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A New Approach for Approximating Solution of Continuous Semi-Infinite Linear Programming

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Abstract. This paper describes a new optimization method for solving continuous semi-infinite linear problems. With regard to the dual properties, the problem is presented as a measure theoretical optimization problem, in which the existence of the solution is guaranteed. Then, on the basis of the atomic measure properties, a computation method was presented for obtaining the near optimal solution by means of famous and simple simplex method. Some numerical results are reported to indicate the efficiency of the new method.

Keywords. Atomic measure, Linear programming, Radon measure, Semi-infinite linear programming, Weak* topology.

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1 Introduction

A semi-infinite linear program (SILP) is an optimization problem with a linear objective and constraint functions where either the number of constrains or the dimension of the variable space, but not both, is allowed to be infinite. Since, the most natural linear phenomena is continuous, the SILP models occur in a wide variety of scientific and engineering applications such as signal processing, filter designing [24], computing solutions of monotonic linear boundary value problems, experimental design in regression [13], air and water pollution [8], Chebyshev approximation [10], lapidary cutting problems [28], solution methods for linear programming problems under uncertainty (like fuzzy systems [15]), transportation problems [26] and geometrical applications [9].

Among the recent studies in this area, we emphasize on using first order approximation of the feasible set corresponding to constraints qualifications [25], non-smooth semi-infinite programming (SIP) problems with a feasible set defined by inequality and equality constrains [17], bivariate interval method for SILP problems [11], and an application of interior point for solving SILP [19] and SIP to lapidary cutting problems [28].

The primal form of a SILP problem can be formulated mathematically as follows:

$$\textbf{(P)} \qquad min \qquad c'x$$
$$subject \;\; to: \quad a_t'x \geq b_t \;, t \in T$$

where $c \in \mathbf{R}^n$, T is the index set, $a_t \equiv a(t) = (a_1(t), a_2(t), ..., a_n(t))'$ maps T onto $\mathbf{R}^n$ and $b_t \equiv b(t)$ is a scalar function on T. If T is a countable infinite set, then the SILP problem is called a countable semi-infinite linear program; also, on the other hand, if T is a compact Hausdorff space and the functions $a : T \longrightarrow \mathbf{R}^n$, $a(t) = a_t$ and $b : T \longrightarrow \mathbf{R}$, $b(t) = b_t$ are continuous functions, the problem is called a continuous semi-infinite linear program. It is also argued in [8] that the natural dualality of a continuous problem (P) is as follows:

$$\textbf{(D)} \qquad max \qquad \int_T b(t)\, d\mu$$
$$subject \;\; to: \quad \int_T a(t)\, d\mu = c, \, \mu \in M^+(T),$$

where $M^+(T)$ is the set of all positive Radon measures on T. It should be reminded that a Radom measure is a positive regular Borel measure (which defined on a compact Hausdorff space) and $M^+(T)$ is the topology space of all positive Radon measures on T. Indeed, a different duality can be defined for an LSIP problem. Actually (D) coincides with the dual problem of the general primal LSIP problem by means of Kretschmer duality theory for infinite linear programming (for more detail and how this dual problem is structured in brief, please see the Notes in chapter 2 of [8], or more explanations, in the related chapters of the book [18]).

Remark. Throughout the paper, it is supposed that (P) is consistent and

$$M^+(T) \neq \phi.$$

Moreover, $\int_T b(t)d\mu - c'x$ is so-called *duality gap* [8, 10]. The relations and properties of the (P) and (D) are discussed in detail in [8, 10] In the absence of duality gap (See [8]), different methods are available for solving the primal and dual forms of the SILP problems, including local reduction and discretization Methods [8], three-phase method [7], primal and dual exchange methods [20], perturbation method [27] and directions method [16]. However, for existing the duality gap there is not an identified solution method especially for dual problem.

In this paper, a method is discussed for solving (D) in a different mathematical point of view. In the second section, the underlying space of the problem is equipped with the weak* topology. Then, the existence of the optimal solution is proved and the problem is converted into a measure theoretical optimization one. It is shown in Section 3 that the optimal measure can be identified as a finite linear combination of atomic measures. Moreover, regarding the unitary atomic measure's properties, it is shown that the new problem could be still left linear and constrains can be shown in a very simple form. Then, by considering the ability of the linear analysis, one can use the simplex method for finding the optimal solution. After presenting the algorithm path of finding optimal solution by the new method, some numerical tests are done in section 4 and the results are compared with some other methods.

One of the other important advantages of this method is that, for this theoretical measure optimization problem, it is possible to characterize and represent the optimal measure in a very simple way. Hence, to compare it with other methods, there is no complexity on the ambiguous dimensional and integral forms of the problem; in addition, in general, the optimal solution can be easily found.

2 Existence

The aim of this section is to show that problem (D) has a solution, independent of (P). This important aim is achieved by transferring the problem to a new space. To this end, let T be a bounded, closed, and hence compact subset of $\mathbf{R}^k$; for every $F \in C(T)$ (the set of all continuous functions $F : T \longrightarrow \mathbf{R}^k$) and every positive measure $\mu \in M^+(T)$, $\int_T F d\mu$ is denoted by $\mu(F)$. Moreover, $\phi \neq Q \subset M^+(T)$ is defined as the set of all regular measures with the compact support (say, Radon measures as [12, 22]) on T which satisfies the following equation:

$$\mu(a_t) = c. \tag{1}$$

Herein, Q is considered just as a subset of $M^+(T)$ and no topology is considered (or used) on it. In this section, this set is considered as a topological space in order to be able to prove the existence of the optimal solution.

As Rubio did in [22], the set Q is endowed with weak* topology which has some benefits in terms of the propositions that will be explained in the following sections. A weak*topology on Q can be defined by the family of semi-norms $\mu \mapsto |\mu(F)|, \forall F \in C(T)$; it gives rise to the basis of neighborhoods of zero in $M^+(T)$ as follows:

$$U_\varepsilon = \{\mu \in M^+(T) : |\mu(F_j)| < \varepsilon, \quad j = 1, 2, ..., r\},$$

for every $\varepsilon > 0$ and for all the finite subsets $\{F_j : \ j = 1, 2, ..., r\}$ of $C(T)$ and for every positive integer r.

$\mu(b_t)$ as a function is continuous; to show this, we present the following proposition in which to prove it we followed Rubio in [22] chapter 2.

Proposition 1. The function $\mu(b_t)$ that maps Q into the real line, is continuous in the sense of weak*topology.

Proof. We know that function $\mu \in Q \mapsto \mu(b_t) \equiv \int_T b_t \, d\mu \in \mathbf{R}$ is continuous if the inverse image of every neighborhood of a basis of neighborhoods of $\mu(b_t)$ in $\mathbf{R}$, is a neighborhood of μ in Q. This is equivalent to the fact that for every $\varepsilon > 0$, the set $\{\nu \in Q : |(\mu - \nu)(b_t)| < \varepsilon\}$ is a neighborhood of μ in Q. This follows from two facts; one is that the weak*topology on $M^+(T)$ is defined by the neighborhoods of zero. The other one is that the neighborhoods of a point with respect to the set Q, are the intersection of this set with the neighborhoods of the point in $M^+(T)$. □

The following proposition helps us to prove that the solution space of (D) is computed.

Proposition 2. The set Q, defined as those measures in $M^+(T)$ which are satisfied in (1), is compact in the sense of weak*topology on $M^+(T)$.

Proof. Since T is compact, for any positive number α, the set $\{\mu : \mu \in M^+(T), \mu(1) = \alpha\}$ is also compact (See [3]) (here, the function 1 equals 1 on T) and we have $\mu(1) = \int_T d\mu \equiv \mu(T)$; thus, the set Q is a subset of a compact set $M^+(T) \equiv \{\mu : \mu \in M^+(T), \mu(1) = \mu(T)\}$. Now it is enough to prove that Q is closed; then, the fact that it is compact which follows readily. We know that Q is defined as a set of all measures $\mu \in M^+(T)$ that $\mu(a_t) = c$; hence, this set is the inverse image of the singleton set $\{c\} \subset \mathbf{R}^n$. Thus Q is closed by the continuity of the function $\mu \mapsto \mu(b_t)$ (See Proposition 1). □

By considering the two above propositions, the set Q, the solution space of (D), is compact and the objective function of (D) is a continuous function; since each upper semi-continuous function on a nonempty compact set attains its maximum [14], the following existence theorem can be presented.

Theorem 1. There exists an optimal positive Radon measure μ^* in the nonempty subset Q of $M^+(T)$ which satisfies $\mu^*(b_t) \geq \mu(b_t)$ for all $\mu \in Q$.

It is remarked herein that, by the above theorem, when $Q \neq \phi$, even if there is the duality gap, the dual problem (D) has a solution. This is one of the important advantages of this method in comparison with the others. Of course, it would be better to have a way for identifying the optimal measure μ^* as well, which is the main goal in the next section.

3 Determination via the Atomic Measure

The idea of using atomic measures for a new representation of a theoretical measure optimization problems was started by Rosenbloom in 1956 [21]. Rubio used this fact to solve the optimal control problems in many papers such as [23, 5, 6]; they also determined the optimal control as a piecewise constant function by using the properties of atomic measures. In addition they have been used to determine the optimal shapes and domains (See for instance [5, 6]). Now, in SILP we are going to introduce a new method to determine measures μ^*, the maximizer of the functional

$$\mu \mapsto \mu(b_t), \tag{2}$$

in the nonempty set Q of positive Radon measures on T which satisfies the equalities

$$\mu(a_i(t)) = c_i, \quad i = 1, 2, ..., n, \tag{3}$$

where it is a representation of (1) since $a_t = (a_1(t), a_2(t), \dots, a_n(t))$ and $c = (c_1, c_2, \dots, c_n)$.

In another point of view, even the problem defined by (2) and (3) is linear according to the unknown variable μ; however, the underlying space is an infinite dimensional one. It is very suitable if one can develop the problem into a finite dimensional one, even approximately. Moreover, it would be so convenient if the solution of this new problem could be able to identify μ^* perfectly well.

Let $\delta(t)$ be a unitary atomic measure with the support of the singleton set $\{t\}$; this means that for each $F \in C(T)$ we have $\delta(t)(F) = F(t)$ (See [4]). Then, by regarding a proposition of Rosenbloom in [21], which is also mentioned in [22], the following important result can be presented.

Proposition 3. The maximizer measure of (D), $\mu^* \in Q$, is in the form of

$$\mu^* = \sum_{k=1}^{N} \alpha_k^* \delta(t_k^*), \tag{4}$$

where t_k^* belongs to a dense subset of T and coefficient α_k^* is positive for all $k = 1, 2, ..., N$.

The above proposition has allowed us to build a new formulation for problem (D) as follows in which the coefficients α_k^* and the supporting points t_k^* are unknowns:

$$\begin{aligned} &max && \textstyle\sum_{k=1}^{N} \alpha_k^* b(t_k^*) \\ &subject\ \ to: && \textstyle\sum_{k=1}^{N} \alpha_k^* a_i(t_k^*) = c_i, \quad i = 1, 2, ..., n; \\ & && \alpha_k^* \geq 0. \end{aligned} \tag{5}$$

In the first view, it may seem that using atomic measures made the solution path very rough, since they change the linear problems (2) and (3) into a strong nonlinear one (5). However, indeed, the mentioned structural result paves the way for determining the optimal solution very easily. It is known that if (5) is maximized with respect to only the coefficients α_k^*, it would be more convenient; this could transform the problem into a finite linear programming one. The answer lies in approximating the supporting points by introducing a dense subset in T.

Proposition 4. Let W be a given countable dense subset of T; then:
(i) for a given $\varepsilon > 0$, a measure

$$\nu = \sum_{k=1}^{N} \alpha_k^* \delta(t_k) \in M^+(T) \tag{6}$$

can be found such that:

$$| (\mu^* - \nu) b_t | < \varepsilon \quad , | (\mu^* - \nu) a_i(t) | < \varepsilon, \ i = 1, 2, ..., n, \tag{7}$$

where the coefficients α_k^* are the same as the ones in the optimal measure (4) and $t_k \in W$ for $k = 1, 2, ..., N$.
(ii) If $N \to \infty$ then $\nu \to \mu^*$.

Proof. To prove the first item, by using of Rubio's method in [22] chapter 3, let $f_i = a_i(t)$ $(i = 1, 2, ..., n)$; since the functions $f_i, i = 1, 2, ..., n$, are continuous and the number of them is finite, we have:

$$\begin{aligned} |(\mu^* - \nu) f_i| &= |\sum_{k=1}^{N} \alpha_k^* (f_i(t_k^*) - f_i(t_k))| \\ &\leq Max_{i,k} |f_i(t_k^*) - f_i(t_k)| |\sum_{k=1}^{N} \alpha_k^*| \\ &= \mu(T) Max_{i,k} |f_i(t_k^*) - f_i(t_k)|. \end{aligned}$$

Since W is a countable dense set of T, then for a given $\varepsilon > 0$, by choosing $t_k \in W, k = 1, 2, ..., N$, sufficiently near to t_k^*, one can make $Max_{i,k}|f_i(t_k^*) - f_i(t_k)|$ less than $\varepsilon/\mu(T)$; therefore, the second inequalities in (7) are satisfied. Moreover, by applying the same way, one can show that the first inequality of (7) is also satisfied.
To prove the second item, since W is a countable set, when $N \to \infty$, we conclude that $\{t_1, t_2, ..., t_N\} \to W$. Thus by density property of W in T, the result is deduced. □

Now, a computational method for solving (5) can be presented. First, a countable dense subset of T is chosen. Then, by selecting a finite number of elements in this set as the candidates of supporting points of (5), the problem can be replaced by a finite linear programming one. The optimal solution of this problem can produce a suitable near optimal solution for (D). As a result of density property, by increasing the number of selected supporting points (N in (5)), one can obtain an approximated solution with more precision.

In summary, we present the executive procedure of the method as the following algorithm:

Step 1: For the give primal SLIP problem (P), set up its dual problem (D);
Step 2: Choose the positive integer number N and a countable dense subset in T;
Step 3: Select N member element of W, say $t_1, t_2, ..., t_N$;

Step 4: Set up measure ν as (7), substitute it instead of μ in (D) to construct a finite linear programming (FLP).

Step 5: By solving the obtained FLP from Step 4, determine the optimal α^* and the approximation optimal value of objective function. Also, set up the near optimal measure by (7).

We remind that by proposition 3.2., this algorithm is convergent, yet not definite in an decreasing manner; we should emphasize that the idea of converting the problem into a finite linear programming, is a direct application of this method. One may use some indirect techniques such as artificial neural network and so on. The following examples show the way that the new method is applied and the level of its accuracy in comparison with others.

4 Numerical Results

To describe the application of the mentioned method for solving continuous SILP problems, two test problems are presented in this section; these problems have been used in many references like [1, 8] as test examples to compare various presented methods for solving SILP problems. Therefore, the reader has the opportunity to compare the obtained results from the presented method in this paper with ones in the literatures.

Problem 1. (Example in [1, 8]) Consider the following SILP problem:

$$\begin{aligned} &min && 2x_1 + x_2 \qquad (P) \\ &subject\ \ to: && tx_1 + (1-t)x_2 \geq t - t^2, \quad t \in T = [0,1]; \\ & && x \in \mathbf{R}^2. \end{aligned}$$

By following the presented algorithm in the previous section, this problem has the following dual form:

$$\begin{aligned} &max && \int_T (t - t^2)\, d\mu \\ &subject\ \ to: && \int_T t\, d\mu = 2; \\ & && \int_T (1-t)\, d\mu = 1; \\ & && \mu \in M^+(T),\, \mu \geq 0. \end{aligned}$$

To discretize T, 200 rational points were chosen for t in $[0,1]$. Therefore, the related linear programming problem similar to (5), with 200 variables and 2 constrains was setup. This problem was solved by the revised simplex method using the software Maple 12. The obtained optimal value of objective function is 0.66665624999994, while the actual amount of the optimal value of the objective function is $0.\bar{6}$ as mentioned in [1, 8]. Moreover, the optimal measure of the dual problem can be represented by $\mu^* = 0.499999999982166042\ \delta(0.66250) + 2.50000000001770050\ \delta(0.66750)$.

Problem 2. (Example in [8]) Consider the problem of evaluating the amount of the function $\tan(t)$ for $t \in [0,1] = T$, with respect to the polynomials of the degrees less than a given n. It was shown in [8], that it is possible to represent this problem as the minimization of the function $c'x$, where

$$c_i = \int_0^1 t^{i-1} dt = i^{-1}, \quad i = 1, 2, ..., n$$

for all the feasible polynomials $p_n(t) = \sum_{i=1}^n x_i t^{i-1}$. This means that the following semi-infinite linear problem should be solved in order to obtain the best coefficients for approximating the function $\tan(t)$ as a linear combination of the functions $1, t, t^2, t^3, ..., t^{n-1}$:

$$\begin{aligned} &min && \textstyle\sum_{i=1}^n i^{-1} x_i \\ &subject\ \ to: && \textstyle\sum_{i=1}^n t^{i-1} x_i \geq tan(t), \quad t \in [0,1] \\ & && x \in \mathbf{R}^n. \end{aligned} \tag{8}$$

The duality of this problem in the sense of (D), which is also mentioned in [8], can be shown as follows:

$$\begin{aligned} &max && \textstyle\int_T (tan(t))\, d\mu \\ &subject\ \ to: && \textstyle\int_T d\mu = 1; \\ & && \vdots \\ & && \textstyle\int_T t^{n-1}\, d\mu = \frac{1}{n}; \\ & && \mu \in M^+(T),\ \mu \geq 0. \end{aligned} \tag{9}$$

To solve (9), as mentioned in the previous section, the optimal measure was considered as $\mu^* = \sum_{k=1}^N \alpha_k^* \delta(t_k^*)$, where $t_k^* \in T$; hence, the problem was converted to the following nonlinear one in which its unknowns include coefficients α_k and the supporting points t_k's. To determine the optimal measure, a discretization was put on $[0,1]$ by dividing it to N equal sections and then selecting a node in each subsection. In this example, the middle point of each subsection was chosen as a node; these nodes are denoted by z_i, $i = 1, 2, ..., N$. Thus, the following finite LP problem was established:

$$\begin{aligned} &max && \sum_{k=1}^{N}\alpha_k(\tan(z_k)) \\ &subject\ \ to: && \sum_{k=1}^{N}\alpha_k = 1; \\ & && \vdots \\ & && \sum_{k=1}^{N}\alpha_k z_k^n = \tfrac{1}{n}; \\ & && \alpha_k \geq 0,\ k = 1,2,...,N. \end{aligned}$$

It should be noted here in this rock example that, we have to select another parameter, n, which is effective in the solution, together with the t_k. This fact in agreement with the related atomic measures may cause some switching like, in the sense of piecewise control function or wallets, which may cause some inconvenient result (but not at all bad) in the approximation scheme (See the 3th row of the following table); one may chose suitable n with by trial and error. Also for a large number of N, since the huge amount of α_i in the linear programming problem (5) should be zero, the error of rounding in digital computers may cause the above-mentioned inconvenient (such as the results of the row $n = 4$ in the following table, when the result for a small number of N is more suitable). It is important to mention that numerical works show that, for large N (more than 10000), the amount of objective function would be fixed, which is a good reason for convergence. The problem was solved for different values of $n \in \{2, 3, 4, ..., 8\}$ with the revised simplex method applied by the software Maple 15. The optimal solution for different numbers of nodes was obtained and presented in Table 1 together with the real optimal values, which are mentioned in [8].

Table 1: Results with respect to the number iterations for problem 2.

N	5	100	100000	Op. Solution
n=2	0.77870386	0.77270618	0.77719207	0.77870386
n=3	0.64483765	0.64789969	0.64875694	0.64904209
n=4	0.63042552	0.63077503	0.63125513	0.62376961
n=5	0.61880620	0.619412723	0.61941442	0.77870386
n=6	0.61635715	0.61676698	0.61676793	0.64904209
n=7	0.61594095	0.61620649	0.61620685	0.62376961
n=8	0.61575436	0.61584115	0.61584140	0.62376961

Our computation experiments accompanied by a brief comparison with [8], reveal that the presented method in this paper occurs sufficiently and, of course, simply; moreover, it is important to note that such as example 1, the results are so close to the optimal solution when we choose less points in discretizing. In addition, the optimal measure can be presented by a positive combination of atomic measure in the sense of (4). For instance, by assuming $n = 3$ and $N = 200$, one can obtain that:

$$\mu^* = \quad 0.006218906180692209\ \delta(0.32750) + 0.747493734265609\ \delta(0.33250) \\ +0.25188437511706\ \delta(0.99750).$$

5 Conclusion

In this paper, a new theoretical measure view was applied to solve continuous SILP problems. This paper, showed that measure theory is a useful methodology for dealing with SILP problems.The existence theorem indicated that in this method, if the solution space is nonempty, then the optimal solution definitely exists, even when the duality gap exists. Moreover, it was shown, that by applying the unitary atomic measure properties, this optimal solution could be identified in a simple manner just by solving a finite linear programming problem, sufficiently well. In brief, some main advantages of this new method regarded in an automatic existence theorem, simplicity, obtaining the dual optimal solution directly independent of the primal solution and the duality gap, and the suitable enough accuracy.

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چکیده

این مقاله یک روش جدید بهینه سازی برای حل مسایل خطی نیمه-نامتناهی پیوسته را شرح می دهد. با در نظر گرفتن خواص دوگان، ابتدا مساله به صورت یک مساله بهینه سازی در نظریه اندازه ها ارایه شده است به طوری که وجود جواب آن تضمین شده می باشد. آنگاه بر پایه خواص اندازه های اتمی و بهره گیری از روش مشهور سیمپلکس، یک روش محاسباتی برای تعیین جواب نزدیک بهینه ارایه شده است. به منظور نمایاندن کارآیی روش جدید، چندین نتیجه عددی نیز گزارش شده است.

کلمات کلیدی

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