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# Non-Linear Modeling of Pressure-Sinkage Behaviour in Soils Using the Finite Element Method

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## ABSTRACT

**A non-linear finite element model could be a useful tool in the development of a method of predicting soil pressure-sinkage behaviour, and can be used to investigate and analyze soil compaction. This study was undertaken to emphasize that the finite element method (FEM) is a proper technique to model soil pressure-sinkage behaviour. For this purpose, the finite element method was used to model soil pressure-sinkage behaviour and a two-dimensional finite element program was developed to perform the required numerical calculations. This program was written in FORTRAN. The soil material was considered as an elastoplastic material and the Mohr-Coulomb elastoplastic material model was adopted with the flow rule of associated plasticity. In order to deal with material non-linearity, incremental method was adopted to gradually load the soil and a total Lagrangian formulation was used to allow for the geometric non-linear behaviour in this study. The FEM model was verified against previously developed models for one circular footing problem and one strip footing problem and the finite element program was used to predict the pressure-sinkage behaviour of the footing surfaces. Statistical analysis of the verification confirmed the validity of the finite element model and demonstrated the potential use of the FEM in predicting soil pressure-sinkage behaviour. However, experimental verification of the model is necessary before the method can be recommended for extensive use.**

**Keywords:** Elastoplastic, Finite Element Method (FEM), Mohr-Coulomb, Non-linear modeling, Pressure-sinkage.



## INTRODUCTION

Agronomists are concerned about the effects of heavy tractors and agricultural machines on agricultural soils due to the possibly of excessive soil compaction that impedes root growth and hence reduces crop yields (Al-Adawi and Reeder, 1996).

Soil compaction under tractors and farm machinery wheels or tracks is of special concern because the weight of these machines has increased dramatically in the last few years (Abu-Hamdeh and Reeder, 2003) and this equipment creates persistent subsoil compaction (Çakir *et al*., 1999).

One of the most important causes of soil compaction is the soil response to pressure and sinkage imposed by wheels and tracks (Abu-Hamdeh and Reeder, 2003). Therefore, the prediction of soil sinkage under load is very important for determining the level of soil compaction. Furthermore, the ability to predict soil sinkage can enable agricultural engineers to till or traffic the soil when it is not in a highly malleable state or to estimate the damage being done to the

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soil structure due to their excessive loading when tillage or traffic is necessary (Raper and Erbach, 1990a).

Most studies dealing with soil sinkage have been experimental. One disadvantage of the experimental procedure is that it is laborious, time consuming, and expensive. An alternative approach is to develop a numerical technique that can predict soil pressure-sinkage behaviour. One such technique that can be used to predict soil sinkage is the finite element method (FEM).

FEM is now firmly accepted as a most powerful general technique for the numerical solution of a variety of problems encountered in engineering. Applications range from the stress analysis of solids to the solution of acoustic, neutron physics and fluid mechanics problems (Hinton and Owen, 1979). Indeed, FEM is now established as a general numerical method for the solution of problems subjected to known boundary and/or initial value conditions. The basic concept of FEM is the idealization of the continuum as an assemblage of a finite number of elements or small segments interconnected at nodal points. The behavior of the continuum when loaded is then predicted by approximating the behaviour of the elements. A solution of this set of equations constitutes a solution of the finite element system (Owen and Hinton, 1980; Bathe, 1996).

For almost last 35 years this method has been touted as a powerful way to solve soil mechanics problems (Girijavallabhan and Reese, 1968; Duncan and Chang, 1970; Perumpral *et al*., 1971; Deasi and Phan, 1980; Pollock *et al*., 1985; Raper and Erbach, 1990a,b; Bailey *et al*., 1992; Shen and Kushwaha, 1994; Arya and Gao, 1995; Fielke, 1999; Mouazen and Nemenyi, 1999; Defossez and Richard, 2002; Abu-Hamdeh and Reeder, 2003).

Furthermore, FEM offers significant promise for modeling of soil pressure-sinkage behaviour. This method can accurately model complex loading geometries (tires, tracks, etc.), and the analysis can be performed easily on microcomputers. However, additional work is required to refine FEM before it can be used to accurately predict soil behaviour and challenges remain for agricultural engineers seeking to solve the sinkage problem. These problems stem from the complex nature of agricultural soils. Agricultural soil experiences much greater strain than other materials (steel, concrete, etc.) that have typically been modeled by civil and mechanical engineers using FEM. The nonlinear nature of agricultural soils is also a complicating factor because it does not obey linear elastic theory and it exhibits elastoplastic behaviour (Raper and Erbach, 1990b).

Recent advances in development of constitutive relationship and theory of plasticity can make FEM a more successful technique for modeling soil behaviour. Therefore, the overall objective of this study was to develop a numerical procedure to predict the soil sinkage. The specific objectives of the study were:

a) To develop a finite element program capable of predicting soil pressure-sinkage behaviour, and

b) To verify the nonlinear finite element model by comparing its results with those of the verified finite element models.

## MATERIALS AND METHODS

### Material Model Development

Two sources of non-linearity are to be expected when a soil is under external loads, namely material and geometrical non-linearity (Naylor and Pande, 1981; Mouazen and Nemenyi, 1999; Abu-Hamdeh and Reeder, 2003).

Material non-linearity can be fully described by the stress-strain relationship. For an elastoplastic material behavior the incremental stress tensor can be related to the incremental strain tensor as (Mouazen and Nemenyi, 1999):

$$d\sigma_{ij} = C^{ep}_{ijkl} d\varepsilon_{kl} \quad (1$$

where:

$C^{ep}_{ijkl}$ = Elastoplastic constitutive matrix

$d\sigma_{ij}$ = Incremental stress tensor

$d\varepsilon_{kl}$ = Incremental strain tensor which is the summation of the incremental elastic strain tensor and incremental plastic strain tensor as (Shen and Kushwaha, 1998):

$$d\varepsilon_{ij} = d\varepsilon^{e}_{ij} + d\varepsilon^{p}_{ij} \quad (2$$

The incremental elastic strain tensor $d\varepsilon^{e}_{ij}$ can be expressed by Hooke's law as (Arya and Gao, 1995):

$$d\varepsilon^{e}_{ij} = \frac{(1+\upsilon)}{E} d\sigma_{ij} - \frac{\upsilon}{E} d\sigma_{kk}\delta_{ij} \quad (3$$

where:

$\upsilon$ = Poisson's ratio

$E$ = Modulus of elasticity

$d\sigma_{kk}$ = Incremental volumetric stress tensor

$\delta_{ij}$ = Kronecker delta.

The incremental plastic strain tensor $d\varepsilon^{p}_{ij}$ can be expressed by the classical theory of plasticity as (Arya and Gao, 1995; Mouazen and Nemenyi, 1999):

$$d\varepsilon^{p}_{ij} = d\lambda \frac{\partial F}{\partial \sigma_{ij}} \quad (4$$

where:

$d\lambda$ = Plastic multiplier

$F$ = Yield function.

The incremental plastic strain tensor is actually a vector perpendicular to the tangent of the yield surface. This definition of the plastic strain is usually designated as associated plasticity (Mouazen and Nemenyi, 1999).

The yield function of Mohr-Coulomb for an elastoplastic material can be expressed as follows (Shen and Kushwaha, 1998):

$$F = \frac{1}{3} J_1 \sin\varphi + (J_{2D})^{1/2}\left(\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\cos\varphi\right) - c\cos\varphi \quad (5$$

where:

$c$ = Soil cohesion

$\varphi$ = Angle of soil internal friction

$J_1$ = The first invariant of the stress tensor

$J_{2D}$ = The second invariant of the deviatoric stress tensor

and:

$$\theta = -\frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}}{3}\frac{J_{3D}}{J_{2D}^{3/2}}\right) \quad , \quad \frac{-\pi}{6} \le \theta \le \frac{\pi}{6} \quad (6$$

where:

$J_{3D}$ = The third invariant of the deviatoric stress tensor.

From Equations (5) and (6) it can be concluded that the Mohr-Coulomb yield criterion accounts for both volumetric and shear behaviour.

## Finite Element Model Development

The governing equations of the finite element method (FEM) can be obtained by using the principle of virtual work. Consider a solid, in which the internal stresses σ, the distributed loads/unit volume *b* and external applied forces $f$ form an equilibrium field, to undergo an arbitrary virtual displacement pattern $\delta d^*$ which result in compatible strains $\delta\varepsilon^*$ and internal displacement $\delta u^*$. Then the principle of virtual work requires that (Owen and Hinton, 1980):

$$\int_{\Omega} (\delta\varepsilon^{*T}\sigma - \delta u^{*T} b)d\Omega - \delta d^{*T} f = 0 \quad (7$$

where:

$\Omega$ = The domain of interest.

Then the normal finite element discretising procedure leads to the following expressions for the displacement and strains within any element (Shen and Kushwaha, 1998):

$$\delta u^* = N\delta d^* \quad (8$$

$$\delta\varepsilon^* = B\delta d^* \quad (9$$

where:

$N$ = Matrix of the shape function

$B$ = Sum of the geometric linear and geometric non-linear strain-displacement matrix

Then the element assembly process gives us:

$$\int_{\Omega} \delta d^{*T}(B^T\sigma - N^T b)d\Omega - \delta d^{*T} f = 0 \quad (10$$

Where, the volume integration over the solid is the sum of the individual element contributions. Since this expression must be true for any arbitrary $\delta d*$ value, it follows that:

$$\int_{\Omega} B^T \sigma d\Omega - f - \int_{\Omega} N^T b d\Omega = 0 \qquad (11$$

For solution of non-linear problems, Equation (11) will not generally be satisfied at any stage of the computation, and:

$$\psi = \int_{\Omega} B^T \sigma d\Omega - (f + \int_{\Omega} N^T b d\Omega) \neq 0 \qquad (12$$

where:

$\psi$ = The residual force vector.

For an elastoplastic situation the material stiffness varies continually, and instantaneously the incremental stress-strain relationship is given by Equation (1). For the purpose of evaluating the element tangential stiffness matrix at any given stage, the incremental form of Equation (12) must be employed. Thus, within an increment of load we have:

$$\Delta\psi = \int_{\Omega} B^T \Delta\sigma d\Omega - (\Delta f + \int_{\Omega} N^T \Delta b d\Omega) \quad (13$$

Substituting for $\Delta\sigma$ from Equation (1) result in:

$$\Delta\psi = K_T d - (\Delta f + \int_{\Omega} N^T \Delta b d\Omega) \qquad (14$$

where:

$K_T$ = Element stiffness matrix associated with the geometric linear and geometric non-linear strain-displacement matrix and can be expressed as:

$$K_T = \int_{\Omega} B^T C_{ep} B d\Omega \qquad (15$$

## Finite Element Program Development

A plane-stress, plane-strain and axisymmetric finite element program (Owen and Hinton, 1980) was modified and a finite element program, entitled PRESSINK, was developed using all the techniques, models, equations and assumptions previously discussed to take into account the material and geometrical non-linearity of soil.

This program was written in FORTRAN for use on a personal computer and the necessary additional subroutines were formulated and assembled to form a working program for a two-dimensional elastoplastic geometrically non-linear analysis of plane-stress, plane-strain and axisymmetric problems. A modular approach was adopted for the program, in that separate subroutines were employed to perform the various operations required in a non-linear finite element analysis.

In order to deal with material non-linearity, obtain stress and strain information at different steps of a loading process, an incremental method was adopted in this study. In addition, soil usually undergoes large deformation and strain, and as we know the stiffness matrix of an element is dependent upon its geometric position and the equilibrium equations must be described by the geometric position after deformation (Shen and Kushwaha, 1998). To model the geometric non-linear behavior using FEM, a total Lagrangian formulation was employed in the program. The modification of the strain-displacement matrix and the evaluation of the strains using a deformation jacobian matrix were the main changes required to account for geometrically non-linear effects.

The flow chart of the program is self-explanatory and is presented in Figure 1 without further comments. In this flow chart:

$\sigma_e^{r-1}$ = Effective stress in the $(r-1)^{th}$ iteration of non-linear solution

$\sigma_e^{r}$ = Effective stress in the $(r)^{th}$ iteration of non-linear solution

$\sigma_Y$ = Equivalent yield stress

## Finite Element Model Verification

Footing problems are one of the most common verification techniques used in engineering application (Raper and Erbach, 1990a). Because the intent of the study was to evaluate the potential use of the finite element method for prediction of soil pressure-sinkage behaviour, it was decided that this goal could be met with published data.

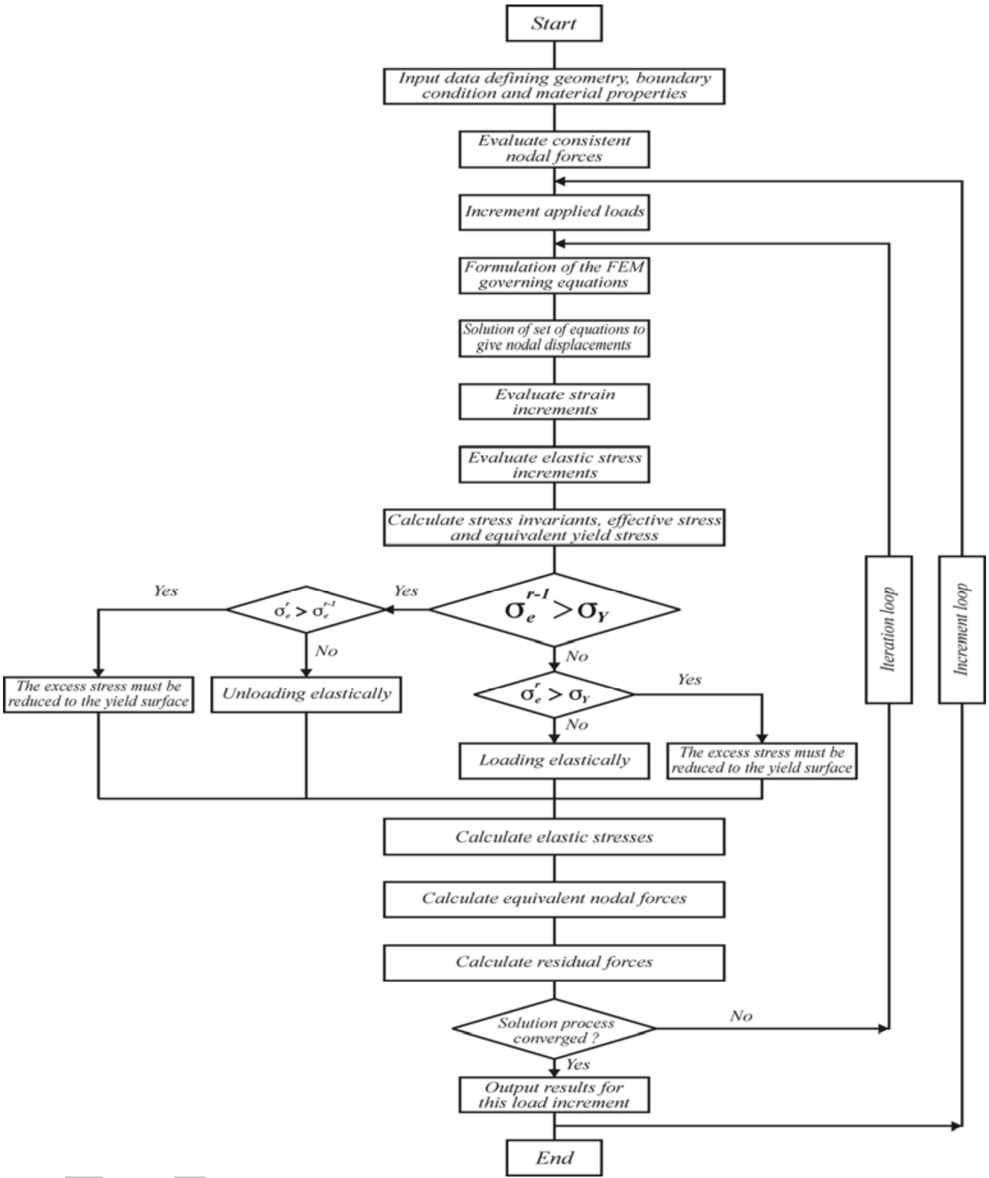


**Figure1.** Flow chart of the non-linear finite element analysis program.

## Verification of the FEM Model using a Circular Footing Problem

Zienkiewicz and Humpheson (1977) have given an application of the finite element method for the analysis of pressure-sinkage behaviour of soil beneath a circular footing. They used a two-dimensional finite element procedure in their investigation. Details of their investigation are given (Zienkiewicz and Humpheson, 1977), and only representative results are presented here.

Our finite element model was firstly verified by using this circular footing problem. In order to verify the finite element model, a two-dimensional FEM mesh was generated within a rectangle 7.32 m long and 3.66 m wide. The FEM mesh that was used to

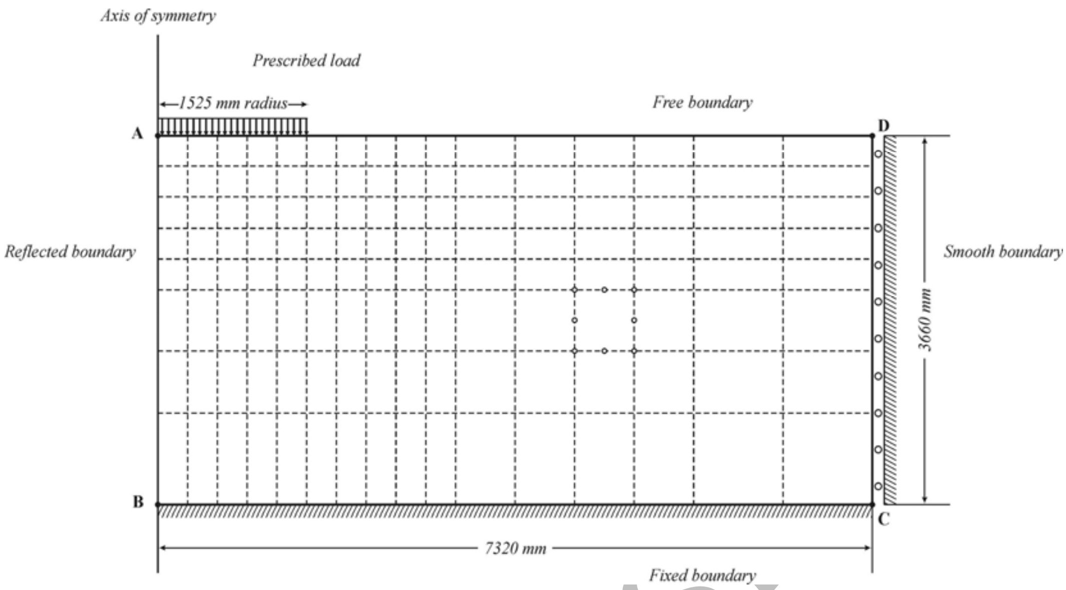


**Figure 2.** Two-dimensional finite element mesh of the soil-circular footing system.

model the axisymmetric geometry of the soil-circular footing system in two-dimensional view is shown in Figure 2. The total number of nodal points and elements were 433 and 128, respectively.

The eight-node serendipity quadrilateral elements were used to represent the soil material. These elements were chosen as it was claimed that they give a more accurate answer for larger mesh sizes (Fielke, 1999) and also they use numerical integration to determine their volume and surface area. These elements are easily numerically integrated by using the Gauss-Legendre rule (Hinton and Owen, 1979; Bathe, 1996). For the elements used in this study, Hinton and Owen advised using 2-point integration, even though our program allowed 2- or 3-point integration.

Since the problem was symmetric about the vertical axis AB, only one half of the system was meshed and considered during the analysis. From Figure 2 it can be seen that the left-side boundary line AB was considered as a reflected boundary and the nodes on the bottom boundary line BC were constrained in both the horizontal and vertical directions. The nodes on the right-side boundary line CD were constrained in a horizontal direction, whilst the nodes on the top boundary line AD were free of any constraints. The circular footing was assumed to be a rigid body and the loading was distributed evenly over the centermost five elements at the top of the finite element mesh.

Soil parameters used for the non-linear finite element analysis of the soil-circular footing system (adopted from the report by Zienkiewicz and Humpheson, 1977) are shown in Table 1. For the finite element analysis, appropriate boundary conditions information, material properties, and nodal and elemental data were entered as required. The load application on the finite element model was simulated in an incremental manner and the total load of 1400 kPa was applied monotonically in increments of 280 kPa.

**Table 1**. Soil parameters used for the finite element analysis of the soil-circular footing system.

| Parameter | Symbol | Value |
|---|---|---|
| Young's Modulus, MPa | E | 207.0 |
| Poisson's ratio, non-dimensional | υ | 0.3 |
| Cohesion, kPa | c | 69.0 |
| Angle of Internal Friction, deg | φ | 20.0 |

## Verification of the FEM Model using a Strip Footing Problem

Siriwardane and Desai (1983) have given another application of the finite element method for the analysis of pressure-sinkage behaviour of soil beneath a strip footing, using a three-dimensional finite element procedure in their investigation. Details of their investigation are given in Siriwardane and Desai (1983), and so only representative results are presented here.

Our finite element model was further verified by using this strip footing problem. As before, in order to verify the finite element model, a two-dimensional FEM mesh was generated within a rectangle 9.0 m long and 4.5 m wide. Figure 3 shows the FEM mesh that was used to model the plane-strain geometry of the soil-strip footing system from a two-dimensional view. The total number of nodal points and elements were 454 and 135, respectively. In this problem, the eight-node serendipity quadrilateral elements were used to represent the soil material and the Gauss-Legendre 2-point integration rule was used to determine their volume and surface.

Since the problem was symmetric about the vertical axis AB, only one half of the system was meshed and considered during the analysis. From Figure 3 it can be seen that the left-side boundary line AB was considered as a reflected boundary and the nodes on the bottom boundary line BC were constrained in both the horizontal and vertical directions. The nodes on the right-side boundary line CD were constrained in a horizontal direction and the nodes on the top boundary line AD were free of any constraints. The strip footing was assumed to be

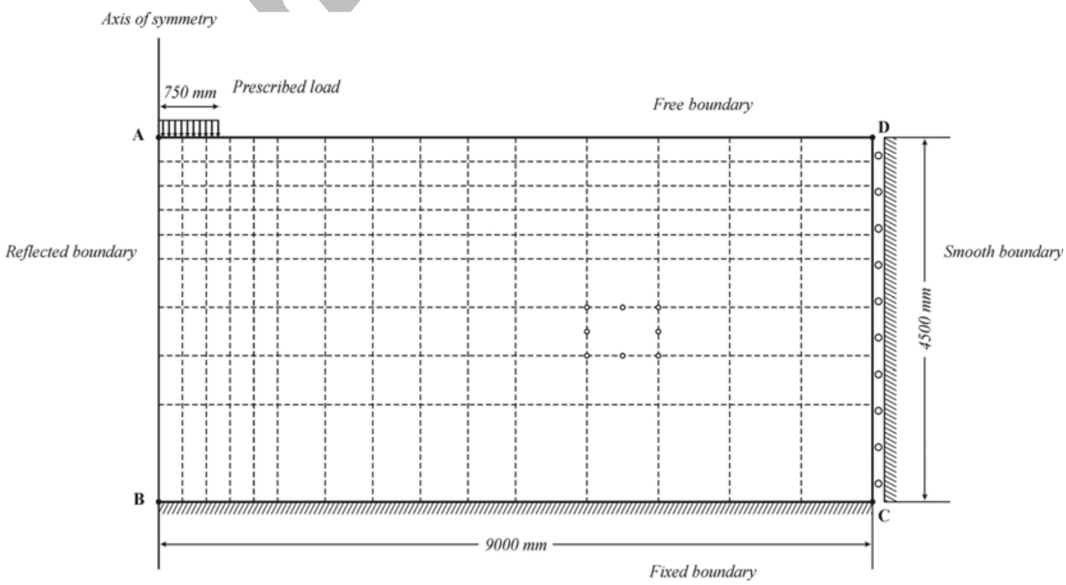


**Figure 3**. Two-dimensional finite element mesh of the soil-strip footing system.

**Table 2.** Soil parameters used for the finite element analysis of the soil-strip footing system.

| Parameter | Symbol | Value |
|---|---|---|
| Young's Modulus, MPa | E | 69.0 |
| Poisson's ratio, non-dimensional | υ | 0.3 |
| Cohesion, kPa | C | 103.5 |
| Angle of Internal Friction, deg | φ | 20.0 |

a rigid body and the loading was distributed evenly over the left-side three elements at the top of the finite element mesh.

The soil parameters used for the non-linear finite element analysis of a soil-strip footing system (adopted from the report by Siriwardane and Desai, 1983) are shown in Table 2. For the finite element analysis, appropriate boundary conditions information, material properties, and nodal and elemental data were input as required. The load application on the finite element model was simulated in an incremental manner and the total load of 1900 kPa was applied monotonically in increments of 380 kPa.

## RESULTS AND DISCUSSION

Results of the finite element analyses included information on the displacement of each nodal point. For each incremental load, the displacement of each nodal point was computed. This process was continued until the total load amount was applied. At this point, the incremental loading was stopped to complete the simulation of soil pressure-sinkage behavior.

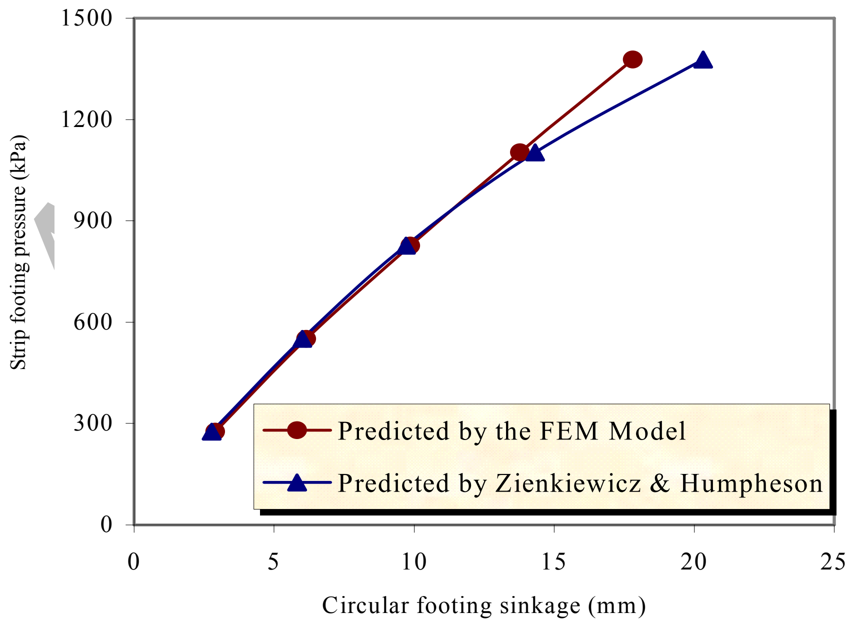


**Figure 4**. Pressure-sinkage curve of the circular footing predicted using the FEM model in compared with that predicted previously by Zienkiewicz and Humpheson.

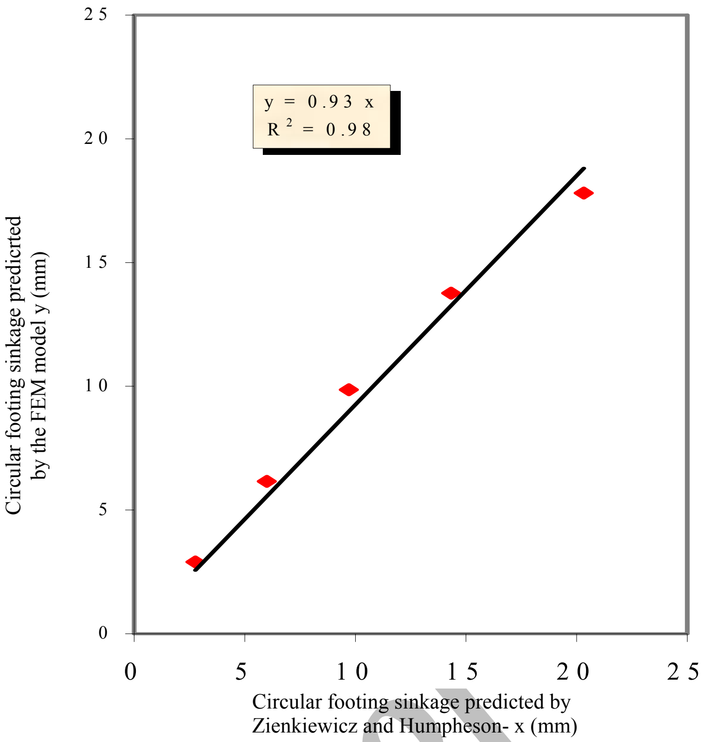


**Figure 5**. Circular footing sinkage values predicted using the FEM model and circular footing sinkage values predicted previously by Zienkiewicz and Humpheson are plotted against each other and fitted with a linear equation with zero intercept.



## Results of the FEM Analysis of the Circular Footing Problem

Figure 4 shows the predicted soil pressure-sinkage curve at the center of the footing surface, which was developed from the results of the finite element analysis. A maximum soil sinkage value was predicted at the footing surface beneath the central axis of the circular footing for all load increments and additional loadings yielded larger increments in soil sinkage. These large values clearly showed that large strain theory could not be used without the incremental loading approach.

Figure 4 also shows the predicted soil pressure-sinkage curve at the center of the footing surface, which was developed from the results obtained previously by Zienkiewicz and Humpheson, (1977).

From comparison of the two curves, it could be concluded that both the analyses gave almost similar results.

A linear regression was performed to verify the validity of the FEM model. Figure 5 shows that the circular footing sinkage values predicted using the FEM model and those predicted previously by Zienkiewicz and Humpheson were plotted against each other and fitted with a linear equation with zero intercept. The slope of the line of best fit and its coefficient of determination were 0.93 and 0.98, respectively.

The root of mean squared errors (RMSE) and the mean relative percentage deviation (MRPD) were used to check the discrepancies between the predicted results using the FEM model and those predicted previously by Zienkiewicz and Humpheson. The amounts of RMSE and MRPD were 1.10 mm and 5 % respectively and, regarding the statistical analysis, the validity of the FEM model was confirmed.

A more likely reason for such negligible discrepancies between the predicted results using the non-linear geometric and material FEM model and those predicted previously by Zienkiewicz and Humpheson is probably the fact that, for this problem, the deformations in the soil are governed predominantly by the material non-linearity rather than by geometric and material non-linearity.

increments in soil sinkage. These large values again confirmed use of the large strain theory in conjunction with the incremental loading approach.

Figure 6 also shows the soil pressure-sinkage curve at the axis of symmetry of the footing surface that was developed from the results obtained previously by Siriwardane and Desai.

From a comparison of the two curves, it could be concluded that both analyses gen-

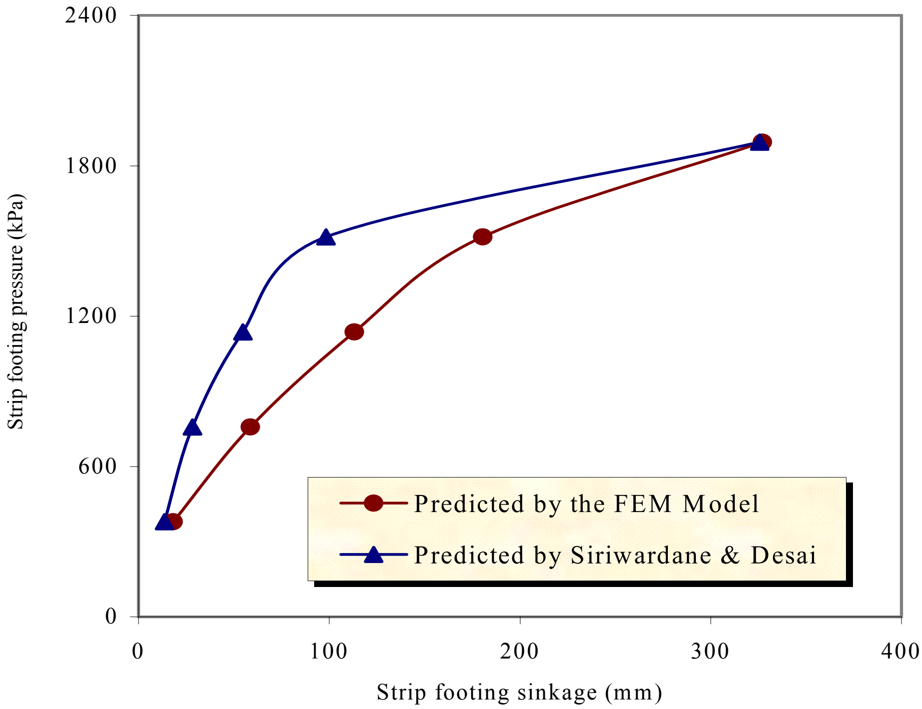


**Figure 6.** Pressure-sinkage curve of the strip footing predicted using the FEM model compared with that predicted previously by Siriwardane and Desai.

## Results of the FEM Analysis of the Strip Footing Problem

Figure 6 shows the predicted soil pressure-sinkage curve at the axis of symmetry of the footing surface that was developed from the results of the FEM analysis. Again, a maximum soil sinkage value was predicted at the footing surface beneath the axis of symmetry of the strip footing for all load increments and additional loadings yielded larger

erally represent similar curves, but the sinkage values predicted by the FEM model are relatively greater than those predicted previously by Siriwardane and Desai.

As before, a linear regression was performed to verify the validity of the FEM model. Figure 7 shows that the strip footing sinkage values predicted using the FEM model and those predicted previously by Siriwardane and Desai were plotted against each other and fitted with a linear equation

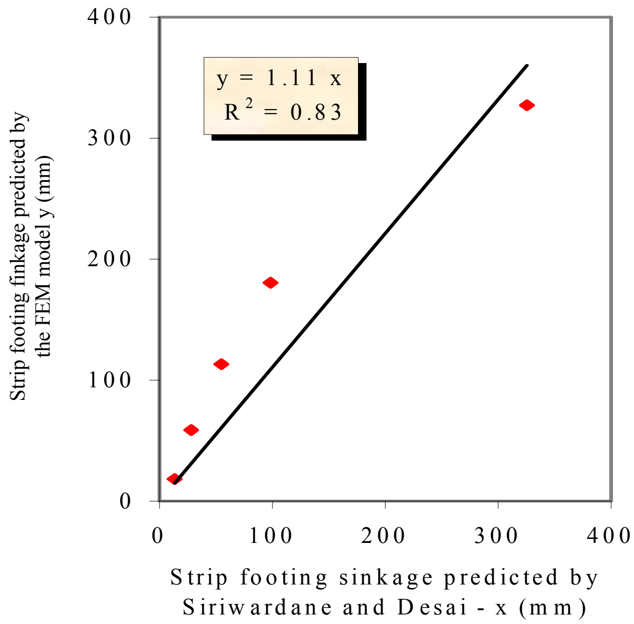


**Figure 7.** Strip footing sinkage values predicted using the FEM model and strip footing sinkage values predicted previously by Siriwardane and Desai are plotted against each other and fitted with a linear equation with zero intercept.

with zero intercept. The slope of the line of best fit and its coefficient of determination were 1.11 and 0.83, respectively.

Again, RMSE and MRPD were used to check the discrepancies between the predicted results using the FEM model and those predicted previously by Siriwardane and Desai. The amounts of RMSE and MRPD were 47.0 mm and 35 %, respectively.

A likely reason for such discrepancies between the predicted results using the non-linear geometric and material FEM model and those predicted previously by Siriwardane and Desai probably stem, from our modeling and is due to the geometric non-linearity assumption. It can be due to the fact that, for this problem, the soil deformations are governed by material and geometrical non-linearity and that to reasonably predict soil pressure-sinkage behaviour in soil problems, both material and geometrical non-linearity should be taken into account over the entire soil volume being modeled. With respect to this fact, statistical analysis confirmed the validity of the FEM model again and demonstrated the potential use of the FEM in predicting soil pressure-sinkage behavior. However, experimental verification of the model is necessary before the model can be recommended for extensive use.

## CONCLUSIONS

The finite element analysis of soil pressure-sinkage behaviour has led to the following conclusions:

- The Finite Element Method proved to be a proper tool in the prediction of soil pressure-sinkage behaviour.
- The likely reason for discrepancies between the predicted results using the geometric and material non-linear FEM model and those predicted previously by other FEM models probably stems from our modeling and is due to the geometric non-linearity assumption and can be due also to

the fact that soil deformations are governed predominantly by material and geometrical non-linearity.

● To reasonably predict soil pressure-sinkage behaviour using the FEM models, both material and geometrical non-linearity should be taken into account for the entire soil volume being modeled.

● The statistical analysis of the verification confirmed the validity of the FEM model and demonstrated the potential use of the FEM in prediction of the soil pressure-sinkage behaviour. However, experimental verification of the model is necessary before the model can be recommended for extensive use.

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## تعیین مدل غیرخطی رفتار فشار- نشست در خاک با استفاده از روش اجزاء محدود

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### چکیده

یک مدل غیرخطی اجزاء محدود برای پیش‌بینی رفتار فشار- نشست در خاک ابزاری سودمند بوده و می‌تواند برای تحلیل و بررسی مساله تراکم در خاکهای کشاورزی مورد استفاده قرارگیرد.این تحقیق به منظور تاکید بر معرفی روش اجزاء محدود (FEM) به عنوان شیوه‌ای مناسب برای مدل‌سازی رفتار فشار- نشست در خاک انجام پذیرفت. به این منظور، روش اجزاء محدود برای مدل‌سازی رفتار غیرخطی فشار- نشست در خاک مورد استفاده قرارگرفت و یک برنامه دو بعدی اجزاء محدود به زبان FORTRAN برای انجام محاسبات عددی مورد نیاز تهیه‌گردید. در این مطالعه خاک به عنوان یک ماده الاستوپلاستیک در نظرگرفته شد و از مدل ماده الاستوپلاستیک مور-کلمب به همراه قانون جریان در تئوری وابسته پلاستیسیته استفاده گردید. به منظور تحلیل رفتار غیرخطی مادی از روش بارگذاری افزایشی و به منظور تحلیل رفتار غیرخطی هندسی و محاسبه تغییرشکل‌ها وکرنش‌های بزرگ از فرمول‌بندی دستگاه مختصات لاگرانژی کل استفاده‌گردید. مدل اجزاء محدود با استفاده از یک مساله شالوده مدور و یک مساله شالوده نواری مورد ارزیابی قرارگرفت و برنامه کامپیوتری مذکور برای پیش‌بینی رفتار فشار- نشست خاک در زیر این دو شالوده به کار گرفته شد. نتایج حاصل از تحلیل آماری اعتبار مدل اجزاء محدود را به اثبات رساند و نشان داد که استفاده از روش اجزاء محدود در پیش‌بینی رفتار فشار- نشست در خاک امکان‌پذیر می‌باشد. با وجود این، ارزیابی تجربی مدل قبل از توصیه برای استفاده گسترده‌تر از آن ضروری می‌باشد.