

Identification of Mechanical Properties in Laminated Composite Plates Using Genetic Algorithm

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ABSTRACT

The elastic properties of composite materials depend on diverse factors like the configuration of the laminates, constituent materials used, production method adopted etc.. Hence, it is generally impossible to find these properties in standard tables or databases. The determination of material parameters of the complex materials, such as fibre reinforced composites, present serious difficulties when applying traditional test methods. To overcome these difficulties, an optimized search procedure, viz., genetic algorithm (GA), has been used to provide an additional technique of evaluating material properties from the measured natural frequencies. In general, GA performs directed random searches through a given set of criteria. These criteria are required to be expressed in terms of objective function, which is usually referred to as a fitness function. This paper describes the capability, reliability and modelling of GA for indirect (non-destructive) prediction of mechanical properties regarding the model development in the finite element method (FEM) and the analytical formulation (Exact Solution). A few examples are presented to identify the material properties of homogeneous isotropic, orthotropic, and anisotropic plates with various stacking sequences, boundary conditions and aspect ratio for showing the ability and performance of this technique.

Key Words:

optimization technique;
material properties;
finite element method;
genetic algorithm;
composite materials.

INTRODUCTION

In the past decade, many researchers have continuously devoted their efforts to predicting the mechanical properties of laminated composite structures by making various assumptions. Although the properties of certain types of composites, mod-

elled as homogeneous, linear and anisotropic, can be assessed analytically from the elastic properties of its constituents, considerable uncertainty remains due to the approximations involved in the derivation of the relevant formulae as well as the effect of

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the manufacturing processes to geometrical arrangement and properties of its two phases. Recently, the non-destructive evaluation of elastic properties of composite structures based on the modal test data is more commonly used. This is an inverse problem with the parameters in the mathematical model being adjusted repeatedly until its analytical responses match satisfactorily with those associated with the physical structure. DeWilde et al. [1, 2, 3] used Kirchhoff assumptions and Bayesian parameter estimation method for the determination of linear elastic characteristics of composite sample through its experimentally measured natural frequencies has been proposed. Sol et al. [4] presented a method, which determines the elastic properties of a composite material plate, using experimentally measured resonant frequencies. Only thin plates, subjected to small lateral deflections were considered as Love-Kirchhoff model. More recently, two alternative methods both requiring experimental eigen frequency data have been published by Sol [5] and Wilde [6] which do not follow the sensitivity analysis approach proposed here. Pedersen et al. [7] presented an indirect identification technique to predict the mechanical properties of composite plate specimens. This technique makes use of experimental eigen frequencies, the corresponding numerical eigen value evaluation, sensitivity analysis and optimization. Lai et al. [8] developed a method to revise the elastic properties of a thin composite plate vibration model in an iterative manner such that its modified analytical responses eventually matched those obtained experimentally. Araujo et al. [9] proposed a numerical/experimental method for the identification of material parameters of the composite materials. GAs are used by Cunha et al. [10] as a complementary technique to perform the initial estimation of the elastic parameters and then refining the solution by classical updating methods. Mariana et al. [11] presented a technique to identify elastic parameters of composite materials using GA to solve the optimization problem. They used, a FEM solution base on a triangular elements with three degrees of freedom per node. Base on the review, most of the investigations considered only isotropic and orthotropic laminated plates using a FEM.

In this paper, an indirect identification technique to predict the elastic properties of isotropic, orthotropic, and anisotropic plates with various stacking sequences,

boundary conditions and aspect ratios, is presented. The proposed method combines experimental works for measuring the resonant frequencies of laminated composite plates along with a FEM or analytical formulation for the determination of the corresponding analytical resonant frequencies. The natural frequencies of the composite plates are experimentally obtained using an FFT analyzer. The identification procedure makes use of a search method based on a GA technique, through the minimization of an error measurement that estimates the deviation between analytical and experimental resonant frequencies, for a set of initial approximation of the material elastic constants. The accuracy of this method is discussed through test cases.

The Linear Finite Element Analysis

In the finite element model, the laminated composite plate is discretized by eight-node quadrilateral elements with six degrees of freedom per node: three rotations and three transversal displacements. Thus, the associate eigen value problem is represented by:

$$(-\omega_i^2 \mathbf{M} + \mathbf{K}) \phi_i = 0 \quad (1)$$

where ω_i is the i_{th} natural frequency and ϕ_i is its vibration mode. \mathbf{M} and \mathbf{K} are, respectively, the mass and stiffness matrix of the finite element model. It is important to remark that the stiffness matrix depends on the linear elastic parameters that are to be identified, i.e.

$$\mathbf{K} = \mathbf{K}(E_1, E_2, G_{12}, \nu_{12}) \quad (2)$$

Classical Lamination Theory (CLT)

The numerical model best on the classical lamination theory underlying the finite element method for the calculation of the dynamic behaviour of laminated composites was outlined. It will be verified that this mathematical model produces accurate results. In this paper only thin plates, subjected to small transverse deflections (w_c), are considered. Only two types of rectangular laminated composite plates simply supported along all four edges, will be considered namely, specially orthotropic plates and anti-symmetric angle ply [12], which are useful for the identification of material properties using measured plates frequencies of structures. Here, the dimensions of the plates are a and b along the x and y axes, and h is the total thickness of

plate in the direction. For especially orthotropic plates, the following form for the natural frequencies may be written:

$$\omega_{mn}^2 = \frac{\pi^4}{b^4} [D_{11} (\frac{mb}{a})^4 + 2(D_{11} + 2D_{66}) (\frac{mnb}{a})^2 + D_{22} n^4] \tag{3}$$

Also, the fundamental frequency for the plate is ω_{11} , which, for the case of an isotropic plate can be shown to be:

$$\omega_{11} = \frac{\pi^2}{b^2} (\frac{D}{\rho h})^{1/2} [(\frac{b}{a})^2 + 1] \tag{4}$$

where $D = Eh^3/12 (1 - \nu^2)$ and E is modulus of elasticity.

For anti-symmetric angle ply plates configuration, the class of laminates with simply supported boundary conditions, the free vibration problem may be solved to determine the natural frequencies as formulated:

$$\omega_{mm}^2 = \pi^4 / \rho \{ F_{23} + (2F_{12} F_{23} F_{13} - F_{22} F_{13}^2 - F_{11} F_{23}^2) / (F_{11} F_{22} - F_{12}^2) \} \tag{5}$$

where $F_{11} = A_{11}(m/a)^2 + A_{66}(n/b)^2$
 $F_{12} = (A_{12} + A_{66})(m/a)(n/b)$
 $F_{13} = - [3B_{16}(m/a)^2 + B_{26}(n/b)^2](n/b)$
 $F_{22} = A_{66}(n/b)^2 + A_{22}(m/a)^2$
 $F_{23} = - [B_{16}(m/a)^2 + 3B_{26}(n/b)^2](m/a)$
 $F_{33} = D_{11}(m/a)^4 + 2(D_{12} + 2D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4$

Genetic Algorithm

GA is an unorthodox search or optimization algorithm, which was first suggested by John Holland in his book *Adaptation and Artificial Systems* [13]. As the name suggests, the GA was inspired by the processes observed in natural evolution. It attempts to mimic these processes and utilize them for solving a wide range of optimization problems. In general, GA performs directed random searches through a given set of criteria. These criteria are required to be expressed in terms of an objective function, which is usually referred to as a fitness function.

The GA method requires that the set of alternatives to be searched be finite. To apply them to an optimization problem where this requirement is not satisfied, the

set involved must be discretized and appropriate finite subset be selected. It is further required that the alternatives be coded in chromosomes of some specific finite length which consist of symbols from some finite alphabet. These are called chromosomes; the symbols that form them are called genes, and their set is called a gene pool.

The GA method searches for the best alternative (in the sense of a given fitness function) through chromosome evolution. The basic steps in the GA analysis are shown in Figure 1. First, an initial population of chromosomes is randomly selected. Then each of the chromosomes in the population is evaluated in terms of its fitness (expressed by the fitness function). Next, a new population of chromosomes is selected from the given population by giving a greater chance to select chromosomes with higher fitness. This is called the reproduction operation. The new population may contain duplicates. If given stopping criteria (e.g., no change in the old and new population, specified computing time, etc.) are not met, some specific, genetic-like operations are performed on chromosomes of the new population. These operations produce new chromosomes, called offsprings. The same steps of this process, evalua-

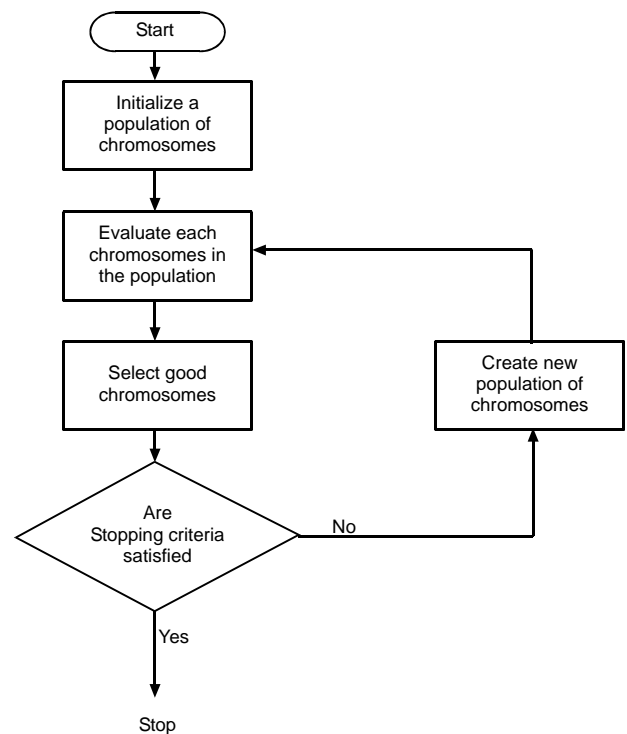


Figure 1. High level description of the GA.

tion and reproduction operation are then applied to chromosomes of the resulting population. The whole process is repeated until the given stopping criteria are met. The solution is expressed by the best chromosome in the final population.

Description of the Identification Optimization Problem

The identification technique aimed at finding the four elastic material properties of laminated composite structures through optimization techniques as GA, by minimizing the difference between experimental and numerical eigen values. The numerical model is based on a finite element method and analytical solution for GA is used to connect the prediction and experimental values through an updating process of natural frequencies followed by the determination of the frequency response. This is an inverse problem with the parameters in the mathematical model being adjusted repeatedly until its analytical response match satisfactorily with those associated with the physical structure. During the iteration process, the GA utilizes an error function to compare the theoretical and measured behaviour as well as the initial and revised property estimates. The objective function is defined as an error functional (f), which depends on the eigen frequencies as follows are:

$$f = |\bar{\omega}_i - \omega_i| \quad i = 1,2,\dots, n \quad (6)$$

The optimization problem is formulated as the identification of the set of material parameters, which minimize the error function:

$$\text{Min } f(E_x, E_y, G_{xy}, V_{xy}) \quad (7)$$

Since GA requires a maximization objective function, then above error function is transformed to a maximization objective function, which is expressed as:

$$F_i = 1 - \frac{|\bar{\omega}_i - \omega_i|}{\omega_i} \quad i = 1,2,\dots, n \quad (8)$$

However, two computer programmes (Fortran) have been developed to implement GA for identification of material properties of laminated composite structures. Figure 2 illustrates the flow chart for integration of GA that can be adopted to analytical or numerical method to analysis of parameters.

RESULTS AND DISCUSSION

Estimation of Mechanical Properties in Isotropic Plates

As a trial measure it was decided to apply this technique to a set of isotropic plates, one made of steel (C12). Experimental results obtained by Sol [14] from tensile tests on specimens and from vibrational measurements are compared with those generated by the present algo-

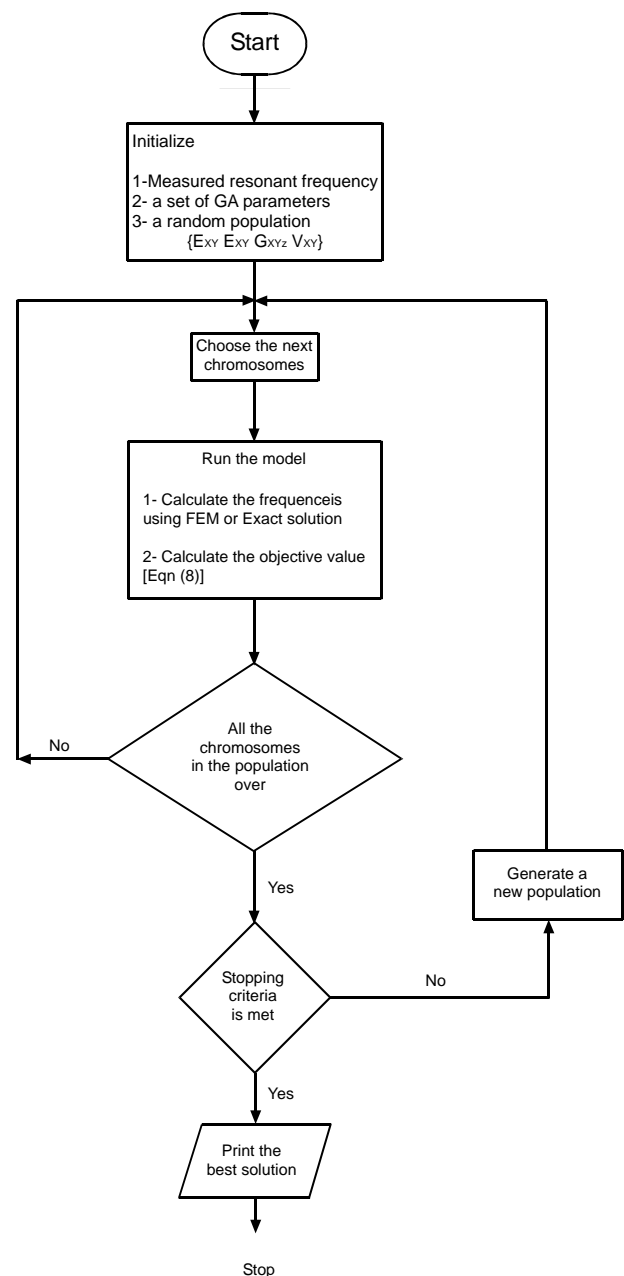


Figure 2. Integration of genetic algorithm.

Table 1. Plate properties and experimental measurement of resonant frequencies of steel plate, B.C's (FF-FF).

Geometry of steel		Mode	Frequency
		No.	(Hz)
Length (m)	0.31985±0.00005	1	214 ± 2
Width (m)	0.28012±0.00005	2	292 ± 2
Mass (kg)	4.0032±0.001	3	404 ± 4
Thickness (m)	0.00569±0.00002	4	524 ± 8
Density (kg/m ³)	7850±30	5	580 ± 8

rihm. The geometric parameters and the experimentally determined frequencies are given in Table 1. The results of four trials using the GA that linked to FEM and experimental measurement of mechanical properties (E & v) are presented in Table 2 and can be seen that the agreement of results is excellent.

Another analysis was performed on a simply supported isotropic square plate using the GA that linked to exact analytical solution [12,15] to identify the mechanical properties. The following plate geometry and analytical frequencies (instead of experimental frequencies) were taken by Sol [14]:

a & $b = 0.3$ m; $h = 0.005$ m; $\rho = 1000$ (kg/m³); $f_1 = 493.65$ (Hz); $f_{2\&3} = 1234$ (Hz); $f_4 = 1974.62$ (Hz) where f_i are measurement resonant frequencies. The estimation of the material properties of this plate is presented in Table 3.

Estimation of Mechanical Properties in Orthotropic Plate

Some laminated composite orthotropic plates whose

material properties are to be identified are considered in this section. Each set of results identified is compared with those of other authors and with strain gauge measurements where available. The first problem with a carbon fibre reinforced epoxy plate made of uni-directional fibre plies with stacking sequence $(\bar{0})_8$ was studied by Araujo et al. [9] and Frederiksen [16]. They identified the mechanical properties of this plate using the combined numerical as well as a higher order theory of finite element, and an experimental method using strain gauges and resonant frequency measurement. The plate under consideration had the dimensions given below and the study used the first five frequencies (Hz) obtained experimentally:

$a = 0.24275$ m; $b = 0.1525$ m; $h = 0.00339$ m; $f_1 = 161.23$; $f_2 = 347.14$; $f_3 = 476.93$; $f_4 = 495.69$; $f_5 = 589.09$ (Hz)

The comparison of the present results and those of the above authors are presented in Table 4. Identified results are in good agreement with those obtained by Araujo et al. [9] and Frederiksen [16] (the percentage error can be calculate as: $f_{i,trial} - f_{i,exp} / f_{i,exp} \times 100$).

Estimation of Mechanical Properties of Anisotropic Plates

At this stage it is interesting to investigate the prediction of mechanical properties of laminated composite plates and show the performance and sensitivity of GA techniques to evaluate these properties. This is the most important stage in the optimal analysis of laminated composite structures. The combination of experimental and numerical analysis, an adequate choice of objective function and constraints, with the correct selec-

Table 2. Estimated mechanical properties of isotropic plate [steel], B.C's (FF-FF), units (N/m²).

Material property	Genetic algorithm (FEM solution)				Experimental work
	Trial 1	Trial 2	Trial 3	Trial 4	
E	2.12×10^{11}	2.14×10^{11}	2.12×10^{11}	2.10×10^{11}	$(2.08 \pm 0.02) \times 10^{11}$
v	0.252	0.268	0.284	0.292	0.287 ± 0.008

Table 3. Estimated mechanical properties of isotropic plate, B.C's (SS-SS), units (N/m²).

Material property	Genetic algorithm (FEM solution)				Experimental work
	Trial 1	Trial 2	Trial 3	Trial 4	
E	87.40×10^9	87.00×10^9	87.20×10^9	87.35×10^9	87.36×10^9
v	0.31	0.305	0.305	0.30	0.30

Table 4. Estimated mechanical properties of orthotropic (0_o)₈ plate, comparison between GA and other results, B.C's (FF-FF), units (N/m²).

Material property	Genetic algorithm (FEM solution)		Araujo	Frederiksen	Frederiksen
	Trial 1	Trial 2	(Analytical)	(Analytical)	(Strain gauge)
E _x	106.75 × 10 ⁹	107.0 × 10 ⁹	107.8 × 10 ⁹	107.1 × 10 ⁹	113.2 × 10 ⁹
E _y	7.50 × 10 ⁹	8.00 × 10 ⁹	8.30 × 10 ⁹	8.30 × 10 ⁹	7.40 × 10 ⁹
G _{xy}	4.5 0 × 10 ⁹	4.25 × 10 ⁹	4.20 × 10 ⁹	4.20 × 10 ⁹	-
ν _{xy}	0.275	0.325	0.421	0.246	0.348
Error (%)	-1.90	0.780	0		

Table 5. Estimated mechanical properties of 10 layers (0, +60₂)_s plate, comparison between GA and other results, B.C's (FF-FF), unit (N/m²).

Material property	Genetic algorithm (FEM solution)			Frederiksen results	Araujo results
	Trial 1	Trial 2	Trial 3		
E _x	118.00 × 10 ⁹	117.75 × 10 ⁹	117.75 × 10 ⁹	118.50 × 10 ⁹	118.80 × 10 ⁹
E _y	8.75 × 10 ⁹	9.5 × 10 ⁹	8.75 × 10 ⁹	9.10 × 10 ⁹	9.10 × 10 ⁹
G _{xy}	4.75 × 10 ⁹	4.5 × 10 ⁹	4.75 × 10 ⁹	5.40 × 10 ⁹	5.60 × 10 ⁹
ν _{xy}	0.275	0.300	0.325	0.304	0.318
Error (%)	0.34	0.304	0.0054	-	0

Table 6. Estimated mechanical properties of two layers (45, -45) laminated plate, B.C's (SS-SS), unit (N/m²).

Material property	Genetic algorithm (FEM solution)				Araujo results
	Trial 1	Trial 2	Trial 3		
E _x	97.50 × 10 ⁹	97.50 × 10 ⁹	97.70 × 10 ⁹	98.00 × 10 ⁹	98.00 × 10 ⁹
E _y	7.90 × 10 ⁹	7.90 × 10 ⁹	7.95 × 10 ⁹	7.90 × 10 ⁹	7.90 × 10 ⁹
G _{xy}	6.59 × 10 ⁹	6.30 × 10 ⁹	5.50 × 10 ⁹	5.60 × 10 ⁹	5.60 × 10 ⁹
ν _{xy}	0.334	0.334	0.270	0.280	0.280
Error (%)	0.0043	0.006	-0.040	0	0
Genetic algorithm (FEM solution)					
E _x	98.20 × 10 ⁹	98.50 × 10 ⁹	97.70 × 10 ⁹	98.20 × 10 ⁹	98.00 × 10 ⁹
E _y	7.85 × 10 ⁹	7.85 × 10 ⁹	7.85 × 10 ⁹	7.85 × 10 ⁹	7.90 × 10 ⁹
G _{xy}	6.66 × 10 ⁹	6.57 × 10 ⁹	5.30 × 10 ⁹	5.50 × 10 ⁹	5.60 × 10 ⁹
ν _{xy}	0.335	0.260	0.329	0.318	0.28
Error (%)	-0.039	0.059	0.113	0.012	0

tion of design variables and optimization algorithms are the basic requirements for efficient optimal structural design. The first problem was for a carbon fibre reinforced epoxy plate made of uni-directional fibre

plies with stacking sequence (0, ±60₂)_s which was studied by Araujo et al. [9] and Frederiksen [16]. The plate dimensions and first five measured resonant frequencies are given below:

$a = 0.2227$ m; $b = 0.1851$ m; $h = 0.00126$ m; $f_1 = 96.310$; $f_2 = 163.18$; $f_3 = 202.66$; $f_4 = 252.17$; $f_5 = 285.23$ (Hz)

The results for three trials are compared with Araujo et al. [9] and Frederiksen [16] and presented in Table 5 indicating good agreement.

The second problem in this paper considers the identification of material properties of four layers laminated composite square plate, using the GA techniques, which linked to exact solution [12, 15] as well as finite element analysis. These plates had two layers (45, -45) with all edges simply supported (SS-SS). Table 6 list the estimated mechanical properties of the above plates, respectively. These results compared well with the actual results. The error estimates of fundamental frequencies obtained by these properties and the actual properties are presented in the tables, which are in good agreement with those results.

$a = 0.27$ m; $b = 0.27$ m; $h = 0.00536$ m; $\rho = 1520$ Kg/m²; $f_1 = 26.692$; $f_2 = 63.635$; $f_3 = 63.635$; $f_4 = 106.77$ (Hz)

CONCLUSION

In this investigation a non-destructive optimization search technique for identification of the four major (E_x , E_y , G_{xy} , ν_{xy}) mechanical properties of laminated composite plates based on the GA has been established. The finite element method as well as analytical solution when associated with experimentally measured free vibration resonant frequencies enable the prediction of the mechanical properties of composite plate specimens within acceptable limits of accuracy through an error function. One program has been developed based on finite element and analytical solution to show the capability and efficiency of GA search technique to identify the optimum results. Of course, the results obtained by the analytical solution give maximum accuracy and even exact results, but analytical solution with appropriate boundary conditions, may not be available for all types of structures. Then a finite element analysis can be used although in some cases, estimation errors are higher than those for analytical solutions. GA is one optimization search technique, which can be used with confidence for identification of mechanical properties of laminated composite materials.

NOTATION

- a, b, h Dimensions of the plate
 E_{ij}, G_{ij}, ν_{ij} Young's modulus, shear modulus, Poisson ratio
 $[K], [M]$ Stiffness and mass matrices
 ϕ_i Mode shape
 ρ Density
 A_{ij}, B_{ij}, D_{ij} Extensional, coupling and bending material stiffness matrices
 ω_i, f_i Natural and measurement frequencies
 w_c Displacement vector

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