Fault-Tolerant Control of a Nonlinear Process with Input Constraints

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ABSTRACT: *A Fault-Tolerant Control (FTC) methodology has been presented for nonlinear processes being imposed by control input constraints. The proposed methodology uses a combination of Feedback Linearization and Model Predictive Control (FLMPC) schemes. The resulting constraints in the transformed process will be dependent on the actual evolving states, making their incorporation in the design context a non-trivial task. A feasible direction method has been integrated in the design procedure based on active set technique to resolve the challenging constraint–based FLMPC problem. The formulated FLMPC design method is utilized to develop a FTC scheme by providing a set of backup control configurations for a CSTR benchmark process. The successful performance of the proposed FTC methodology has been demonstrated via a category of common fault scenarios by exercising an arbitrary replacement of control configurations through a supervisor to maintain the CSTR operation at an unstable desired steady-state point.*

KEY WORDS: *Model predictive control, Feedback linearization, Fault tolerant control, Constraints, Feasible direction method.*

INTRODUCTION

Considerable effort has been devoted to development of Fault Tolerant Control (FTC) systems for chemical plants to minimize their susceptibility against potential failures in their operations and equipments. If no corrective action is accommodated to counteract the consequent degradation effects due to these occurring failures, serious impacts will ultimately influence the overall plant safety and productivity. Thus, it is crucially desirable to explore for FTC schemes to avoid disaster upon a fault occurrence and yet minimize plant performance against its invertible degradation, leading to smooth repair to nominal operation.

Over the last decade, FTC has been received a significant amount of attention. The proposed methods can be categorized within the robust and reconfiguration-based approaches. Robust control approach essentially relies to the robustness of the control strategy to tolerate

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faults which are being treated as disturbance for some prescribed degree. This passive approach, however, is not effective to deal with some faults which significantly erode the available control action, leading to closed-loop stability problem. Another motivating FTC approach attempts to reconfigure a process control structure upon detection of a fault in order to preserve closed-loop system stability and performance [1-6]. Apposed to the passive methods, the key elements of this approach include multiple control configurations with well-defined regions of closed-loop stability and a supervisor to enable switching between faulty and well-functioning control configurations. However, this approach is restricted to the assumption of availability of sufficient backup or redundant control configurations to be appropriately activated so as to preserve closed-loop stability at nominal equilibrium point. Therefore, the approach may exhibit some potential disadvantages. The main issue concerns the critical reliance of its successful operation upon the existence of adequate number of backup control configurations. At the time any failure occurrence, it is probable that the states of system to be out of stability regions. Therefore, no backup control configuration can be accommodated to maintain the stability of the closed-loop system. This has motivated the research work in this paper to address this issue by devising a FLMPC control strategy to design each backup control configuration. This can lead to a more reliable condition in which a supervisor is able to take a proper action following the observation of a failure occurrence. The proposed approach can provide different back up control configuration options without doubting about their stability considerations. Thereby, the supervisor can select the most suitable control configuration in the case of any probable failure to realize a FTC objective while being assured that any possible imposed restriction has already been managed via the FLMPC design scheme.

The rest of the paper has been configured into four sections. In Theoretical Section, the proposed FLMPC technique is presented to develop a constrained nonlinear control system. Next section addresses the stability analysis of the developed control system. The proposed FTC control methodology is implemented in Case Study Section on a continuous Stirred Tank Reactor (CSTR) benchmark to evaluate its performance. Finally concluding remarks are summarized in conclusions section.

THEORETICAL SECTION

This section is devoted to design and development of the proposed FLMPC controller for constrained nonlinear systems in the context of control input constraint.

Development of FLMPC Controller

Process description

Consider the class of continuous-time, single-input nonlinear process with input constraint, described by the following affine state-space model equations:

$$\begin{cases} \dot{x}(t) = f_k(x(t)) + g_k(x(t)) \cdot u_k(t) \\ y(t) = h(x(t)) \\ u_{k,min} \le u_k \le u_{k,max}, k \in \{1, 2, ..., l\} \end{cases} \tag{1}$$

Where $x(t) \in R^n$ denotes the vector of state variables and $u_k(t)$ indicates the constrained manipulated input associated with the kth control configuration being confined between $u_{k,min}$ and $u_{k,max}$ as the respective lower and upper constraints. $f_k(.)$ and $g_k(.)$ represent the vector field functions, constituting different configurations for different manipulated inputs belonging to each value of $k \in \{1, 2, ..., l\}$ and $h(.)$ is the output function. Functions $f_k(.)$, $g_k(.)$ and $h(.)$ are assumed to be sufficiently smooth on their domains of definition.

For simplicity of representation, the subscript k, indicating the kth control configuration, is neglected in the subsequent sections.

Full state feedback linearization

It is considered that the nonlinear process in Eq.(1) is observable and all the states measurements are available. The first step in the controller design involves the application of full state feedback linearization technique to introduce a new input variable *v* and a nonlinear transformation that uses state feedback to compute the original input u(.) as follows [7,8]:

$$u = \frac{v - \sum_{i=0}^{r=2} \beta_i L_f^i \varepsilon_1}{\beta_2 L_g L_f \varepsilon_1} \tag{2}$$

Where the relative degree of the system has been assumed to be r = 2 for sake of simplicity. The notion

$L_f^i h$ denotes a lie derivative of order i of a scalar function $h(.)$ with respect to the vector function $f_k(.)$. L_f, L_g and $\varepsilon=[\varepsilon_1,\varepsilon_2]^T$ show the parameters of the resulting linear state-space model, given by:

$$\begin{cases}\dot{\varepsilon}_1=\varepsilon_2=L_f\varepsilon_1, L_g\varepsilon_1=0\\ \dot{\varepsilon}_2=L_f\varepsilon_2+L_g\varepsilon_2.u, L_g\varepsilon_2\neq 0,\end{cases} \quad (3)$$
$$y=h(x)=\varepsilon_1$$

This linearization strategy makes the system described by Eq.(1) behaves linearly from the new input *v* to the output y, represented by:

$$\frac{y}{v}=\frac{1}{\beta_2 S^2+\beta_1 S+\beta_0} \quad (4)$$

Where β_0, β_1 and β_2 indicate tunable parameters that can be selected arbitrary to force the linearized system to be stable. It is of considerable importance to assess which systems can be input-output linearizable whose necessary and sufficient conditions for the existence of a feedback control law have already been presented in [9].

Substituting Eq.(2) in Eq.(3) yields the following discrete-time state-space based on a suitable sampling time:

$$\begin{cases}\varepsilon(k+1)=A.\varepsilon(k)+B.v(k)\\ y(k)=C.\varepsilon(k)\end{cases} \quad (5)$$

Where

$$A=\begin{bmatrix}0 & 1\\ \frac{-\beta_0}{\beta_2} & \frac{-\beta_1}{\beta_2}\end{bmatrix} \quad B=\begin{bmatrix}0\\ \frac{-\beta_1}{\beta_2}\end{bmatrix} \quad C=[1 \quad 0]$$

Model predictive control

The linearized system can be controlled through an MPC algorithm, formulating in discrete-time by solving an online finite horizon optimal control problem at each sample time k, being specified by the following objective function:

$$J=\sum_{j=1}^{p}\delta(j)[y(k+j\,|\,k)-w(k+j\,|\,k)]^2+\sum_{j=0}^{m-1}\gamma(j)\Delta v^2(k+j) \quad (6)$$

Where p and m indicate prediction and control horizons, respectively. δ, γ represent the respective vectors of output error and input-rate weights with appropriate dimensions. w(.) denotes the future trajectory.

Solution of the foregoing MPC problem leads to the following control decision vector at each time step:

$$V(k\,|\,k)=[v(k\,|\,k) \ \ldots \ v(k+m-1\,|\,k)]' \quad (7)$$

Therefore, a state feedback control law is obtained by implementing only the first calculated input i.e. *v*(k)= *v*(k | k), and then resolving the control problem at the next time step with new state measurements x(k+1). The actual input $u(k)$ is finally calculated from *v*(k) via the Feedback Control Law (FCL) described in Eq.(2).

Handling system input constraints

The optimization problem in Eq.(6) should be solved subject to system input constraints of the form:

$$v_{min}(k+j\,|\,k)\le v\ (k+j\,|\,k)\le v_{max}(k+j\,|\,k) \quad (8)$$
$$0\le j\le(m-1)$$

The input constraints in Eq.(8) can be determined by mapping the original system constraints on u(k) into the feedback linearized space using the nonlinear transformation relationship, given in Eq.(2), yielding :

$$v_{min}(k+j\,|\,k)=\underset{u}{Min}\begin{Bmatrix}\beta_2 L_g L_f h(x(k+j\,|\,k)).u\\ +\sum_{i=0}^{2}\beta_i L_f^i h(x(k+j\,|\,k))\end{Bmatrix} \quad (9)$$
$$v_{max}(k+j\,|\,k)=\underset{u}{Max}\begin{Bmatrix}\beta_2 L_g L_f h(x(k+j\,|\,k)).u\\ +\sum_{i=0}^{2}\beta_i L_f^i h(x(k+j\,|\,k))\end{Bmatrix}$$

Where $u_{min}\le u\le u_{max}$ and $x(k+j\,|\,k)$ represents an estimate of the future state vector $x(k+j)$ whose non-trivial solution in the control context has been addressed in [10].

A much simpler approach can be realized on the basis of using inputs calculated at the previous time step to approximate the future constraints, yielding [10]:

$$V(k\,|\,k-1)=[v(k\,|\,k-1) \ \ldots \ v(k+m-2\,|\,k-1) \ \ 0]' \quad (10)$$

This approach makes it feasible to calculate $u(k-1)$ from the first input of *V*(k-1|k-1) which is already available from the previous time step optimization. Meanwhile, the control sequence at the current time step in Eq.(10) can be estimated using the remaining inputs of *V*(k-1|k-1).

The current measurements x(k) can then be used to calculate the transformed state variable $\varepsilon(k)$ The result can be employed as initial to estimate predicted values of the transformed state variables with the input sequence $V(k \mid k-1)$ to yield:

$$\begin{cases} \hat{\varepsilon}(k+j \mid k) = A.\hat{\varepsilon}(k+j-1 \mid k) + B.v(k+j-1 \mid k-1) \\ \hat{\varepsilon}(k \mid k) = \varepsilon(k); 0 \le j \le (m-1) \end{cases} \tag{11}$$

Where $\hat{\varepsilon}(k+j \mid k)$ denote predicted state vector associated to linearized system. This enables the calculation of the upper and lower limits of system input constraints in Eq.(9) using the predicted state values.

The preceding procedure is repeated at the next time step with the input sequence $V(k \mid k)$ and the new measurement $x(k+1)$. The resulting linear MPC problem is said to be feasible [10] if there exists a feasible input sequence V (k | k), implicating that all the sequence elements remain within the input constraints in Eq.(8).

Solution of the constrained optimization problem using feasible directions method

Consider an optimization problem which is defined via solution of the following cost function (J) subject to the given set of constraints:

$$\begin{cases} J = \frac{1}{2}\Delta v'.H.\Delta v + E'.\Delta v + f \\ \text{s.t.} \quad A_c.\Delta v \le a \end{cases} \tag{12}$$

Where A_c and a represent a $n \times m$ matrix and a $n \times 1$ vector, respectively. Δv indicates the control input vector having a dimension of $m \times 1$.

The optimization problem can be solved using an active set method realized through feasible directions [11]. The key idea behind this method is to minimize the cost function (J) by moving from a feasible point to an improved feasible point until the optimum is reached.

For this purpose, the given constraint set in Eq.(12) is divided into active constraint set ($A_{c1}.\Delta v = a_1$) and inactive constraints set ($A_{c2}.\Delta v < a_2$). Then, cost function (J) can be minimized along an improved feasible direction (S_k) with a suitable step length (λ_k), giving:

$$\Delta v(k+1) = \Delta v(k) + \lambda_k.S_k \tag{13}$$

Via satisfying the following two conditions:

(i) $\nabla J(\Delta v(k)).S_k < 0$ indicating that the cost function is minimized along S_k, leads to the following selection:

$$S_k = -P.\nabla J(\Delta v(k)) \tag{14}$$

Where P is a positive definite matrix.

(ii) $A_{c1}.S_k \le 0$ to guarantee that the new point is feasible. To satisfying this condition, P can be defined as:

$$P = I - A'_{c1}\left(A_{c1}A'_{c1}\right)^{-1}A_{c1} \tag{15}$$

The value of λk can be computed from λk = Min(λmax, λopt) where λmax indicates the maximum possible λ to ensure the new point remain within feasible region, determined by:

$$\lambda_{max} = \text{Min}_j \frac{a_{2j} - A_{c2j} \cdot \Delta v_k}{A_{c2j} \cdot S_k} \tag{16}$$

Where A_{c2j} and a_{2j} denote the rows of the inactive constraint set and the bound set, respectively. λ_{opt} is the value of λ which minimizes the cost function via $dJ(\Delta v_{k+1})/d\lambda_k = 0$, leading to the following solution :

$$\lambda_{opt} = \frac{S'_K - H \cdot \Delta v_k + E' \cdot S_K}{S'_K - H \cdot S_K} \tag{17}$$

Fig. 1 shows the descriptive flowchart of the adopted method. As shown, solution to the optimization problem is terminated via 'maxiter' or η test observation. 'maxiter' defines the maximum number of iterations and η denotes a threshold parameter [11]. When S_k is found to be equal to zero and the active constraints set exists, the minimum is verified based on Kuhn-Tucker criterion. Therefore, Lagrange function is defined as $L = J - W'\left(A_{c1}\Delta v_k - a_1\right)$ where Lagrange multiplier (W) is computed by $dL/d\Delta v_k = 0$, giving :

$$\nabla J(\Delta v_k) - A'_{c1}.W = 0 \Rightarrow \tag{18}$$
$$W = (A_{c1}A'_{c1})^{-1}A_{c1}.\nabla J(\Delta v_k)$$

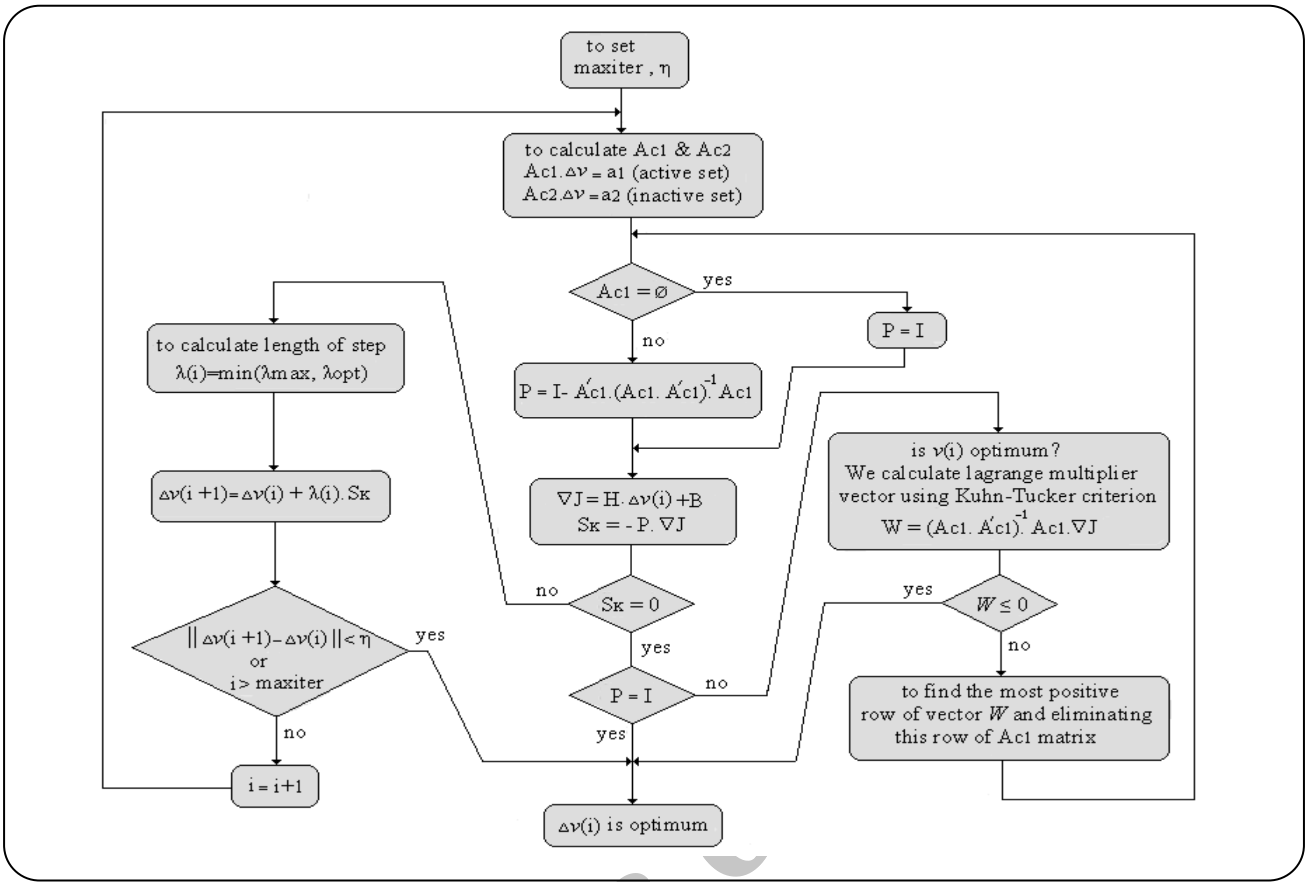

Fig. 1: Flowchart of Feasible Directions Method.

This criterion implies that Δv is minimum provided that coefficients W to be non-positive [11].

STABILITY ANALYSIS

It can be shown that global asymptotic stability is ensured under the following assumptions:

Assumption1: The nonlinear algebraic equation $u(k)= \varphi(x(k),v(k))$ has a unique solution for all $x(k) \in R^n$ and all $v(k) \in R$.

Assumption 1 can be held provided that the nonlinear system possesses a well defined relative degree throughout the state space.

Assumption 2: There exists a globally defined diffeomorphism $\varepsilon(k)=\psi(x(k))$ such that the nonlinear dynamic is transformed to the linear form.

This means that function ψ (.) should be invertible and inverse of ψ(.) to be sufficiently smooth.

Since the goal is to achieve full state stabilization, process output can selected arbitrary such that function ψ (.) to be diffeomorphism.

When following two conditions are satisfied, then there exist a diffeomorphism function $\psi(x(k))$ with relative degree *n* for system represented by Eq.(1).

I) the matrix $G(x) = [\ g(x),\ ad_f g\ (x),\ \dots\ ad_f^{n-1}g(x)\ ]$ to be full rank in Ω region that is in a neighborhood of operating point.

II) the set $\{g(x),\ ad_f g(x),\ \dots,\ ad_f^{n-2}g(x)\ \}$ to be involutive in Ω region.

Where $ad_f g(x)$ is defined by $ad_f g(x) = [g,g](x)$ and $[f,g](x)$ is expressed as $L_f g(x)-L_g f(x)$ and $ad_f^k g(x) = \left[f, ad_f^{k-1} g(x)\right]$.

Assumption3: internal dynamic of the process should be stable. This assumption is not relevant for the cases in which internal dynamic don't exist due to using the full state feedback linearization technique.

Assumption4: it is necessary that all the elements of input sequence $V(k|k-1)$ to be satisfied in (8) for all $k \geq 1$.

Theorem: when the above assumptions are satisfied and MPC problem is feasible for all $k \geq 1$, then $x(k) = 0$

is treated as a globally asymptotically stable fixed point of the closed-loop system comprising Eqs. (1), (2), (8) and (9).

Proof: since MPC problem is feasible in k=0, so MPC solution is feasible for all $k \geq 0$ by virtue of assumption 4. Supposing that the cost function is $\hat{J}(k)$ for the input sequence defined by V(k|k-1), the elements of this sequence, represented by $\hat{v}(k+j|k)$, together with the state variables $\hat{\varepsilon}(k+j|k)$ can be derived by Eq. (11).

Now, it is possible to show that $\hat{\varepsilon}(k+j|k)$ is equal to $\varepsilon(k+j|k-1)$. For this purpose, $\varepsilon(k|k-1)$ can be solved directly via Eq.(11) using $v(k-1|k-1)$ as the control input. On the other hand, $u(k-1|k-1)$ can uniquely be obtained through Eq.(2) on the basis of assumption 1 using the same control input. Having x(k) and ε(k) as already known via Eq(1) and assumption2, it can be concluded that $\varepsilon(k|k-1) = \hat{\varepsilon}(k|k)$. This consequently leads to $\varepsilon(k+j|k-1) = \hat{\varepsilon}(k+j|k)$ for all values of $j \geq 1$ and $v(k+j|k-1) = \hat{v}(k+j|k)$ for $\forall j \geq 0$. Assuming that the control horizon is taken to be equal to prediction horizon, the cost function $\hat{J}(k)$, corresponding to a Multi-Input, Multi-Output (MIMO) process, can be expressed as follows:

$$\hat{J}(k) = \sum_{j=0}^{m-1}[(\hat{\varepsilon}(k+j|k)'\delta\hat{\varepsilon}(k+j|k) + \gamma\Delta\hat{v}(k+j|k)^2] \Rightarrow \hat{J}(k) = \sum_{j=0}^{m-1}[(\varepsilon(k+j|k-1)'\delta\varepsilon(k+j|k-1) + \gamma\Delta v(k+j|k-1)^2] \quad (19)$$

Optimization at time k yields $J(k) \leq \hat{J}(k)$ .Thus, it can be written:

$$J(k) - J(k-1) \leq \hat{J}(k) - J(k-1) = -\varepsilon(k-1)'\delta\varepsilon(k-1) - \gamma\Delta v(k-1)^2 \quad (20)$$

The cost function J(k) is non-negative and deceasing with respect to time due to assumption of $\delta \geq 0, \gamma \geq 0$. This implies that the sequence J(k) will be asymptotically converged to zero, requiring that $\lim_{k\to\infty} v(k) = 0$. Whereas the linearized process in Eq.(11) is stable due to considering βi tunable parameters, $\lim_{k\to\infty} \varepsilon(k) = 0$ will be satisfied. Accordingly, it can be deducted that $\lim_{k\to\infty} x(k) = 0$ using assumption 2. This is mainly due to the fact that stability is preserved under diffeomorphism. This theorem can hence be generalized to every point using transformation of coordinates, i.e. $\lim_{k\to\infty} x(k) = x_s$.

Implementation of the Proposed Fault Tolerant Control

This section introduces the implementation of the proposed FTC scheme on a CSTR benchmark to explore its performances under two fault scenarios.

CASE STUDY

In this paper, a Continuous Stirred Tank Reactor (CSTR) is utilized to illustrate the design and implementation of the proposed FTC approach. Three parallel irreversible exothermic reactions are carried out in reactor as $A \xrightarrow{K_3} R$, $A \xrightarrow{K_2} U$, $A \xrightarrow{K_1} B$ where A is input reactant, B is the desired output and U, R represent undesired output. Input reactant A is fed to CSTR at flow rate F, molar concentration C_{A0} and temperature T_{A0}. A jacket has been considered to provide heat transfer to reactor [4]. The nonlinear differential equations of the reactor can be described as:

$$\begin{cases} \dfrac{dT}{dt} = \dfrac{F}{Vol}(T_{A0} - T) + \sum_{i=1}^{3}\dfrac{(-\Delta H_i)}{\rho.C_P}K_{i0}\exp(\dfrac{-E_i}{RT}).C_A + \dfrac{Q}{\rho.C_P.Vol} \\ \dfrac{dCA}{dt} = \dfrac{F}{Vol}(C_{A0} - C_A) - \sum_{i=1}^{3} K_{i0}\exp(\dfrac{-E_i}{RT}).C_A \\ |Q| \leq 2.7\times10^6 \,{}^{kj}/_{hr}, |C_{A0}| \leq 4 \,{}^{kmol}/_{m^3} \\ , |T_{A0}| \leq 100°k \end{cases} \quad (21)$$

Where, C_A, T, Q and Vol are inlet concentration, temperature of reactor , rate of heat input to reactor and volume of reactor, respectively. E_i, ΔH_i, K_{i0} for i =1, 2, 3 denote activation energies, the enthalpies and pre-exponential constant of three reaction, respectively. The parameters and operating conditions of the process have been summarized in Table 1.

Table 1: The parameters and nominal values of the process.

$F = 4.998$	m^3 / h
$Vol = 1.0$	M^3
$R = 8.314$	kJ/kmol . K
$T_{A0} = 300.0$	K
$C_{A0} = 4.0$	kmol / m^3
$\Delta H_1 = -5.0 \times 10^4$	kJ / kmol
$\Delta H_2 = -5.2 \times 10^4$	kJ / kmol
$\Delta H_3 = -5.4 \times 10^4$	kJ / kmol
$k_{10} = 3.0 \times 10^6$	h^{-1}
$k_{320} = 3.0 \times 10^5$	h^{-1}
$K_{30} = 3.0 \times 10^5$	h^{-1}
$E_1 = 5.0 \times 10^4$	kJ / kmol
$E_2 = 7.53 \times 10^4$	kJ / kmol
$E_3 = 7.53 \times 10^4$	kJ / kmol
$\rho = 1000.0$	kg / m^3
$c_\rho = 0.231$	kJ / kg . K
$T^s = 388.57$	K
$C^s_A{}^{= 3.59}$	kmol / m^3

The process with Q=0 has three equilibrium points in which two points are stable and the third point, specified by the equilibrium point (T^S, C_A^S)= (388.57°k, 3.5907 kmol/m³), is unstable. The aim of controller design is to stabilize reactor operation around the unstable point and in the presence of faults. For this purpose, Q, T_{A0}, C_{A0} are considered as the three manipulated inputs to independently realize three possible control configurations.

The design of control configurations

As discussed, each of the three inputs Q, T_{A0}, C_{A0} can be utilized to stabilize the process. Hence, three constrained controllers should be designed independently to realize each possible control configuration. When a failure occurs, the supervisor can immediately activate any of the already designed control configurations. The three control configurations have been introduced individually in the following sections. Having determined the candidate backup control configuration, the corresponding FLMPC controller can be derived individually. The supervisor mechanism can then switch between any of backup control configuration arbitrarily to maintain the closed-loop stability in the event of failure.

Q-control configuration

In this section, the process is controlled through q-input. Hence, the affine model of CSTR can be presented as:

$$\dot{x}(t) = f_{(1)}(x) + g_{(1)}(x).u_{(1)}(t), x(t) = \begin{bmatrix} T(t) \\ C_A(t) \end{bmatrix}$$

$$y = C_A(t), u_{(1)}(t) = Q, |Q| \le 2.7 \times 10^6 \text{ kJ/h}$$

$$f_{(1)}(x) = \begin{bmatrix} \frac{F}{Vol}(T_{A0} - T) + \sum_{i=1}^{3} \frac{(-\Delta H_i)}{\rho.C_P} K_{i0} \exp(\frac{-E_i}{RT}).C_A \\ \frac{F}{Vol}(C_{A0} - C_A) - \sum_{i=1}^{3} K_{i0} \exp(\frac{-E_i}{RT}).C_A \end{bmatrix}$$

$$g_{(1)}(x) = \begin{bmatrix} \frac{1}{\rho.C_P.Vol} \\ 0 \end{bmatrix}$$

The main objective is to control the output concentration $y=C_A(t)$ via full-state stabilization scheme. This choice yields a relative degree of $r = 2$.

T_{A0} -control configuration

For this configuration, the process is stabilized by T_{A0}-input. So, the CSTR affine model is described as:

$$\dot{x}(t) = f_{(2)}(x) + g_{(2)}(x).u_{(2)}(t), x(t) = \begin{bmatrix} T(t) \\ C_A(t) \end{bmatrix}$$

$$y = C_A(t), u_{(2)}(t) = T_{A0}, |T_{A0}| \le 100\,^\circ k$$

$f_{(2)}$, $g_{(2)}$ are derived similar to the previous section. $y=C_A(t)$ is chosen as the output, yielding a relative degree of r = 2.

C_{A0} -control configuration

In this control configuration, C_{A0} is used to stabilize of the process, resulting into the following affine model with a relative degree of r = 2:

$$\dot{x}(t) = f_{(3)}(x) + g_{(3)}(x).u_{(3)}(t)$$

$$y = T(t), u_{(3)}(t) = C_{A0}, |C_{A0}| \le 4\,\text{kmol/m}^3$$

$$f_{(3)}(x) = \begin{bmatrix} \frac{F}{Vol}(T_{A0} - T) + \sum_{i=1}^{3} \frac{(-\Delta H_i)}{\rho.C_P} K_{i0} \exp(\frac{-E_i}{RT}).C_A \\ + \frac{Q}{\rho.C_P.Vol} \\ -\sum_{i=1}^{3} K_{i0} \exp(\frac{-E_i}{RT}).C_A - \frac{F}{Vol} C_A \end{bmatrix}$$

$$g_{(3)}(x) = \begin{bmatrix} 0 \\ \frac{F}{Vol} \end{bmatrix}$$

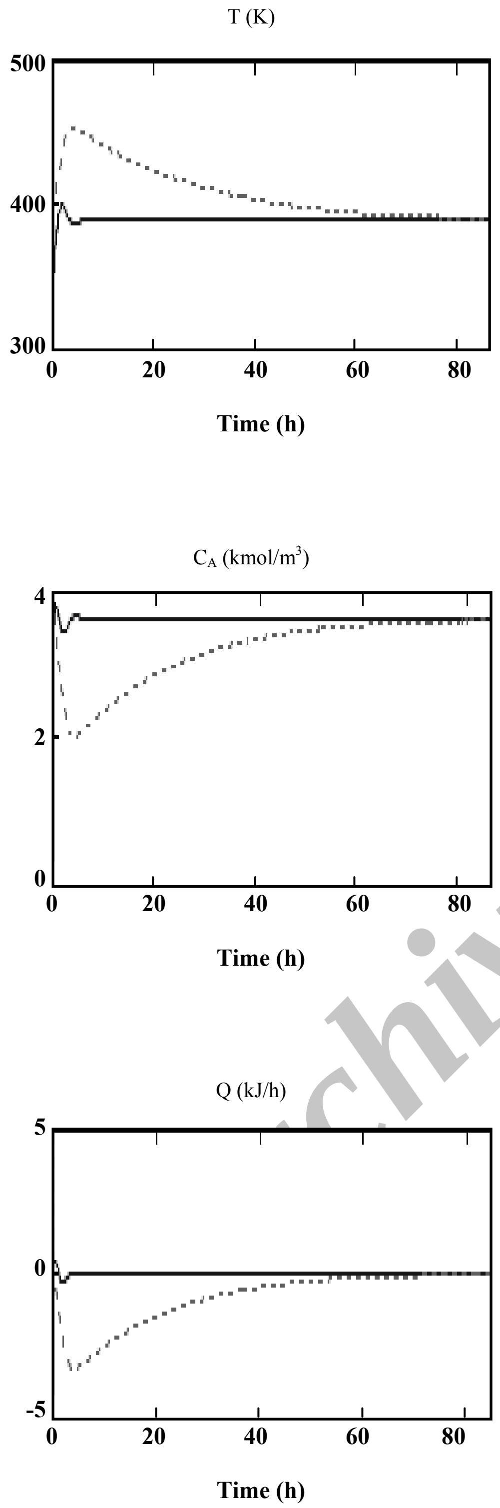

Fig. 2: Closed-loop output trajectories (C_A) with control input Q based on FLMPC controller (solid) and based on PID controller (dashed).

As indicated, y = T(t) has been considered as the process output.

Implementation and simulation of FTC controller

In this section, the proposed FTC controller can be implemented on the CSTR benchmark process being simulated in the Matlab software environment. For this purpose, the following design parameters have been considered to realize the FTC controller:

T_{sp} (Sampling Time) =0.05hr, m (Control horizon) =3, p (Prediction horizon) =20, Q (output error weight) =I, λ (input-rate weight) =1, β_0=1, β_1=10, β_2=2, (coefficients related to feedback linearization method)

The control objective is to stabilize the CSTR process around its unstable steady state in the presence of faults.

Healthy test scenario

First, the CSTR process is assumed to be controlled under the Q-control configuration option. Fig. 2 illustrates the resulting Closed-loop output trajectories corresponding to T and C_A together with the control effort representation. A comparative study has been conducted to demonstrate the efficiency of the proposed FLMPC controller with respect to a well-tuned PID controller. As shown, it can easily be observed that the FLMPC controller yields a much better response compared to the PID controller.

Figs. 3 and 4 demonstrate the respective performance of the two alternatives T_{A0} and C_{A0} control configurations in the context of the proposed FLMPC control strategy. A white noise with variance of o.o4 has been included in Fig. 3 as process noise to explore the resulting performance under noisy condition. As illustrated, the CSTR process can successfully be controlled via each of these two control configurations.

This is in contrast to the works reported in [4,5] where the same CSTR process has not been able to be controlled individually via T_{A0} and C_{A0} as manipulated inputs using a lyapunov-based control approach.

The required computing time to implement the preceding FLMPC controller was found to be about two seconds. Thus, it can easily be observed that the proposed FLMPC controller is practically feasible, considering the three minutes sampling time of the CSTR process.

Faulty test scenarios

Two fault scenarios are adopted to evaluate the proposed FTC performance. For this test study, the

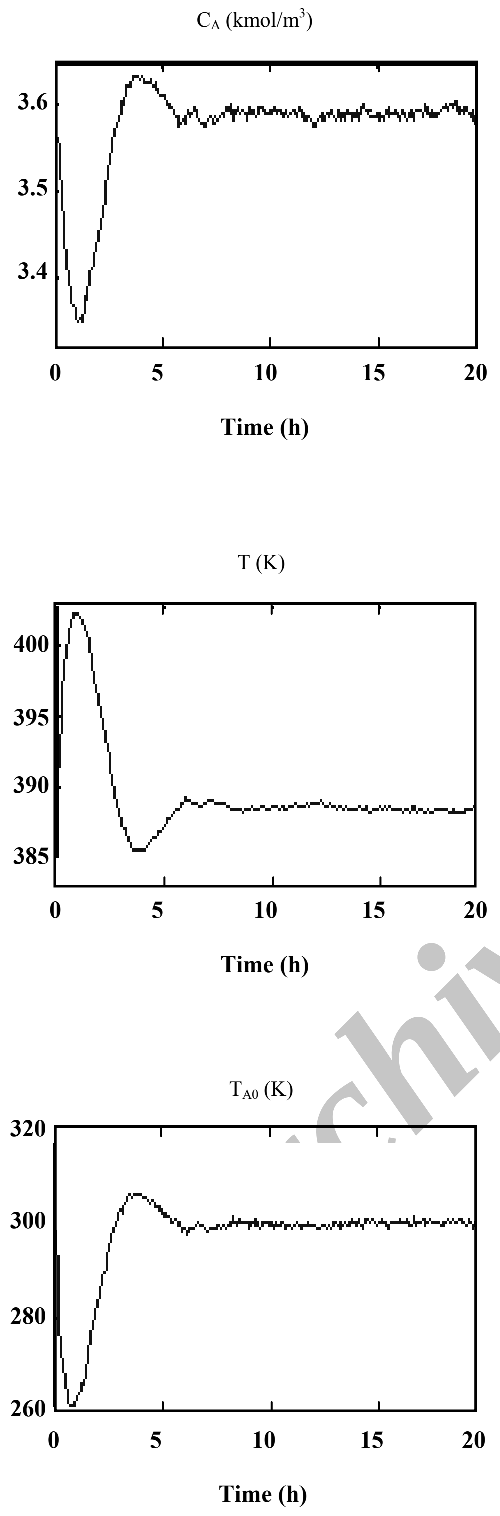

Fig. 3: Output response (T, C_A) with manipulated input T_{A0} under white noise with variance of 0.04 and no failure condition.

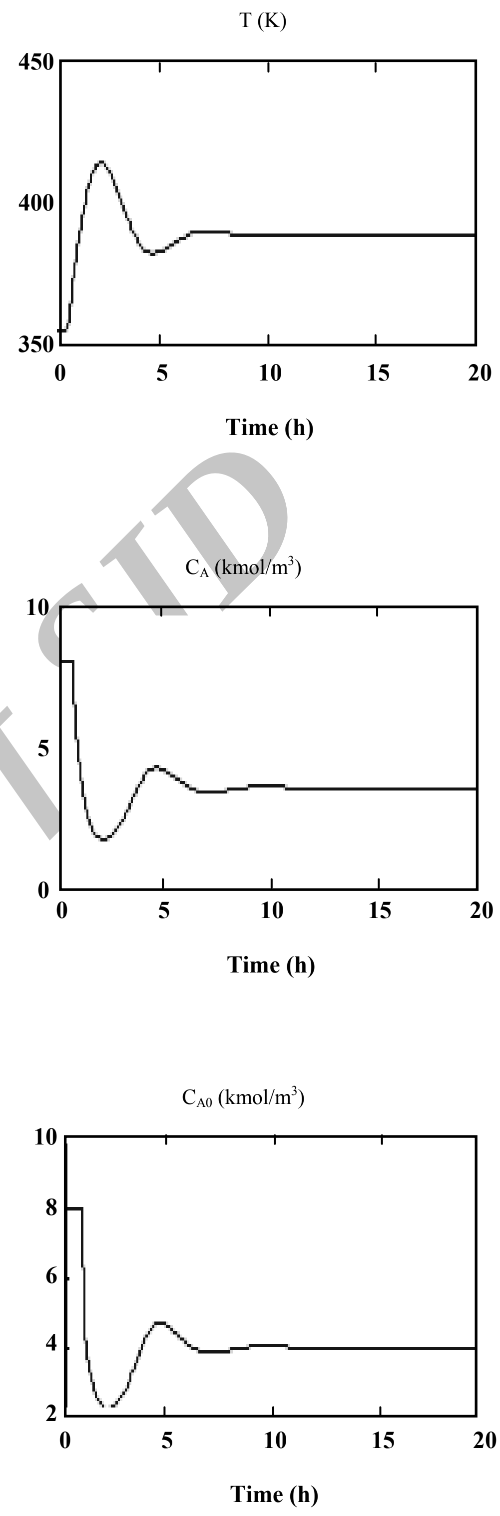

Fig. 4: Output response (T, C_A) with manipulated input C_{A0} under condition no failure.

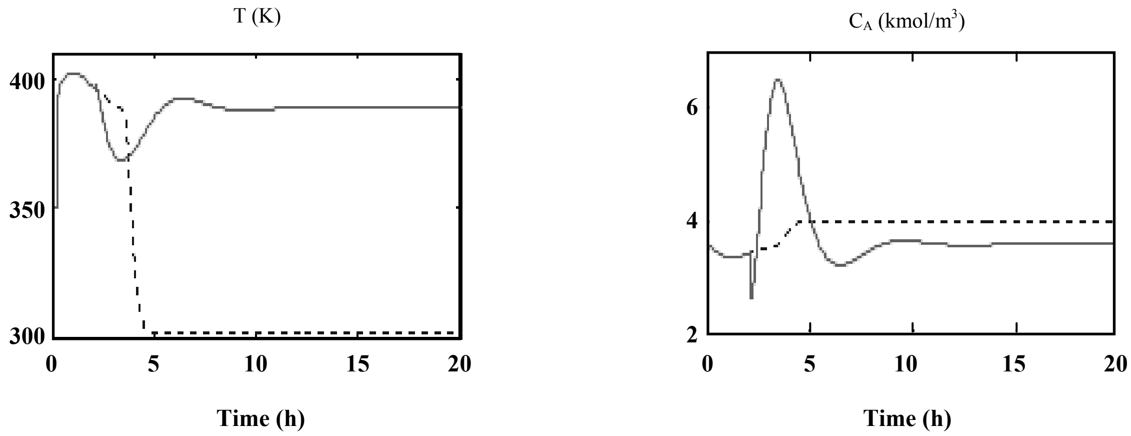

Fig. 5: Closed-loop output trajectory under conditions of failure in T_{A0} configuration with replacing C_{A0} backup configuration (solid) and without replacing backup configuration (dashed).

process is first stabilized via T_{A0} control configuration. After 2 hours of startup, a fault is artificially introduced by gradual fixing of the manipulated input T_{A0} at 300K, as the process output is illustrated in Fig.5 by the dashed trajectory. Therefore, the supervisor should immediately activate the candidate backup control configuration to properly manage the CSTR operation. Otherwise, the induced failure causes the control system to operate in an open-loop manner, resulting into the convergent of the CSTR process operation to the stable equilibrium point, given by (T, C_A) = (301K, 3.99kmol/m^3). This is treated as an undesirable operating point due to its insufficient efficiency and low product concentration. At the time of failure event, the supervisor replaces T_{A0} backup configuration with C_{A0} backup configuration. Fig. 5 clearly demonstrates the successful operation of the substituted C_{A0} backup control configuration to stabilize the CSTR operation at the desired unstable steady-state operating point, represented by the solid trajectories.

In the second test scenario, a series of cascaded possible failures have been introduced in the CSTR operation. Fig. 6 illustrates the obtained results. As shown, the CSTR process is initially running normally under the control of Q-configuration strategy. After 3 hours, the Q-configuration is impaired by fixing its manipulated input as depicted in Fig.6 (a). The supervisor switches the control to the T_{A0}-control configuration as shown in Fig.6 (b). Figs.6 (d) and 6(e) show that the CSTR process states can be managed to be guided towards the desired equilibrium point at (T^S, C_A^S) = (388.57°k, 3.5907kmol/m³).Then, the second failure is organized to be occurred after 6 hours of operation. As depicted in Fig. 6 (b), this failure is realized through fixing the manipulated input T_{A0}. Afterwards, the C_{A0}-control configuration is immediately taken over the control responsibility by the supervisor decision, as represented in Fig. 6 (c).

This is, in fact, the second switching action made by the supervisor to maintain the CSTR operation at the desired unstable steady-state point. Figs.6 (d) and 6(e) clearly illustrate the successful operation of the proposed FTC control methodology to robustly manage the overall CSTR control objective in the face of a cascade of occurred failures. Fig.6 (f) verifies the feasibility of the proposed FTC methodology to be implemented in practical applications, requiring computation time much less the sampling time.

CONCLUSIONS

Fault Tolerant Control (FTC) of nonlinear processes poses an open challenging issue especially when the process inputs face constraints. The proposed FLMPC methodology in this paper makes use of feedback linearization and model predictive control in an integrated framework to tackle this demanding control issue. For this purpose, an active set method has been incorporated to resolve the dependency of the transformed process constraints to the actual evolving process states. A CSTR benchmark process has been adopted to explore the performance of the proposed FLMPC methodology. Two sets of fault scenarios were organized to address the two

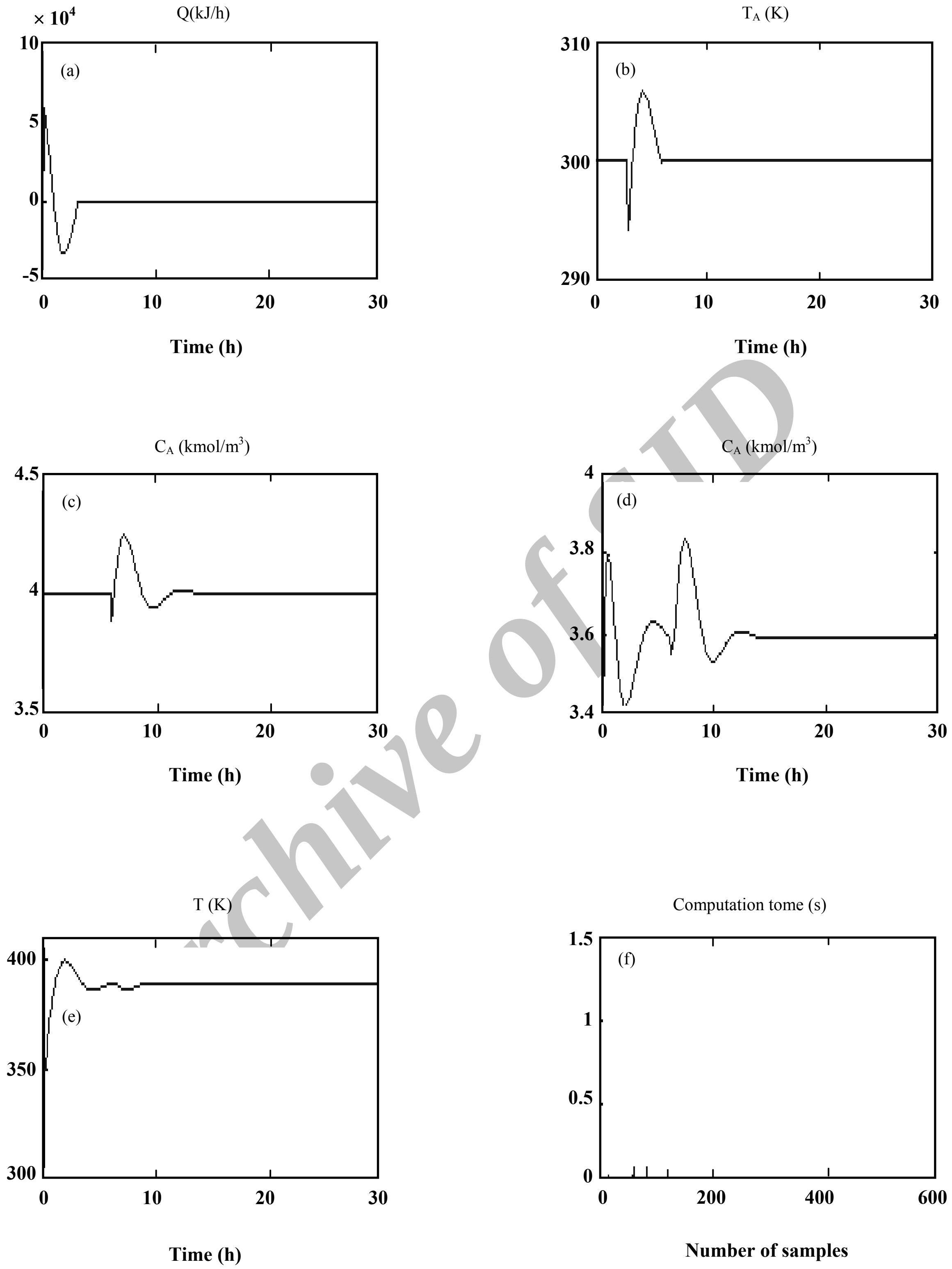

Fig.6: Closed-loop output trajectories a Fault tolerant system with manipulated inputs under conditions of failure.

critical input constraints, corresponding to T_{A0} and Q control configurations. The obtained results demonstrated the successful activation of the proposed method to maintain the desired CSTR operation of an unstable steady-state point by replacing the candidate control configurations via the dedicated supervisor.

Nomenclatures

T	Output temperature as the controlled variable
C_A	Effluent concentration as the controlled variable
T^S	Output temperature in steady state
C_A^S	Effluent concentration in steady state
Q	Rate of heat input to reactor as the manipulated variable
C_{A0}	Feed concentration as the manipulated variable
T_{A0}	Feed temperature as the manipulated variable
Ei	Activation energy of three reactions (i=1, 2, 3)
ΔH_i	Enthalpy of three reactions (i=1, 2, 3)
K_{i0}	Pre-exponential constant of three reactions (i=1, 2,3)
ρ	Density of fluid in the reactor
C_p	Heat capacity of fluid in the reactor
Vol	Volume of reactor
F	Flow rate of fluid

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