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A Fully Fuzzy Method of Network Data Envelopment Analysis for Assessing Revenue Efficiency Based on Ranking Functions

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Abstract. The purpose of this paper is to evaluate the revenue efficiency in the fuzzy network data envelopment analysis. Precision measurements in real-world data are not practically possible, so assuming that data is crisp in solving problems is not a valid assumption. One way to deal with imprecise data is fuzzy data. In this paper, linear ranking functions are used to transform the full fuzzy efficiency model into a precise linear programming problem and, assuming triangular fuzzy numbers, the fuzzy revenue efficiency of decision makers is measured. In the end, a numerical example shows the proposed method.

Keywords. Network data envelopment analysis, Revenue efficiency, Full fuzzy linear programming, Ranking function.

MSC. 90C34; 90C40.

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1 Introduction

Data envelopment analysis (DEA), initially developed by [3], is a non-parametric technique for evaluating the relative efficiencies of homogeneous decision-making units (DMUs) in terms of multiple inputs and multiple outputs. The basic DEA models and their numerous theoretical and methodological extensions have been reported in [6]. Unlike the black box model, the Network Data Envelopment Analysis (NDEA) model considers all internal processes in performance evaluation. For example, many companies are composed of several sections that have linked activities such as Figure 1. In this example, the company has 3 sections. Each section uses its input resources to generate its output. In either case, there are links or intermediate products that are shown by the link $1 \longrightarrow 2$ and $1 \longrightarrow 3$, and the link $2 \longrightarrow 3$. The link $1 \longrightarrow 2$ shows that part of the outputs of section 1 are used as inputs in section 2. In the current DEA models, each activity must belong to an input or output, and not both, so these models cannot be formulated with intermediate products. For the first time in the year 2000, Fare and

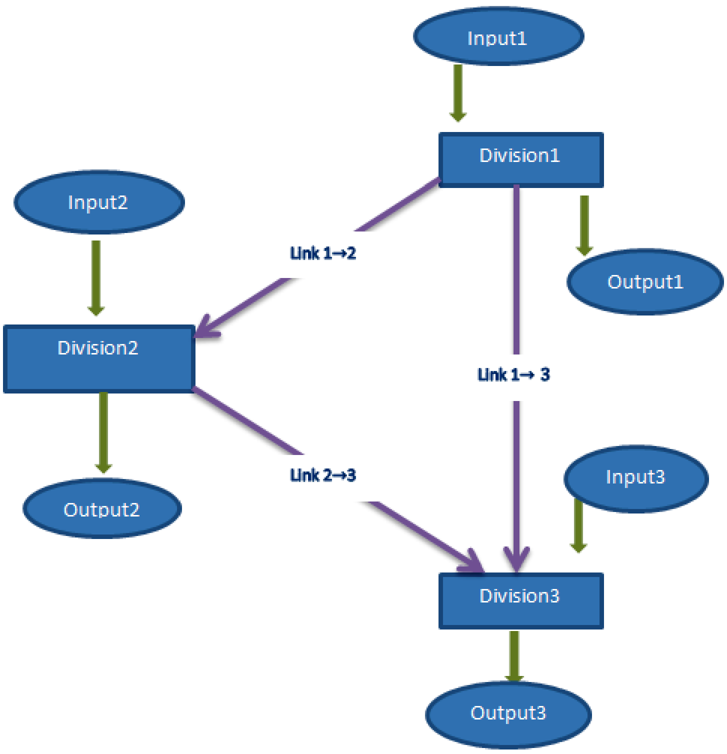

Figure 1: A company with three linked activities

Grosskopf [7] provided network data envelopment analysis models. Their models were expanded by several authors. Sexton and Lewis presented a multi-stage network data envelopment analysis model in 2004 as an extension of the Lewis and Sexton two-step data envelopment analysis model [9]. This article solves a dea model independently for each NODE. Tone and Tsutsui [16] presented a network-based data envelopment analysis model in 2009 based on the SBM model.

The Revenue Efficiency Model (RE) seeks to find a unit that receives the highest revenue from inputs equal to the inputs of the unit under consideration, from the sales of non-less than the outputs of the unit under evaluation. Revenue Efficiency is defined as the ratio of observed revenue to the maximum possible revenue. Given the fact that in the real world we are dealing with network data envelopment analysis, it is important for managers to evaluate the revenue efficiency in NDEA. In 2013, Bani Hashemi and Tohidi [2] presented a model for assessing the revenue efficiency of network data envelopment analysis models.

Classical DEA models assume that all data is crisp. However, crisp data is not always available because the nature of data can be vague and unclear. In this case, one of the important methods for dealing with inaccurate data is to consider fuzzy data. Only in [12] and [13] the fuzzy revenue efficiency (FRE) with input- outputs fuzzy and fuzzy input prices is discussed. Aghayi [1] is examined revenue efficiency measurement with undesirable data in fuzzy DEA and also Kordrostami and Jahani Sayyad Noveiri [8] are studied fuzzy revenue efficiency in sustainable supply chains.

However, in none of these studies, the measurement of fuzzy revenue efficiency has not been mentioned in Full Fuzzy Network Data Envelopment Analysis (FFNDEA). In this paper, we examine full-fuzzy models of network data envelopment analysis (fuzzy input-outputs and fuzzy input prices) to evaluate fuzzy revenue efficiency. Here, the method of ranking functions is used. Therefore, the ranking functions transform the full fuzzy model of network revenue efficiency into a crisp linear programming problem for measuring the fuzzy network revenue efficiency. The rest of the article will be as follows. In section 2, we refer to fuzzy clauses. In the next section, the problem of fuzzy linear programming and its transformation into a crisp problem is studied. section 4 addresses the measurement of revenue efficiency in the DEA, and in Sections 5 and 6 is examined network data envelopment analysis based on SBM model and revenue efficiency in it. Section 7,the proposed method for measuring fuzzy revenue efficiency in FFNDEA is presented and, based on the proposed method, a numerical example is solved in the last section.

2 Fuzzy Premises

2.1 Basic Definitions of Fuzzy

In this section, the basic definitions and the symbols of the fuzzy sets [17, 18], fuzzy Numbers [4], Ranking function [10], and the FFLP concept used in this article.

Definition 1. [17] A fuzzy set $\tilde{A}$ is defined in the reference set X with $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \longrightarrow [0,1]$ is the membership function and $\mu_{\tilde{A}}(x)$ is the degree of x in A.

Definition 2. [18] Regarding X as the reference set, then fuzzy set A will be convex if and only if for every $x_1, x_2 \in X$:

$$\mu_{\tilde{A}}\big(\lambda x_1 + (1-\lambda)x_2\big) \geq \min\big(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big) \qquad \forall \lambda \in [0,1]$$

Definition 3. [18] Assuming that X is the reference set, then the fuzzy set A is called normal provided that there exist $x \in X$ so that $\mu_{\tilde{A}}(x) = 1$.

Definition 4. [18] A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line R such that

1. it exists exactly one $x_0 \in R$ $\mu_{\tilde{A}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{A}$).
2. $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 5. [18] A triangular fuzzy number (TFN), $\tilde{A} = (a^l, a^m, a^u)$ is a fuzzy number with the given membership function $\mu_{\tilde{A}}$

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a^l)/(a^m-a^l) & a^l < x \leq a^m \\ (x-a^u)/(a^m-a^u) & a^m \leq x < a^u \\ 0 & otherwise. \end{cases}$$

Definition 6. A triangular fuzzy number $\tilde{A} = (a^l, a^m, a^u)$ is called a nonnegative number if and only if a $a^l \geq 0,\ a^m - a^l \geq 0,\ a^u - a^m \geq 0$ and it is a positive number if and only if $a^l > 0,\ a^m - a^l \geq 0,\ a^u - a^m \geq 0$.

Definition 7. The support of a fuzzy set $\tilde{A}$, $S(\tilde{A})$ is the crisp set of all $x \in X$ such that . $\mu_{\tilde{A}}(x) > 0$. The (crisp) set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α-cut set: $A_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}$

Definition 8. [10] Suppose $\mathfrak{F}$ a set of all triangular fuzzy numbers. If $\tilde{A} \in \mathfrak{F}$, $[A^1_\alpha, A^u_\alpha]$, $\alpha \in [0,1]$ the α- cut is $\tilde{A}$. Then, the ranking function of a function $\mathfrak{R} : \mathfrak{F} \longrightarrow \mathbb{R}$ is:

$$\mathfrak{R}(\tilde{A}) = \frac{1}{2}\int_0^1 (A^l_\alpha + A^u_\alpha)d\alpha$$

If $\tilde{A} = (a^l, a^m, a^u)$ is a triangular fuzzy number, then $\mathfrak{R}(\tilde{A}) = \dfrac{1}{4}(a^l + 2a^m + a^u)$.

Definition 9. [10] If $\tilde{A} = (a^l, a^m, a^u)$ and $\tilde{B} = (b^l, b^m, b^u)$ are two triangular fuzzy numbers, then order of $\tilde{A}$ and $\tilde{B}$ based on the ranking function $\mathfrak{R}$ will be:

(i) $\tilde{A} \preccurlyeq \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) \leq \mathfrak{R}(\tilde{B})$

(ii) $\tilde{A} \succcurlyeq \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) \geq \mathfrak{R}(\tilde{B})$

(iii) $\tilde{A} \approx \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$

And the features of Linearity will be:

$$\mathfrak{R}(k\tilde{A} + \tilde{B}) = k\mathfrak{R}(\tilde{A}) + \mathfrak{R}(\tilde{B}), \qquad k \in \mathbb{R}$$

2.2 Math Operations on Triangular Fuzzy Numbers

If $\tilde{A} = (a^l, a^m, a^u)$ and $\tilde{B} = (b^l, b^m, b^u)$ are two triangular fuzzy numbers, then the mathematical operations on triangular fuzzy numbers will be as follows:

(i)	Addition	$\tilde{A} \oplus \tilde{B} \approx (a^1 + b^1, a^m + b^m, a^u + b^u)$
(ii)	Subtraction	$\tilde{A} \ominus \tilde{B} \approx (a^1 - b^u, a^m - b^m, a^u - b^1)$
(iii)	Multiplication	$\tilde{A} \otimes \tilde{B} \approx (a^1 b^1, a^m b^m, a^u b^u), \quad \tilde{A}, \tilde{B} \succcurlyeq \tilde{0}$
(iv)	Division	$\frac{\tilde{A}}{\tilde{B}} \approx \frac{(a^1, a^m, a^u)}{(b^1, b^m, b^u)} \approx \left(\frac{a^1}{b^u}, \frac{a^m}{b^m}, \frac{a^u}{b^1}\right), \quad \tilde{A}, \tilde{B} \succ \tilde{0}$
(v)	Scalar multiplication	$\forall k \in \mathbb{R},\ k\tilde{A} \approx \begin{cases} (ka^1, ka^m, ka^u), & k > 0 \\ (ka^u, ka^m, ka^1), & k < 0 \end{cases}$

3 Fuzzy linear programming problem

A linear programming problem with fuzzy coefficients and variables is called a full fuzzy linear programming problem. A full-fuzzy linear programming problem [11] with m constraints and n fuzzy variables are defined by the following model:

$$\begin{aligned} &\tilde{Z} = \max\,(or\,\min)(\tilde{C}^T \otimes \tilde{X}) \\ &\text{subject to} \quad \tilde{A} \otimes \tilde{X} \preccurlyeq, \approx, \succcurlyeq \tilde{b};\ \tilde{X} \succcurlyeq \tilde{0} \qquad (P1) \end{aligned}$$

where $\tilde{C} = [\tilde{c}_j]_{n\times 1}$, $\tilde{X} = [\tilde{x}_j]_{n\times 1}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{b} = [\tilde{b}_j]_{m\times 1}$, and $\tilde{a}_{ij}$, $\tilde{c}_j$, $\tilde{b}_i \in \mathfrak{F}$, $\tilde{x}_j$ are non-negative fuzzy numbers and $\tilde{0} = (0, 0, 0)$.

Definition 10. [11] The fuzzy optimal solution to the full-fuzzy linear programming problem $(P1)$ will be $\tilde{X} = [\tilde{x}_j]_{n\times 1}$. will apply if the following conditions apply:

1) $\tilde{x}_j$ is a non-negative fuzzy number,

2) $\tilde{A} \otimes \tilde{X} \preccurlyeq, \approx, \succcurlyeq \tilde{b}$,

and 3) If there exist any non-negative fuzzy number such as $\tilde{Y} = [\tilde{y}_j]_{n\times 1}$, to the point where $\tilde{A}\otimes\tilde{X} \preccurlyeq, \approx, \succcurlyeq \tilde{b}$, then $\Re(\tilde{C}^T\otimes\tilde{X}) \geq \Re(\tilde{C}^T\otimes\tilde{Y})$ for the maximization problem and $\Re(\tilde{C}^T\otimes\tilde{X}) \leq \Re(\tilde{C}^T\otimes\tilde{Y})$ for the minimization problem.

Definition 11. [11] Suppose that $\tilde{X} = [\tilde{x}_j]_{n\times 1}$ is the fuzzy optimal solution for full fuzzy linear problem $(P1)$. If there exist any non-negative fuzzy number such as $\tilde{Y} = [\tilde{y}_j]_{n\times 1}$, then $\tilde{A}\otimes\tilde{Y} \preccurlyeq, \approx, \succcurlyeq \tilde{b}$, and $\Re(\tilde{C}^T\otimes\tilde{X}) = \Re(\tilde{C}\otimes\tilde{Y})$, then $\tilde{Y} = [\tilde{y}_j]_{n\times 1}$ is called a fuzzy optimal solution of $(P1)$. Suppose that $\tilde{c}_j = (c_j^1, c_j^m, c_j^u)$, $\tilde{x}_j = (x_j^1, x_j^m, x_j^u)$, $\tilde{a}_{ij} = (a_{ij}^1, a_{ij}^m, a_{ij}^u)$ and $\tilde{b}_j = (b_i^1, b_i^m, b_i^u)$ represents triangular fuzzy numbers. Then, the fuzzy decision parameters and variables in the model $(P1)$ are converted as follows:

$$\begin{aligned} &\tilde{Z} = \max\,(or\ \min)\Big(\sum_{j=1}^{n}(c_j^1, c_j^m, c_j^u)\otimes(x_j^1, x_j^m, x_j^u)\Big) \\ &\text{subject to} \quad \sum_{j=1}^{m}(a_{ij}^1, a_{ij}^m, a_{ij}^u)\otimes(x_j^1, x_j^m, x_j^u) \preccurlyeq, \approx, \succcurlyeq\ (b_i^1, b_i^m, b_i^u)\ \forall i; \\ &(x_j^1, x_j^m, x_j^u) \succcurlyeq \tilde{0} \quad \forall j \qquad (p2) \end{aligned}$$

After performing the mathematical operations discussed in Section 2-2, the model $(P2)$ is converted to the following form:

$$\begin{aligned} &\tilde{Z} = \max\,(or\ \min)\ \Big(\sum_{j=1}^{n} c_j^l x_j^l, \sum_{j=1}^{n} c_j^m x_j^m, \sum_{j=1}^{n} c_j^u x_j^u\Big) \\ &\text{subject to} \quad \Big(\sum_{j=1}^{n} a_{ij}^l x_j^l, \sum_{j=1}^{n} a_{ij}^m x_j^m, \sum_{j=1}^{n} a_{ij}^u x_j^u\Big) \preccurlyeq, \approx, \succcurlyeq (b_i^l, b_i^m, b_i^u)\ \forall i; \\ &(x_j^l, x_j^m, x_j^u) \succcurlyeq \tilde{0} \quad \forall\ j \qquad (P3) \end{aligned}$$

Now, using Nasseri et al.'s algorithm [11] and the ranking method, the FFLP $(P2)$ turns into a precise linear programming problem. The steps in the algorithm are briefly summarized below:
Step 1: Transform full fuzzy objective function using its ranking function $\Big(\Re\big(\sum_{j=1}^{n} c_j^l x_j^l, \sum_{j=1}^{n} c_j^m x_j^m, \sum_{j=1}^{n} c_j^u x_j^u\big)\Big)$ into the crisp format.
Step 2: Full fuzzy constraints of the model $(P2)$ using the following ranking functions are:

$$\begin{aligned} &\sum_{j=1}^{n} a_{ij}^l x_j^l \leq, =, \geq b_i^1 \quad \forall i \\ &\sum_{j=1}^{n} a_{ij}^m x_j^m \leq, =, \geq b_i^m \quad \forall i \\ &\sum_{j=1}^{n} a_{ij}^u x_j^u \leq, =, \geq b_i^u \quad \forall i \end{aligned}$$

Step 3: The non-negative Fuzzy constraints, that is, $(x_j^1, x_j^m, x_j^u) \succcurlyeq \tilde{0}\ \forall j$ in the model $(P2)$, which guarantees the decision variables assessment as non-triangular fuzzy numbers, will be as follows:

$$x_j^1 \geq 0,\ x_j^m - x_j^1 \geq 0,\ x_j^u - x_j^m \geq 0,\ \forall j$$

Therefore, using the above steps, the model $(P2)$ turns into the exact linear programming problem:

$$\begin{aligned} Z = \max\,(or\ \min)\Re\Big(\sum_{j=1}^{n} c_j^l x_j^l, \sum_{j=1}^{n} c_j^m x_j^m, \sum_{j=1}^{n} c_j^u x_j^u\Big) \\ \text{subject to} \quad & \sum_{j=1}^{n} a_{ij}^l x_j^l \leq,=\geq b_i^l \quad \forall i \\ & \sum_{j=1}^{n} a_{ij}^m x_j^m \leq,=\geq b_i^m \quad \forall i \qquad\qquad (P4) \\ & \sum_{j=1}^{n} a_{ij}^u x_j^u \leq,=\geq b_i^u \quad \forall i \\ & x_j^l \geq 0,\ x_j^m - x_j^l \geq 0,\ x_j^u - x_j^m \geq 0, \quad \forall\, j \end{aligned}$$

Theorem 1. Each feasible solution in the model $(P4)$ is also a feasible solution in the model $(P3)$. Argument in [13].

Theorem 2. The optimal solution of the model $(P4)$ is the optimal solution for the model $(P3)$ Argument in [13].

4 Revenue Efficiency in DEA

The output-oriented DEA model under the assumption of variable return to scale can be used for calculation of output-oriented technical efficiency and revenue efficiency. Output-oriented model under the assumption of variable return to scale can be written in the following form:

$$\begin{aligned} \max \quad & \varphi_0 \\ \text{subject to} \quad & x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}, \qquad i = 1,\cdots,m \\ & \varphi_0 y_0 \leq \sum_{j=1}^{n} \lambda_j y_j \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0 \qquad\qquad \forall j \end{aligned}$$

Where φ_0 is output-oriented technical efficiency of DMU_o in the output-oriented DEA model. To calculate revenue efficiency the following revenue maximisation DEA problem is necessary to solve [5]:

$$\begin{aligned} \max \quad & p_o y \\ \text{subject to} \quad & x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}, \qquad i = 1, \cdots, m \\ & y \leq \sum_{j=1}^{n} \lambda_j y_j \\ & \lambda_j \geq 0 \qquad \forall j \end{aligned}$$

Where p_o is vector output prices for DMU_o. The overall revenue efficiency is defined as the ratio of observed revenue to maximum revenue for the DMU_o [5]:

$$\alpha^* = p_o y_o / p_o y_o^*$$

where y_o^* is an optimal solution for model [*Revenue*].

4.1 Single output case

In this section, we deal with n DMUs with m inputs $\boldsymbol{x} = (x_1, x_2, \cdots, x_m)$ to produce one output of $y(> 0)$. For a $DMU_o(o = 1, \cdots, n)$, let the inputs and output be $\boldsymbol{x_o} = (x_{1o}, x_{2o}, \cdots, x_{mo})$ and y_o (> 0)respectively, and the unit price of output y_o be p_o (> 0).

Between the two efficiency measures (technical efficiency φ^* and revenue efficiency α^*) we have the following theorem.

Theorem 3. For the single output case, $\alpha^* = 1/\varphi^*$.

Proof. Let us denote y as φy_o in [Revenue] and change the variable from y to φy_o. Then, noting $y_o > 0$ and $p_o > 0$, [Revenue] becomes:

$$\begin{aligned} \max \quad & p_o \varphi y_o \\ \text{subject to} \quad & x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}, \qquad i = 1, \cdots, m \\ & \varphi y_o \leq \sum_{j=1}^{n} \lambda_j y_j \\ & \lambda_j \geq 0 \qquad \forall j \end{aligned}$$

This program is equivalent to [CCR] and its optimal objective value is $\varphi^* p_o y_o$. Thus we have

$$\alpha^* = \frac{p_o y_o}{\varphi^* p_o y_o} = \frac{1}{\varphi^*}$$

□

Definition 12. (Allocative efficiency): The allocative efficiency γ^* of DMU_o is defined as the ratio of revenue efficiency to technical efficiency, ie, $\gamma^* = \frac{\alpha^*}{\varphi^*}$. The allocative efficiency γ^* is less than or equal to one, and DMU_o is called allocatively efficient when $\gamma^* = 1$ holds

4.2 General case

Here we observe a more general case where we have m inputs $\boldsymbol{x} = (x_1, x_2, \cdots, x_m)$ and s outputs $\boldsymbol{y} = (y_1, y_2, \cdots, y_s)$. Suppose that DMUs A and B have the same amount of inputs and outputs, ie, $\boldsymbol{x}_A = \boldsymbol{x}_B$ and $\boldsymbol{y}_A = \boldsymbol{y}_B$. Assume further that the unit price of DMU A is twice that of DMU B for each output, ie, $\boldsymbol{p}_A = 2\boldsymbol{p}_B$. Under these assumptions, we have the following theorem:

Theorem 4. Both DMUs A and B have the same price (overall) and allocative efficiencies.

Proof. Since DMUs A and B have the same inputs and outputs, they have the same technical efficiency, ie, $\varphi_A^* = \varphi_B^*$.

The revenue efficiency of DMU A (or DMU B) can be obtained by solving the following LP:

$$\begin{aligned} \max \quad & \boldsymbol{p}_A\boldsymbol{y}(= 2\boldsymbol{p}_B\boldsymbol{y}) & \\ \text{subject to} \quad & x_{iA}(= x_{iB}) \geq \sum_{j=1}^{n} \lambda_j x_{ij}, & i = 1, \cdots, m \\ & y_r \leq \sum_{j=1}^{n} \lambda_j y_{rj}, & r = 1, \cdots, s \\ & \lambda_j \geq 0 & \forall j \end{aligned}$$

Apparently, DMUs A and B have the same optimal solution (outputs) $\boldsymbol{y}_A^* = \boldsymbol{y}_B^*$, and hence the same revenue efficiency, since we have:

$$\alpha_A^* = \boldsymbol{p}_A\boldsymbol{y}_A/\boldsymbol{p}_A\boldsymbol{y}_A^* = 2\boldsymbol{p}_B\boldsymbol{y}_B/2\boldsymbol{p}_B\boldsymbol{y}_B^* = \boldsymbol{p}_B\boldsymbol{y}_B/\boldsymbol{p}_B\boldsymbol{y}_B^* = \alpha_B^*.$$

□

They also have the same allocative efficiency by definition 1. This also sounds very strange, since DMUs A and B have the same revenue and allocative efficiencies even though the price of DMU B is half that of DMU A.

4.3 A new scheme

The previous two sections reveal the shortcomings and irrationality of the revenue and allocative efficiencies proposed thus far.

These shortcomings are caused by the structure of the supposed production possibility set P as defined by:

$$P = \{(x, y) \big| x \geq X\lambda,\ y \leq Y\lambda,\ \lambda \geq 0\}$$

The production possibility set P is defined only on the basis of the technical factors $X = (x_1, \cdots, x_n) \in \mathbb{R}^{m\times n}$ and $Y = (y_1, \cdots, y_n) \in \mathbb{R}^{s\times n}$ and has no concern with the prices of the outputs $P = (p_1, \cdots, p_n)$. Banihashemi and Tohidi [2] define a set of new production possibility set based on revenue as follows:

$$P_p = \{(x, \bar{y}) | x \geq X\lambda,\ \bar{y} \leq \bar{Y}\lambda,\ \lambda \geq 0\}$$

where $\bar{Y} = (\bar{y}_1, \cdots, \bar{y}_n)$ and $\bar{y}_j$ assuming that the matrices P and Y are non-negative, and all inputs are revenue-oriented. Another assumption is that the elements $\bar{y}_{ij} = (p_{ij}, y_{ij})\ \forall(i,j)$ are in homogeneous units, e.g., \$, so that the multiplication of these elements is significant. Based on the definition of the set of new possible generation P_p, the new technical efficiency $\bar{\varphi}^*$ is given as the optimal solution to the linear programming problem:

$$\begin{aligned} \bar{\varphi}^* = \max \quad & \bar{\varphi} \\ \text{subject to} \quad & x_o \geq X\lambda \\ & \bar{\varphi}\bar{y}_o \leq \bar{Y}\lambda \\ & \lambda \geq 0 \end{aligned}$$

The new revenue efficiency $\bar{\alpha}^*$ is as follows:

$$\bar{\alpha}^* = e\bar{y}_o / e\bar{y}_o^*$$

where $e \in \mathrm{R}^m$, is a row vector with the elements 1 and $\bar{y}_o^*$ is the solution to the linear programming problem below:

$$\begin{aligned} [Nrevenue] \quad \max \quad & e\bar{y} \\ \text{subject to} \quad & x_o \geq X\lambda \\ & \varphi\bar{y} \leq \bar{Y}\lambda \\ & \lambda \geq 0 \end{aligned}$$

5 Network Data Envelopment Analysis Based on SBM Model

The common DEA models which measure the relative efficiency of multiple input/ output decision-maker units may experience drawbacks such as neglecting intermediate products or linked activities. In this section, the network data envelopment analysis and the parameters of its production probability set are discussed.

Suppose n is the decision maker available in Section K. m_k and r_k are the numbers of inputs and outputs in the k^{th} section. The link from division k to division h is represented by (h, k) and the set of all links is shown by L. The observed data is $\{x_j^k \in R_+^{m_k}\}(j = 1, \cdots, n, k = 1, \cdots, K)$, $\{y_j^k \in R_+^{r_k}\}(j = 1, \cdots, n, k = 1, \cdots, K)$ and $\{z_j^{(k,h)} \in R_+^{t_{(k,h)}}\}(j = 1, \cdots, n, (k, h) \in L)$.

Thus, the production possibility set in network data envelopement analysis will be:

$$P = \Big\{(x^k, y^k, z^{(h,k)}) \big| x^k \geq X^k\lambda^k, y^k \leq Y^k\lambda^k, z^{(k,h)} = z^{(k,h)}\lambda^k\,(as\,outputs\,k), z^{(k,h)} = z^{(k,h)}\lambda^h(as\,inputs\,h), \lambda \geq 0\Big\}$$

Assume that the following model (with input nature) has a variable returns to scale and DMU_o, $(o = 1, \cdots, n)$ unit under evaluation. Since the SBM model needs to have positive data, this paper assumes that all data are positive.

$$
\begin{aligned}
[NSBM] \quad \theta_0 = \min \ & \sum_{k=1}^{K} w^k \left[1 - \frac{1}{m_k}\Big(\sum_{i=1}^{m_k} \frac{s_i^{k-}}{x_{io}^k}\Big)\right] \\
\text{subject to} \quad & x_o^k = X^k \lambda^k + s^{k-} \\
& y_o^k = Y^k \lambda^k - s^{k+} \\
& \lambda^k, \lambda^h, s^{k-}, s^{k+} \geq 0 \\
& z_o^{(k,h)} = z^{(k,h)} \lambda^k \ \big(\forall (k,h)\big), \qquad (a) \\
& z_o^{(k,h)} = z^{(k,h)} \lambda^h \ \big(\forall (k,h)\big), \\
& \text{or} \\
& z^{(k,h)} \lambda^k = z^{(k,h)} \lambda^h \ \big(\forall (k,h)\big), \qquad (b)
\end{aligned}
$$

Where z Where $z^{(k,h)} = \big(z_1^{(k,h)}, \cdots, z_n^{(k,h)}\big) \in R^{t_{(k,h)} \times n}$, $X^k = (x_1^k, \cdots, x_n^k) \in R^{m_k \times n}$, $y^k = (y_1^k, \cdots, y_n^k) \in R^{r_k \times n}$, s^{k-} (s^{k+}) are slacks vectors of the input (output). Given the link constraints, there are several choices that can be made in two possible ways:

(a) In the first case, the values of fixed intermediate current are taken into account.

$$
\begin{aligned}
& z_o^{(k,h)} = z^{(k,h)} \lambda^k \ \big(\forall (k,h)\big), \qquad (a) \\
& z_o^{(k,h)} = z^{(k,h)} \lambda^h \ \big(\forall (k,h)\big)
\end{aligned}
$$

(b)In the second case, the values of the average flow in the link can be freely reduced or increased.

$$
z^{(k,h)} \lambda^k = z^{(k,h)} \lambda^h \ \big(\forall (k,h)\big), \qquad (b)
$$

6 Revenue Efficiency in Network DEA

In this section we deal New Network Revenue Efficiency (NNRE) on Network Slack Based Measure (NSBM) that prices play a role in the PPS on output. The production possibility set based on price for the network data envelopment analysis is [2]:

$$
\begin{aligned}
P_p = \Big\{ & (x^k, \bar{y}^k, \bar{z}^{(k,h)}) \big| x^k \geq X^k \lambda^k, \bar{y}^k \leq \bar{Y}^k \lambda^k, \ \bar{z}^{(k,h)} = \bar{z}^{(k,h)} \lambda^k (as\, outputs\, k), z^{(k,h)} \\
& = z^{(k,h)} \lambda^h (as\, inputs\, h), \ e\lambda^k = 1, \ \lambda \geq 0 \Big\}
\end{aligned}
$$

where

$$
\begin{aligned}
& \bar{Y}^k = (\bar{y}_1^k, \cdots, \bar{y}_n^k), && \bar{y}_j^k = (p_{1j}^k y_{1j}^k, \cdots, p_{r_k j}^k y_{r_k j}^k) \\
& \bar{z}^{(k,h)} = \big(\bar{z}_1^{(k,h)}, \cdots, \bar{z}_n^{(k,h)}\big), && \bar{z}_j^{(k,h)} = \big(c_{1j}^k z_{1j}^{(k,h)}, \cdots, c_{r_k j}^k z_{r_k j}^{(k,h)}\big)
\end{aligned}
$$

Based on this set, a new production possibility, $\bar{\alpha}^{*k}$, is obtained from the following linear programming problem:

$$
\begin{aligned}
[NNRE] \quad & \max \sum_{k=1}^{K} \bar{y}^k + \sum_{h} \bar{z}^{(k,h)} \\
\text{subject to} \quad & x_o^k \geq X^k \lambda^k, && k = 1, \cdots, K \\
& \bar{y}^k \leq \bar{Y}^k \lambda^k, && k = 1, \cdots, K \\
& \bar{z}_o^{(k,h)} = \bar{z}^{(k,h)} \lambda^h \ \big(\forall (k,h)\big) && (a) \\
& z_o^{(k,h)} = z^{(k,h)} \lambda^k \ \big(\forall (k,h)\big), \\
& \text{or} \\
& \bar{z}^{(k,h)} \lambda^k = \bar{z}^{(k,h)} \lambda^h \ \big(\forall (k,h)\big) && (b) \qquad (P5) \\
& e\lambda^k = 1, \\
& \lambda^k, \ \lambda^h \geq 0
\end{aligned}
$$

and

$$
\bar{\alpha}^{*k} = \sum_{k=1}^{K} \bar{y}_o^{*k} + \sum_{h} \bar{z}_o^{(k,h)} \Big/ \sum_{k=1}^{K} \bar{y}_o^{*k} + \sum_{h} \bar{z}_o^{*(k,h)}
$$

Where $e \in \mathbb{R}^m$, a row vector with elements, equals 1 and $\bar{y}_o^*$, $\bar{z}_o^*$ are optimal solutions for model $(P5)$.

7 Proposed Fuzzy Revenue Efficiency Method in Fully Fuzzy Network Data Analysis

In the real world, input-output data and their corresponding prices are not accurately observed and may be available in inappropriate forms such as fuzzy numbers, in particular triangular fuzzy numbers. Many researchers investigated the revenue efficiency with fuzzy and intermediate data. In these studies only, the decision parameters are considered as fuzzy and the decision variables are precise quantifiers. However, in this paper, we use full-fuzzy models of network data envelopment analysis to measure the revenue efficiency in a fully fuzzy environment in which all decision-making parameters and variables are represented by triangular fuzzy numbers.

To measure fuzzy revenue efficiency in network data envelopment analysis, we extend the model (4) to a completely fuzzy environment. Suppose that the decision maker unit is available in Section K. m_k and r_k are the number of fuzzy inputs and outputs in the k-section. The link from section k to part h is represented by (k, h) and the set of all links with L. The observed fuzzy data $j = 1, \cdots, n$, $k = 1, \cdots, K$ $\tilde{x}_j^k$, $\tilde{y}_j^k$, $\tilde{z}_j^{(k,h)}$ and $\tilde{p}_j^k$ respectively contain the input and Fuzzy outputs in each section, fuzzy link activities from section k to section h as well as the revenue of the fuzzy input units in each section. If these data are triangular fuzzy numbers, we will have:

$$\begin{aligned}
&\tilde{x}_j^k = \left(x_j^{l,k}, x_j^{m,k}, x_j^{u,k}\right), && j=1,\cdots,n, \quad k=1,\cdots,K\\
&\tilde{y}_j^k = \left(y_j^{l,k}, y_j^{m,k}, y_j^{u,k}\right), && j=1,\cdots,n, \quad k=1,\cdots,K\\
&\tilde{p}_j^k = \left(p_j^{l,k}, p_j^{m,k}, p_j^{u,k}\right), && j=1,\cdots,n, \quad k=1,\cdots,K\\
&\tilde{z}_j^{(k,h)} = \left(z_j^{l,(k,h)}, z_j^{m,(k,h)}, z_j^{u,(k,h)}\right), && j=1,\cdots,n, \quad (k,h)\in L
\end{aligned}$$

According to the above, the model $(P5)$ will become a fully fuzzy model as follows:

$$\begin{aligned}
[FFNNRE] \quad & \min \sum_{k=1}^{K} \tilde{\bar{y}}^k \oplus \sum_h \tilde{\bar{z}}^{(k,h)} \\
\text{subject to} \quad & \tilde{x}_o^k \succcurlyeq \sum_{j=1}^n \tilde{X}_j^k \otimes \tilde{\lambda}_j^k, && k=1,\cdots,K \\
& \tilde{\bar{y}}^k \preccurlyeq \sum_{j=1}^n \tilde{\bar{y}}_j^k \otimes \tilde{\lambda}_j^k, && k=1,\cdots,K \qquad (P6)\\
& \tilde{\bar{z}}_o^{(k,h)} \approx \sum_{j=1}^n \tilde{z}_j^{(k,h)} \otimes \tilde{\lambda}_j^k, && \forall (k,h) \qquad (a)\\
& \tilde{\bar{z}}_o^{(k,h)} \approx \sum_{j=1}^n \tilde{z}_j^{(k,h)} \otimes \tilde{\lambda}_j^h, && \forall (k,h)\\
& \text{or} \\
& \sum_{j=1}^n \tilde{\bar{z}}_j^{(k,h)} \otimes \tilde{\lambda}_j^k \approx \sum_{j=1}^n \tilde{\bar{z}}_j^{(k,h)} \otimes \tilde{\lambda}_j^h, && \forall (k,h) \qquad (b)\\
& \sum_{j=1}^n \tilde{\lambda}_j^k \approx \tilde{1} \\
& \tilde{\lambda}_j^k,\ \tilde{\lambda}_j^h \succcurlyeq \tilde{0} && \forall j,k
\end{aligned}$$

The model $(P6)$ is a fuzzy revenue envelopment model in the Fuzzy Network Data Envelopment Analysis. After replacing the triangular fuzzy variables and parameters in model $(P6)$ and using mathematical operations on triangular fuzzy numbers and steps of the Nasseri algorithm, the full-fuzzy linear programming model $(P6)$ becomes the crisp linear programming:

$$\max \ \frac{1}{4}\left[\sum_{k=1}^{K} \bar{y}^{l,k} + \sum_h \bar{z}^{l,(k,h)} + 2\Big(\sum_{k=1}^{K} \bar{y}^{m,k} + \sum_h \bar{z}^{m,(k,h)}\Big) + \sum_{k=1}^{K} \bar{y}^{u,k} + \sum_h \bar{z}^{u,(k,h)}\right]$$

$$\begin{aligned}
\text{subject to} \quad & x_o^{l,k} \geq \sum_{j=1}^n X_j^{l,k}\lambda_j^{l,k}, && k=1,\cdots,K\\
& x_o^{m,k} \geq \sum_{j=1}^n X_j^{m,k}\lambda_j^{l,k}, && k=1,\cdots,K\\
& x_o^{u,k} \geq \sum_{j=1}^n X_j^{u,k}\lambda_j^{l,k}, && k=1,\cdots,K
\end{aligned}$$

$$\bar{y}^{l,k} \leq \sum_{j=1}^{n} \bar{y}_j^{l,k} \lambda_j^{l,k}, \qquad k = 1, \cdots, K \qquad (P7)$$

$$\bar{y}^{m,k} \leq \sum_{j=1}^{n} \bar{y}_j^{m,k} \lambda_j^{m,k}, \qquad k = 1, \cdots, K$$

$$\bar{y}^{u,k} \leq \sum_{j=1}^{n} \bar{y}_j^{u,k} \lambda_j^{u,k}, \qquad k = 1, \cdots, K$$

$$\bar{z}_o^{l,(k,h)} = \sum_{j=1}^{n} \bar{z}_j^{l,(k,h)} \lambda_j^{l,k}, \qquad \forall (k,h)$$

$$\bar{z}_o^{m,(k,h)} = \sum_{j=1}^{n} \bar{z}_j^{m,(k,h)} \lambda_j^{m,k}, \qquad \forall (k,h)$$

$$\bar{z}_o^{u,(k,h)} = \sum_{j=1}^{n} \bar{z}_j^{u,(k,h)} \lambda_j^{u,k}, \qquad \forall (k,h)$$

$$z_o^{l,(k,h)} = \sum_{j=1}^{n} z_j^{l,(k,h)} \lambda_j^{l,h}, \qquad \forall (k,h) \qquad (a)$$

$$z_o^{m,(k,h)} = \sum_{j=1}^{n} z_j^{m,(k,h)} \lambda_j^{m,h}, \qquad \forall (k,h)$$

$$z_o^{u,(k,h)} = \sum_{j=1}^{n} z_j^{u,(k,h)} \lambda_j^{u,h}, \qquad \forall (k,h)$$

or

$$\sum_{j=1}^{n} \bar{z}_j^{l,(k,h)} \lambda_j^{l,k} = \sum_{j=1}^{n} \bar{z}_j^{l,(k,h)} \lambda_j^{l,h}, \qquad \forall (k,h)$$

$$\sum_{j=1}^{n} \bar{z}_j^{m,(k,h)} \lambda_j^{m,k} = \sum_{j=1}^{n} \bar{z}_j^{m,(k,h)} \lambda_j^{m,h}, \qquad \forall (k,h) \qquad (b)$$

$$\sum_{j=1}^{n} \bar{z}_j^{u,(k,h)} \lambda_j^{u,k} = \sum_{j=1}^{n} \bar{z}_j^{u,(k,h)} \lambda_j^{u,h}, \qquad \forall (k,h)$$

$$\sum_{j=1}^{n} \lambda_j^{l,k} = 1, \quad \sum_{j=1}^{n} \lambda_j^{m,k} = 1, \quad \sum_{j=1}^{n} \lambda_j^{u,k} = 1, \qquad k = 1, \cdots, K$$

$$\lambda_j^{l,k} \geq 0,\ \lambda_j^{m,k} - \lambda_j^{l,k} \geq 0,\ \lambda_j^{u,k} - \lambda_j^{m,k} \geq 0 \qquad \forall j, k$$

$$\bar{y}_j^{l,k} \geq 0,\ \bar{y}_j^{m,k} - \bar{y}_j^{l,k} \geq 0,\ \bar{y}_j^{u,k} - \bar{y}_j^{m,k} \geq 0 \qquad \forall j, k$$

$$\bar{z}_j^{l,k} \geq 0,\ \bar{z}_j^{m,k} - \bar{z}_j^{l,k} \geq 0,\ \bar{z}_j^{u,k} - \bar{z}_j^{m,k} \geq 0 \qquad \forall j, k$$

Theorem 5. Model $(P7)$ is a feasible model.

Proof. This model has a feasible solution as follows :

$$\begin{array}{lll}
\lambda_o^{l,k}=1, & \lambda_j^{l,k}=0, & j\neq o\\
\lambda_o^{m,k}=1, & \lambda_j^{m,k}=0, & j\neq o\\
\lambda_o^{u,k}=1, & \lambda_j^{u,k}=0, & j\neq o\\
\lambda_o^{l,h}=1, & \lambda_j^{l,h}=0, & j\neq o\\
\lambda_o^{m,h}=1, & \lambda_j^{m,h}=0, & j\neq o\\
\lambda_o^{u,h}=1, & \lambda_j^{u,h}=0, & j\neq o\\
\bar{y}^{l,k}=\bar{y}_o^{l,k} & \bar{y}^{m,k}=\bar{y}_o^{m,k} & \bar{y}^{u,k}=\bar{y}_o^{u,k}
\end{array}$$

And with considering (b)

$$\bar{z}_o^{l,(k,h)}=\bar{z}_o^{l,(k,h)} \qquad \bar{z}_o^{m,(k,h)}=\bar{z}_o^{m,(k,h)}, \qquad \bar{z}_o^{u,(k,h)}=\bar{z}_o^{u,(k,h)}$$

□

Theorem 6. The optimal solution for the model (P7) will be a model optimization solution (P6). The proof of this is similar to the proof of Theorem 1.

Definition 13. The fuzzy cost efficiency of the i^{th} DMU in the FFDEA is defined as the ratio of the minimum fuzzy cost to the observed fuzzy cost of DMU_i:

$$\begin{aligned}
\tilde{\alpha}_i^{*k} &= \frac{\sum_{k=1}^{K}\tilde{\bar{y}}_i^{*k}\oplus\sum_h\tilde{\bar{z}}_i^{*(k,h)}}{\sum_{k=1}^{K}\tilde{\bar{x}}_i^{k}\oplus\sum_h\tilde{\bar{z}}_i^{*(k,h)}}\\
&= \frac{\left(\sum_{k=1}^{K}\bar{y}_i^{l,k}+\sum_h\bar{z}_i^{l,(k,h)},\sum_{k=1}^{K}\bar{y}_i^{m,k}+\sum_h\bar{z}_i^{m,(k,h)},\sum_{k=1}^{K}\bar{y}_i^{u,k}+\sum_h\bar{z}_i^{u,(k,h)}\right)}{\left(\sum_{k=1}^{K}\bar{y}_i^{l,k*}+\sum_h\bar{z}_i^{l,(k,h)*},\sum_{k=1}^{K}\bar{y}_i^{m,k*}+\sum_h\bar{z}_i^{m,(k,h)*},\sum_{k=1}^{K}\bar{y}_i^{u,k*}+\sum_h\bar{z}_i^{u,(k,h)*}\right)}\\
&= \left(\frac{\sum_{k=1}^{K}\bar{y}_i^{l,k}+\sum_h\bar{z}_i^{l,(k,h)},\sum_{k=1}^{K}\bar{y}_i^{m,k}+\sum_h\bar{z}_i^{m,(k,h)},\sum_{k=1}^{K}\bar{y}_i^{u,k}+\sum_h\bar{z}_i^{u,(k,h)}}{\sum_{k=1}^{K}\bar{y}_i^{u,k*}+\sum_h\bar{z}_i^{u,(k,h)*},\sum_{k=1}^{K}\bar{y}_i^{m,k*}+\sum_h\bar{z}_i^{m,(k,h)*},\sum_{k=1}^{K}\bar{y}_i^{l,k*}+\sum_h\bar{z}_i^{l,(k,h)*}}\right)
\end{aligned}$$

where $(\bar{y}_i^{l,k*},\bar{y}_i^{m,k*},\bar{y}_i^{u,k*} \quad \forall i,k,h)$ $(\bar{z}_i^{l,(k,h)*},\bar{z}_i^{m,(k,h)*},\bar{z}_i^{u,(k,h)*})$ are the optimal solutions obtained from model (p6).

Definition 14. i^{th} DMU in the network data envelopment analysis is called Fuzzy Cost Efficiency if the observed Fuzzy Cost and the minimum Fuzzy Cost equal DMU_i, that is,

$$\sum_{k=1}^{K}\tilde{\bar{y}}_i^{k}\oplus\sum_h\tilde{\bar{z}}_i^{(k,h)}\approx\sum_{k=1}^{K}\tilde{\bar{y}}_i^{*}\oplus\sum_h\tilde{\bar{z}}_i^{*(k,h)}$$

$$\Re\Big(\sum_{k=1}^{K}\tilde{\bar{y}}_i^{k}\oplus\sum_h\tilde{\bar{z}}_i^{(k,h)}\Big)\approx\Re\Big(\sum_{k=1}^{K}\tilde{\bar{y}}_i^{*}\oplus\sum_h\tilde{\bar{z}}_i^{*(k,h)}\Big)$$

8 Numerical example

In this section, an illustrative example of electric power companies are presented for describing network DEA. As we know, the vertically integrated electric power companies consist of several divisions such as generation, transmission and distribution. For illustrative purpose, ten

vertically integrated electric power companies in the U.S in 1994 [16]. The inputs, outputs and links are as follows:
Generation (Div1):
Input1 = Labor input (number of employees)
Transmission (Div2):
Input2 = Labor input (number of employees)
Output2 = Electric power sold to large customers
Distribution (Div3):
Input3 = Labor input (number of employees)
Output3 = Electric power sold to small customers
Link (1-2) = Electric power generated (output from Generation Devision and input to Transmission Devision)
Link (2-3) = Electric power sent (output from Transmission Devision and input to Distribution Devision) Here, it is assumed that the intermediate flow rates are able to rise or fall freely in the link, so that the proposed model for evaluating the fuzzy revenue efficiency will be as follows:

$$\max \ \frac{1}{4}\left[\sum_{k=1}^{K}\bar{y}^{l,k}+\sum_{h}\bar{z}^{l,(k,h)}+2\Big(\sum_{k=1}^{K}\bar{y}^{m,k}+\sum_{h}\bar{z}^{m,(k,h)}\Big)+\sum_{k=1}^{K}\bar{y}^{u,k}+\sum_{h}\bar{z}^{u,(k,h)}\right]$$

$$\begin{aligned}
\text{subject to}\quad & x_o^{l,k} \geq \sum_{j=1}^{n} X_j^{l,k}\lambda_j^{l,k}, && k=1,\cdots,K\\
& x_o^{m,k} \geq \sum_{j=1}^{n} X_j^{m,k}\lambda_j^{m,k}, && k=1,\cdots,K\\
& x_o^{u,k} \geq \sum_{j=1}^{n} X_j^{u,k}\lambda_j^{u,k}, && k=1,\cdots,K\\
& \bar{y}^{l,k} \leq \sum_{j=1}^{n} \bar{y}_j^{l,k}\lambda_j^{l,k}, && k=1,\cdots,K\\
& \bar{y}^{m,k} \leq \sum_{j=1}^{n} \bar{y}_j^{m,k}\lambda_j^{m,k}, && k=1,\cdots,K\\
& \bar{y}^{u,k} \leq \sum_{j=1}^{n} \bar{y}_j^{u,k}\lambda_j^{u,k}, && k=1,\cdots,K\\
& \sum_{j=1}^{n} \bar{z}_j^{l,(k,h)}\lambda_j^{l,k} = \sum_{j=1}^{n} z_j^{l,(k,h)}\lambda_j^{l,h}, && \forall (k,h)\\
& \sum_{j=1}^{n} \bar{z}_j^{m,(k,h)}\lambda_j^{m,k} = \sum_{j=1}^{n} z_j^{m,(k,h)}\lambda_j^{m,h}, && \forall (k,h)\\
& \sum_{j=1}^{n} \bar{z}_j^{u,(k,h)}\lambda_j^{u,k} = \sum_{j=1}^{n} z_j^{u,(k,h)}\lambda_j^{u,h}, && \forall (k,h)\\
& \lambda_j^{l,k} \geq 0,\quad \lambda_j^{m,k}-\lambda_j^{l,k} \geq 0,\quad \lambda_j^{u,k}-\lambda_j^{m,k} \geq 0 && \forall j,k\\
& \bar{y}_j^{l,k} \geq 0,\quad \bar{y}_j^{m,k}-\bar{y}_j^{l,k} \geq 0,\quad \bar{y}_j^{u,k}-\bar{y}_j^{m,k} \geq 0, && \forall j,k
\end{aligned}$$

$$\bar{z}_j^{l,(k,h)} \geq 0, \quad \bar{z}_j^{m,(k,h)} - \bar{z}_j^{l,(k,h)} \geq 0, \quad \bar{z}_j^{u,(k,h)} - \bar{z}_j^{m,(k,h)} \geq 0, \quad \forall j,k,h$$

Table 1 contains the fuzzy inputs, fuzzy outputs, and fuzzy revenues of each division.

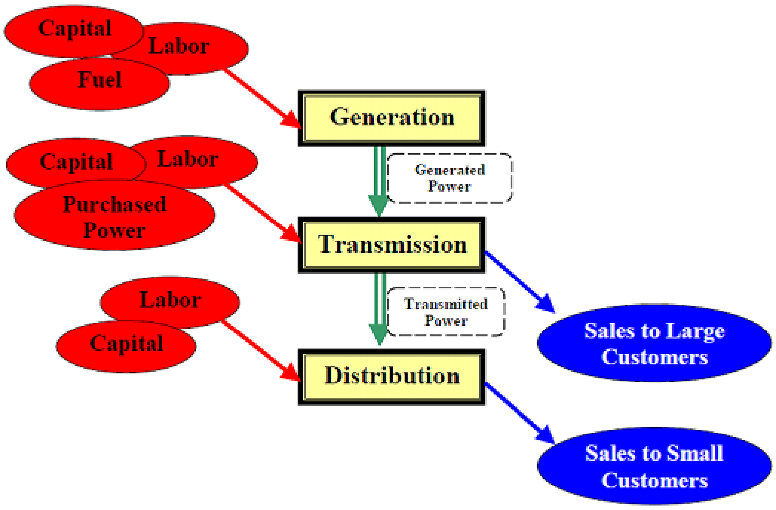

Figure 2: Vertically integrated electric power companies

The revenue of the input and output links is also given in Table 2.

Table 1: Fuzzy inputs, fuzzy outputs, fuzzy input cost in three divisions

	Div1	Div2		Div3			
DMU	Input1	Input2	Output3	P2	Input3	Output3	P3
A	(0.836,0.838,0.840)	(0.275,0.277,0.279)	(0.876,0.879,0.881)	(896,900,903)	(0.960,0.962,0.965)	(0.335,0.337,0.340)	(685,687,689)
B	(1.231,1.233,1.235)	(0.130,0.132,0.133)	(0.535,0.538,0.540)	(737,739,742)	(0.440,0.443,0.445)	(0.15,0.18,0.20)	(190,194,196)
C	(0.318,0.321,0.323)	(0.042,0.045,0.048)	(0.909,0.911,0.914)	(138,142,145)	(0.482,0.485,0.487)	(0.195,0.198,0.200)	(280,285,287)
D	(1.480.1.483,1.485)	(0.110,0.111,0.113)	(0.55,0.57,,0.59)	(860,863,865)	(0.465,0,467,0.470)	(0.488,0.491,0.495)	(398,401,404)
E	(1.590,1.592,1.595)	(0.205,0.208,0.211)	(1.085,1.086,1.089)	(305,307,310)	(1.070,1.073,1.075)	(0.370,0.372,0.375)	(175,179,182)
F	(0.76,0.79,0.81)	(0.136,0.139,0.141)	(0.720,0.722,0.724)	(1198,1200,1203)	(0.543,0.545,0.548)	(0.250,0.253,0.255)	(1052,1054,1056)
G	(0.449,0.451,0.454)	(0.073,0.075,0.077)	(0.507,0.509,0.511)	(268,270,273)	(0.365,0.366,0.368)	(0.238,0.241,0.244)	(390,394,396)
H	(0.405,0.408,0.410)	(0.072,0.074,0.076)	(0.617,0.619,0.621)	(985,987,990)	(0.226,0.229,0.231)	(0.095,0.097,0.099)	(272,276,280)
I	(1.860,1.864,1.865)	(0.059,0.061,0.063)	(1.021,1.023,1.025)	(354,356,358)	(0.689,0.691,0.693)	(0.35,0.38,0.40)	(838,840,843)
J	(1.220,1.222,1.225)	(0.147,0.149,0.151)	(0.765,0.769,0.771)	(467,470,472)	(0.336,0.337,0.339)	(0.175,0.178,0.180)	(159,161,164)

The above model is solved using GAMS software and the results are shown in Table 3.

As Table 3 shows none of the decision making units are revenue efficiency. Indeed, one of the major drawbacks of the network models is that the full efficiency cannot be achieved in most of the cases. To solve this issue, efficiency of each unit can be devided to the maximum efficiency, resulting to deriving the relative efficiency (Table 3, column 4). In this case, unit H is the relative revenue efficiency and units A, C, D, F and G have the relative revenue efficiency more than half.

Table 2: Fuzzy unit input link revenue

Link			
Link12	Lp1	Link23	Lp2
(0.891,0.894,0.897)	(945,947,950)	(0.360,0.362,0.365)	(1031,1034,1036)
(0.675,0.678,0.780)	(680,682,685)	(0.185,0.188,0.190)	(986,989,992)
(0.835,0.836,0.838)	(700,705,708)	(0.205,0.207,0.210)	(750,752,755)
(0.865,0.869,0.872)	(1125,1128,1130)	(0.514,0.516,0.520)	(1109,1111,1113)
(0.690,0.693,0.695)	(490,492,495)	(0.405,0.407,0.410)	(850,852,855)
(0.961,0.966,0.970)	(665,670,673)	(0.265,0.269,0.273)	(640,642,645)
(0.645,0.647,0.650)	(1085,1087,1090)	(0.255,0.257,0.259)	(820,824,826)
(0.752,0.756,0.760)	(924,926,930)	(0.101,0.103,0.105)	(970,973,975)
(1.189,1.191,1.194)	(630,634,638)	(0.400,0.402,0.405)	(910,913,915)
(0.790,0.792,0.795)	(775,779,782)	(0.185,0.187,0.190)	(645,647,650)

Table 3: Evaluating and ranking revenue efficiency

$DMUs$	$\tilde{\tilde{\alpha}}^{*k}$	$R(\tilde{\tilde{\alpha}}^{*k})$	Relative Efficiency	Rank
A	(0.433,.0.648,0.734)	0.648	0.733	4
B	(0.130,0.331,0.450)	0.331	0.374	8
C	(0.435,0.680,0.872)	0.680	0.769	3
D	(0.435,0.553,0.754)	0.553	0.625	6
E	(0.125,0.263,0.365)	0.263	0.297	10
F	(0.534,0.709,0.845)	0.709	0.802	2
G	(0.456,0.647,0.745)	0.647	0.732	5
H	(0.534,0.884,0.915)	0.884	1	1
I	(0.234,0.403,0.478)	0.403	0.456	7
J	(0.25,0.33,0.56)	0.33	0.373	9

9 Conclusion

Given the importance of revenue efficiency in the management and economic sectors as well as inaccuracies in real-world data, this paper proposes a new idea of the extension of classical NNRE model to fully fuzzy environments for dealing with the practical situations more realistically. A FFNNRE model has been developed where input–output data and their corresponding prices are taken in triangular membership forms. A method based on ranking function approach is presented to transform FFNNRE model into the crisp linear programming problem. The final FFNNRE measures are then defined as TFNs. Finally, using the presented ranking function in the article, the DMUs are ranked based on revenue efficiency.

Since revenue efficiency sensitivity analysis helps the manager or decision maker to modify the amount of outputs under evaluation to maximize revenue . Therefore, future work can include sensitivity analysis of performance, as well as finding the appropriate stability area to maintain revenue efficiency in precise and imprecise network data envelopment analysis.

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چکیده

هدف این مقاله، ارزیابی کارایی درآمد در تحلیل پوششی داده‌های شبکه‌ای تمام فازی می‌باشد. اندازه‌گیری دقیق داده‌ها در دنیای واقعی عملا امکان‌پذیر نمی‌باشد، بنابراین فرض دقیق بودن داده‌ها در حل مسائل، فرض درستی نمی‌باشد. یکی از راه‌های مواجهه با داده‌های نادقیق، داده‌های فازی می‌باشد. در این مقاله از توابع رتبه‌بندی خطی، برای تبدیل مدل تمام فازی کارایی درآمد به یک مسئله برنامه‌ریزی خطی دقیق استفاده می‌شود و با فرض اعداد فازی مثلثی، کارایی درآمد فازی تصمیم‌گیرنده‌ها اندازه‌گیری می‌شود. در پایان، یک مثال عددی روش پیشنهادی را نشان می‌دهد.

کلمات کلیدی

تحلیل پوششی داده‌های شبکه‌ای، کارایی درآمد، برنامه‌ریزی خطی تمام فازی، تابع رتبه‌بندی.