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A NEW APPROACH IN SYSTEM RELIABILITY EVALUATION, SHORTEST PATH OF E-NETWORKS *

A. AZARON[1] AND M. MODARRES[2**]

[1]Dept. of Industrial Engineering, University of Bu-Ali-Sina, Hamadan, I. R. of Iran
Email: aazaron@is.dal.ca
[2]Dept. of Industrial Engineering, Sharif University of Technology, Tehran, I. R. of Iran

Abstract – By applying shortest path analysis in stochastic networks, we introduce a new approach to obtain the reliability function of time-dependent systems. We assume that not all elements of the system are set to function from the beginning. Upon the failure of each element of the active path in the reliability graph, the system switches to the next path. Then, the corresponding elements are activated, and consequently, the connection between the input and the output is established. It is also assumed that each element exhibits a constant hazard rate and its lifetime is a random variable with exponential distribution. To evaluate the system reliability, we construct a directed stochastic network called E-network, in which each path corresponds with a minimal cut of the reliability graph. We also prove that the system failure function is equal to the distribution function of the shortest path of E-network. The shortest path of this new constructed network is determined analytically by using continuous time Markov processes.

Keywords – System reliability, standby redundancy, shortest path, Markov processes

1. INTRODUCTION

Many researchers in the second half of the last century have investigated system reliability evaluation and have developed a variety of methods in this regard [1, 2]. Yet, the existing analytical methods are usually constructed on the basis of some assumptions, which are quite restrictive and are not capable of analyzing all real systems. Thus, to calculate the systems reliability with special structures, it is still necessary to design new methods.

In this paper, a new approach is introduced to determine the reliability function of time-dependent systems with standby redundancy. The existing methods for time-dependent systems are developed with the assumption that all elements are set to function concurrently from the beginning. However, this assumption is not true for many real cases. In fact, in practice not all elements are functioning at time zero, but whenever an element fails then another one is activated. To illustrate that this assumption is not true for all real cases, an example is presented in section 4.

To calculate the reliability function of time-dependent systems when all elements are set to function at time zero, it is customary to apply the joint density or distribution functions techniques. The other approach is to apply state-transition models such as Markov chains. The major obstacle in solving these models is the complexity, which arises from the large size of the first order differential equations. For example, a system with n elements, modeled as a Markov process, may require a solution of as many as 2^n first-order differential equations (see Shooman [3] for the details).

In this paper, we consider a time-dependent reliability system with standby redundancy. At the beginning, only the main elements work. No standby element is set to function unless one active element fails. In terms of graphs, at time zero only the elements of the first path are functioning. In other words, the reliability graph works because its input and output are connected through this path. As soon as one element

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**Corresponding author

of this path fails, the system is switched to the second path and consequently all elements of the second path are set to function. This process continues until all connections between the input and output are interrupted, and as a result the system fails.

We assume the lifetime of each element follows an exponential distribution function. Throughout its development, the theory of reliability has been based heavily on the exponential failure law, primarily because of its mathematical tractability. It is the appropriate model for used-good-as-new components like fuses and many other electronic parts because of the memoryless property of the exponential distribution (see Grosh [2] for more details). The objective is to determine the reliability function of this system.

The shortest path analysis of stochastic networks is applied in order to analyze this system. As a matter of fact, we first construct a directed network from the reliability graph of the system. This network is a stochastic one and we call it E-network (or equivalent network). In this network, each path corresponds with a minimal cut of the reliability graph. Then, we obtain the distribution function of the shortest path from the source to the sink node of this E-network by using the method developed by Kulkarni [4]. It is also proved that the mean time to failure of the systems with the standby nature is greater than that of the ordinary systems, in which all elements are set to function concurrently at time zero. Consequently, this system clearly works better compared with the ordinary one. Therefore, what distinguishes our research from the previous ones are the following:

1. We relax the assumption that all elements start working concurrently from the beginning.
2. The method is a new one, on the basis of the shortest path of stochastic E-networks.

For computing the probability between 2 given nodes in the reliability graph of the system, there exists at least 1 operation path, Fishman [5] proposed a Monte Carlo sampling plan, which uses lower and upper bounds to increase its accuracy and efficiency. Manzi et al. [6] provided a detailed, clear exposition of the Fishman method and its extension for computing the global network reliability (probability that the network is connected).

Exact evaluation of system reliability is extremely difficult and sometimes impossible. Once one obtains the expression for the structure function, the system reliability computations become straightforward. Attempts have been made to compute the exact system reliability of complex systems. For example, the algorithm in Aven [7] is based on minimal cut sets. Chaudhuri *et al*. [8] overcame the problem of calculating system reliability in complex systems through a new representation of the structure function of a coherent system and demonstrated that the well-known systems considered state-of-art, follow this new representation. English *et al*. [9] presented a discretizing procedure for reliability prediction of complex systems.

System reliability depends not only on the reliabilities of components in the system, but also on their interactions or the dependencies among them. In recent years, studies on the dependent failure theories have been widely developed. The main elements in research are the common-cause failures in redundant systems. Lin *et al*. [10] described the parallel redundant systems. Lesanovsky [11] proposed a multiple-state Markov model of the system with the dependent components, in which the system is a homogeneous continuous-time Markov process with discrete states. Humphreys and Jenkins [12] summarized developments of techniques for dealing with the dependent failures.

In the area of determining the shortest path of stochastic networks, Martin [13] introduced a method to obtain its distribution function, as well as its expected value. Frank [14] computed the probability that the duration of the shortest path in a stochastic network is smaller than a specific value when the arc lengths are continuous random variables. Mirchandani [15] developed another method with the advantage that it is not necessary to solve multiple integrals. However, this method works only if the arc lengths are discrete random variables. Kulkarni [4] presented an algorithm for obtaining the distribution function of the shortest path in directed stochastic networks, when the arc lengths are independent random variables with exponential distributions. This method is constructed in the framework of continuous time Markov processes. Sigal *et al.* [16] used the uniformly directed cuts in their analysis of shortest paths.

The advantage of the proposed algorithm from the point of management implication roots is in its assumptions. This new analytical approach was developed for obtaining the reliability function of time-dependent systems by considering the standby nature of the structure and it is not required that all elements start working concurrently at time zero. Since we relax this restrictive assumption, the proposed approach can be applied for many real world reliability systems, which can not be solved by the existing methods.

The remainder of this paper is organized in the following way: In section 2, we describe the reliability graphs and introduce the equivalent E-networks, which is the basis of the proposed model. In section 3, a method for obtaining the distribution function of shortest path in stochastic networks is presented. In section 4 the method is illustrated through a numerical example, and finally we draw the conclusion of the paper in section 5.

2. RELIABILITY GRAPHS

A very efficient method to compute the reliability of a system is to express it as a graph. Reliability graphs consist of a set of arcs. Each arc represents an element of the system, while the nodes of the graph tie the arcs together and form the structure. Corresponding with the *i*th arc of the reliability graph, i=1,2,…,n, there is an exponential random variable T_i , with parameter λ_i, which represents the lifetime of this element. These random variables are independent due to the fact that the elements work independently. If a system has i path, denoted by P_1, P_2,…, P_i, then it has a connection between its input and output nodes, if at least one path is intact.

By definition, a cut of the graph is a set of arcs, which interrupts all connections between input and output when removed from the graph. A minimal cut is a cut with the minimum number of terms. Each system failure can be represented by the removal of at least one minimal cut from the graph.

We assume the system functions by its real nature. In other words, not all of its elements are set to function at time zero. Initially, only the elements of the first path of the reliability graph work. Upon failure of one element of this path, the system is switched to the next path and the connection between input and output is established through this second path. This process continues until the connection between the input and the output of the graph no longer exists. In that case, the system fails.

Notation:

T_i **:** lifetime of the *i*th element of the system;
T **:** system lifetime;
C_j **:** *j*th minimal cut of the reliability graph, j=1,2,…,m;
X_j **:** failure time of the *j*th minimal cut of the reliability graph.

Lemma 1. For j=1,2,…, m, the following relation holds

$$X_j = \sum_{i \in C_j} T_i \qquad (1)$$

Proof. Taking into account the standby nature of the structure, upon failure of each element of the jth minimal cut, the system is switched to the next path. Consequently, at any moment only one element of this minimal cut is activated. Therefore, the failure time of this cut is the sum of all its elements.

Equivalent network: Now, we construct a new directed network called equivalent network (or E- network). There are m paths in this network, in which the jth path of this directed network is corresponding with the jth minimal cut of the reliability graph of the system, j=1,2,…, m. Clearly, by lemma 1 the length of each path is equal to the failure time of the corresponding cut. The following rule describes how to construct E-network:

Rule 1. Arc i belongs to the *j*th path of the directed network, if and only if $i \in C_j$.

Let $F(t)$ represent the distribution function of the shortest path (from the source to the sink node) in E-network, and $R(t)$ represent the reliability function of the system. The relation between $F(t)$ and $R(t)$ can be expressed by the following theorem:

Theorem 1. The system lifetime, T, is a random variable as follows:

$$T = \min_{j=1,2,\dots,m} \{X_j\} \tag{2}$$

Consequently

$$R(t)=1-F(t) \tag{3}$$

Proof. Upon the failure of the first minimal cut of the reliability graph of the system, all connections between input and output are interrupted, and consequently the system fails. Therefore, the lifetime of the system is equal to the failure time of the first minimal cut which results in (2). Relation (3) follows from the definition of $R(t)$ and $F(t)$. Later, in section 3, we present an analytical method to obtain the distribution function of the shortest path of E-network.

The following lemma shows the mean time to failure in the systems with the standby structure when is greater than that of a system where all elements are concurrently set to function at time zero.

Lemma 2. The system lifetime with the standby structure is stochastically greater than that of the system with parallel structure.

Proof. Let $R_i(t)$ and $\overline{R_i(t)}$ represent the reliability function of the ith element of the system at time t, with standby and parallel structure, respectively. On the other hand, let us also assume this element starts functioning at time x_i, when the system is considered as a standby. Then

$$R_i(t) = P[T_i > t - x_i]$$

and

$$\overline{R_i(t)} = P[T_i > t]$$

It is clearly concluded that

$$R_i(t) > \overline{R_i(t)} \tag{4}$$

Since the reliability of each element of the standby system is greater than that of the parallel structure, then the desired result is obtained.

Consequently, from lemma 2, it is also concluded that the mean time to failure in the systems with standby structure is greater than that of ordinary systems.

3. DISTRIBUTION FUNCTION OF SHORTEST PATH IN STOCHASTIC NETWORKS

In this section, we present an analytical method for obtaining the distribution function of the shortest path of E-network or, in fact, the distribution function of the path from the source to the sink node of a directed stochastic network, in which arc lengths are exponentially distributed. To do that, we need to apply a shortest path algorithm for stochastic networks.

Although there are many simple algorithms for solving the shortest path problem in deterministic networks, there are not so many analytical algorithms for this problem when the arc lengths are random variables. Clearly, the nature of stochastic networks causes the algorithms of the shortest path to become much too complicated. Consequently, we apply the method developed by Kulkarni [4].

Let $G=(V,A)$ be a directed network in which V and A represent the set of nodes and arcs of the network, respectively. We also assume s and t represent the source and the sink nodes of this network, respectively. The length of arc $(u,v) \in A$ is indicated by $T_{(u,v)}$, which is an exponential random variable with parameter $\lambda_{(u,v)}$.

For analyzing the stochastic process properly, it is convenient to visualize the network as a communication one. In this network, the nodes are considered as stations capable of receiving and transmitting messages and arcs as one-way communication links connecting pairs of nodes. As soon as a node receives a message over one of the incoming arcs, it transmits it along all the outgoing arcs and then disables itself. Now, let *X(t)* be the set of all disable nodes at time *t*.

Definition 1: To describe the evolution of the stochastic process $\{X(t), t \geq 0\}$, for each $X \subset V$, where $s \in X$ and $t \in \overline{X} = V - X$, we define the following sets:

1. $\overline{X}_1 \subset \overline{X}$, the set of nodes not included in *X* with the property that each path which connects any node of this set to the sink node *t*, contains at least one member of *X*.
2. $S(X)=X \cup \overline{X}_1$.

Example 1. In the network depicted in Fig. 1, if we consider $X=\{1,2\}$, then $\overline{X}_1 = \phi$ and $S(X)=\{1,2\}$. However, if we consider X={1,4}, then the only path that connects node {2} to node {5} passes through node {4}, but node {3} does not belong to $\overline{X}_1$ because it can be connected to {5} directly and the path *3-5* does not include any nodes of *X*. Therefore, $\overline{X}_1=\{2\}$, and $S(X)=\{1,2,4\}$.

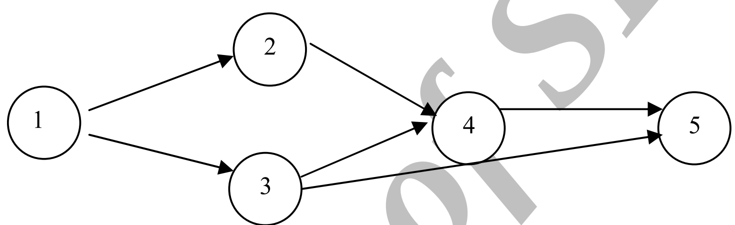

Fig. 1. Graph of example 1

Definition 2:

$$\Omega = \{X \subset V / X = S(X)\} \tag{5}$$

$$\Omega^* = \Omega \cup V \tag{6}$$

In the above example, $\Omega^*=\{(1),(1,2),(1,3),(1,2,3),(1,2,4),(1,2,3,4),(1,2,3,4,5)\}$. The first six elements of Ω^* are the members of Ω, while the last element of this set is *V*.

Definition 3:

If $X \subset V$ such that $s \in X$ and $t \in \overline{X}$, then a cut is defined as

$$C(X,\overline{X}) = \{(u,v) \in A / u \in X, v \in \overline{X}\} \tag{7}$$

There is a unique minimal cut contained in $C(X,\overline{X})$, denoted by *C(X)*. If $X \in \Omega$, then

$$C(X,\overline{X}) = C(X)$$

It can be shown that $\{X(t), t \geq 0\}$ is a continuous time Markov process with state space Ω^*. The infinitesimal generator matrix of this process is denoted by $Q=[q(X,Y)]$ $(X,Y \in \Omega^*)$, (see Kulkarni [4] for details), where

$$q(X,Y)= - \begin{cases} \sum_{(u,v) \in C(X)} \lambda_{(u,v)} & if\ Y = S(X \cup \{v\}) \\ -\sum_{(u,v) \in C(X)} \lambda_{(u,v)} & if\ Y = X, \\ 0 & otherwise \end{cases} \tag{8}$$

We sort the states of Ω^* such that if $s_1 < s_2$, then s_1 comes before s_2. Consequently, the infinitesimal generator matrix turns out to be an upper triangular one. Thus, the state which is denoted by *1* represents the source node, while *V* is denoted by *N*.

Let *T* represent the length of the shortest path in this E-network. Then, it is clear that

$$T=min\ \{t>0: X(t)=N/X(0)=1\} \tag{9}$$

Therefore, the length of the shortest path in the network is equal to the time until $\{X(t), t \geq 0\}$ gets absorbed in the final state N, starting from state *1*. The objective is to compute $F(t)=P\{T \leq t\}$ or the distribution function of the shortest path in the stochastic network.

Chapman-Kolmogorov backward or forward equations can be applied to compute $F(t)$. We define

$$P_i(t)=P\{X(t)=N/X(0)=i\} \quad i=1,2,\ldots,N \tag{10}$$

Therefore, $F(t)=P_1(t)$. By using the backward algorithm, the system of differential equations for the vector $P(t)=[P_1(t), P_2(t),..., P_N(t)]^T$ is given by

$$\dot{P}(t)=Q \cdot P(t), \qquad P(0)=[0,0,\ldots,1]^T \tag{11}$$

In (11), $P(t)$ represents the state vector of the system and Q is the infinitesimal generator matrix of the stochastic process $\{X(t), t \geq 0\}$. By taking advantage of the upper triangular nature of Q, the differential Eqs. (11) can be easily solved. An analytical or a numerical method can be applied to solve these equations.

After computing the distribution function of the shortest path in this directed stochastic E-network, $F(t)$, we can compute the reliability function of the system from Eq. (3).

4. NUMERICAL EXAMPLE

To operate the accounting activities of a firm, either one computer or one calculator is needed. The calculator needs one battery to do the required operations. However, there are two batteries available in the system to function as standby. At the beginning, the system may start with the computer. If it fails, then the calculator with one battery does the necessary operations. In that case, if the calculator fails so does the system. However, if the battery fails, the calculator works with the standby one. In fact, if either calculator or the second battery fails, the operation comes to the end.

This system can be represented by a reliability graph as depicted in Fig. 2, in which arc *1* represents the lifetime of the computer, arc *2* represents the lifetime of the calculator, arc *3* and arc *4* represent the lifetime of the first and second battery, respectively. Let T_i, $i=1,2,3,4$, be a random variable which represents the lifetime of the ith arc. Furthermore, we assume that T_i, $i=1,2,3,4$, are independent random variables with exponential distributions and the following parameters:

$$\lambda_1=2, \quad \lambda_2=3, \quad \lambda_3=2, \quad \lambda_4=1$$

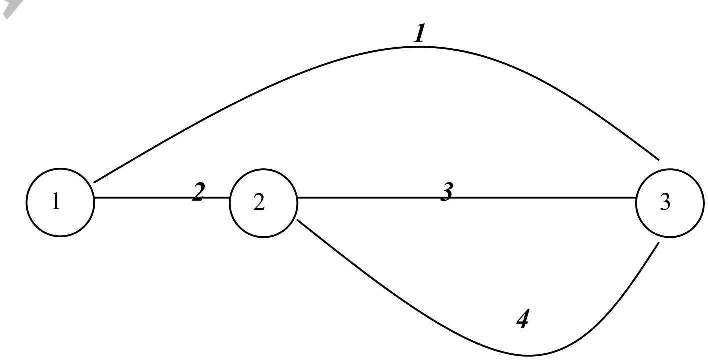

Fig. 2. Reliability graph of the system

The objective is to obtain the reliability function of this system. Two minimal cut sets of the reliability graph are

$$C_1=(1,2), \qquad C_2=(1,3,4) \tag{12}$$

From lemma 1, the failure times of the minimal cuts are

$$X_1=T_1+T_2, \qquad X_2=T_1+T_3+T_4 \tag{13}$$

Therefore, we construct the directed E-network following Rule 1, as depicted in Fig. 3. This network has two paths

$$P_1=(1,2), \qquad P_2=(1,3,4) \tag{14}$$

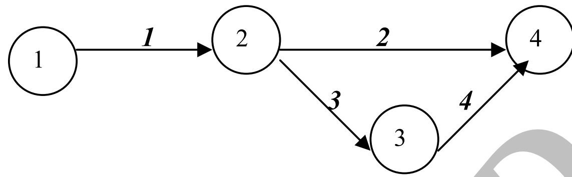

Fig. 3. E-network corresponding to the reliability graph

Now, we compute the distribution function of the shortest path from node 1 to node 4 in the directed E-network. The indicated stochastic process $\{X(t), t \geq 0\}$ has 4 states in the order of $\Omega^*=\{(1),(1,2),(1,2,3),(1,2,3,4)\}$. Table 1 shows Q, the infinitesimal generator matrix.

Table 1. Matrix Q corresponding to the numerical example

State	1	2	3	4
1	-2	2	0	0
2	0	-5	2	3
3	0	0	-4	4
4	0	0	0	0

Then, we solve these related differential equations

$$\dot{P}_4(t)=0, \quad \dot{P}_3(t)=-4P_3(t)+4P_4(t), \quad \dot{P}_2(t)=-5P_2(t)+2P_3(t)+3P_4(t)$$
$$\dot{P}_1(t)=-2P_1(t)+2P_2(t), \quad P_i(0)=0 \quad i=1,2,3, \quad P_4(0)=1 \tag{15}$$

Finally, $F(t)=P_1(t)$, and consequently $R(t)$ or the reliability function of the system is obtained as follows:

$$F(t)=1+2e^{-4t}-0.6667e^{-5t}-2.3333e^{-2t}$$
$$R(t)=1-F(t)=-2e^{-4t}+0.6667e^{-5t}+2.3333e^{-2t} \tag{16}$$

We can also compute the mean time to failure of the system from Eq. (17)

$$MTTF=\int_0^\infty R(t)dt=0.8 \tag{17}$$

We can compare this quantity with the mean time to failure of the ordinary system in which four elements are set to function at time zero. For a four-element system, the states are as follows:

$$\begin{aligned}
&S_0=Y_1Y_2Y_3Y_4, \quad S_1=\overline{Y_1}Y_2Y_3Y_4, \quad S_2=Y_1\overline{Y_2}Y_3Y_4, \quad S_3=Y_1Y_2\overline{Y_3}Y_4,\\
&S_4=Y_1Y_2Y_3\overline{Y_4}, \quad S_5=\overline{Y_1}\overline{Y_2}Y_3Y_4, \quad S_6=\overline{Y_1}Y_2\overline{Y_3}Y_4, \quad S_7=\overline{Y_1}Y_2Y_3\overline{Y_4},\\
&S_8=Y_1\overline{Y_2}\overline{Y_3}Y_4, \quad S_9=Y_1\overline{Y_2}Y_3\overline{Y_4}, \quad S_{10}=Y_1Y_2\overline{Y_3}\overline{Y_4}, \quad S_{11}=\overline{Y_1}\overline{Y_2}\overline{Y_3}Y_4,\\
&S_{12}=\overline{Y_1}\overline{Y_2}Y_3\overline{Y_4}, \quad S_{13}=\overline{Y_1}Y_2\overline{Y_3}\overline{Y_4}, \quad S_{14}=Y_1\overline{Y_2}\overline{Y_3}\overline{Y_4}, \quad S_{15}=\overline{Y_1}\overline{Y_2}\overline{Y_3}\overline{Y_4}
\end{aligned} \tag{18}$$

where Y_i, $i=1,2,3,4$, means that the ith element works, and $\overline{Y_i}$ means that the ith element fails. $\overline{R(t)}$, or the reliability function of this system is given by

$$\overline{R(t)}=P_{S_0}(t)+P_{S_1}(t)+P_{S_2}(t)+P_{S_3}(t)+P_{S_4}(t)+P_{S_6}(t)+P_{S_7}(t)+P_{S_8}(t)+P_{S_9}(t)+P_{S_{10}}(t)+P_{S_{14}}(t) \tag{19}$$

The system of differential equations for the vector $P(t)=[P_{S_O}(t), P_{S_1}(t),\ldots,P_{S_{15}}(t)]^T$ is given by

$$\dot{P}(t)=P(t)\cdot \overline{Q}, \qquad P(0)=[1,0,\ldots,0]^T \tag{20}$$

where, $\overline{Q}$ has 16 states from S_0 to S_{15}. Table 2 shows matrix $\overline{Q}$.

Table 2. Matrix $\overline{Q}$ corresponding to the numerical example

State	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_O	-8	2	3	2	1	0	0	0	0	0	0	0	0	0	0	0
S_1	0	-6	0	0	0	3	2	1	0	0	0	0	0	0	0	0
S_2	0	0	-5	0	0	2	0	0	2	1	0	0	0	0	0	0
S_3	0	0	0	-6	0	0	2	0	3	0	1	0	0	0	0	0
S_4	0	0	0	0	-7	0	0	2	0	3	2	0	0	0	0	0
S_5	0	0	0	0	0	-3	0	0	0	0	0	2	1	0	0	0
S_6	0	0	0	0	0	0	-4	0	0	0	0	3	0	1	0	0
S_7	0	0	0	0	0	0	0	-5	0	0	0	0	3	2	0	0
S_8	0	0	0	0	0	0	0	0	-3	0	0	2	0	0	1	0
S_9	0	0	0	0	0	0	0	0	0	-4	0	0	2	0	2	0
S_{10}	0	0	0	0	0	0	0	0	0	0	-5	0	0	2	3	0
S_{11}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1
S_{12}	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	2
S_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	3
S_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2
S_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Finally, $\overline{R(t)}$ is obtained in this manner

$$\overline{R(t)}=e^{-2t}+e^{-4t}+e^{-5t}-2e^{-6t}-e^{-7t}+e^{-8t} \tag{21}$$

The mean time to the failure of this system is computed from Eq. (22).

$$\overline{MTTF}=\int_0^\infty \overline{R(t)}dt=0.599 \tag{22}$$

It is clear that the mean time to failure of this system is smaller than that of the previous system with the standby structure (about 33 percent) as proved.

5. CONCLUSION

In this paper, we developed a new approach for obtaining the reliability function of time-dependent systems by considering the standby nature of the structure. The lifetime of all elements is assumed to be independent random variables and exponentially distributed.

In this type of system, all elements are not set to function concurrently from the beginning. It is assumed that only the elements of the first path of the reliability graph of the system work at time zero. Upon failure of each element in one path, the system is switched to the next path. The system works until all connections between the input and output are interrupted.

We introduced a new directed network, called E-network, constructed on the basis of the reliability graph. In this stochastic network each path is in correspondence with a minimal cut of the reliability graph of the system. We also proved the distribution function of the shortest path of E-network is equal to the distribution function of the system lifetime. Therefore, the reliability function of the system should be equal to one minus this distribution function.

We also developed an algorithm based on Kulkarni's method for obtaining the distribution function of shortest path in directed stochastic networks, in which the arc lengths are independent random variables with exponential distributions. In this section, we constructed a continuous time Markov chain with a single absorbing state from the directed network such that the time until absorption into this absorbing state starting from the initial state is equal to the length of the shortest path in the network.

The limitation of the proposed method is that the state space of the continuous time Markov process can grow exponentially with the network size. As a worst case example, for a complete constructed network with l nodes and l(l-1) arcs, the size of the state space would be $2^{l-2}+1$. One must also note that for very large networks, any method of producing reasonably accurate answers will be prohibitively expensive.

If we compare the systems with the standby structure with the ordinary systems, we can conclude these results:

1. The mean time to failure of the systems with the standby structure is greater than the ordinary systems.
2. In the ordinary systems with *n* elements, we require a solution of 2^n first-order differential equations, but in the systems with the standby structure, the differential equations which we need to solve are at most $2^{n-2}+1$, which is less than the ordinary systems.

This model can be extended to the time-dependent systems when the elements have non-constant hazard rates. It is also possible to optimize the system in which the hazard rates of the elements can be controlled. If the purchasing price of each element depends on its lifetime, then it is possible to maximize the mean time to failure of the system with respect to the total purchasing costs of the elements.

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