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# Comparison of Mathematical Models for Describing the Growth of Baluchi Sheep

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## ABSTRACT

**The objectives of this study were to identify a suitable mathematical model for describing the growth curve of Baluchi sheep based on monthly records of live weight from birth to yearling; and to evaluate the efficacies of nonlinear mixed effect model (NLMM) and the nonlinear fixed effect model (NLM) methodologies. Growth models were fitted to a total of 16,650 weight–age data belonging to 2071 lambs. Five nonlinear growth functions of von Bertalanffy, Gompertz, Brody, Logistic, and Richards and two linear polynomial functions were applied. The growth models were compared by using the Akaike's information criterion (AIC) and residual mean square (MSE). Among all nonlinear fixed effect models, the Brody function had the smallest *AIC* and *MSE* values, indicating the best fit for both sexes. The Brody fixed effect model compared with NLMM including one random effect of asymptotic mature weight. The model evaluation criteria indicated that the Brody mixed effect model fitted the data better than the corresponding fixed effect model. It can be concluded that, among the linear models, the polynomial of the third order and, among nonlinear models, Brody mixed model were found to best fit the Baluchi sheep growth data.**

**Keywords:** Body weight, Growth model, Nonlinear regression.

## INTRODUCTION

About half of the livestock production in Iran is accounted for by the 27 different breeds of sheep (Kamalzadeh and Shabani, 2007). The most numerous of these breeds are fat-tailed local carpet-wool sheep, which are well adapted to the prevalent extensive, migratory, or semi-pastoral production systems, using the poor rangelands condition as the major source of feed (Farid *et al*., 1977). According to Kamalzadeh and Shabani (2007) the Baluchi fat-tailed sheep is the most important native breed. It accounts for slightly more than 12% of the total sheep population. During the winter period, this small breed is mainly pastured on arable farms in the lowlands and around villages. During the hot season, the animals migrate to the higher mountain grazing areas.

Growth is a trait of interest in domestic animals. The primary definition of growth is given by the increase in size, number, or mass with time. However, this does not include the phenomenology and etiology of growth. Growth should be evaluated by growth rate or by weight and size increases during different stages of life, because it is a continuous function during an animal's life, from the first embryonic stages up to adult age (mature weight)

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(Arango and Van Vleck, 2002). Analyzing growth curves - that is, the acquisition of data for the same animal or plant over a certain period - is a basic task in biological research (Spilke *et al*., 2009). Growth models are designed for exploring longitudinal data on individuals over time. In order to model growth of biological systems, numerous mathematical models have been used. According to Karkach (2006) the curves can be classified according to the type of growth: determinate or indeterminate. Most of the curves used nowadays describe determinate growth, because this is easily observed in animals such as sheep and cattle. These curves are also called asymptotic, because this kind of growth is determined by a maximum size, which is reached with the growth rate diminishing.

Recently, nonlinear mixed effect models have become very popular in the analysis of growth data because these models quantify the population mean as well as population variation of the structural parameters. Nonlinear mixed effect models provide a powerful extension of traditional regression models for longitudinal growth data. These models involve both fixed effects and random effects. Within and between individual variations are at least two sources of variation for longitudinal growth data. The fixed effect growth models do not account for variability between individuals (Craig and Schinckel, 2001).

Because of the permanent increase of consumers' demand for sheep meat and limited natural sources in Iran, there is a need for improvement of productivity using appropriate breeding strategies. Therefore, the objectives of this study were to determine the best mathematical model to explain the body weight-age relationship in Baluchi sheep under an extensive feeding system and evaluate the efficacies of nonlinear mixed effect model (NLMM) and the nonlinear fixed effect model (NLM) methodologies.

## MATERIALS AND METHODS

### Data

The data used in this study were obtained from 2071 Baluchi lamb. A total of 16650, out of 17531, body weight records were analyzed in the study. The field records were obtained from the Abbas-Abad Baluchi sheep breeding station, northeast region of Iran during 2002 to 2009.

In general, the animals in the Abbas-Abad Baluchi sheep breeding station were managed by following the conventional industry practices. The mating period started between late summer (August) and early autumn (September) and included at most three estrous cycles (51 days). Lambing commenced in the beginning of February and ended at the end of March. The lambs were creep fed on natural pasture and kept together with their mothers until the mean weaning age of all lambs at about 3 months. The animals were kept on pasture during spring and summer and were kept indoors for about 4 months in winter. All animals were weighed at birth, 1, 2, 3, 4, 6, 9, and 12 months of age and, for most of the animals, these records were documented. Numbers of records for these body weights were 2071, 2071, 2041, 1987, 1932, 1912, 1903, and 1807, respectively. The data sets for 5-, 7-, 8-, 10-, and 11-month weight included 178, 177, 190, 192, and 189 records, respectively. On average, there were 8.04 body weight recordings per individual, with a maximum of 13 recordings in data set used in the present study.

### Statistical Analysis

Outliers of the data set were detected using the influence diagnostics recommended by Ryan (1997). This method was incorporated into the REG procedure of SAS by using the INFLUENCE option in the MODEL statement. The standardized deletion residuals or RSTUDENT ($e_i^*$) were used for analyzing the influence of each weight record and was calculated as follows:

$$e_i^* = \frac{e_i}{s_{(i)}\sqrt{(1-h_i)}}$$

Where, $e_i$ is the residual; $s_{(i)}$ is the estimated standard deviation in the absence of the i'th observation; and $h_i$ is the hat matrix, which is the $i^{th}$ diagonal of the projection matrix for the predictor space. The leverages, i.e. the diagonal elements of the hat matrix, describe the effect of observations on their own fitted values. Belsley *et al*. (1980) suggest paying special attention to all observations with RSTUDENT larger than an absolute value of 2. This resulted in a total of 16650 body weight records of 2,071 animals (1,053 males and 1,018 females) that were used for the growth analysis. Of these body weights, male and female records were 8,404 and 8,246, respectively. The growth curves were modeled separately for each sex.

To model weight–age relationship, five nonlinear growth functions of von Bertalanffy, Gompertz, Brody, Logistic, and Richards as well as two linear polynomial functions of second and third order of fit were fitted to the Baluchi growth data. These nonlinear functions were previously described by Malhado *et al*. (2009). The equations for the applied growth models are given in Table 1. The biological interpretation of the parameters in these models is as follows: (1) Asymptotic mature weight is represented by the variable *A* in the equation. The asymptotic limit of each model, as age (t) approached infinity, does not approximate the heaviest weight attained by the animal. It is an asymptotic mean weight (Brown *et al*., 1976); (2) The Y-intercept (B variable) depicts weight at time zero, which is the initial weight of animal. This value is called the integration constant and has no biological interpretation (Richards, 1959; Fitzhugh Jr., 1976); (3) Slope of a non-linear growth curve is represented by k variable in the equation. The slope in such growth curves is also a measure of the rate of approximation for its asymptotic value and represents the postnatal maturing rate. The large k value indicates the early maturity for animal (Taylor, 1965); (4) *M* is the parameter shaping the curve (Malhado *et al*., 2009).

The estimation of the nonlinear fixed and mixed effect growth models were carried out using the NLMIXED procedure of SAS (SAS Institute, 2009). The linear growth curves were calculated with the MIXED procedure. The models were compared by using residual mean square error (MSE) and Akaike's information criterion (AIC; Akaike, 1974). By using the NLMIXED procedure for fitting fixed and mixed effect growth models, the AIC values obtained from the MIXED and the NLMIXED (fixed and mixed models) procedures can be compared. The model with the smallest MSE and AIC values was chosen to be the best for fitting growth data.

**Table 1**. Functions considered for modeling the growth curve of Baluchi sheep.

| Model | Equation [a] | No. of parameters |
|---|---|---|
| von Bertalanffy | $W_t = A(1-Be^{-kt})^3 + \varepsilon$ | 3 |
| Gompertz | $W_t = Ae^{(-Be^{-kt})} + \varepsilon$ | 3 |
| Brody | $W_t = A(1-Be^{-kt}) + \varepsilon$ | 3 |
| Logistic | $W_t = A(1+Be^{-kt})^{-1} + \varepsilon$ | 3 |
| Richards | $W_t = A(1-Be^{-kt})^M + \varepsilon$ | 4 |
| Polynomials | $W_t = d_0 + \sum_{i=1}^{r} d_i \times t^i + \varepsilon$ | 3 to 4 |

[a] $W_t$= Weight at time *t*; *A*= Asymptotic mature weight; *B*= Integration constant; *k*= Maturity rate; *M*= The parameter shaping the curve; *e*= Euler's number; $d_0$= Intercept; *r*= Second to third order of fit; $d_i$= Regression coefficients, $\varepsilon$= Residual.

$$AIC = -2\log L + 2p$$

Where, *-2logL* is the minus two times the maximized log-likelihood and p is the number of parameters in the model. The MSE calculated by dividing the residual sum of square by the number of observations, which represents the estimator of the maximum likelihood of the residual variance. *AIC* is a criterion for model selection based on the likelihood function. The basic idea behind this criterion is penalizing the likelihood for the model complexity, i.e. the number of explanatory variables used in the model. It is more advantageous than the $R^2$, which increases with increasing numbers of parameters in the models and, thus, is not useful for the comparison of models with different numbers of parameters. The comparison of different non-nested models with this method determines which model is more likely to be correct and accounts for differences in the number of degrees of freedom.

After selecting the best nonlinear fixed effect model, growth parameter estimations of this model were obtained with mixed effect model (NLMM), using NLMIXED procedure of SAS software, for both sexes. The nonlinear mixed effect model can be expressed as follows:

$$Y_{ij} = A_{ij}\beta + B_{ij}b_i + \varepsilon_{ij},$$

$$b_i \sim N(0,\sigma_u^2), \varepsilon_{ij} \sim N(0,\sigma_e^2),$$

Where, $Y_{ij}$ is the $j_{th}$ observation of body weight on the $i_{th}$ individual; $\beta$ is a vector of fixed population parameters; $b_i$ is a random effect vector associated with individual i; $\varepsilon_{ij}$ is the residual term and $A_{ij}$ and $B_{ij}$ are design matrices for the fixed and random effects, respectively. $\sigma_u^2$ and $\sigma_e^2$ are the variance of the random effect and the residuals, respectively. Fixed effect model was compared with NLMM containing one random effect of asymptotic mature weight using the residual error variance ($\sigma_e^2$), $-2logL$, and *AIC* as the criteria for evaluating this alternative model.

The absolute growth rate (AGR) was calculated by taking the first derivative of the best function with respect to time (dW/dt), then, the result was compared with the AGR of Baluchi sheep which were kept under intensive feeding system (Bahreini Behzadi and Aslaminejad, 2010). The AGR represents the weight gain per time unit and equals the daily weight gain in this study. Therefore, it corresponds to the mean animal growth rate within the period of study for a population (Malhado *et al*., 2009).

## RESULTS AND DISCUSSION

Comparison of the models by AIC values and residual mean square error (MSE) are presented in Table 2. *F*-Statistic *SS* type 1 showed that the polynomials of the second and third order of fit had significant (P< 0.01) influence on the estimation of the growth curves. Regarding the whole growth curves, the linear polynomial of the third order of fit had the smallest *AIC* and *MSE* values for both sexes, indicating the best fit for the Baluchi growth data. Among all nonlinear models, the smallest *AIC* and *MSE* values were calculated for the Brody

**Table 2.** Values of -2logL, Akaike's information criterion (AIC), and residual mean squares (MSE) of functions modeling growth of male and female Baluchi sheep.

| | Number of | Male | | | Female | | |
|---|---|---|---|---|---|---|---|
| Models | parameters | -2logL | AIC | MSE | -2logL | AIC | MSE |
| von Bertalanffy | 3 | 47416.03 | 47424.03 | 16.51 | 45004.57 | 45012.57 | 13.73 |
| Gompertz | 3 | 47657.14 | 47665.14 | 16.99 | 45213.19 | 45221.19 | 14.09 |
| Brody | 3 | 47205.11 | 47213.11 | 16.10 | 44881.12 | 44889.12 | 13.53 |
| Logistic | 3 | 48465.64 | 48473.64 | 18.71 | 45960.03 | 45968.03 | 15.42 |
| Polynomial, 2$^{nd}$ order | 3 | 47942.61 | 47950.61 | 17.58 | 45568.10 | 45576.10 | 14.71 |
| Richards | 4 | 47659.76 | 47669.76 | 17.00 | 45215.51 | 45225.51 | 14.09 |
| Polynomial, 3$^{rd}$ order | 4 | 47006.46 | 47016.46 | 15.73 | 44762.50 | 44772.50 | 13.34 |

function for both sexes. The logistic function was the worst model with the greatest *AIC* and MSE values. Comparison of the Brody and the Logistic functions is given in Figure 1. The growth curves estimated were typically sigmoid. Comparing the collected weight data with the estimated Logistic function shows that the Logistic growth curve clearly underestimated the growth in the early (40 to 90 days) phase and the latest (315 to 365 days) phase of growth. However, it overestimated growth from 150 to about 300 days of age.

Parameter estimation of nonlinear growth models and correlation coefficients between A and k parameters for both sexes are represented in Table 3. The importance of the relationship between A and k has been discussed by several authors (Brown *et al*., 1976; López de Torre and Rankin, 1978; López de Torre *et al*., 1992). The negative association between these two parameters indicates that the sheep with smaller mature weight will be maturing faster. Negative correlations between the parameters A and k were obtained in all nonlinear models in this study (Table 3). These results are in agreement with the reports of Taylor and Fitzhugh Jr. (1971), DeNise and Brinks (1985), Bathaei and Leroy (1998), Eyduran *et al*. (2008), and Malhado *et al*. (2009).

As noted before, the third order polynomial regression results in best fit (Table 2), which could be applied as fixed regression with random regression models. However, the estimated parameters in a linear growth model are not biologically meaningful and interpretable. Therefore, the application of a nonlinear model in growth analysis in regard to biologically interpretable parameters is advisable, especially in genetic studies of growth. The

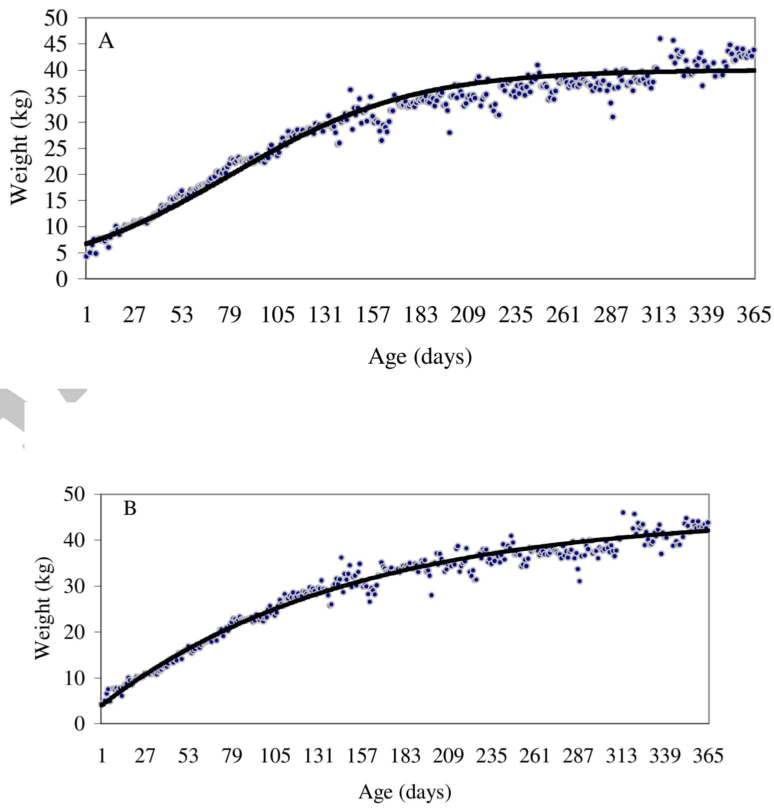


**Figure 1.** Growth curves of Baluchi sheep as predicted by the Logistic (A) and Brody (B) functions.

**Table 3.** Parameters and correlation coefficient between A and k parameters of nonlinear growth models of male and female Baluchi sheep[a].

| Models | Sex | *A* | *B* | *k* | *M* | $r_{Ak}$ |
|---|---|---|---|---|---|---|
| von Bertalanffy | Male | 43.98 | 0.52 | 0.011 | - | -0.80 |
| | Female | 40.44 | 0.52 | 0.011 | - | -0.79 |
| Gompertz | Male | 43.07 | 2.04 | 0.013 | - | -0.75 |
| | Female | 39.68 | 2.04 | 0.014 | - | -0.75 |
| Brody | Male | 47.62 | 0.92 | 0.007 | - | -0.92 |
| | Female | 43.45 | 0.92 | 0.007 | - | -0.91 |
| Logistic | Male | 41.54 | 5.09 | 0.020 | - | -0.63 |
| | Female | 38.40 | 5.06 | 0.021 | - | -0.63 |
| Richards | Male | 43.06 | -0.007 | 0.013 | -299.55 | -0.75 |
| | Female | 39.67 | -0.007 | 0.014 | -299.55 | -0.75 |

[a] *A*= Asymptotic mature weight; *B*= Integration constant; *k*= Maturity rate; *M*= The parameter shaping the curve, $r_{Ak}$= Correlation coefficient between *A* and *k*.

Brody is the simplest and easiest function for biological interpretation compared with the other growth curves (Nelder, 1961). In this study, Brody was the best fitting model among the nonlinear models.

The estimates of Brody growth parameters obtained with fixed (NLM) and mixed (NLMM) effects models for both sexes are presented in Table 4. In NLMM, the between-animal variation was modeled by varying the asymptotic mature weight. In this model, the error variation has partitioned into within-animal variation ($\sigma_e^2$) and between-animal variation ($\sigma_u^2$). The variance-covariance partitioning associated with the random effect allowed for the separation of the between- and within-animal variation. The log-likelihood and *AIC* criteria were lower in NLMM compared with NLM, suggesting a better fit of the Brody mixed effect model to the data of Baluchi sheep. The residual variance in the males and females were reduced by about 51% in NLMM compared with NLM, indicating an improvement in the accuracy of estimating the growth parameters. The growth parameter estimates were not different between the fixed and the mixed effect models. The result of this study concerning better fitting of mixed effect model is in accordance with those reported in the literature. Nonlinear mixed effect models allow a more accurate estimation of animal growth functions than the corresponding fixed effect models (Craig

**Table 4.** Growth parameter estimates from fixed and mixed Brody growth models for male and female Baluchi lambs.

| | Fixed Model | | Mixed Model | |
|---|---|---|---|---|
| Parameter[a] | Male | Female | Male | Female |
| *-2logL* | 47205 | 44881 | 43586 | 41236 |
| *AIC* | 47213 | 44889 | 43596 | 41246 |
| *A* | 47.62 | 43.45 | 47.80 | 42.68 |
| *B* | 0.92 | 0.92 | 0.92 | 0.92 |
| *k* | 0.007 | 0.007 | 0.007 | 0.007 |
| $\sigma_e^2$ | 16.10 | 13.53 | 7.86 | 6.56 |
| $\sigma_u^2$ | - | - | 25.48 | 19.23 |

[a] *A*= Asymptotic mature weight; *B*= Integration constant; *k*= Maturity rate; $\sigma_e^2$ = Residual variance, $\sigma_u^2$ = Random effect variance.

and Schinckel, 2001; Schinckel and Craig, 2002; Schinckel *et al*., 2005; Aggrey, 2009).

In order to reveal the effect of environmental conditions on the growth rate, the result of this study is compared with the study by Bahreini Behzadi and Aslaminejad (2010) carried out on the Baluchi sheep kept under an intensive feeding regime. In contrast to the present study, Bahreini Behzadi and Aslaminejad (2010) concluded that the Gompertz and von Bertalanffy models were the most appropriate for describing growth trajectory of Baluchi lambs under intensive feeding conditions. In addition, the absolute growth rate (AGR) based on the first derivative of the Brody function and the third order polynomial from birth to 147 days of age in the current study is compared with the *AGR* of Brody function estimated from data provided by Bahreini Behzadi and Aslaminejad (2010). In both studies the *AGR* declines gradually in relation to increasing time (Figure 2). In general, the course of the *AGR* under the two different management systems is similar in both studies, indicating that the degree of reduction of daily gain over time is approximately equal. However, the magnitude of daily gain in intensive feeding system is distinctively higher. The relatively low *AGR* estimates found in our study can be explained by the low nutritional level and poor quality of the pasture at the Abbas-Abad sheep breeding station. Therefore, good nutritional programs are required in order to minimize the effects of diet changes, reducing the *AGR* losses after weaning. Sarmento *et al*. (2006) pointed out that the decrease in the *AGR* might result from improper management practices that fail to match the increasing nutritional demands as long as the animals grow.

In the literature (Eyduran *et al*., 2008), a 4-parameter logistic model is fitted which now will be shown to be fully equivalent to the Brody model. The model equation for this 4-parameter logistic model is as follows:

$$W_t = A^* + (A^* - M^*)(1 + B^* e^{-kt}) + \varepsilon$$

Which can be written as:

$$W_t = A^* + (A^* - M^*) + (A^* - M^*)B^* e^{-kt} + \varepsilon$$

. The asterisk at the parameters indicates that they are defined specific to this model.

By substituting $C_1 = 2A^* - M^*$ and $C_2 = -(A^* - M^*)B^*$ the model has the form

$$W_t = C_1 - C_2 e^{-kt} + \varepsilon$$

The Brody function in turn can be written as

$$W_t = A(1 - Be^{-kt}) + \varepsilon = A - ABe^{-kt} + \varepsilon$$

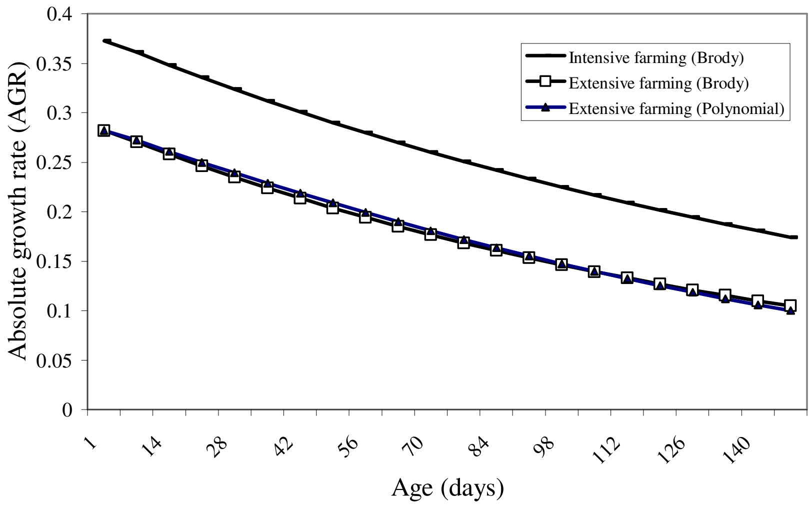


**Figure 2.** Absolute growth rate (AGR) of Baluchi sheep under extensive (present study) and intensive farming (data provided by Bahreini Behzadi and Aslaminejad, 2010) from birth to 147 days based on Brody and third order polynomial functions.

Obviously these two models are fully equivalent, if $C_1 = A$ and $C_2 = AB$.

Fitting this 4-parameter Logistic model to the data resulted in estimates of $A^* = 32.62$, $M^* = 19.70$, and $B^* = -3.24$, and it is easy to show that the above equivalence holds with $A = 45.54$ and $B = 0.92$ obtained for the Brody model (Table 3). Naturally, the fit and the likelihood obtained with the two models were identical. It, therefore, must be stated that this 4 parameter logistic model is in fact only a 3 parameter model, because it is fully equivalent to the 3 parameter Brody model. Therefore, it is also inappropriate to account for 4 degrees of freedom in calculating the AIC for the 4 parameter logistic model, since only 3 parameters can be chosen freely.

The results of published studies carried out on growth and weight development in sheep are summarized in Table 5. Bathaei and Leroy (1998) evaluating the growth in Mehraban Iranian fat-tailed sheep, selected the Brody function because of simplicity of interpretation and ease of estimation. Lewis *et al*. (2002) chose the Gompertz function to analyze the growth curve in Suffolk sheep. They suggested that this model would present desirable properties for a growth function. Goliomytis *et al*. (2006) used the Richards function to investigate the growth potential and carcass characteristics of the Karagouniko sheep from birth to maturity at 720 day of age. Lambe *et al*. (2006) indicated that the Gompertz and Richards models fit the observed data well for body weights of Texel and Scottish Blackface breeds. They used exponential models besides other nonlinear models in their study. Tekel *et al*. (2005) reported that the best growth models in predicting change of body weight of Awassi male lambs was Gompertz and Logistic models.

Akbas *et al*. (1999) stated that Gompertz growth model in Kivircik and Daglic breeds was the best growth model. In addition to the mentioned models in Table 5, they used ten other linear models. McManus *et al*. (2003) and Malhado *et al*. (2008b) determined that the Logistic function would be more suitable than Richards and Brody to adjust the growth curves in Bergamasca sheep and in crossbred Santa Inês×Texel lambs, respectively. This function has also shown the best fit for describing the growth of scrotal circumference in Awassi male lambs (Bilgin *et al*., 2004a). Keskin *et al*. (2009) compared Quadratic, Cubic, Gompertz, and Logistic functions in order to describe the growth of Konya Merino lambs and concluded that the Quadratic and Gompertz models showed the best fit. The variation in results obtained from determination of the most appropriate model can be due to genetic background of the population used in the study and environmental factors, particularly nutritional situation.

In the last few years, growth curves approaches have been used to analyze the growth of farm animals for improvement in their husbandry. Many studies related to modeling of growth curves have been applied to pigs (Wellock *et al*., 2004; Köhn *et al*., 2007), cattle (Nešetřilovà, 2005), and poultry (Schinckel *et al*., 2005; Ersoy *et al*., 2006; Ahmadi and Mottaghitalab, 2007). The growth curves are being used in different aspects of management in producing meat type animals such as designing optimum feeding programs and determining optimum slaughtering age.

They also can be used in evaluating the effect of selection on growth curve parameters and weight at certain ages (Blasco and Gomes, 1993).

## CONCLUSIONS

This study showed that the addition of random effect of mature weight to the Brody fixed effect model resulted in a decrease in the estimates of log-likelihood, *AIC*, and residual variance criteria for both sexes, indicating that the Brody mixed effect model fitted the data better than the corresponding fixed effect model. In conclusion, among the linear models, the polynomial of third order and, among nonlinear models, Brody mixed model were found to best fit the Baluchi sheep growth data. More studies are needed

**Table 5.** Comparison of best nonlinear models in present study and literature[a].

| Reference | Breed | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Present study | Baluchi | × | × | ✓ | × | | | × |
| Akbas *et al*., 1999 | Kivircik | × | ✓ | × | × | × | | |
| | Daglic | × | ✓ | × | × | × | | |
| Bilgin and Esenbuga, 2003 | Morkaraman | × | × | ✓ | | × | | × |
| Daskiran *et al*., 2010 | Norduz | × | × | × | ✓ | × | | |
| Bilgin *et al*., 2004b | Awassi | × | × | ✓ | × | | | ✓ |
| | Morkaraman | × | × | ✓ | × | | | ✓ |
| Eyduran *et al*., 2008 | morkaraman | | ✓ | | × | | × | × |
| | Kivircik | | ✓ | | × | | × | × |
| Gbangboche *et al*., 2008 | West African Dwarf | × | × | ✓ | × | | | |
| Karakus *et al*., 2008 | Norduz | | × | × | ✓ | | | × |
| Keskin *et al*., 2009 | Konya Merino | | ✓ | | × | | | |
| Kum *et al*., 2010 | Norduz | | ✓ | | × | | × | |
| Lambe *et al*., 2006 | Texel | | ✓ | | × | | | ✓ |
| | Scottish Blackface | | ✓ | | × | | | ✓ |
| McManus *et al*., 2003 | Bergamasca | | | × | ✓ | | | × |
| Malhado *et al*., 2008b | Santa Inês×Texel | × | × | × | ✓ | | | × |
| Malhado *et al*., 2009 | Dorper×Morada Nova | × | ✓ | × | ✓ | | | × |
| | Dorper×Rabo Largo | × | ✓ | × | ✓ | | | × |
| | Dorper×Santa Inês | × | ✓ | × | ✓ | | | × |
| Sarmento *et al*., 2006 | Santa Inês | × | ✓ | × | × | | | × |
| Tekel *et al*., 2005 | Awassi | × | ✓ | × | ✓ | × | | |
| Topal *et al*., 2004 | Morkaraman | × | ✓ | × | × | | | |
| | Awassi | ✓ | × | × | × | | | |
| Keskin and Daskiran, 2007 | Norduz kids | | ✓ | | × | | × | |
| Malhado *et al*., 2008a | Anglo-Nubian goats | ✓ | × | × | × | | | × |
| Oliveira *et al*., 2009 | Anglo-Nubian goats | × | × | × | ✓ | | | |

[a] ✓: Selected model; ×: Models used in study; 1: von Bertalanffy; 2: Gompertz; 3: Brody; 4: Logistic; 5: Negative exponential; 6: Monomolecular,7: Richards.

to determine the characteristics of the growth curve until the adulthood age, including other features such as carcass composition and meat quality in Baluchi sheep.

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## مقایسه مدل های ریاضی تشریح کننده رشد در گوسفند بلوچی

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## چکیده

اهداف این تحقیق، تعیین یک مدل ریاضی مناسب تشریح کننده منحنی رشد در گوسفند بلوچی بر اساس رکوردهای ماهیانه وزن زنده از تولد تا یکسالگی و ارزیابی مدلهای رشد غیر خطی مختلط و ثابت بود. مدلهای رشد به ۱۶۶۵۰ رکورد وزن بدن متعلق به ۲۰۷۱ بره بلوچی برازش داده شد. از پنج تابع رشد غیر خطی، ون برتالانفی، گومپرتز، برودی، لجستیک و ریچاردز و دو تابع چند جمله ای خطی استفاده شد. برای مقایسه مدلهای رشد از میانگین مربعات خطا (MSE) و شاخص اطلاعات آکائیک (AIC) استفاده شد. بین مدلهای غیر خطی ثابت، برای هر دو جنس کمترین مقادیر میانگین مربعات خطا و شاخص اطلاعات آکائیک برای تابع برودی به عنوان بهترین مدل غیر خطی محاسبه شد. مدل ثابت و مختلط برودی با هم مقایسه شدند. در مدل مختلط برودی، تنوع بین حیوانات در وزن بلوغ به عنوان اثر تصادفی در نظر گرفته شد. شاخصهای ارزیابی مدل نشان داد که مدل مختلط برودی بهتر از مدل ثابت برودی قادر به برازش داده های وزن است. می توان نتیجه گیری کرد که بین مدلهای خطی، چند جمله ای با درجه برازش سه و بین مدلهای غیر خطی، مدل مختلط برودی بهترین برازش را با داده های رشد گوسفند بلوچی داشت.