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Regeneralized London free-energy for high-T_c vortex lattices

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Abstract
The London free-energy is regeneralized by the Ginsburg-Landau free-energy density in the presence of both d and s order parameters. We have shown that the strength of the s-d coupling, ε, makes an important rule to determine the form of the lattice vortex. Appearance of the ratios of the coherence length to penetration depth in the higher order corrections of the free-energy density will truncate these corrections for even large values of ε.

Keywords: superconductivity, vortex lattice, free-energy

1. Introduction

One of the most striking properties of type II superconductors is a mixed state, characterized by a regular array of magnetic flux lines penetrating the material. The vortex structure of the d-wave superconductors has attracted much attention because it may have a different structure from that of conventional s-wave superconductors. According to considerations based on the two-component Ginsburg-Landau (GL) theory, it is possible that the s-wave component is coupled with the d-wave component through the gradient terms. Therefore, the s-wave components, may be induced when the d-wave order parameter spatially varies such as near the vortex or interface under certain restricted consideration [1]. The vortex structure of a d-wave superconductor is of great interest because it might be relevant to high T_c superconductors [2]. It is expected that the structure of a d-wave vortex is very different from that of s-wave [3] or p-wave [4]. The original pioneering work of Abrikosov [5], based on the solution of GL equations near the upper critical field, predicts a triangular flux lattice. In some compounds neutron scattering experiments revealed deviation from perfect triangular lattices in strong field [6] which is attributed to anisotropies in the electronic band structure and other effects. These were modeled by theory through additional higher order derivative terms reflecting the material anisotropies [6].

Ian Affleck et al. [7] based on a generalization of the London free-energy to include anisotropy of fourfold symmetry, presented a simple and general approach to the effects of vortex lattice in an extreme type II superconductor. Their model breaks down when the strength of the s-d coupling, ε, appearing in the GL theory and the external magnetic field, H, are large enough to make the minimum angle between unit vectors (β) differs by 60°. In this regime, Affleck et al. proposed that higher order correlations are important in the GL theory. In this paper, we reconsidered the free-energy density of both states in powers of the strength of the s-d coupling, ε, and show that the higher order in GL theory can be neglected.

2. The superfluid velocity

The free-energy density of a superconductor within the GL theory with both d and s order parameter may be written as [7]

$$f = \alpha_s |s|^2 + \alpha_d |d|^2 + \gamma_s |\vec{\Pi} s|^2 + \gamma_d |\vec{\Pi} d|^2 + f_4 + \frac{h^2}{8\pi} + \gamma_v [(\Pi_y s)^* (\Pi_y d) - (\Pi_x s)^* (\Pi_x d) + c.c], \quad (1)$$

where $\vec{\Pi} = -i\vec{\nabla} - \frac{e^* \vec{A}}{\pi c}$ and f_4 contains the quadratic terms.

The GL theory involves both the d-wave order parameter and s-wave order parameter, which arise in inhomogeneous states through a mixed gradient coupling and identically vanishes at zero magnetic field. A small s component with highly anisotropic spatial distribution is nucleated in the vicinity of a vertex near H_{C1} and giving rise to nontriangular equilibrium lattice structure [1].

By using Euler-Lagrange equation s-wave order parameter, can be expressed to the leading order in $\left(1-T/T_c\right)$ as

$$s=\left(\gamma_v/\alpha_s\right)\left(\vec{\Pi}_{x^2}-\vec{\Pi}_{y^2}\right)d. \tag{2}$$

By substituting eq. (2) in eq. (1) we get

$$f=\gamma_d[\left|\vec{\Pi}d\right|^2-\varepsilon\frac{\xi^2}{3}\left|\left(\Pi_x^2-\Pi_y^2\right)d\right|^2]+\cdots, \tag{3}$$

Where $\varepsilon=3\left(\alpha_d\gamma_v^2/\alpha_s\gamma_d^2\right)$ is strength of the s-d coupling and $\xi=\sqrt{\gamma_d/\left|\alpha_d\right|}$ is the GL coherence length. Ian Affleck [7] by assuming that the penetration depth λ, is much larger than coherent length, ξ, and $\left|d(\vec{r})\right|\cong d$, obtained the generalized London equation as

$$\frac{e}{4\pi}\vec{\nabla}\times\vec{B}=\left(\frac{2e^*}{\hbar c}\right)\gamma_d d_0^2\left\{\vec{v}-\frac{2\varepsilon\xi^2}{3}\left\{\left(\hat{y}v_y^{(0)}-\hat{x}v_x^{(0)}\right)\left[\left(v_y^{(0)}\right)^2-\left(v_x^{(0)}\right)^2\right]-\left(\hat{y}\partial_y-\hat{x}\partial_x\right)\left(\partial_y v_y^{(0)}-\partial_x v_x^{(0)}\right)\right\}\right\}. \tag{4}$$

They used the perturbative method to expand the above equation in terms of small parameter ε. To zero, first, second and third order corrections, the superfluid velocity, $\vec{V}=\vec{V}^{(0)}+\vec{V}^{(1)}+\vec{V}^{(2)}+\vec{V}^{(3)}$, may be written respectively as

$$\vec{V}^{(0)}\equiv\frac{\vec{\nabla}\times\vec{B}}{B_0}, \tag{5}$$

$$\vec{V}^{(1)}=\varepsilon\frac{2\xi^2}{3}\left\{\left(\hat{y}v_y^{(0)}-\hat{x}v_x^{(0)}\right)\left[\left(v_y^{(0)}\right)^2-\left(v_x^{(0)}\right)^2\right]-\left(\hat{y}\partial_y-\hat{x}\partial_x\right)\left(\partial_y v_y^{(0)}-\partial_x v_x^{(0)}\right)\right\}, \tag{6}$$

$$\vec{V}^{(2)}=\varepsilon^2\frac{2\xi^2}{3}\left\{\left(\hat{y}v_y^{(1)}-\hat{x}v_x^{(1)}\right)\left[\left(v_y^{(0)}\right)^2-\left(v_x^{(0)}\right)^2-2v_y^{(0)}v_y^{(1)}+2v_x^{(0)}v_x^{(1)}\right]-\left(\hat{y}\partial_y-\hat{x}\partial_x\right)\left(\partial_y v_y^{(1)}-\partial_x v_x^{(1)}\right)\right\}, \tag{7}$$

and

$$\vec{V}^{(3)}=\varepsilon^3\frac{2\xi^2}{3}\left\{\left(\hat{y}v_y^{(0)}-\hat{x}v_x^{(0)}\right)\left[\left(v_y^{(1)}\right)^2-\left(v_x^{(1)}\right)^2+2v_y^{(0)}v_y^{(2)}-2v_x^{(0)}v_x^{(2)}\right]+\left(\hat{y}v_y^{(1)}-\hat{x}v_x^{(1)}\right)\left[2v_y^{(0)}v_y^{(1)}-2v_x^{(0)}v_x^{(1)}\right]+\left(\hat{y}v_y^{(2)}-\hat{x}v_x^{(2)}\right)\times\left[\left(v_y^{(0)}\right)^2-\left(v_x^{(0)}\right)^2\right]-\left(\hat{y}\partial_y-\hat{x}\partial_x\right)\left(\partial_y v_y^{(1)}-\partial_x v_x^{(1)}\right)\right\}, \tag{8}$$

where $B_0=\frac{\Phi_0}{2\pi\lambda^2}$ is the order of H_{C1} and $\Phi_0=\frac{2\pi\hbar c}{e^*}$ is the quantum flux and the zero order penetration depth may be written as $\lambda_0^{-2}=8\pi\gamma_d\left(\frac{e^*d_0}{\hbar c}\right)^2$.

3. One dimension model

Since the magnitude of the applied field is nearly equal to H_{C1}, we simply suppose that magnetic field depends only on x. In the next section the two dimension model will be considered. The free-energy density may be expanded as $f=f^{(0)}+f^{(1)}+f^{(2)}+f^{(3)}$. By using eqs. (5), (6), (7) and (8) the free-energy density finally is given by

$$f=\frac{1}{8\pi}B^2+\frac{\lambda_0^2}{8\pi}\left(\frac{\partial B}{\partial x}\right)^2+\varepsilon\frac{\lambda_0^2\xi^2}{8\pi\ B_0^2}\left(\frac{\partial B}{\partial x}\right)^4+\varepsilon^2\frac{5}{18\pi}\frac{\xi^4\lambda_0^2}{B_0^4}\left(\frac{\partial B}{\partial x}\right)^6+\varepsilon^3\frac{7}{9\pi}\frac{\xi^6\lambda_0^2}{B_0^6}\left(\frac{\partial B}{\partial x}\right)^8. \tag{9}$$

To tackle the problem we may further write

$$\vec{B}(x)=B(0)e^{-\frac{x}{\lambda}}\hat{z}. \tag{10}$$

Hence, the total free-energy of the system is

$$F=\frac{A}{8\pi}\left[B^2(0)\frac{\lambda}{2}+\frac{\lambda_0^2}{2\lambda}B^2(0)\right]+\varepsilon\frac{A\lambda_0^2s^2}{32\pi\ B_0^2\lambda^3}B^4(0)+\varepsilon^2\frac{5A}{108\pi}\frac{\lambda_0^2\xi^4}{\lambda^5B_0^4}B^6(0)+\varepsilon^3\frac{7A}{72\pi}\frac{\lambda_0^2\xi^6}{\lambda^7B_0^6}B^8(0), \tag{11}$$

where A is the surface of the sample. By finding the derivative of eq. (11) with respect to λ we can obtain the penetration depth, λ, which obeys the following equation

$$\lambda^2=\lambda_0^2\left(1+\varepsilon\frac{3}{2}\frac{\xi^2}{\lambda_0^2}+\varepsilon^2\frac{157}{108}\frac{\xi^4}{\lambda_0^4}+\varepsilon^3\frac{35}{36}\frac{\xi^6}{\lambda_0^6}\right), \tag{12}$$

It is obvious from the above equation that the corrected terms contain the ratio $\frac{\xi}{\lambda_0}$, which is much smaller than one in high-transition temperature superconductors. Therefore, the strength of the s and d-wave coupling can be taken as large as one, and the generalized London penetration depth in one dimension, may be written as

$$\lambda=\lambda_0\left(1+\varepsilon\frac{3}{4}\frac{\xi^2}{\lambda_0^2}\right). \tag{13}$$

4. Two dimension model

The direction of the vortex line is parallel to the magnetic field, $\vec{B}=B\hat{z}$, hence the lattice of the vortices

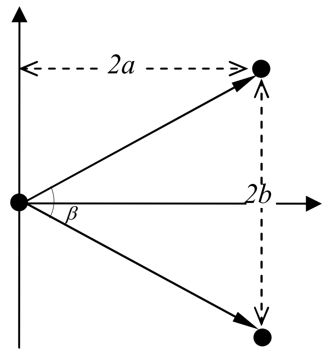

Figure 1. Primitive vectors of the vortex lattice.

is in the xy plane. In general the magnetic field of a vortex line is a function of variables x and y. To determine vortex lattice structure in two dimensions, we followed the procedure in Ref. [7] and generalized their result up to the second order of ε. After lengthy and tedious calculations, we found the magnetic field distribution of the vortex,

$$B(\vec{r}) = \bar{B}\sum_{\vec{k}} \frac{e^{i\vec{k}.\vec{r}}\, e^{-k^2\xi^2/2}}{1+k^2\lambda_0^2+4\varepsilon\lambda_0^2\xi^2k_x^2k_y^2+\frac{8}{9}\varepsilon^2\lambda_0^2\xi^4k^2k_x^2k_y^2}, \tag{14}$$

where $\bar{B}=\dfrac{\varphi_0}{\Omega}$, Ω is the area of the unit cell in the vortex lattice and $\vec{k}$ is the reciprocal lattice vector which can be written in terms of angle between the unit vectors, β, (see figure 1) of the cell as

$$\vec{k} = m_1\vec{k}_1 + m_2\vec{k}_2\,, \tag{15}$$

where

$$\vec{k}_1 = \frac{\pi}{a}\hat{i}+\frac{\pi}{b}\hat{j}, \qquad \vec{k}_2 = \frac{\pi}{a}\hat{i}-\frac{\pi}{b}\hat{j}, \tag{16}$$

and

$$a=\sqrt{\frac{\Phi_0}{2\bar{B}\tan(\beta/2)}}, \qquad b=\sqrt{\frac{\Phi_0\tan(\beta/2)}{2\bar{B}}}, \tag{17}$$

It is noted that in deriving eq. (14) we neglect the terms which are proportional inversely to B_0^2. eq. (14) of Ref. [7] is the same as our eq. (14) if ε^2 term is neglected.

The lattice symmetry is determined by minimizing the Gibbs free-energy ($G(\beta) = f_L - BH/4\pi$). In figure 2 we plot $G(\beta)-G_0$ versus β. Here for brevity we do not write the details of calculations and we only mention that the same denominator and exponential term, $e^{-k^2\xi^2}$ in the eq. (14) will appear in $G(\beta)-G_0$ too. Since the exponential function $e^{-k^2\xi^2/2}$ truncate the contribution of the reciprocal lattice vectors $\vec{k}$ of the order $\sim\dfrac{\pi}{\xi}$, the major contribution comes from the $\vec{k}\sim\dfrac{\pi}{\lambda}$. Hence in both one and two-dimensional calculations, terms like $\lambda_0^2\xi^4k^2k_x^2k_y^2 \sim \xi^4/\lambda^4$ do not contribute seriously and one may take ε as large as one (Actually, the maximum value of ε is about 0.4 for $H = 6.8\,\mathrm{T}$; see figure 3). The numerical calculations also prove these results. In figure 4 the β_{MIN} is plotted versus H and the results are the same as the results of Ref. [7]. This is another evidence that the terms proportional to ε^2 and higher order pertarbative terms have not any contributions to the results.

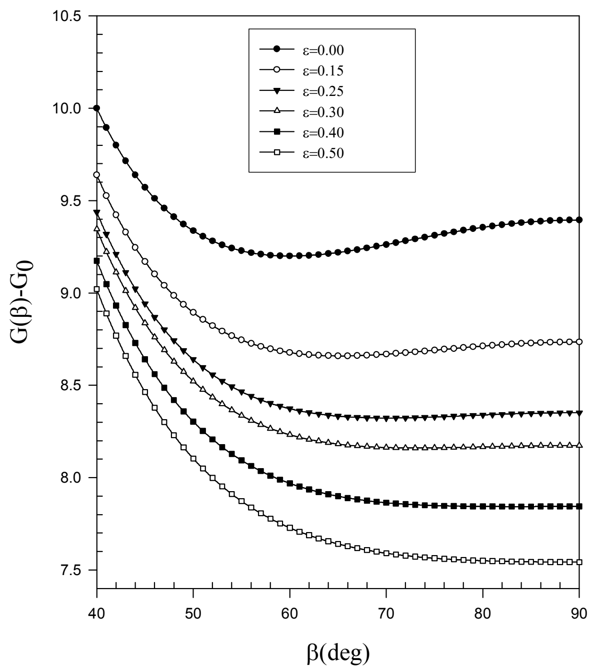

Figure 2. Gibbs free-energy as a function of β for $H = 6.8\,\mathrm{T}$, $\lambda_0 = 1400$ Å, $\kappa = \lambda_0/\xi = 68$ and various values of ε ($G_0 = -H^2/8\pi$). We have rescaled the vertical axis (arbitrary units) which is defferent from Ref. [7].

5. Discussion

Our analysis simply does not take into account the anisotropy in the coherence length ξ and penetration depth λ in the x and y axis. This effect would stretch flux lattice shape in the x axis by the factor $\dfrac{\lambda_x}{\lambda_y}$ [7], and the degeneracy between flux lattice and ionic lattice will be removed.

Neutron-scattering experiments on YBCO [8] suggest that the vortex lattice is well-aligned with the twin boundaries, whereas STM imaging of YBCO [9] does not suggest this alignment. However, both experimental groups suggest that the vortex lattice has approximately centered rectangular symmetry with $\beta \approx 73^\circ$ and 77° respectively. If one could neglect the

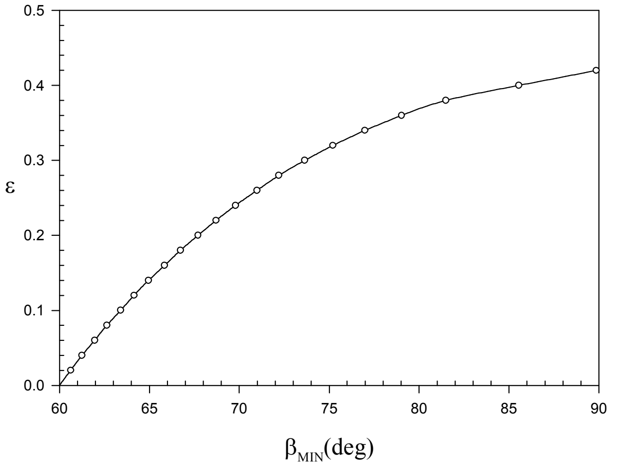

Figure 3. Equilibrium angle β_{MIN} versus the values of ε for $H = 6.8\,\mathrm{T}$.

effect of the boundaries in YBCO [8] in the neutron-scattering experiments and accept $\beta \approx 73^{\circ}$, then the best value of the ε is 0.30 for $H = 6.8$ T . Any way for determining the exact value of ε one needs more experimental data in a clean tetragonal material such as $Tl_2Ba_2CuO_{6+d}$.

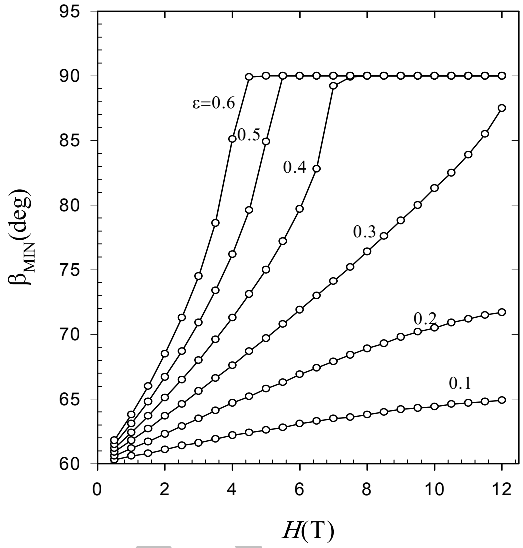

Figure 4. Equilibrium angle β_{MIN} versus the H.

In conclusion in one and two dimensional vortex lattice we clarify and show that the strength of the s-d wave coupling is not a perturbative parameter and the new experimental data, especially on tetragonal material such as $Tl_2Ba_2CuO_{6+d}$ can estimate its value.

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