده علوم ریاضی، دانشگاه مازندران.
yazdani@umz.ac.ir

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چکیده

مساله تخصیص یکی از مدل های کاربردی مهم در ادبیات مسائل برنامه ریزی خطی است. داده های ورودی در ماتریس هزینه مساله تخصیص اغلب دقیق نیستند، از این رو در کاربردهای بسیاری پارامترهای فرموله شده مدل ها با نوعی عدم قطعیت در نظر گرفته می شوند و در برخی موارد اعداد خاکستری هستند. در این راه، محققان بسیاری از تکنیک سفیدسازی برای حل مساله تخصیص خاکستری استفاده می کنند. از آنجایی که روش سفیدسازی تنها یک مدل معادل قطعی را مهیا می کند و خاصیت تکاملی مجموعه خاکستری را منعکس نمی کند، آن نمی تواند دامنه ای از مقادیر بهینه و جواب های بهینه را ایجاد کند. بر این اساس، در این مقاله یک رویکرد مستقیم برای حل مساله تخصیص در محیط خاکستری معرفی شده است. برای ساختن روش اشاره شده، برخی نتایج نظری به منظور تقویت روش شناسی موضوع ارائه شده است. در پایان، مثال عددی به منظور آزمون درستی روش پیشنهادی حل شده است. بر اساس روش پیشنهادی، تاکید می شود که رویکرد مشابه می تواند هنگامی که هر مدل برنامه ریزی خطی در محیط خاکستری فرموله شده باشد، استفاده شود.

کلمات کلیدی

مساله تخصیص، نظریه سیستم های خاکستری، عدد خاکستری، عدم قطعیت، روش سفید سازی.