Fuzzy Real-Time Optimization of the Tennessee Eastman Challenge Process

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ABSTRACT: *A Real-Time Optimization (RTO) strategy incorporating the fuzzy sets theory is developed, where the problem constraints obtained from process considerations are treated in fuzzy environment. Furthermore, the objective function is penalized by a fuzzified form of the key process constraints. To enable using conventional optimization techniques, the resulting fuzzy optimization problem is then reformulated into a crisp programming problem. The crisp programming problem is solved using both Sequential Quadratic Programming (SQP) and Heuristic Random Optimization (HRO) techniques for comparison purposes. The proposed fuzzy RTO strategy is demonstrated via the Tennessee Eastman benchmark process, and is also compared with a crisp RTO strategy from the literature. Remarkable economical improvement is found over the crisp RTO. In spite of the fuzzified constraints, the proposed strategy yields smooth operation of the process, while maintaining the product quality within the acceptable range.*

KEY WORDS: *Process control, Real-time optimization, Fuzzy logic, Tennessee Eastman process.*

INTRODUCTION

The ever increasing competition, high cost and limitation of energy resources along with tough environmental regulations have motivated manufacturing companies and academia to cooperate on lowering the production costs. As a result, the Real-Time Optimization (RTO) of processes has gained considerable attention over the past few years. Generally, the RTO refers to any control system designed for economic optimization of processes [1]. It improves process economics by successively moving the operating conditions toward the plant optimum, in spite of model uncertainties [2].

The implementation of an RTO system on a chemical plant may be subject to various constraints. The constraints may be upper and lower limits for some process variables that if violated, the plant may undergo difficulties concerning safe and stable operation of equipment, product quality, or even shutdown. However, there would not be crisp criteria for the violation of the constraints. In other words, it is the *extent* to which a constraint is violated that affects the process operation rather than exceeding a specified value. This supports the fact that real-world situations are not so definite, and decision-making in such circumstances could be rather subjective [3]. The idea of fuzzy sets proposed by *Zadeh* [4] serves to address the aforementioned issue.

On the other hand, it is likely that the optimum points coincide with the problem constraints. Therefore,

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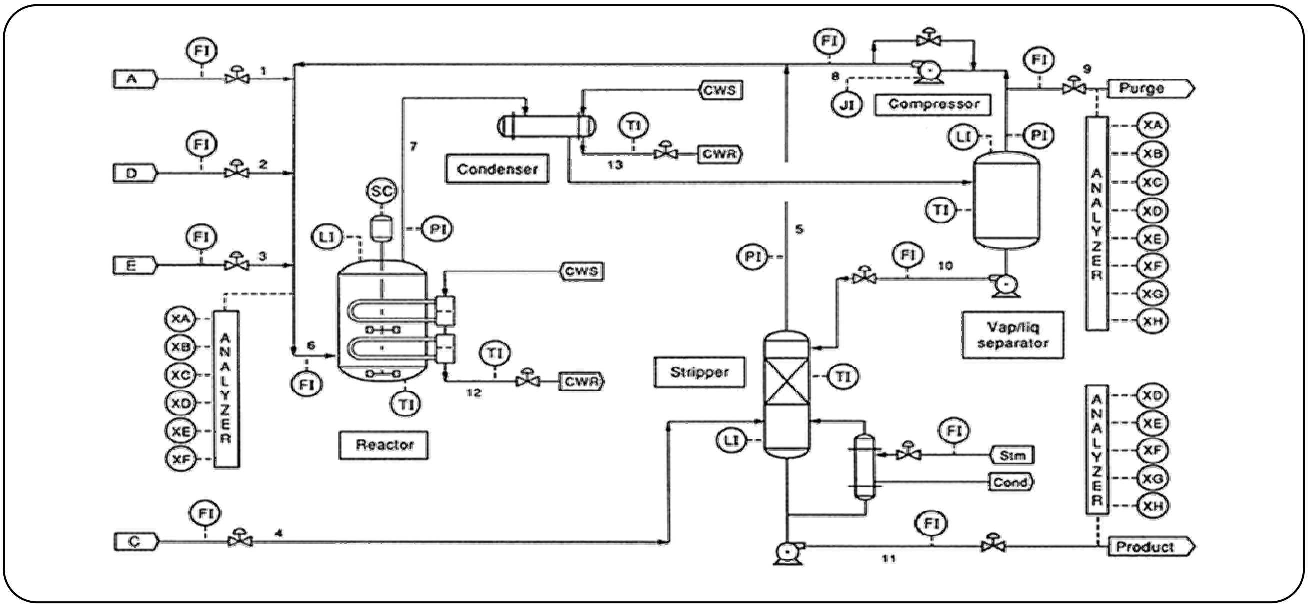

Fig. 1: Flowsheet of the Tennessee Eastman process.

the incorporation of the fuzzy logic into the optimization problems could potentially improve the optimization results. This is the basic idea that motivates the use of fuzzy decision making in the RTO systems.

To date, several studies have been focused on the concept and application of RTO in chemical processes. *Duvall & Riggs* [5] developed an RTO strategy for the Tennessee Eastman (TE) process proposed by [6]. The RTO strategy effectively decreased the process operating cost from the base case value of 170 ($\$ h^{-1}$) [6] to 120 ($\$ h^{-1}$). However, it was still higher than the optimum value of 114 ($\$ h^{-1}$) reported by *Ricker* [7], who used the exact process model to solve the optimization problem in an off-line manner. The value reported by *Ricker* [7] was considered the minimum value that can be achieved via an on-line optimization system. Later on, *Golshan et al.* [8] obtained the minimum achievable operating cost in an on-line manner through their RTO algorithm. However, all these results have been obtained according to a crisp interpretation of the process constraints, which may be rather subjective in nature.

In this paper, the fuzzy decision-making is incorporated into the crisp RTO algorithm proposed by *Golshan et al.* [8]. The TE process has been chosen as the benchmark to compare the results against the crisp RTO algorithm. In fact, the two studies follow a similar procedure, unless the optimization problem has been fuzzified in this work. The fuzzy RTO is demonstrated through minimization of the plant operating cost where remarkable improvement is achieved over the crisp RTO. The rest of the paper is organized as follows. In next section, the TE process is briefly described. This is followed by control structure section which explains the control structure used to stabilize the process. Fuzzy real time optimization section details the fuzzy RTO algorithm. In results and discussion section, the results of the algorithm are shown and compared against the work of *Golshan et al.* [8] so as to demonstrate the effectiveness of the proposed strategy. The paper is concluded with summarizing the main points in the last section.

PROCESS DESCRIPTION

The TE process was introduced by *Downs & Vogel* [6] to serve as a complex control benchmark for educational and research purposes. It consists of five major units including a two-phase reactor, a product condenser, a separator, a stripper, and a recycle compressor. Fig. 1 illustrates the schematic flow and instrumentation diagram of the process.

The following exothermic irreversible reactions take place in the reactor in the presence of a nonvolatile catalyst drops dissolved in the liquid phase.

$$A(g)+C(g)+D(g) \longrightarrow G(liq) \qquad \text{Product 1}$$

$$A(g)+C(g)+E(g) \longrightarrow H(liq) \qquad \text{Product 2}$$

$$A(g)+E(g) \longrightarrow F(liq) \qquad \text{Byproduct}$$

$$3D(g) \longrightarrow 2F(liq) \qquad \text{Byproduct}$$

Table 1a: Measured variables in the TE Plant.

Variable number	Variable name	Variable number	Variable name
1	A feed (stream1)	22	Separator cooling water outlet temperature
2	D feed (stream2)	23	Component A in stream 6
3	E feed (stream3)	24	Component B in stream 6
4	A and C feed (stream4)	25	Component C in stream 6
5	Recycle flow (stream8)	26	Component D in stream 6
6	Reactor feed rate (stream6)	27	Component E in stream 6
7	Reactor pressure	28	Component F in stream 6
8	Reactor level	29	Component A in stream 9
9	Reactor temperature	30	Component B in stream 9
10	Purge rate (stream9)	31	Component C in stream 9
11	Product separator temperature	32	Component D in stream 9
12	Product separator level	33	Component E in stream 9
13	Product separator pressure	34	Component F in stream 9
14	Product separator underflow(stream10)	35	Component G in stream 9
15	Stripper level	36	Component H in stream 9
16	Stripper pressure	37	Component D in stream 11
17	Stripper underflow (stream 11)	38	Component E in stream 11
18	Stripper temperature	39	Component F in stream 11
19	Stripper steam flow	40	Component G in stream 11
20	Compressor work	41	Component H in stream 11
21	Reactor cooling water outlet temperature		

where G and H are the desired products; B is an inert; F is a byproduct. These components together with the reactants (A, C, D, and E) make up the total of eight components existing in the process. The whole un-reacted gases get out of the reactor along with the products. This is due to the fact that the reactor has no effluent stream other than the gas outlet (Fig. 1). The partial condenser makes the reactor effluent a two-phase stream which is then separated into gas and liquid streams in the flash separator. The gas stream is pressurized by the compressor and recycled to the reactor. A purge stream is provided to avoid accumulation of the non-condensable component B, excess reactants, and the byproduct F. The separator bottom stream goes to the stripper where the reactants D and E are separated and recycled back to the reactor. The stripper underflow is the product of the plant containing mainly G and H.

Downs & Vogel [6] offered an intentionally obscure FORTRAN code to simulate the process and asked the research community not to modify the code for the sake of meaningful comparisons. The code provides 12 inputs (as manipulating variables) and 41 outputs (as measurements) corrupted by zero mean white noise. The measurements and manipulating variables are listed in Tables 1a and 1b, respectively.

CONTROL STRUCTURE

A possible control structure for the TE process should be able to effectively maintain the main process variables (i.e., the production rate and product quality defined in terms of G/H ratio) during normal operations and upsets [6]. Moreover, it should prevent the system from violating the safety and process constraints as described by *Downs & Vogel* [6].

McAvoy & Ye [9] proposed a control structure for the TE process. It consists of single-input-single-output controllers with most of them being cascade loops. This structure, while featured with PI controllers, was claimed to be appropriate for upper-level control strategies such as online optimization, as it is successfully implemented by *Golshan et al.* [8,10] in their RTO studies. To allow meaningful comparisons with the foregoing studies, the same control structure with PI controllers is used in this work.

Table 1b: Manipulating Variables in the TE Plant.

Variable Number	Variable name
1	D feed flow (stream 2)
2	E feed flow (stream 3)
3	A feed flow (stream 1)
4	A and C feed flow (stream 4)
5	Compressor recycle valve
6	Purge valve (stream 9)
7	Separator pot liquid flow (stream 10)
8	Stripper liquid product flow (stream 11)
9	Stripper steam valve
10	Reactor cooling water flow
11	Condenser cooling water flow
12	Agitator speed

Application of other control strategies such as model predictive control for set-point tracking is investigated by *Jockenhövel et al.* [11].

Table 2 provides the configuration and tunings of the loops whose setpoints were used in the fuzzy RTO. More details on the control structure can be found in [9].

FUZZY REAL-TIME OPTIMIZATION

As mentioned previously, the general procedure used in this study is similar to the one developed by *Golshan et al.* [8]. This procedure is summarized here for the sake of clarity. Then, the fuzzification phase will be described in the following subsections.

The RTO algorithm encompasses several parts, namely the nonlinear state space model of the TE process proposed by *Ricker & Lee* [12], an Extended Kalman Filter (EKF) used to update the model online, and an optimizer which is composed of the objective function, constrains, and an optimization method. Fig. 2 shows the schematic diagram of the algorithm. As shown in Fig. 2, the plant data are sent to the EKF where the model parameters and states are estimated online. Each time the optimizer is triggered, a set of initial guesses for the decision variables (setpoints) are sent to the optimizer where the updated model and the plant measurements are used to evaluate the objective function and constraints. Once the optimizer is converged, the optimum setpoints are sent to the plant through first order filters with time constants of 7 h. This is due to the fact that the plant may go unstable if the setpoints change rapidly [8].

The application of fuzzy logic in optimization algorithms is appreciated in recent studies [13-14]. The fuzzy RTO system developed in this study benefits from a fuzzy framework in which the fulfillment of constraints are fuzzy variables qualifying the extent to which the constraints are satisfied. In this framework, the objective function is modeled in fuzzy environment as well. Then, the problem is reformulated to be solved via a crisp optimization method. The fuzzy RTO algorithm and its constituting elements are detailed in the next subsections.

Nonlinear process model

Ricker & Lee [12] developed a nonlinear mechanistic model for the TE process based solely on the material balance of the process. The model was aimed at capturing the key characteristics of the process without including unnecessary details. To do this, they added PI temperature-control loops so that the reactor and separator temperatures can be converted from *dependent* variables to independent variables. Thus, the two variables may be input to the model rather than be model outputs, removing the need for heat calculations. The nonlinear model is expressed in the following general form.

$$\dot{\varphi} = f(\varphi, u, \theta) \ , \ y = g(\varphi, u, \theta) \qquad (1)$$

where φ, u, θ, y are states, inputs, parameters, and outputs of the model, respectively. The model consists of 10 inputs, 26 states, 15 parameters, and 23 outputs which are provided in [12]. The state variables are the molar holdups of the eight components calculated for different parts of the process.

The model updating required for the optimization algorithm (Fig. 2) is accomplished using an Extended Kalman Filter (EKF) developed by *Ricker & Lee* [12]. The EKF uses the plant measurements for continuous estimation of the states and parameters. The model outputs are then obtained from the EKF using the estimated states and parameters. It is worth noting that the efficiency of the RTO algorithm severely depends on the validity of the model states and parameters obtained from the EKF. This is addressed in [8]

Table 2: Configuration and tunings of the PI controllers whose setpoints are used in the fuzzy RTO.

Controlled variables	Manipulated variables	K_c	τ_I (min.)
Reactor level	E-Feed Flow Set point	500 (kg / h / %)	200
Stripper level	Product Flow Set point	-0.5 (m^3/ h / %)	300
Separator level	Its Bottom Flow Set point	-2.5 (m^3/ h / %)	200
Product rate	C-Feed Flow Set point	3.57 (kmol/m^3)	40
Product G/H Ratio	D/E Feed ratio	0.05	40
Reactor pressure	A-Feed Flow Set point	-0.143 (kmol/h/kPa)	300
Reactor temperature	Reactor Cooling Set point	1.0	50
Stripper temperature	Steam Flow Set Point	10.0 (kg / h /º C)	10
Compressor power	Recycle Valve	0.08 (% / KW)	20
Mole fraction of B in purge	Purge Rate	-1.34 (kmol/h/%)	100
Compressor outlet Flow	Condenser Cooling Set Point	0.034 (º C/ kmol/h)	50
Mole fraction of E in Product	Stripper Temperature Set Point	-0.5 (º C / %)	100

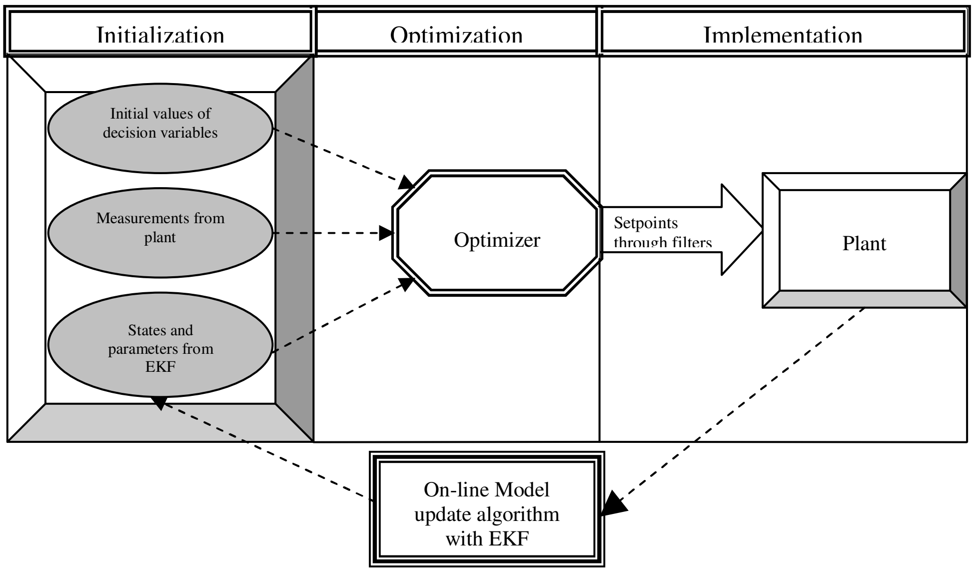

Fig. 2: Schematic diagram of the RTO algorithm.

where the EKF outputs are shown to follow the real process very well. A comprehensive description of the EKF combined with the nonlinear model can be found in [12]. The detailed description of the nonlinear model and EKF has also been presented in the two preceding work [8,10]. Since this study follows exactly the same model and updating method, they are not presented here.

Problem formulation

A fuzzy optimization problem can be stated in the following form [15-17]:

$$\begin{aligned}&\text{Minimize} \quad f(X)\\ &\text{Subject to} \quad g_j(X) \lesssim b_j \qquad j = 1,...,m \qquad (2)\\ &X_i \geq 0 \quad i = 1,...,n\end{aligned}$$

where f(X) is the objective function to be minimized; $g_j(X) \lessapprox b_j$ are the problem constraints that are generally nonlinear; X_i are the decision variables that must essentially be positive in this work. It is noted that equality constraints are converted to the above inequality constraints, as discussed later. Thus, g(X) in Eq. (2) includes the equality constraints as well. Assuming an aspiration level, *z*, for the optimum value of the objective function, Eq. (2) can be rewritten as [17]:

$$\begin{array}{ll} \text{Find X such that} & f(X) \gtrapprox z \\ g_j(X) \lessapprox b_j & j=1,...,m \\ X_i \geq 0 & i=1,...,n \end{array} \quad (3)$$

In Eqs. (2),(3), $\gtrapprox$ and $\lessapprox$ denote the fuzzified version of $\geq$ and $\leq$ that bring the linguistic interpretation. In order to make more similarity between fuzzy objective function inequality and fuzzy constraints, it is possible to multiply the objective function inequality by (-1). This forms the following fuzzy optimization problem:

$$\begin{array}{l} \text{Find X such that} \\ B(X) \lessapprox D \qquad X \geq 0 \\ B = \begin{pmatrix} -f(X) \\ g(X) \end{pmatrix}, \quad D = \begin{pmatrix} -z \\ b \end{pmatrix} \end{array} \quad (4)$$

Each of the (m+1) rows of the matrix B (see Eq. (3)) can be represented by a fuzzy set, whose membership function is $\mu_k(X)$. Then, the membership function corresponding to the optimality of a potential decision in Eq. (4) can be obtained as [17]:

$$\mu_D(X) = \min_k \{\mu_k(X)\}, \quad k=1,...,m+1 \quad (5)$$

$\mu_k(X)$ may be interpreted as the degree to which X satisfies the kth row of the fuzzy inequality of $B_k(X) \lessapprox D_k$. The decision maker chooses the X which maximizes $\mu_D(X)$:

$$\max_{X\geq 0} \min_k \{\mu_k(X)\} = \max_{X\geq 0} \mu_D(X) \quad k=1,2,...m+1 \quad (6)$$

In this work, linear-type membership functions are used, that is, 0 if the constraints are strongly violated and 1 if they are fully satisfied, with monotonic increase in between as:

$$\mu_k(X) = \begin{cases} 0 & \text{if } B_k(X) > D_k + P_k \\ \dfrac{D_k + P_k - B_k(X)}{P_k} & \text{if } D_k < B_k(X) \leq D_k + P_k \\ 1 & \text{if } B_k(X) \leq D_k \end{cases} \quad (7)$$

P_k is the element of the vector *P* that indicates the upper limit of acceptable violations of the constraints and objective function. It has been shown that substituting Eq. (7) into Eq. (6), with some assumptions and rearrangements, would lead to the following optimization problem [17]:

$$\max_{X\geq 0} \min_K \left(\frac{B_K(X) - D_k}{\mu_K} \right) \quad k=1,2,...m+1 \quad (8)$$

Introducing one new variable, λ, which indicates the satisfaction degree of the constraints, the problem becomes as follows:

$$\begin{array}{l} \max_{\lambda,X} \text{imize } \lambda \\ \text{Such that } \lambda P_k + B_k(X) \leq D_k + P_k \\ k=1,...,m+1 \quad X \geq 0 \end{array} \quad (9)$$

If the optimal solution to Eq. (9) is the vector [λ, X_0], X_0 is the maximizing solution of Eq. (6) [17]. The values of the vectors *P* and *D* used in this work are given in the Appendix.

Decision Variables

As mentioned before, the decentralized structure of *MeAvoy & Ye* [9] was used in this study. Therefore, the available manipulating variables are the setpoints of the master loops in the structure, or that of the single loop (the compressor power). It should be noted that since one of the setpoints corresponds to the product composition, which is to be constant for Mode 1 [6,8], it cannot be used as a decision variable. Furthermore, the agitator speed was set to its maximum value since it has a positive effect on the reaction rates by maximizing the range of cooling duties [7,8]. Thus, the remaining 10 setpoints are considered the decision variables which are the same as those used in [8].

Fuzzification of constraints

In order to ensure safe operation of the plant and maintaining key process variables as stipulated by the designer, some constraints must be placed on the optimization algorithm. The constraints include lower and upper bounds for the decision variables (setpoints), constituting the inequality constraints. Also, an equality constraint is required for the product flowrate variability whose valid range is ±5%, and its frequency should be in the range of 8-16 h^{-1} [6]. Another equality constraint is the steady-state material balance for the reactor. All the constraints considered in this study are the same as [8], therein more details are also provided.

As mentioned before, each equality constraint is split into two inequality constraints, indicating the left and right margins of acceptance for the original equality constraint. In crisp terms, it can be expressed as:

$$C_{eq,k} \le \alpha_k \quad \text{and} C_{eq,k} \ge -\beta_k \ , \ \alpha_k, \beta_k > 0 \qquad (10)$$

$$C_{eq,k} = 1 - \frac{PV_k}{PV_{k,desired}}$$

where $C_{eq,k}$ is the original equality constraint that should be ideally zero; α_k and β_k are the acceptable margins of deviation from zero; PV_k is the variable for which the constraint has been placed. This way, the equality constraints can be treated fuzzily along with the inequality ones.

The next step is to fuzzify the constraints. There are 10 decision variables, resulting in 20 inequality constraints as their lower and upper bounds. Furthermore, the two equality constraints add four additional inequality constraints (Eq. (10)). These constraints along with the objective function inequality make up a total of 25 inequality constraints, which are to be fuzzified. The constraints are fuzzified according to Eq. (7) where linear-type membership functions are employed. The parameters used in Eq. (7) are provided in the Appendix.

Fuzzification of objective function

The objective function is the plant operating cost provided by *Downs & Vogel* [6]. It represents the reactants and products losses in the purge stream, the reactants losses in the product stream, and the cost of steam consumption and compressor work. The operating cost is defined as follows:

$$C_{tot.} = F_9 \sum_{\substack{i=A \\ i \ne B}}^{H} C_{i,cst} x_{i,9} + \qquad (11)$$

$$F_{11} \sum_{i=D}^{F} C_{i,cst} x_{i,11} + 0.0536 W_{cmp} + 0.0318 F_{Steam}$$

where F_i is the molar flow of component i (kmol h^{-1}); $C_{i,cst}$ is the cost of component i ($ $kmol^{-1}$) provided by *Downs & Vogel* [6]; $x_{i,j}$ represents the mol fraction of component i in stream j; W_{cmp} and F_{steam} are the compressor power (kW) and stripper steam flowrate (kg h^{-1}), respectively.

The objective function is penalized through adding some fuzzified constraints. The constraints include only the 10 original inequality plus the two equality constraints obtained from the process considerations, as given in Table A3 of the Appendix. The resulting fuzzified objective function is as follows:

$$OF_p = \eta_1 OF_n - \left(\eta_2 \sum_{i=1}^{10} \mu_{C_{in,i}} + \eta_3 \mu_{C_{eq,1}} + \eta_4 \mu_{C_{eq,2}} \right) \qquad (12)$$

$$OF_n = \frac{C_{tot.} - C_{tot.min}}{C_{tot.max} - C_{tot.min}}$$

$$\eta_1 = 10, \ \ \eta_2 = 1, \ \ \eta_3 = 50, \ \ \eta_4 = 1$$

where OF_p and OF_n are the penalized and normalized objective functions, respectively; η_i are the weight factors; μ_{cin} and μ_{ceq} are the membership functions for the inequality and equality constraints, respectively. The minimum and maximum operating costs ($C_{tot.min}$ and $C_{tot.max}$) were set to 20 ($ h^{-1}) and 500 ($ h^{-1}), respectively. The inequality constraints used in Eq. (12) are expressed in normalized form such that they are fully satisfied if they are negative:

$$C_{in,i} = \begin{cases} 1 - \dfrac{PV_i}{PV_{min,i}} & \text{(if lower bound)} \\ \dfrac{PV_i}{PV_{max,i}} - 1 & \text{(if upper bound)} \end{cases} \qquad (13)$$

where $C_{in,i}$ is the inequality constraint; PV_i is the variable for which the inequality constraint is defined. Therefore, the corresponding membership functions are defined monotonically as:

$$\mu_{C_{in,i}} = \begin{cases} 1 & C_{in,i} \le 0 \text{ (fully satisfied)} \\ \dfrac{d_i - C_{in,i}}{d_i} & 0 < C_{in,i} \le d_i \\ -100 & C_{in,i} > d_i \text{ (strongly violated)} \end{cases} \quad , i = 1, 2, ..., 10 \quad (14)$$

where $\mu_{cin,i}$ is the membership function for the ith inequality constraint; d_i is the acceptable tolerance in normalized form provided in Table A3 of the Appendix. Also, the equality constraints used in Eq. (12) are fuzzified as:

$$\mu_{eq,1} = \begin{cases} 1 & \left|C_{eq1}\right| \le 0.005 \\ 10.5C_{eq1} + 1 & -0.1 < C_{eq1} < -0.005 \\ -10.5C_{eq1} + 1 & 0.005 < C_{eq1} < 0.1 \\ 0 & \text{Otherwise} \end{cases} \quad (15)$$

$$\mu_{eq,2} = \begin{cases} 1 & \left|C_{eq2}\right| \le 0.005 \\ 200C_{eq2} + 1 & -0.01 < C_{eq2} < -0.005 \\ -10.5C_{eq2} + 1 & 0.005 < C_{eq2} < 0.1 \\ 0 & \text{Otherwise} \end{cases}$$

where C_{eq1} and C_{eq2} correspond to the reactor material balance and product flowrate, respectively. The settings of the above functions (Eqs. (12), (14), (15)) were obtained using ad hoc methods and were deemed to be appropriate through extensive simulations. The penalty terms in Eq. (12) affect the optimality of the objective function by increasing its value as the constraints are violated or not fully satisfied. Combining Eqs. (4), (9), (12), (14), and (15) constitutes the final optimization problem to be solved.

RESULTS AND DISCUSSION

In this section, the fuzzy RTO strategy is demonstrated via computer simulations. The simulations are performed on a PC with Intel Core 2 Duo 2.4GHz CPU and 4GB RAM.

Optimization Method

The fuzzy optimization problem described in the previous subsections can be solved by any crisp optimization methods [17]. In this study, the optimization problem was solved using two methods, namely Sequential Quadratic Programming (SQP) and Heuristic Random Optimization (HRO) proposed by *Li & Rhinehart* [18]. In effect, the HRO method was used to verify the results obtained from the SQP technique and ensure that the obtained results represent global optima. This is due to the fact that the SQP method, like other gradient-based optimization techniques, may be trapped in local optima as discussed in [10].

The execution frequency of the fuzzy RTO is chosen to be every 8 hours of the process operation, which is the same as in [8,10]. Also, no optimization is executed during the first 10 hours of operation when the process is allowed to reach steady state at the base case conditions as given in [6].

Simulation results

As mentioned previously, *Ricker* [7] obtained the minimum achievable operating cost of 114 ($\$ h^{-1}$) for the base case in an off-line manner. This result was obtained subject to crisp process constraints in the problem formulation. This minimum value was later achieved in an on-line manner through the RTO strategy developed by *Golshan et al.* [8]. The results from the present study are reported in Table 3, where the optimum values corresponding to the proposed fuzzy RTO, crisp RTO, and the off-line study in [7] are compared. Interestingly, the SQP and HRO techniques have resulted in the same values for the setpoints, which suggests that a global optimum is found. However, it is important to note that no mathematical guarantee of global optimality can be given with gradient-based and stochastic techniques. Such a guarantee can be provided with a deterministic global optimization algorithm [19], which is not within the scope of this article. In terms of the computational expense, the case with the HRO method is expected to be less efficient than with the SQP technique. It takes about 2.5 seconds for the HRO technique to find an optimum, whereas the SQP method requires only 0.2 second, that is 12 times faster. The higher computational demand by the HRO method is a result of a random search scheme.

From Table 3, the reduction in the operating cost by the fuzzy RTO is 10% compared to the crisp RTO, and 40% compared to the base case value obtained by [6]. The constraint fuzzification results in a reactor level of 61.7% instead of the crisp lower limit of 65% suggested by *Ricker* [7]. This means that economical operation of the process favors a lower reactor level. On the contrary, a slightly higher reactor pressure than the upper limit of 2800 kPa [7] is found by the fuzzy RTO. This indicates that a higher reactor pressure yields a more economical operation. Note that the obtained reactor level and reactor pressure are still within their normal operating ranges defined by *Downs & Vogel* [6]. Another process variable that is noticeably affected by the fuzzification procedure is the compressor work, which is decreased from 277 kW [8,10] to 271 kW in the present study. It is evident from Eq. (11) that the compressor work has a major effect on the operating cost.

The cost functions versus time obtained during the fuzzy RTO and crisp RTO are illustrated in Fig. 3. The final steady state operating costs are those reported in Table 3. In both cases, the operating cost is seen to undergo large fluctuations before reaching its steady-state

Table 3: Comparison of the fuzzy optimization results with the base case values, crisp RTO, and off-line study in [7].

Decision variable	Base case value [6]	Ricker's [7] results	Crisp RTO by HRO and SQP [8,10]	Fuzzy RTO by HRO and SQP
Reactor level (%)	75	65	65	61.70
Stripper Level (%)	50	50	50	42.48
Separator Level (%)	50	50	51.9	50.00
Product Flow ($m^3 h^{-1}$)	22.949	22.89	22.949	21.4
Reactor Press.(kPa)	2705	2800	2799.9	2804.68
Reactor Temp (˚C)	120.4	122.9	123.3	124.34
Compress. Power (KW)	341.43	278.9	277	271.22
B in Purge Flow (%)	13.823	21.83	21.8	21.83
Recycle flow (kmol/h)	1201	1437	1437.5	1437
E in Product (%)	0.83570	0.58	0.56	0.381
Cost Function ($\$ h^{-1}$)	170.6	114.31	114.2	102.91

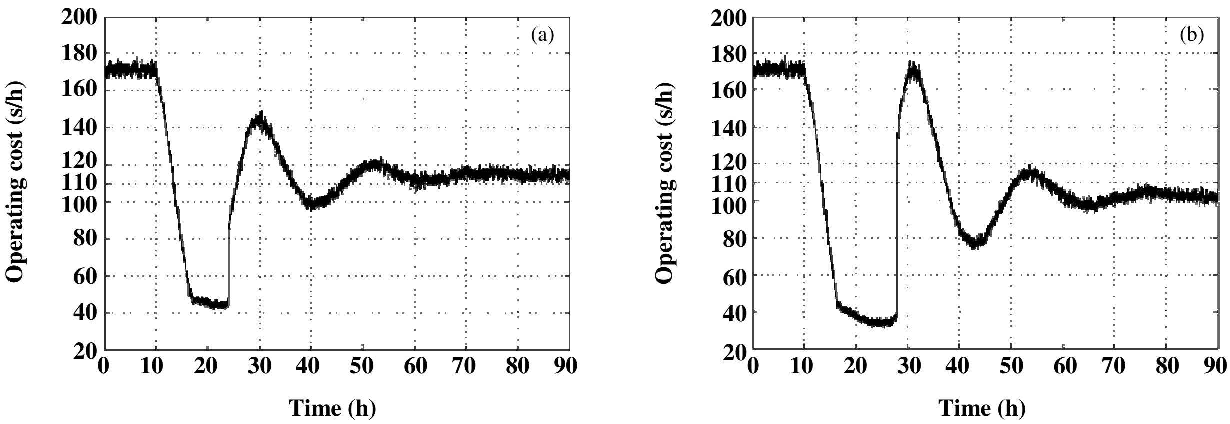

Fig. 3: Cost functions obtained by (a) crisp RTO, and (b) fuzzy RTO.

value (Table 3). This is due to the setpoint changes resulting from successive executions of the RTO algorithm.

The key process variables whose setpoints are determined using the fuzzy RTO are illustrated in Fig. 4. The steady-state values of these process variables are given in Table 3, and discussed earlier.

It is critical for an optimization algorithm to satisfy key process constraints such as product quality. Fig. 5 depicts the product composition, G mol (%), during the execution of the fuzzy RTO. The G mol (%) satisfies the ±5% product variability described by *Downs & Vogel* [6].

CONCLUSIONS

A real-time optimization algorithm incorporating fuzzy logic has been developed and implemented on the Tennessee Eastman benchmark process. The RTO builds upon the algorithm developed by *Golshan et al.* [8]. However, the constraints and objective function have been fuzzified in order to widen the search region while ensuring a feasible solution. The resulting optimization problem has been solved using a gradient-based technique (SQP) and also a heuristic search method (HRO). The conformity of the results from both methods suggests global optimality of the solution. Nevertheless, the HRO

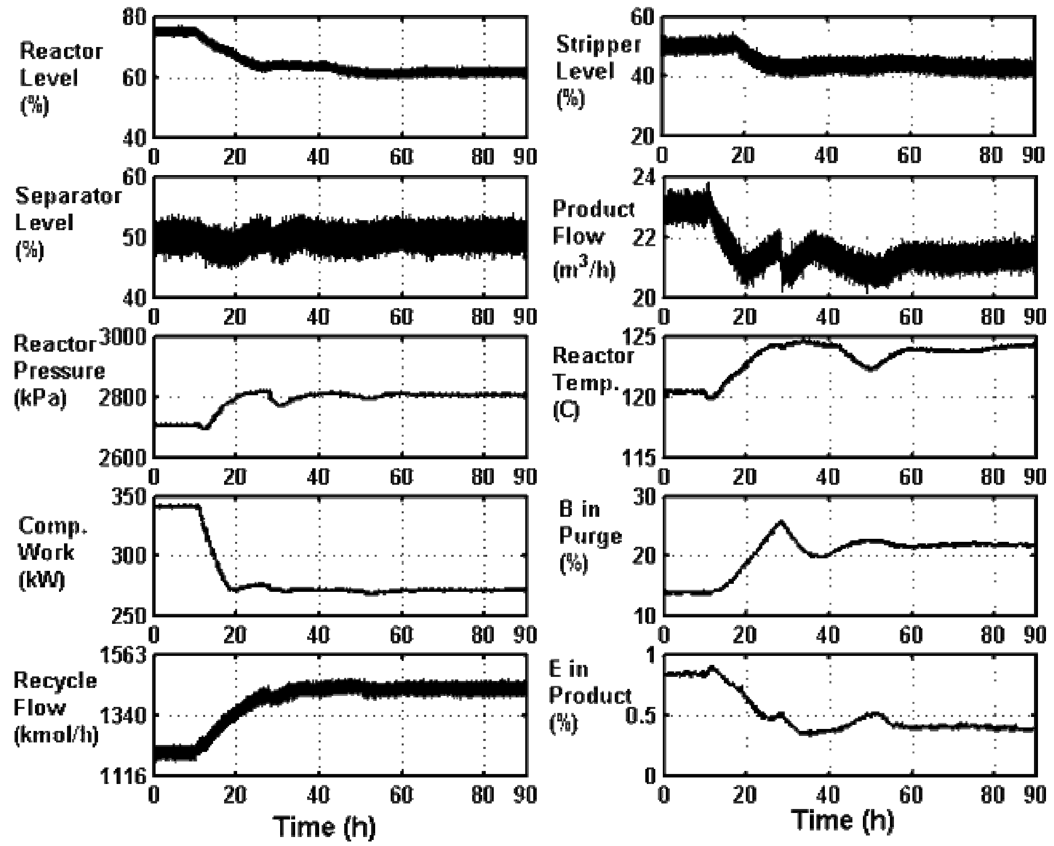

Fig. 4: Key process variables whose setpoints are obtained by the fuzzy RTO.

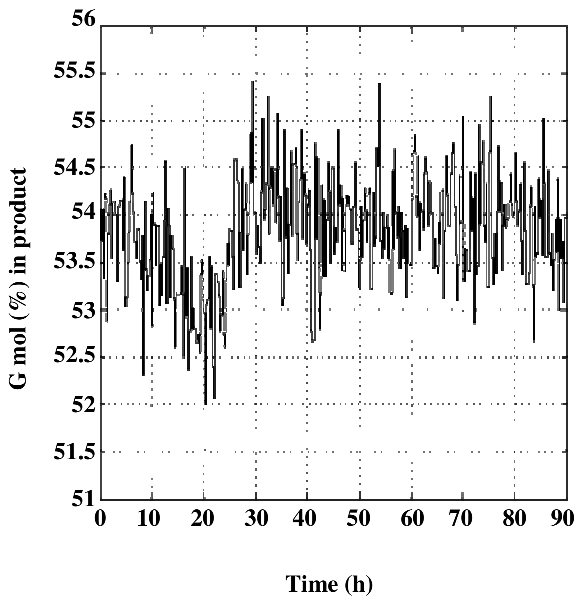

Fig. 5: Variations of G mol (%) in the product stream.

method is found to be computationally more demanding. The fuzzy RTO has been shown to improve the plant economics considerably as compared to the crisp RTO, while maintaining a safe operation and the desired product specification.

Nomenclature

F_i	Molar flow of component i, kmol h^{-1}
$C_{i,cst}$	Cost of component i, \$ $kmol^{-1}$
$x_{i,j}$	Mol fraction of component i in stream j
W_{cmp}	Compressor power, W
F_{steam}	Stripper steam flowrate, kg h^{-1}
C_{tot}	Operating cost, \$ h^{-1}
u	Model input
y	Model output
X	Decision variable
z	Aspiration level
φ	State variables

Appendix. Fuzzification parameters

Table A1: The D vector.

Type of constraint	Element index	Description	Value in the vector D
Process consideration	1	Objective function aspiration level (*z*)	100 ($ h^{-1})
(Inequality constrains)	2	Max. reactor pressure	2800 (kPa)
	3	Min. reactor level	65 (%)
	4	Max. reactor level	80 (%)
	5	Max. reactor temperature	125 (°C)
	6	Min. compressor power	277 (kW)
	7	Min. E(%) in Product	0.56 (%)
	8	Min. stripper level	40 (%)
	9	Max. stripper level	80 (%)
	10	Min. separator level	40 (%)
	11	Max. separator level	80 (%)
Nonnegative decision variables	12	Reactor level	0
	13	Stripper level	0
	14	Separator level	0
	15	Product flow	0
	16	Reactor pressure	0
	17	Reactor temperature	0
	18	Compressor power	0
	19	B(%) in purge	0
	20	Recycle flow	0
	21	E(%) in Product	0
Process considerations	22	Reactor material balance (negative margin)	0.01
(Equality constrains)	23	Product flowrate (left margin)	22.03 (m^3 h^{-1})
	24	Reactor material balance (positive margin)	0.01
	25	Product flowrate (right margin)	23.06 (m^3 h^{-1})

Table A2: The P vector. Any constraint violations beyond these margins make the corresponding membership function zero.

Type of constraint	Element index	Description	Acceptable violation
Process consideration	1	Objective function	50 ($\$$ h^{-1})
(Inequality constrains)	2	Max. reactor pressure	30 (kPa)
	3	Min. reactor level	15 (%)
	4	Max. reactor level	2 (%)
	5	Max. reactor temperature	1.25 (°C)
	6	Min. compressor power	37 (kW)
	7	Min. E(%) in Product	0.21 (%)
	8	Min. stripper level	10 (%)
	9	Max. stripper level	4 (%)
	10	Min. separator level	10 (%)
	11	Max. separator level	4 (%)
Nonnegative decision variables	12	Reactor level	0
	13	Stripper level	0
	14	Separator level	0
	15	Product flow	0
	16	Reactor pressure	0
	17	Reactor temperature	0
	18	Compressor power	0
	19	B(%) in purge	0
	20	Recycle flow	0
	21	E(%) in Product	0
Process considerations	22	Reactor material balance (negative margin)	0.1
(Equality constrains)	23	Reactor material balance (positive margin)	0.1
	24	Product flowrate (max acceptable)	0.23 (m^3 h^{-1})
	25	Product flowrate (min acceptable)	2.3 (m^3 h^{-1})

Table A3: Constraints used in fuzzification of the objective function.

Constraint number	Description	d_i
Inequality constraints		
1	Max. reactor pressure	0.011
2	Min. reactor level	0.2
3	Max. reactor level	0.025
4	Max. reactor temperature	0.01
5	Min. compressor power	0.05
6	Min. E(%) in Product	0.15
7	Min. stripper level	0.25
8	Max. stripper level	0.05
9	Min. separator level	0.25
10	Max. separator level	0.05
Equality constraints		
11	Reactor material balance	
12	Product flowrate	

θ Model parameters

μ_i Membership function of i-th fuzzy set

α_k Right margin of equality constraint violation

β_k Left margin of equality constraint violation

η Weight factor in objective function

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