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Finite Element Analysis of a 6.45–14 Bias Tire Under Contact Load

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ABSTRACT

This research work is devoted to development of a non-linear finite element model for a bias tire under inflation pressure and contact loads. The model was developed by a pre-processor program for NSTAR which is the non-linear module of COSMOS/M software. Three-dimensional isoparametric brick elements with total Lagrangian formulation have been used to construct the finite element mesh of the tire model. Applying appropriate rim displacement toward a rigid surface simulated the contact load. The stress-strain relationships of composite sections (plies, breaker and bead) are described by linear orthotropic constitutive equation while Mooney-Rivlin incompressible model was used for the description of the behaviour of rubbery parts. The analysis was performed in a Lagrangian framework to take the geometrical non-linearity of large deformation of tire into account in conjunction with Newton-Raphson iterative procedure to solve the final set of working equations. Comparisons of the numerical results with laboratory measured tire deformations and also available tire standards, confirm the general suitability of the developed mathematical model

Key Words: tire, finite element, stress analysis, contact load, COSMOS/M

INTRODUCTION

Despite the widespread use of the finite element method for the analysis of pneumatic tires, it is still one of the most challenging problems in computational mechanics. It is a very complex 3D contact analysis involving complicated geometry, composite materials, different loading and boundary conditions as well as large deformations and large strains.

Many attempts have been devoted to analyze of such complex problems with varying degree of sophistication (see for example [1–10]). The main goal of such analyses is to reduce the number of full-scale experiments required to find the optimized constructional parameters before actual manufacturing. This inevitably results in the reduction of time, energy and expenditure of raw materials for the development of new tires. Due to the unique features of this type of problem such as non-linear stress-strain behaviour of the material, large strains and large deformations and also changing of boundary conditions, highly sophisticated non-linear finite element codes must be used for this purpose. There are a number of commercial finite element codes, which can be used for the analysis of tires such as NASTRAN, ADINA, MARC, ABAQUS and COSMOS/M.

In the present work we have used a finite element code (COSMOS/M Ver. 1.75) to analyze a 6.45–14 bias tire subjected to inflation and static contact loading. The main aim was the prediction of the deformed shapes and stress components under loads. A three-dimensional finite element model based on the use of solid (brick) element with a Lagrangian formulation is developed to simulate the structure of the tire. The results of the analysis are also compared with available experimental data. The main assumptions made in the development of this model are as follows:

–Linear orthotropic stress-strain behaviour is assumed for the carcass elements (ply and breaker) and bead region while an isotropic incompressible element is used for the modeling of rubbery parts (tread and side-wall).

–The parameters of the material models are constants and remain unchanged during the application of external loads. It is found that in some cases [3,5] it may be necessary to take into account the effect of cord angle changes between plus and minus ply due to the element deformation and lifting phenomenon. On the other hand, the composite properties generally differ in tension and compression and thus different material constants should be used during the analysis of each case. However, owing to the lack of ability of considering of these two effects in the present version of COSMOS/M, they have not been taken into consideration in our model.

–A smooth tread pattern has been assumed in the model to avoid of excessive computational cost and efforts.

–The applied pressure load is assumed to be deformation-dependent so that the pressure always remains normal to the tire internal surface.

–A frictionless surface is assumed between the tire tread and contact surface

In the following section, we first briefly describe the finite element formulation of geometrically non-linear problems and contact algorithms. Then the tire lay-out and the parameters of the material models are introduced. The finite element model used for this tire and the results of the simulations are presented in the next section and finally the conclusions are drawn for this finite element analysis.

NON-LINEAR FINITE ELEMENT FORMULATION

The basic problem in a general non-linear analysis is to find the state of equilibrium of a body corresponding to the applied loads. Assuming that the applied loads are described as a function of time (a virtual meaning in a static analysis without time effects), the basic equation to be solved at any time step t+Δt is given by:

$$^{t+\Delta t}\{R\} - ^{t+\Delta t}\{F\} = 0 \qquad (1)$$

Where $^{t+\Delta t}\{R\}$ is the vector of externally applied load and $^{t+\Delta t}\{F\}$ is the vector of internally generated nodal forces. It should be noted that, the solution at time t is assumed to be known. Since the internal nodal forces $^{t+\Delta t}\{F\}$ depend on nodal displacement at time t+Δt (i.e., $^{t+\Delta t}\{U\}$), an iterative procedure such as Newton-Raphson, Modified Newton-Raphson or BFGS should be adopted to obtain convergent solution. On the other hand the fact that in a large deformation analysis (geometrical non-linear) the body under study changes continuously with applied loads, requires that appropriate measures should be used for stress, strain and constitutive relation. In the present work the *2nd Piola-Kirchhoff* stress tensor and the *Green-Lagrange* strain tensor are used in conjunction with the total Lagrangian in an incremental formulation for the analysis of our large deformation problem. In a total Lagrangian scheme, all kinematic and static variables (like stress and strain) are referred to initial configuration at time 0 (undeformed configuration). Details of this formulation as well the numerical techniques to solve the non-linear eqn (1) are completely described in reference [7,11].

CONTACT LOADING ALGORITHM

One of the most important areas of the tire analysis is the determination of the state of the stress and strain under contact loading. Several methods have been developed in order to handle such a problem, namely penalty method, using of Lagrange multiplier and hybrid technique [12]. However, the penalty and the Lagrange multiplier methods have some drawbacks. In the penalty method, large numerical values are introduced into the stiffness matrix of the system to simulate the rigidity between two nodes. A very large penalty factor causes numerical difficulties while a small penalty number produces inaccurate solution. On the other method, the problem size increases due to the addition of new variables (Lagrange multiplier) into the working equations. Consequently, more computer time and storage are required which reduce the efficiency of this method. In the third approach, a hybrid technique is used to solve the contact problem. Finite element methods of structural analysis can be generally categorized as the displacement method and the force method. In the first method the unknown quantities are the nodal displacements while in the second method the unknown values are the nodal forces. The hybrid technique combines the displacement and force method to solve the matrix equation [12]. This technique is shown to be more efficient in dealing with geometrical non-linearity than the other methods.

MATERIAL PROPERTIES

The elastic constants of a cord-rubber composite are usually obtained as a function of rubber and cord properties. Various authors [13–15] have expressed analytically the elastic constants of orthotropic laminated layers dependent upon the Young modulus, Poisson's ratio and shear modulus of the cord and rubber according to their volume fractions in composite. In the present work the Clark's equations [15] are selected to determine the elastic constants of an orthotropic cord-rubber composite.

Due to the change in shape between the cylindrical uncured (green) tire on building drum and the molded shape, the cord angle and volume fractions at individual points on the carcass are calculated by the use of the so-called "lift equations". For the tread and side-wall (rubbery parts), the Mooney-Rivlin hyperelastic model has been selected to describe the stress-strain behaviour of the material. Table 1 gives the elastic constants of the cord and rubber as well as the material constants of the Mooney-Rivlin model for the rubber parts.

NUMERICAL RESULTS AND DISCUSSION

The finite element calculations were carried out for a (6.45–14) passenger car tire. The cross sectional view of the initial shape or as it is assumed in this work, the tire lay-out in the mold is shown in Figure 1. Two plies and one breaker, made of nylon 6(1260D/2) and nylon 6,6(840D/2) construct the carcass of the tire. The cured angle of the plies and breaker at crown are equal to 32° and 35° with respect to circumferential direction, respectively. The green-ends count for the plies and breaker are also equal to 1.14 /mm and

Table 1. Material constants for various components of the tire[1].

Component	Young's modulus	C_1[2]	C_2[2]	E_1[3]	E_2[3]	G_{12}[3]
Nylon 6 (1260D/2)	3351	–	–	–	–	–
Nylon 6,6(840D/2)	3053	–	–	–	–	–
Tread	–	0.31	0.38	–	–	–
Side-wall	–	0.1	0.7	–	–	–
Bead	–	–	–	157200	72.4	10.7

(1) All units are in MPa.

(2) First and second constants of the Mooney-Rivlin equation $[W = C_1(I_1 - 3) + C_2(I_2 - 3)]$

(3) Subscripts 1,2 and 3 refer to the longitudinal, lateral and normal to the plane directions in a lamina, respectively.

0.67 /mm, respectively. The finite element model for this tire has been generated for NSTAR program (non-linear module of COSMOS/M software) by GEOSTAR pre-processor program. The 8-nodded, isoparametric brick elements with total Lagrangian large deformation formulation was used for the modeling of the tire. Taking advantage of tire symmetry, only one-half of the tires were modelled. However, due to the out-of-plane deformation in cord-rubber composite used in the tire, it is necessary to take a 360° model into consideration. Figure 2 shows the finite element mesh of the tire. The total number of nodes and elements used in this mesh are 2380 and 1764, respectively. As it can be seen, the elements near the contact area are more refined compared to those away from the contact region. In order to simulate tire contact of the tire with a flat rigid surface, 158 one-nodded gap elements [12] on tread surface have been defined. Finite element analyses were performed in two stages. During the first stage, the rim was assumed to be fixed and an internal pressure of 0.165 MPa (24 psi) has been applied. Five time steps have been selected to obtain convergent results for the inflation step. The second stage was restarted from the results obtained at the previous step and the rim has given a total displacement equal to 25.4 mm (1") toward the contact surface (Figure 3). In this step, 8 times steps are required to obtain convergent solutions. The convergence tolerance for both cases was 0.001. The total solution time was about 80 min on a PII 400 MHz computer.

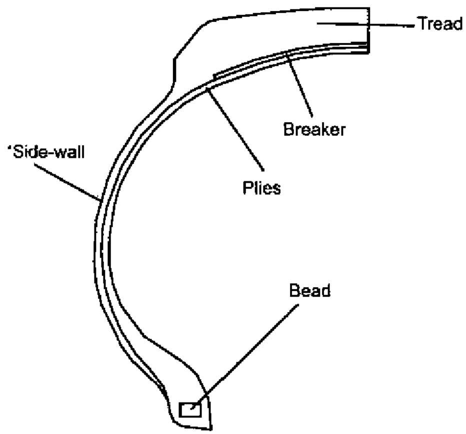

Figure 1: Tire lay-out in the mould.

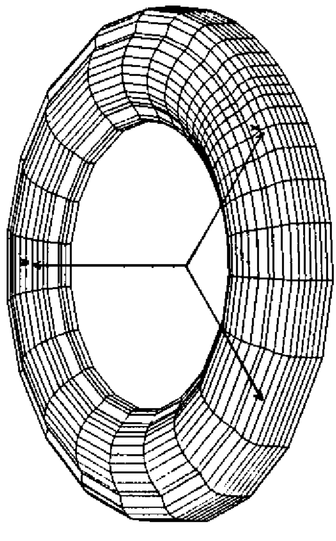

Figure 2. Finite element's mesh of the tire

Figure 4 shows the deformed profile of the tire after application of inflation pressure. The calculated crown displacement for this model is 10.6 mm. Considering the effect of shrinkage upon opening of

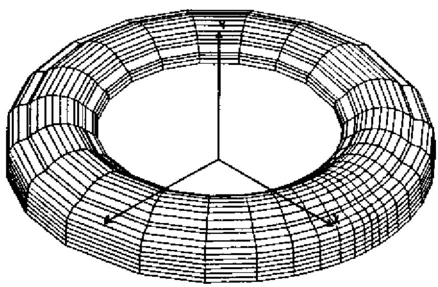

Figure 3: Tire with contact surface.

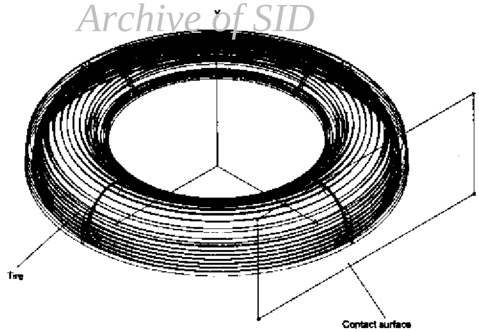

Figure 4: Deformed shape of the tire after application of inflation pressure.

the mold and thus selection of uninflated shape as initial configuration, the experimentally measured values for this parameter are between 7.5 to 10.5 mm which confirms the accuracy of the developed model. It can be also observed that due to the out-of-plane deformation in cord-rubber composite the warping effect takes place even in a symmetric loading such as inflation pressure. The deflected shape of the tire corresponding to four rim displacements, 3.175, 9.525, 19.05 and 25.4 mm are illustrated in Figure 5. The load-deflection curve for this tire is also plotted in Figure 6. The calculated values for this tire are also in close agreement with data recorded in tire standards such as Tire & Rim and ETRTO.

One of the most important features of the finite element analysis of a tire under loads is to examine the state of stress and strain at different locations of the tire carcass. The study of the shear stress and

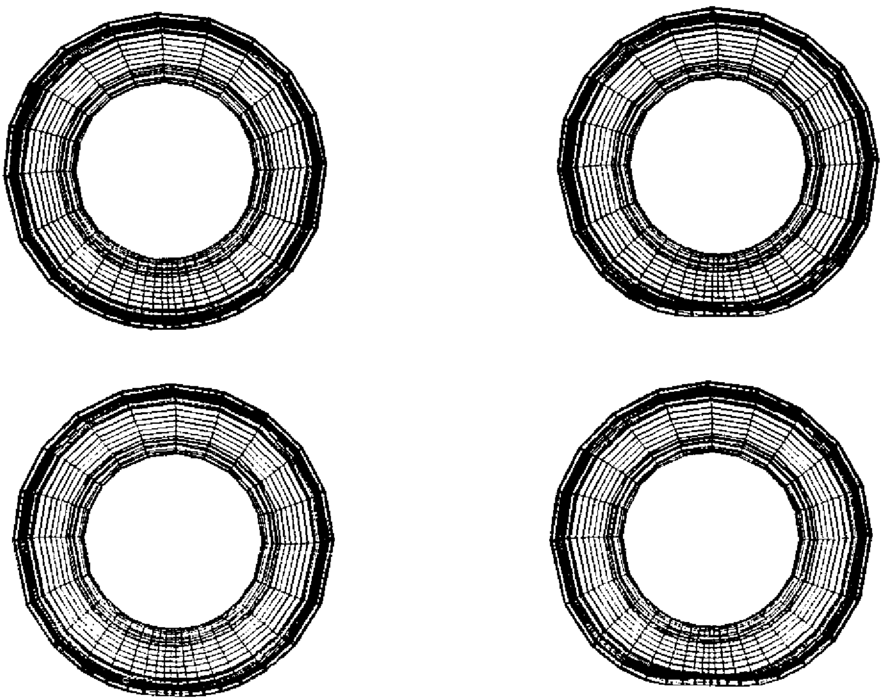

Figure 5: Deformed shapes of the tire under contact load.

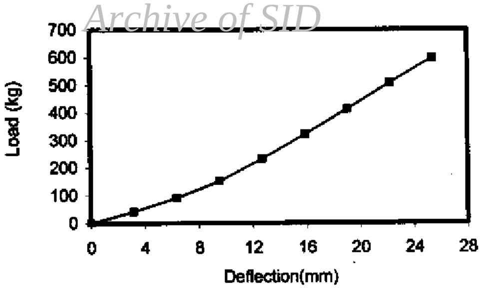

Figure 6: Load deflection curve of the tire under contact load.

shear strain, which is related to tire endurance, can be effectively used to find the optimum construction. The distributions of the three components of shear stress and shear strain (τ_{xy}, τ_{xz}, τ_{yz} and γ_{xy}, γ_{xz}, γ_{yz}) at line between the plies and breaker (side-wall where the end of breaker is reached) for two different load cases (after inflation and complete deflection) are shown in Figures 7 and 8, respectively. In these graphs the distance is a parametric variable between the crown and bead. A distance equal to 0 represents the crown while a value of distance of 1 corresponds

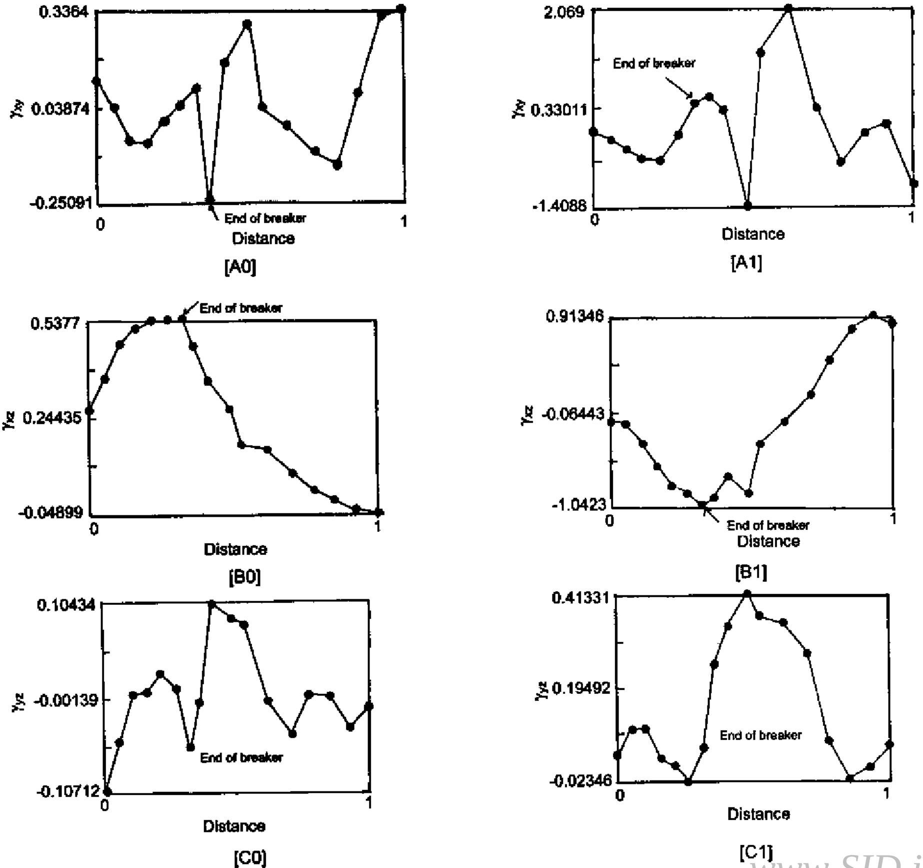

Figure 7: Components of the shear stress in carcass line after inflation and under contact load.

to bead region. The end of breaker is also depicted on each graph. The x, y and z directions in these graphs are local elemental coordinates that correspond to meridian, normal to meridian and circumferential directions on tire profile, respectively. Figures 7_A to 7_C show the distribution of shear stress components after inflation and under contact load with rim displacement equal to 25.4 mm. As it can be seen, there are crucial differences in state of stress for all components between inflated and contact load cases.

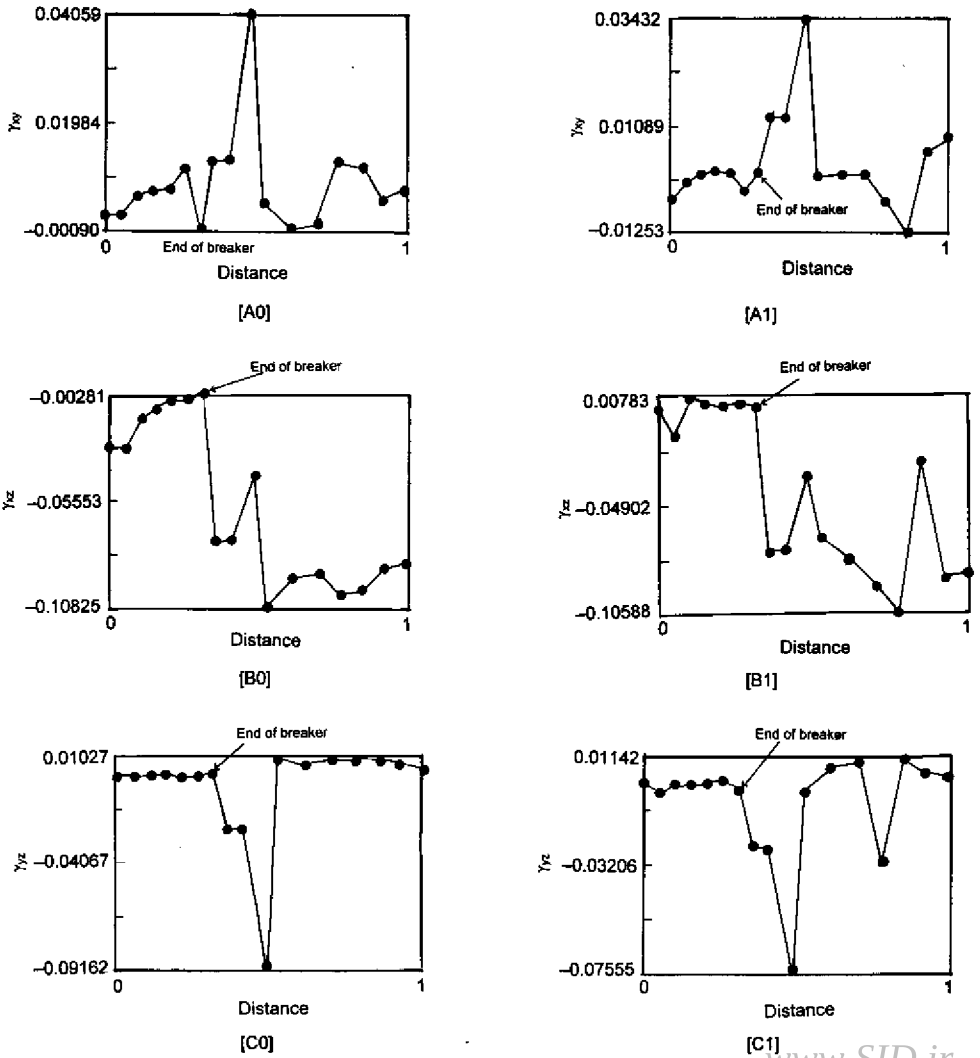

Figure 8: Components of the shear strain in carcass line after inflation and under contact load.

A similar circumstance can also be observed for shear components of strain tensor in Figures 8_A to 8_C. Variations of the shear stress and strain between inflated and contact conditions can be attributed to tire performance. Therefore, such analyses for various designs can be effectively used to optimize the construction of the tire. In particular, the shear strain component (γ_{yz}) is related to durability and fatigue life of the tire, thus minimizing its value and also the difference between two mentioned conditions i.e., inflated and contact load cases, give rise to production of tire with better endurance.

CONCLUSION

A 3D geometrical non-linear finite element model was developed for the simulation of a 6.45–14 bias tire under inflation pressure and contact load. The non-linear capabilities of COSMOS/M software (NSTAR module) were used for the analysis of the tire model. It is shown that by a complete three-dimensional finite element modeling, the components of stress and strain tensors can be accurately computed. The results of the analyses have also been compared with available experimental data, which is shown that they are in good agreement with those acquired from experiments and tire standards. However, it should be revealed that due to the lack of some computational features of COSMOS/M such as bilinear material behaviour and ability to consider the effect of cord-angle change on material direction, the results should be used with some degree of awareness specially under contact load. In addition, in bias tires made of nylon cords, the shrinkage phenomenon on tire configuration is highly important. Consequently, highly accurate experimental procedures should be employed to determine the shrink force for the inclusion in model equations to predict a precise initial configuration of tire.

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