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# Three Dimensional Finite Element Modelling of Truck Tyre Curing Process in Mould

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## ABSTRACT

Owing to the significant changes in tyre geometry especially for truck tyres with complex patterns and high thickness at tread area, the effect of heat transfer in circumferential direction on temperature field development during tyre curing should be taken into account. This work is based on the development of a three dimensional nonlinear finite element model for simulating of tyre curing process of in mould. The heat conduction equation was solved by the use of a standard Galerkin technique in a Cartesian coordinate system. Transient temperature field was modelled using the implicit-θ method. The Kamal and Sourour empirical cure reaction kinetic equation was also used to calculate the state and rate of cure as a functions of time. Based on the developed model, an interactive computer program was written in Visual Basic. This program was used to simulate the curing process of a 12-24 truck tyre in mould. Results of our simulations showed that distributions of both temperature and state of cure in circum ferential direction cannot be ignored. The applicability of the model was also verified by comparison between the temperature profiles at two points inside the tyre with experimental data. It has been shown that there is a very good agreement between the model predictions and actual data.

Key Words:
finite element method;
truck tyre;
mathematical model;
rubber;
curing;
kinetic.

## INTRODUCTION

It is quite obvious that the process of vulcanization of pneumatic tyres has a crucial effect on quality and service performance of final products. This is mainly important because during the manufacturing process at this stage the desired shape and physical properties of various components of the tyre are determined. This step requires high energy consumption and its optimization not only helps to produce high quality tyres but also improves the process cost. Due to the low thermal conduc-

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tivity of the rubber and cord and also using time dependent operating conditions, every point inside the tyre has a unique temperature history during the curing cycle. This leads to non-uniform distributions of temperature and state of cure (degree of cure). Therefore a simple cure evaluation curve at constant temperature is generally inadequate to predict the state of cure in tyres.

There are two methods to tackle this problem. The first technique is to measure directly the temperature as a function of time by using inserted thermocouples into the critical points of a tyre such as its shoulder and under tread areas. The second one is to use an appropriate kinetic equation. These temperature histories are then converted to state of cure and thus the necessary time for the completion of a cure cycle is determined. Since for each experiment at least one tyre must be damaged to find out the exact locations of thermocouples inside it, this method is costly and very time-consuming. Consequently, tyre industries are seeking alternative approaches based on the use of computer simulations to predict the distributions of temperature and state of cure without any requirements for experimental measurement of temperature profiles.

Several studies have been carried out in which both finite difference and finite element methods are used to solve numerically the governing equations [1-6]. All of these works are based on the development of two dimensional models in which a cross section of tyre is selected as the domain of solution. These models cannot predict temperature distribution in circumferential direction and thus variation of temperature is not taken into account. However, due to the complex tread patterns and higher thickness of critical areas in truck tyres (specially in semi-rib and lug types) temperature profile in circumferential direction is not uniform. Therefore three-dimensional models are required to find temperature profile, there.

In our work, we have developed a new mathematical model for the simulation of the vulcanization process of a tyre in mould. The developed model is based on the finite element solution of the three dimensional nonlinear transient heat conduction equation in conjunction with an empirical cure kinetic model for the evaluation of the state of cure in tyre. This model and the developed finite element solution algorithm have been the basis of an in-house written computer code in Visual Basic. To demonstrate the applicability and capability of the model and computer program, the curing process of a heavy-duty truck tyre with semi-rib tread pattern in mould has been simulated. In addition, the calculated temperature profiles for two points inside the tyre were compared with experimentally measured data. This comparison provides evidence for the accuracy of the developed model.

In the following sections, the mathematical model and kinetic equations of cure reaction are first described and then the finite element working equations associated with this problem are introduced. The computational results are presented and finally the conclusions are drawn.

## MATHEMATICAL MODEL

### Heat Transfer Equation

The governing equation of the transient heat conduction in a three dimensional cartesian coordinate system is given as:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) + \dot{Q} \quad (1)$$

In this equation, t, T, $\rho$, $C_p$, and k are time, temperature, density, heat capacity, and thermal conductivity, respectively. For rubbery materials k is assumed to be a linear function of temperature expressed by [2]:

$$k = a - bT \quad (2)$$

Heat capacity of rubber ($C_p$) generally increases with increasing of temperature while density decreases and thus the product of $\rho C_p$ in equation (1) is relatively insensitive to temperature. Consequently, it is assumed that $\rho$ and $C_p$ have constant values [2].

Determination of heat generation rate per unit volume $\dot{Q}$ is not a trivial matter and needs careful consideration. Let $\alpha$ denote the state of cure (degree of cure) defined as:

$$\alpha = Q_t/Q_\infty \quad (3)$$

Where $Q_t$ and $Q_\infty$ are the heat released up to time t, and total heat of reaction, respectively. Therefore, the heat

generation rate per unit volume of the rubber is calculated by:

$$\dot{Q} = Q_\infty (\frac{d\alpha}{dt}) \quad (4)$$

Total heat of reaction $Q_\infty$ is a material property and can be determined by either an isothermal or non-isothermal DSC experiment.

**Cure Reaction Kinetic**

During the curing process of elastomers a chemical reaction resulting in appearance of cross-links between polymer chains takes place. For curing of rubbery materials, the state or degree of cure ($\alpha$), which represents the extent of reaction, have been described by many empirical equations (for example [7-9]). In the present work, the following kinetic model proposed by Kamal and Sourour [7] was used:

$$\alpha = \frac{k(t-t_i)^n}{1+k(t-t_i)^n} \quad (5)$$

Where k and n are kinetic constant and order of reaction, respectively. During the induction period ($t_i$) the chemical reaction does not take place. This parameter is a function of temperature and it is proposed that it can be described by an Arrehenius-type equation:

$$t_i = t_0 \exp(T_0/T) \quad (6)$$

Where $t_0$ and $T_0$ are material constants. Parameter k in equation (5) is a rate constant with an Arrehenius-type temperature dependence form:

$$k = k_0 \exp(-E/RT) \quad (7)$$

Where $k_0$, E, and R are a constant, the activation energy, and the gas constant, respectively.

Vulcanization process of rubbery materials is a highly non-isothermal process and thus a non-isothermal curing kinetic model should be adopted in which not only the effect of time but also the influence of temperature history on cure rate and state of cure have been taken into account. A suitable non-isothermal cure kinetic model should properly consider both induction time period and curing stage. For the induction time, we have used a previously proposed model [7], given by:

$$\bar{t} = \int_0^t \frac{dt}{t_i(T)} \quad (8)$$

Where $t_i(T)$ is the temperature dependence of induction time, described by equation (6). When the value of dimensionless time $\bar{t}$ becomes equal to one, the upper limit of t in integral (8) is considered as induction time. However, for the curing stage, several non-isothermal curing models have been reported [7-9]. The simplest form is a quasi-isothermal model in which rate constant (k) at every time step is calculated at the mean temperature between two successive time steps, i.e.,

$$\alpha_i = \alpha_{i-1} + \int_{t_{i-1}}^{t_i} \left(\frac{d\alpha}{dt}\right)_{T=T_m} dt \quad (9)$$

Where

$$T_m = \frac{T_i + T_{i-1}}{2}$$

and

$$\frac{d\alpha}{dt} = \frac{kn(t-t_i)^{n-1}}{[1+k(t-t_i)^n]^2} \quad (10)$$

In a more rigorous method, Isayev and Deng [7] proposed a total differential approach given as:

$$\alpha_i = \alpha_{i-1} + \int_{T_{i-1}}^{T_i} \left(\frac{d\alpha}{dT}\right)_t dT + \int_{t_{i-1}}^{t_i} \left(\frac{d\alpha}{dt}\right)_T dt \quad (11)$$

Our numerical studies showed that none of these models are capable of considering the effect of temperature decrease on rate and state of cure. Since it is generally known that during the curing process the temperature at some steps (such as cooling stage) decreases thus, these models fail to predict accurately the state of cure. In order to overcome this controversy condition, two approaches are usually adopted. In the first approach, a curing kinetic equation in the form of $d\alpha/dt = k(T)f(\alpha)$ is used where $f(\alpha)$ for kamal and Sourour model (5) is expressed as:

$$\frac{d\alpha}{dt} = nk^{1/n}\alpha^{(n-1)/n}(1-\alpha)^{(n+1)/n} \quad (12)$$

Therefore, the state of cure can be computed by the solution of the ordinary differential equation (12).Since the right hand side of this equation is depended on $\alpha$, either a predictor-corrector technique should be used or the RHS of equation (12) must be evaluated at the pre-

vious time step to solve this nonlinear equation. On the other hand, since $\alpha$ at the start of calculation is equal to zero (initial condition) the RHS of equation (12) cannot be expressed in terms of $\alpha$ and k at the previous time step (i.e., $\alpha_{i-1}$ and $k_{i-1}$) otherwise, a zero profile is obtained for $\alpha$. In the other method proposed by Chan et al. (6), a reduced time approach is used to predict the state of cure. The numerical expression for the cumulative state of cure in this method based on Kamal and Sourour model [8] is given as:

$$(\frac{\alpha_i}{1-\alpha_i})^{1/n} = (\frac{\alpha_{i-1}}{1-\alpha_{i-1}})^{1/n} + (\int_{t_{i-1}}^{t_i} k^{1/n} dt) \quad (13)$$

As it can be seen, in this approach, the initial condition of $\alpha$ (i.e., $\alpha = 0$ at time = $t_i$) does not affect the calculation of the $\alpha$ at next time step. Therefore in the present work we have adopted this technique for the calculation of the state of cure during the simulation of the process. The non-isothermal rate of cure is also computed as:

$$(\frac{d\alpha}{dt})_{non-isothermal} = \frac{\alpha_i - \alpha_{i-1}}{\Delta t} \quad (14)$$

**Finite Element Working Equations**

The above described model is solved by the use of well-known standard Galerkin finite element method. Details of the application of the standard Galerkin method for the derivation of finite element working equations can be found in several textbooks and thus it is not repeated here (Reddy and Gartling [10]). The associated working equation to our problem is given as:

$$[M]\{\dot{T}\} + [K]\{T\} = \{F\} \quad (15)$$

Where [M], [K], and {F} are the mass matrix, stiffness matrix, and load vector, respectively defined as:

$$(M)_{ij} = \iiint \rho C_p \varphi_i \varphi_j dxdydz \quad (16)$$

$$(K)_{ij} = \iiint k(\frac{\partial \varphi_i}{\partial x}\frac{\partial \varphi_j}{\partial x} + \frac{\partial \varphi_i}{\partial y}\frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_i}{\partial z}\frac{\partial \varphi_j}{\partial z})dxdydz \quad (17)$$

$$\{F\}_i = \iiint \varphi_i \dot{Q} dxdydz + \int_\Gamma \varphi_i k(\frac{\partial T}{\partial x}n_x + \frac{\partial T}{\partial y}n_y + \frac{\partial T}{\partial z}n_z)d\Gamma \quad (18)$$

In these equations $\varphi$, and $\{T\}$, and $\{\dot{T}\}$ are the interpolation function, vector of unknowns (nodal temperature), and the first time derivative of the latter, respectively. In order to solve first order ordinary matrix differential equation (15) the implicit-$\theta$ time stepping scheme was used. The final form of the working equations in this scheme is given as:

$$([M]^a + \theta\Delta t[K]^a)\{T\}^{n+1} = ([M]^a - \Delta t[1-\theta][K]^a)\{T\}^n + (1-\theta)\{F\}^n + \theta\{F\}^{n+1})\Delta t \quad (19)$$

Where superscripts n and n+1 refer to the time $t_n$ and $t_{n+1}$, respectively. The superscript, a, also indicates a time level between $t_n$ and $t_{n+1}$ defined as:

$$\{T\}^a = (1-\theta)\{T\}^a + \theta\{T\}^{a+1} \quad (20)$$

In this work $\theta$ was selected to be equal to 1.

**Solution Algorithm**

Using isoparametric mapping, the working equations of the present scheme are cast into a local coordinate system. The members of the stiffness matrix and load vectors are computed for each element by a Gauss quadrature method. The resulting algebraic equations are assembled into global matrix and solved by a frontal solution algorithm [11] after imposing the appropriate set of boundary conditions. Due to the temperature dependent nature of the thermal conductivity and also the dependency of the heat generation rate per unit volume on rate of cure ($d\alpha/dt$ in equation 4), the set of equations are non-linear and thus an appropriate iterative technique is required to solve these equation. In the present work we have adopted Picard's iterative method for this purpose [12]. The following steps summarize the solution strategy based on the mentioned technique.

I. The entire domain of interest is first discretized into a mesh of finite element.
II. An initial estimate for temperature field is selected. The assembled working equations are solved using appropriate boundary and initial conditions without

heat generation rate (i.e., $\dot{Q}= 0$) and the temperature field is obtained.

III. The thermal conductivities are updated based on the calculated temperature field.

IV. The dimensionless induction time ($\bar{t}$ in equation 8) is evaluated for each element.

V. If dimensionless induction time ($\bar{t}$) is found to be grater than unity in an element then the state and rate of cure ($\alpha$ and $d\alpha/dt$, respectively) are computed using curing kinetic equations (13) and (14) for that element.

VI. The heat generation rate is computed based on the values of ($d\alpha/dt$) calculated at the previous step for those element in which the values of ($\bar{t}$) are greater than unity.

VII. The finite element working equations are re-solved with updated values of thermal conductivity (step III) and $\dot{Q}$(step VI).

VIII. Steps II-VII are repeated until a converged solution on temperature filed is obtained. The convergence criterion used in this work is based on the calculation of the second norm of the filed variable that is given by:

$$\sqrt{\frac{\sum_{j=1}^{N}\left|T_i^{r+1}-T_i^{r}\right|^2}{\sum_{j=1}^{N}\left|T_i^{r+1}\right|^2}} \le \delta \qquad (21)$$

Where N, r, and $\delta$ are the total number of nodes, the iteration number, and the convergence tolerance, respectively where the latter value is approximately $10^{-3}$.

IX. Time is then forwarded by a $\Delta t$ (i.e., $t \rightarrow t + \Delta t$) until the final time of the process is obtained; otherwise steps II-VIII are repeated for the new time value.

## RESULTS AND DISCUSSION

Based on the above described mathematical model and solution strategy, a computer code was written in Visual Basic to solve the heat transfer and curing problem in rubber. This code was used to simulate the vulcanization process of a 12-24 truck tyre with semi-rib tread pattern in mould. Figure 1 shows a half portion of this tyre. Due to symmetry and also replication of

**Figure 1.** A 3D CAD generated of the 12-24 truck tyre with semi rib tread pattern.

sequences of tread blocks in circumferential direction, only a 5° section of the tyre, as shown in Figure 2, was considered. The domain of the analysis was divided into 1216 eight noded brick elements with total number of nodes equal to 1762 (Figure 3). Pre-processing step was performed using Geostar software [13].

Density was measured using a density gradient column. A PL DSC system was used to determine the heat of reaction and heat capacity. Thermal conductivities of the materials and the parameters of the curing kinetic equation were also determined by the use of a Taurus

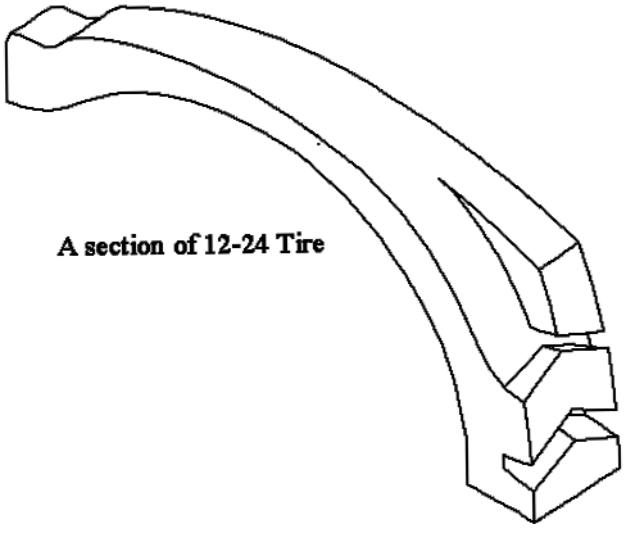


**Figure 2.** Domain of the analysis of 12-24 tyre.

**Table 1.** Parameters of kinetic models of different compounds in tyre section.

| Compound | $t_0$(s) | $T_0$(K) | $k_0$ | n | E (J/m) | $Q_\infty$ (J/m$^3$) |
|---|---|---|---|---|---|---|
| Tread | $4.104\times10^{-10}$ | 11713 | $1.4\times10^{25}$ | 2.91 | 260622 | $1.673\times10^{7}$ |
| Tread-base | $3.37\times10^{-10}$ | 11695 | $4.66\times10^{27}$ | 5.17 | 320644 | $3.062\times10^{7}$ |
| Bead-filler | $9.41\times10^{-10}$ | 11214 | $5.8\times10^{13}$ | 3.88 | 181717 | $4.114\times10^{7}$ |
| Bead | $4.6\times10^{-6}$ | 7223 | $1.19\times10^{22}$ | 2.64 | 225711 | $8.679\times10^{7}$ |
| Side-wall | $1.3\times10^{-7}$ | 9154 | $3.43\times10^{17}$ | 2.31 | 187076 | $3.034\times10^{7}$ |
| Plies | $1.085\times10^{-8}$ | 10205 | $4.85\times10^{24}$ | 2.84 | 250002 | $2.983\times10^{7}$ |

TCA 200 system and a Zwick ODR machine, respectively.

The physical and thermal properties of the different parts of the tyre and the parameters of the curing kinetic model are given in Tables 1 and 2, respectively. The problem was analyzed under transient condition. The simulation time for the vulcanization process in the mould was 4200s (70 min) with time step equal to 42s. Thus, 100 time steps were required to complete the simulation. First type boundary condition with time-varying temperature values given in Figure 4 was applied to the outer surface of the mould (tread surface) and inner surface of the tyre, respectively.

As mentioned earlier, our main goal was to study the effect of heat flow on the distribution of temperature and state of cure in circumferential direction. To show this, we have selected six points for our numerical investigation as shown in Figure 5. The first three points (N613, N101 and N118) are on the left side

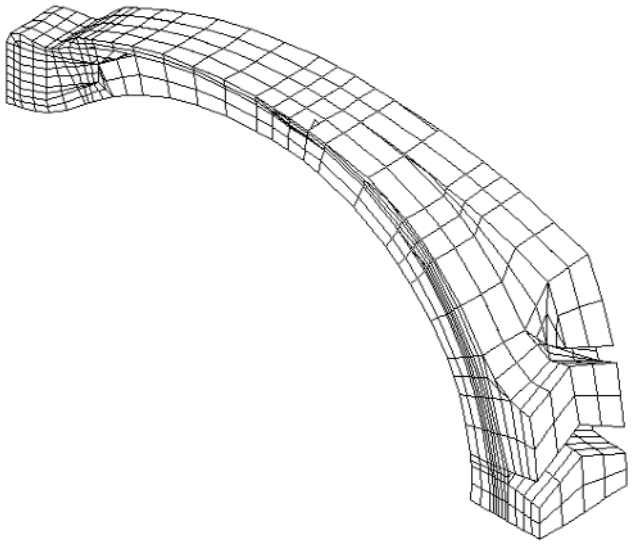

**Figure 3.** Finite element mesh.

**Table 2.** Thermal and physical properties of different compounds in tyre section.

| Compound | a | b | $C_p$ (J/kgK)* | ρ (kg/m$^3$) |
|---|---|---|---|---|
| Tread | 0.1612 | 0.0002 | 717 | 1127 |
| Tread-base | 0.1793 | 0.0003 | 641 | 1106 |
| Bead-filler | 0.3426 | 0.0006 | 664 | 1118 |
| Bead** | 50 | 0 | 460 | 7700 |
| Side-wall | 0.1941 | 0.0004 | 663 | 1134 |
| Plies | 0.07 | 0 | 615 | 1071 |

(*) Average values were used for heat capacity of compounds.

(**) Due to low volume fraction of rubber in bead section, thermal and physical properties of bead section was taken to be equal to bead wire properties.

**Table 3.** Specification of the selected points for results presenting.

| Level | Location | Left hand side node No.* | Right hand side node No.* |
|---|---|---|---|
| 1 | Ply | N613 | N679 |
| 2 | Lower tread | N101 | N287 |
| 3 | Upper tread | N118 | N290 |

(*) See Figure 5

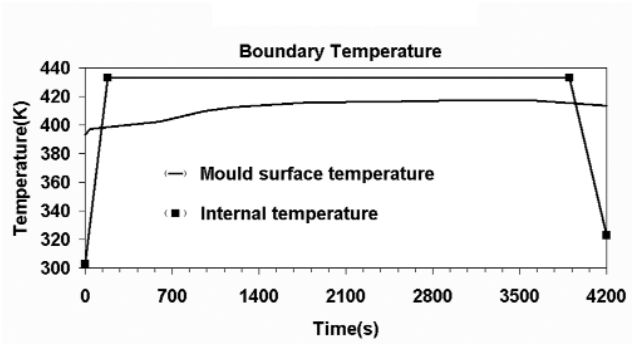


**Figure 4.** Variation of temperature boundary vs. time used in this simulation.

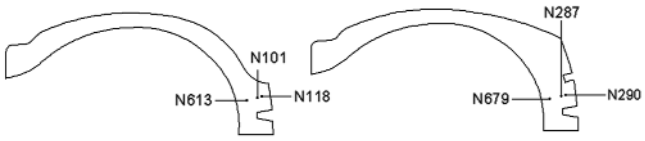


**Figure 5.** Locations of sampling points for results representing.

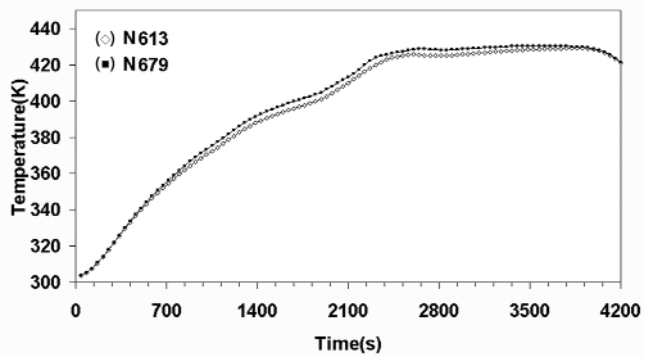


**Figure 6.** Variation of temperature vs. time in nodes located at level 1.

while the second three points (N679, N287 and N290) are on the right side of the model, respectively. These points are arranged in three levels as given in Table 3. As it can be seen, each level indicates a certain location in tyre. Level 1 (N613, N679) is located in ply, level 2 (N101, N287) is located in lower tread part, and level 3 (N118, N290) is located in upper tread part.

Figures 6-8 show the variations of temperature with time for nodes located in level 1, 2, and 3, respectively. Also the variations of state of cure for these levels are shown in Figures 9-11, respectively. In level 1, where

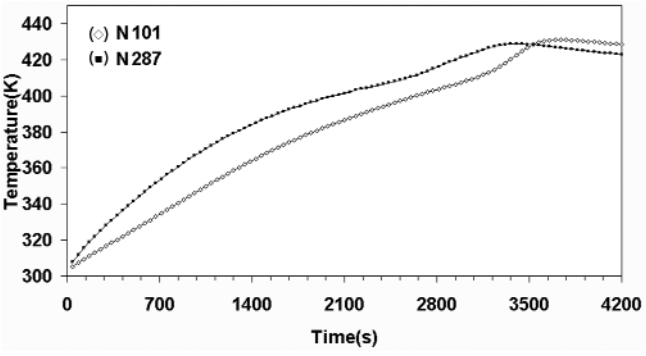


**Figure 7.** Variation of temperature vs. time in nodes located at level 2.

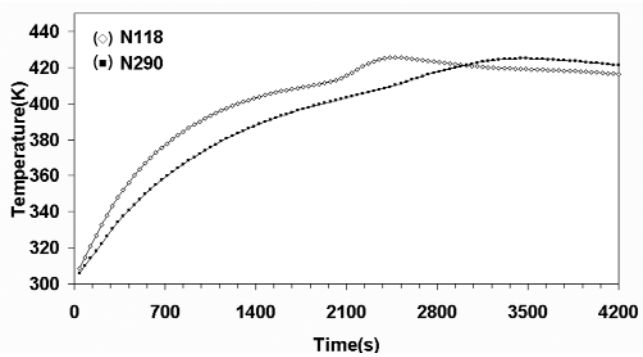


**Figure 8.** Variation of temperature vs. time in nodes located at level 3.

the nodes (N613 and N 679) are located in ply, minor differences in both temperature and state of cure profiles are observed (Figures 6 and 9). This is due to the very negligible variations in tyre geometry in circumferential direction. However, when we move from ply to tread in radial direction (from level 1 to level 2 and 3), the variation in tyre geometry in circumferential direction becomes more prominent. Therefore the difference in profiles of temperature and state of cure for nodes N101 and N287 (level 2) and N118 and N287 (level 3) become greater as shown in Figures 7 and 10 and Figures 8 and 11, respectively. On the other hand, since the thickness of model of the tyre on the left side is lower than right side, the rate of curing for those nodes located on the left side (Figure 5a) is greater than nodes located on the right side (Figure 5b). Also nodes on left side have greater temperature values than nodes on right side.

In order to check the accuracy of the predicted results, the variation of temperature for two points

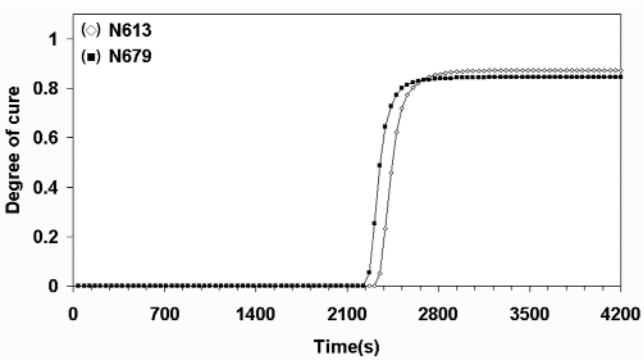


**Figure 9.** Variation of state of cure vs. time in nodes located at level 1.

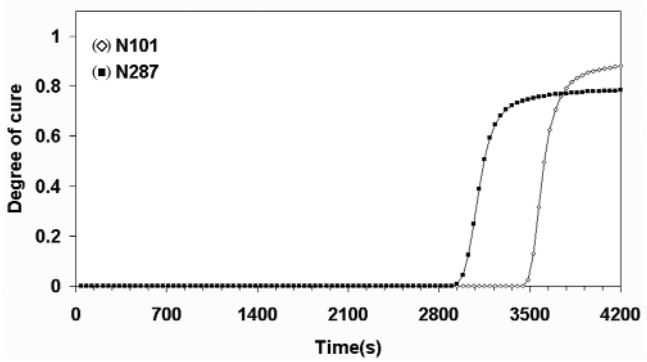


**Figure 10.** Variation of state of cure vs. time in nodes located at level 2.

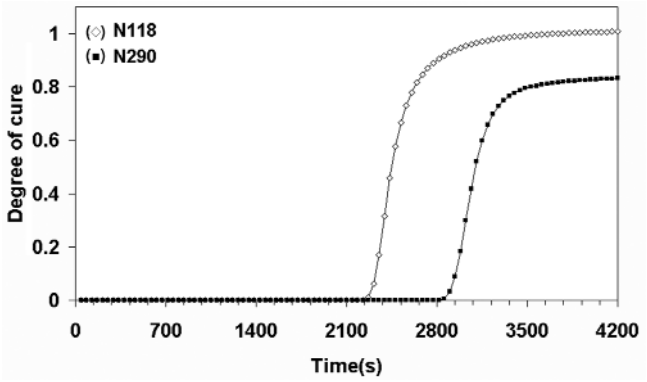


**Figure 11.** Variation of state of cure vs. time in nodes located at level 3.

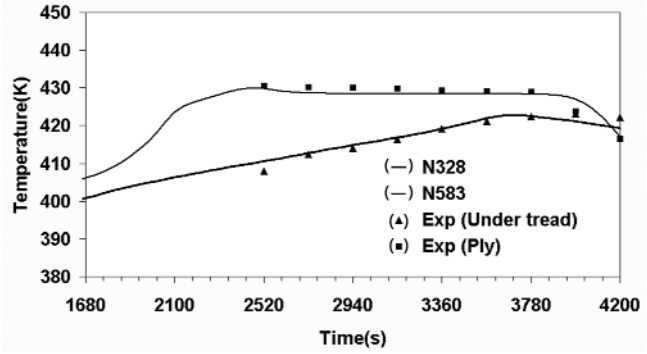


**Figure 12.** Variation of temperature vs. time in measured points.

inside a tyre, located at ply and upper tread sections, were compared with experimentally measured data. This comparison is shown in Figure 12, which confirms the validity and applicability of the developed finite element model and code.

## CONCLUSION

A three dimensional nonlinear finite element model was developed for the simulation of the curing process of rubbery materials in moulds. A computer code was also developed based on this model. This computer model was used to simulate the curing process of a 12-24 truck tyre with semi-rib tread pattern. Results of simulation showed that due to the variation of the tread pattern geometry in circumferential direction and also higher thickness of tread section compared to passenger tyres; the heat flow in this direction cannot be neglected. Therefore a three dimensional model must be used to predict accurately the temperature field and state of cure in truck tyres.

The applicability of the developed model has also been verified by comparing the calculated temperature profile with experimental data. This model can be used for the optimization of curing cycles and thus reducing the time and expenditure, which are normally required to find the optimum curing specifications in tyre industries. The remaining challenge in this area of research is to simulate accurately the post curing stage that needs precise determination of heat transfer coefficient. This is not a simple task and in spite of existing numerical schemes for the inclusion of convection boundary condition, there is no robust technique to find out this parameter for simulation of tyre curing process. We aim to extend our work to develop reliable methods for evaluating of heat transfer coefficient in post curing step.

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