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SOLVING THE LONG-TERM HYDRO-THERMAL COORDINATION PROBLEM WITH A SPECIAL GENETIC ALGORITHM [*]

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Abstract – A special hybrid genetic algorithm (GA) is designed to solve the long-term coordination of hydro-thermal power systems with cascaded reservoirs and stochastic inflows. Since decision variables are continuous, in the proposed GA we employ real number rather than binary encoding. To create superior children we introduce dynamic tuning of the weights of operators. An exponential normalization is also developed such that better chromosomes have more chance to reproduce. To test the proposed method, 16 GAs are investigated which differ based on real or binary encoding, dynamic tuning or fixed weights for operators, inverse or exponential normalization and mixed or pure random initial populations. By applying the data of a real power system, the performance of these algorithms are compared. We also compare the performance of the proposed GA with that of the conventional Lagrangian relaxation method. The results show that the proposed GA gives promising performance for situations, which if not impossible, are very difficult to handle using conventional optimization methods.

Keywords – Hydro-thermal power systems, cascaded reservoirs, genetic algorithms, dynamic tuning

1. INTRODUCTION

The role of the hydro-thermal coordination (HTC) problem is to determine the contribution of each hydro and thermal plant generation to satisfy the demand in a horizon of T periods. The objective is to minimize the total system costs subject to system and unit constraints.

The importance of HTC in significantly reducing system operating costs, as well as its inherent complexity, has motivated extensive research efforts to develop effective solution methods for the problem [1-3]. The HTC problem is, in general, a non-linear, non-convex, dynamic and large-scale problem. In long-term studies additional complexity arises due to the fact that the reservoir inflows are stochastic. Due to the complexity of the problem, particularly in long-term studies, most of the methods reported in the literature are based on some kind of simplification or approximations of the models, such as linearization, convexification, aggregation of hydro power plants and deterministic treatment of stochastic parameters [4-7].

In order to provide flexibility for incorporation of more practical aspects of system operations, we propose a special GA for solving the problem. In our proposed GA, the weights of GA operators are not constant; rather they are determined based on a competition among operators to create better children. Since decision variables of the HTC problem are continuous, real number encoding is employed and a double creep mutation operator is proposed, which randomly sweeps along opposite directions in the solution space. Also, in order to give better chromosomes a higher chance to reproduce, an exponential normalization on fitness values is employed. Furthermore, since the thermal part of the system is solved by an effective nonlinear programming technique, the proposed GA is hybrid. Therefore, what distinguishes this work from the others is the proposal of a special GA with the above characteristics together with a complete comparison of the performance of the proposed GA with those of various binary GAs and conventional Lagrangian relaxation method for solving the long-term HTC problem.

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The two most common methods for solving the long-term HTC problem are the Lagrangian relaxation (LR) [8], and L-shaped methods, which are often referred to as stochastic Benders decomposition [9, 10]. Also stochastic dynamic programming was applied in earlier studies of the HTC problem, which is then extended in [4]. However, its application is practically limited for large-scale systems since it involves discretization of the state variables, which leads to the explosive increase of the number of states commonly known as "curse of dimensionality". This problem has been alleviated to some extent by [11], in which a stochastic dual dynamic programming (SDDP) is introduced and the expected cost-to-go-function is approximated by piecewise linear functions. However in non-linear programming, the assumption of a piecewise linear function for the dual objective as used in [11] can not be guaranteed and therefore the application of [11] is limited to linear cases.

The LR technique is reported in the literature as one of the most successful approaches for solving the short-term HTC problem, where reservoir inflows are deterministic [12-15]. Nevertheless its application in long-term studies, particularly for the case of cascaded reservoirs with stochastic inflows is limited. This is mainly due to the fact that the effectiveness of LR technique stems from its ability to decompose the original problem into a number of subproblems for each hydro and thermal plant. However, when hydro plants are cascaded (the discharge and spillage of upstream hydro plants enters the reservoir of a downstream hydro plant), decomposition of a hydro subproblem into a number of problems for each hydro plant becomes impossible. Therefore, other methods such as stochastic dynamic programming with successive approximations should be applied for the hydro part of the system, where their efficiency decreases rapidly as the number of hydro plants in the system increases. Furthermore, finding an effective method for solving the dual problem (updating Lagrange multipliers) is still under investigation and most effective techniques reported in the literature such as the variable metric method and distance of optimality, are designed for deterministic cases [13, 16].

Stochastic Benders decomposition has become increasingly popular for solving stochastic multistage linear programming problems. This technique applied in [5, 17, 18], obtains the optimal solution by iteratively improving lower and upper bounds on the objective function. Recently an inexact cut algorithm for stochastic Benders decomposition has been proposed which has shown its effectiveness for the HTC problem, however, this improvement is mainly developed for the linear case [19]. The main difficulty with this technique is that the problem should be linearized in order to apply the method efficiently. Although the Benders decomposition has been generalized to nonlinear cases [20], no successful application of generalized Benders decomposition for the HTC problem, particularly for long-term studies, has been reported.

A combination of LR technique and cutting plane method for solving the HTC problem is introduced in [12]. This technique, which is an improvement of [21], obtains the solution of the dual problem for LR by successively shrinking and enlarging the feasible region, and has given promising results for the nonlinear case. However, its application in the case of stochastic inflows is not reported in the literature.

Concerning the above-mentioned limitations in the existing analytical methods for solving the HTC problem with cascaded reservoirs and stochastic inflows; recently there has been an increasing interest applying meta-heuristic methods, especially genetic algorithms (GAs). Nevertheless, most of the GA applications for the HTC problem reported in the literature belong to short term studies [22-24]. In [25], a hybrid genetic algorithm-dynamic program is developed for unit commitment of a hydro-thermal power system with pumped storage plants. However, in those applications and in conventional GAs, decision variables are represented as a set of binary strings. This representation technique causes several difficulties; in particular is the increased probability of premature convergence due to inherent discretization of the problem [26]. In recent years the trend has been toward employing GAs with real number encoding [27]. In this paper we will show that real encoded GA is both more accurate as compared with binary GA, and obtains the optimal (near optimal) solution more rapidly as compared with the classic LR method.

The remainder of the paper is organized as follows: In section 2, a definition of the HTC problem and its mathematical model is presented. Then in section 3, characteristics of both discrete and continuous GAs used in this paper are given. Since GAs are not robust algorithms in terms of their control parameters, a dynamic tuning scheme is proposed which employs the idea of competition among operators for tuning their weights. In section 4, application of three methods (LR, Real and Binary GAs) for a real hydro- thermal power system and a comparison of the results for various cases is illustrated. The LR technique is briefly described in the appendix.

2. DEFINITION AND FORMULATION OF HTC PROBLEM

The purpose of solving a a long-term HTC problem is to determine generation levels of hydro and thermal plants in a horizon of T periods (months) such that the expected total system cost is minimized, while system and unit constraints are satisfied. The power system consists of M thermal and N hydro power plants. Hydro power plants have multi-purpose reservoirs, which perform other functions such as water supply and flood control. The decision variables are generation levels of hydro and thermal plants. However, it is a common practice to consider turbine discharges of hydro plants as decision variables for hydro subsystems. The hydro power plants are cascaded; i.e. the output of a reservoir is the input of one or more subsequent reservoirs. Since the head and volume of reservoirs are related through some non-convex functions, it is implied that the problem is also a non- convex programming. The inflow of water into the reservoirs of hydro plants is assumed to be stochastic. The objective is to minimize the total expected cost of the system.

a) The model

The problem is represented by a mathematical model as follows:

$$\text{Minimize} \quad E\left[\sum_{m=1}^{M}\sum_{t=1}^{T} GC_m(\mathrm{x}_{mt}) + \sum_{n=1}^{N} TC_n(\mathrm{v}_{n,T+1})\right] \tag{1}$$

subject to

$$\mathrm{v}_{n,t+1} = \mathrm{v}_{nt} + \mathrm{Q}_{nt} + \sum_{k\in UP_n}\left(y_{kt} + s_{kt}\right) - \mathrm{y}_{nt} - \mathrm{s}_{nt} \qquad n = 1,\ldots,N; \quad t = 1,\ldots,T \tag{2}$$

$$\underline{x_{mt}} \le x_{mt} \le \overline{x_{mt}} \qquad m = 1,\ldots,M; \quad t = 1,\ldots,T \tag{3}$$

$$\underline{y_{nt}} \le y_{nt} \le \overline{y_{nt}} \qquad n = 1,\ldots,N; \quad t = 1,\ldots,T \tag{4}$$

$$\underline{u_{nt}} \le u_{nt} \le \overline{u_{nt}} \qquad n = 1,\ldots,N; \quad t = 1,\ldots,T \tag{5}$$

$$\underline{v_{nt}} \le v_{nt} \le \overline{v_{nt}} \qquad n = 1,\ldots,N; \quad t = 1,\ldots,T \tag{6}$$

$$\sum_{m=1}^{M}\mathrm{x}_{mt} + \sum_{n=1}^{N}\mathrm{u}_{nt} = D_t \qquad t = 1,\ldots,T \tag{7}$$

where E[·] represents the expected value with respect to reservoir inflows.

b) The objective function and constraints

In this model the objective function consists of two parts. The first and second term of the objective function represent the following costs, respectively:

a) Cost of energy generation by thermal power plants.

b) Terminal cost of reservoirs.

No cost is associated with energy generation of hydro power plants in the objective function, since it is almost expense free (except the fixed cost, which is independent of the output level). The generating cost of a thermal power plant is approximated by a quadratic function with respect to its energy output x as follows:

$$\mathrm{GC(x)} = \mathrm{ax}^2 + \mathrm{bx} + \mathrm{c}$$

where, a, b and c are constant parameters.

The second term of the objective function penalizes the deviations from the desired volume in the last period and is mainly used to prevent excessive usage of water during the last period. To understand why this cost function must be included in the objective function one should note that the excessive usage of water of a reservoir in a period results in less hydro energy generation capability in the next period. In this case, part of the demand in the next period must be satisfied by more expensive thermal generation. Clearly, the optimization model determines the optimal values for turbine discharges such that the total expected cost is minimized. However, if this cost function is not considered then the model forces the reservoir content in the last period to be used completely. Thus, it is necessary to include a penalty cost function in the objective function in order to preserve the water content of reservoirs at some desired level in the last period. To see the general structure of this cost function the reader is referred to [4].

Constraint (2) maintains the balance between water inflow and water outflow of reservoirs, or in fact, the conservation of water in reservoirs. There are bounds on the maximum and minimum allowable generation for thermal and hydro power plants, represented by (3) and (5). On the other hand, as mentioned before, the reservoirs are multi-purpose and the other requirements such as flood control, supply of water for irrigation, and domestic consumption must also be satisfied. Thus, a lower as well as an upper bound on the turbine discharge in each period is considered by constraint (4) to ensure the supply of water and satisfaction of the maximum allowable release from each one of the turbines. The energy output of hydro power plant **n** in period **t**, i.e. u_{nt}, is a function of the discharge from the turbine as well as the average head in period **t**. It is assumed that the water inflow to a reservoir in period **t** is independent of inflow to the other ones, but it depends on its inflow in period **t**-1. More specifically, water inflow to a reservoir follows a Markov Chain pattern [4]. Concerning flood control, it should be noted that determining maximum and minimum allowable values of reservoir volume in each period is usually performed based on a given risk level for flood control and the results are provided as a set of values which are known as "reservoir rule curves". Therefore, lower and upper bounds on the volume of reservoirs in each period are determined by (6). Constraint (7) guarantees the energy balance of each period and makes the demand and generated energy to be equal.

At this point, it is necessary to explain why a system with cascaded reservoirs cannot be decomposed similarly to what we do for non-cascaded reservoirs systems. The main difference between two systems roots in the water balance, constraint (2) of the model. For non-cascaded reservoir systems this constraint is as follows:

$$v_{n,t+1} = v_{nt} + Q_{nt} - y_{nt} - s_{nt}, \quad n = 1,...,N; \quad t = 1,...,T$$

It can be seen that in the mathematical model of section 2.2 constraints 3, 4, 5 and 6 all have a single variable and therefore each can be related to a single plant. Furthermore, when constraint 7 is transferred to the objective function and the dual problem is formed (shown in appendix B of the paper), the objective function can also be decomposed into separable terms for each plant (also shown in appendix B). However, when reservoirs are not cascaded the water balance constraint has the simple form shown above which contains only the variables of the single plant n (for $n=1,...,N$). Hence the LR technique can decompose the original problem into M thermal and N hydro subproblems, which are independent of each other, or in fact each subproblem obtained by decomposition can be solved independent of the other subproblems. This is due to the fact that both the objective function and constraints of each hydro and thermal subproblem depend only on the decision variables for that particular hydro and thermal plant. On the other hand, for cascaded systems in constraint (2), for reservoir n, the decision variables for all upstream reservoirs (i.e. $\sum_{k \in UP_n} y_{kt}$) are included, which prevents a complete decomposition of the hydro subproblem into $\boldsymbol{N}$ independent subproblems for each hydro plant.

3. THE PROPOSED ALGORITHM

a) Genetic Algorithms (GAs)

Genetic algorithms belong to the class of random search or meta-heuristic methods. In these algorithms the search process is inspired by mechanisms of natural evolution. One advantage of GA is that it does not require all of the stringent requirements such as convexity or differentiability of the objective function or constraints of the problem, while many classical optimization methods require these features on problem functions. It is also shown that if GAs are employed, then the probability of finding a solution close to global optimum for non-convex problems is much greater than that of conventional techniques [28]. Furthermore, for very complex problems it is usually quite difficult to obtain an optimal solution by applying conventional techniques, if not impossible. These reasons have made GA an attractive solution procedure for many practical problems.

Holland, who introduced the concept of GAs, essentially used binary strings called *chromosomes* as the basic building blocks of the algorithm [29]. Each chromosome in GA is a solution for the problem under consideration and may contain one or more bits possessing values that are identical with those in the optimal solution. These certain bit positions are, in fact, good features of that particular chromosome and provide the basic building blocks in the search for an optimal solution. Holland calls such a building block a schema [29]. He showed that GAs essentially manipulate schemata in their search for an optimal solution through some operators. The most popular GA operators are called crossover and mutation. The crossover, which is an extremely important operator of GA, allows chromosomes to exchange their genetic material with each other. By mutation operator, one may change genetic material in bit positions of a single chromosome randomly.

As mentioned, in the early stages of development of GAs the elements of chromosomes were binary strings. However, since in conventional optimization techniques a solution is usually identified by real numbers, in order to apply GAs the solution has to be transformed (encoded) into binary strings. To overcome the difficulties caused by using binary chromosomes, some researchers developed GAs which use real numbers to represent decision variables. These are called real encoded or continuous GAs as compared with binary encoded or discrete GAs. There has been considerable debate regarding the relative merits of binary and real encoding of chromosomes, [30].

While binary encoding has a number of advantages over real encoding, some drawbacks are also seen. In order to discuss some of the main advantages and disadvantages of binary GAs, we will give a brief description of the basic concepts of GAs [31]. Initially note that concerning GA operators, the binary encoding has the main advantage that a schema is represented with the highest number of possible positions on a string. That is, binary encoding provides the highest number of hyperplane partitions available in the solution space for schema processing, but it also has the following drawbacks:

When binary GAs are used to find the optimal solution of a problem with continuous variables that are subject to lower and upper bounds on them, crossover operation on two feasible chromosomes (solutions) may result in infeasible solutions. Consider an optimization problem with a single variable x such that $3 \le x \le 11$. If four bits are used to encode x, then application of one point crossover to feasible solutions 4 and 8 would result in infeasible solutions as shown in Fig. 1.

When the number of variables as well as the number of bits used to encode them increases, this problem becomes more severe. However as will be seen, this problem never occurs with real number encoding, since crossover operator only exchanges values which are within upper and lower bounds. This difficulty also occurs with mutation operator for binary encoding more frequently than with mutation operator proposed for real number encoding described in the next section.

Parent (a) = 8 : 1 | 0 0 0 ⇒ **Child (a) = 12 : 1 1 0 0**
Parent (b) = 4 : 0 | 1 0 0 **Child (b) = 0 : 0 0 0 0** } ⇒ **(infeasible)**
Crossover point

Fig. 1. Illustration of infeasible solution generation by binary crossover

The second difficulty caused by binary encoding results from its inherent procedure of converting real numbers into discrete ones. The accuracy of solutions obtained by GA depends on the number of bits used to represent decision variables. Decreasing the number of bits may cause premature convergence, while increasing those results in more computational time. Although this problem has been alleviated by multiple step resolution method to some extent [23], it will not happen by real number encoding. Finally, since users need to interpret the results of optimization expressed in real numbers, some additional computation is required for this purpose if binary GA is employed, which will increase the overall computation time.

These issues have encouraged some researchers to use real number encoding [27, 32]. As Davis reports, performance theorems for GAs with real number encoding have also been proved, which are similar to Schema theorem for binary encoding, [32]. In this paper we will compare the performance of binary (discrete) GA with a special real (continuous) encoded GA for solving long-term HTC problem. In both GAs, the solution of thermal subsystem is obtained by using an effective nonlinear programming method, thus both GAs are hybrid GAs.

b) Characteristics of the proposed hybrid GAs

1. Encoding technique: In our real encoded GA, each chromosome is composed of N·T real numbers as shown in Fig. 2. Each element of chromosome, i.e. y_{nt}, is the discharge of reservoir **n** in period **t** as defined before. The elements consist of the discharges of N hydro power plants for the next T periods. For binary GA, chromosomes have the same structure, but each real number is replaced by a string of 10 bits, therefore in binary GA each chromosome has 10·N·T elements.

$y_{11}\ y_{12} \dots y_{1,T}$	$y_{21}\ y_{22} \dots y_{2,T}$	…	$y_{N1}\ y_{N2} \dots y_{N,T}$

Fig. 2. Proposed real number encoding

It is necessary to mention that since the complexity of long-term HTC is mainly caused by a hydro subsystem, we merely consider hydro plant discharges as decision variables of hybrid GAs. Thermal plant generation levels associated with each chromosome are calculated based on the hydro plant discharges (generations) of the corresponding chromosome subtracted from the demand levels for each period. Then the nonlinear programming method of economic dispatch is applied [33].

2. Initialization technique and population size: Conventionally the initial population is composed of randomly generated chromosomes. However, as recommended in [23], we have included a few chromosomes which are known to be good solutions for the problem. Since these non-random components may cause premature convergence, selecting them needs special care. For a proper choice of population size, several useful guidelines are given in [34]. We have experimented with different population sizes for both binary and real GAs. It is found that a population size of 30 is both accurate enough and computationally acceptable for both binary and real GAs. In order to have comparisons on the same grounds, the population sizes for both types of GAs are taken to be equal and are chosen to be 30 chromosomes.

3. Fitness function and its normalization: As mentioned before, the hydro plant discharges together with the demand level of each period can be used to determine the thermal plant generation levels, from which the expected thermal generation cost can be calculated. The distribution of the storage level of reservoirs at the end of planning horizon can also be determined from hydro plant discharge values from the last period, from which the expected terminal cost of reservoirs can be determined. Since penalizing constraint violations with a penalty term in the objective function may cause numerical instability, we let only the reservoir volume constraints be violated slightly within full and dead storage level of the reservoirs. All the other constraints should always be satisfied by any chromosome in the population. The fitness of each chromosome is then defined as the sum of (1) and penalty costs associated with violation of (6). The penalty cost is chosen such that the cost of any feasible solution be lower than the cost of any other infeasible solutions. The choice of penalty factor can have a great impact on the performance of GAs. If the penalty factor is chosen such that the penalty term dominates the objective function, then the optimal solutions are most likely feasible but tend to be poor. On the other hand, if the objective function dominates the penalty term then the probability that the optimal solutions are infeasible increases. Hence we have chosen the penalty factor so that the penalty term and the objective function are of the same order.

Two types of fitness normalization techniques are applied in order to give the better chromosomes a higher chance of reproduction. The first technique is the inverse transformation method proposed in [22]. We propose the second method as exponential normalization. Both methods normalize fitness values within the range [0,1]. Let Fit_i denote the fitness of the i-th chromosome and Fit_{min} the fitness of the chromosome with the lowest cost in the current population. Then, normalized fitness of the i-th chromosome, $\overline{Fit_i}$, for proposed exponential normalization is given by

$$\overline{Fit_i} = \exp\left(-k\left(\frac{Fit_i}{Fit_{\min}} - 1\right)\right)$$

where k is a constant parameter which should be determined based on problem characteristics.

4. Parent selection, crossover and mutation: The roulette wheel parent selection method using normalized fitness values is employed to select parent chromosomes for crossover and mutation operations. Due to the importance of crossover operation in GAs, three different crossover operators are used for both binary and real encoded GAs, which are one-point, two-point and uniform crossover operators [32]. The crossover operation for real number encoding applies the same concept as in binary crossover, i. e.; it conveys genetic material of the two parent chromosomes to their children. However, the mutation operators for binary and real GAs differ in operation. The binary mutation on a single parent creates one child by randomly changing the values of bit positions on the chromosome if a probability test is passed. The mutation operator for real number encoding used in this work is of creep type. It sweeps along the chromosome and creeps any value up and down a random amount if a probability test is passed [32]. Thus, operating on a single parent it creates two children, one for creeping up and the other for creeping down.

5. Parent replacement: Each child is allowed to enter the population if its normalized fitness is not lower than that of the worst member of the previous generation, and also if it is not identical with any member of that generation. When a child enters into the population, the worst member is deleted to keep the population size constant.

As mentioned before, the crossover operator is the most important operator of GA and hence its weight should be much more than that of the mutation operator. However, note that as the population tends to converge, the effectiveness of the crossover operator tends to reduce due to the fact that similarity of chromosomes in the population increases. Therefore, in this situation the mutation operator should be given more chance to introduce diversity into the population. This implies that operator weights should change as the algorithm proceeds and the basic criterion for changing the weights of operators is based on the degree of success of each operator to introduce superior children into the population.

6. Dynamic tuning of operator weights: Concerning the points mentioned in section 5, in our proposed GAs (both real and binary GAs), operators compete with each other to be selected for operating on parent chromosomes. This is, in fact, an implementation of the basic idea of GA on the operators of GA itself. Thus operator weights are dynamically tuned in each iteration of GA based on the performance of each operator, measured by qualities of the children produced by the corresponding operator. Also, in order to prevent complete elimination of operators in competition, a lower bound is considered such that operator weights never reach below this level. The steps followed in the dynamic tuning of operator weights are as follows:

1- Initialize the weights of operators.

2- If an operator is selected and introduces a child into the population, increase its weight by an amount Inc obtained from the following formulae:

$$Inc = \frac{Cost_{\max} - Cost_{chld}}{Cost_{\min}} k_0$$

where $Cost_{max}$, $Cost_{min}$ and $Cost_{chld}$ are the total expected costs associated with the worst and the best members of the current population and newly introduced child into the population, respectively. k_0 is a parameter which should be determined such that neither rapid dominance of a weak operator nor negligible increment for a good operator occurs. If an operator can introduce two children into the population, the above increment is calculated for each of them and the sum of increments is added to the weight of the corresponding operator.

3- The increase in the weight of the selected operator is equally subtracted from the weights of the other operators, provided that their weights have not reached their lower bounds. This is done to keep the sum of the weights of operators equal to unity. If the weights of all the other operators are equal to their lower bounds, which implies that the weight of the selected operator has reached its upper bound, then the weight of the selected operator is not increased.

The initial weights for operators were given as 0.75 for three crossover operators (one-point, two-point and uniform crossover), each a weight of 0.25 and 0.25 for the weight of the mutation operator. The value of k_0 is chosen equal to 0.01. Finally, the adaptive operator fitness technique presented in [35] can be used to determine the probability test values for uniform crossover and mutation operators. These are 0.5 for uniform crossover passing test and 0.09 for mutation passing test. The flow diagram of the proposed method is shown in Fig. 3.

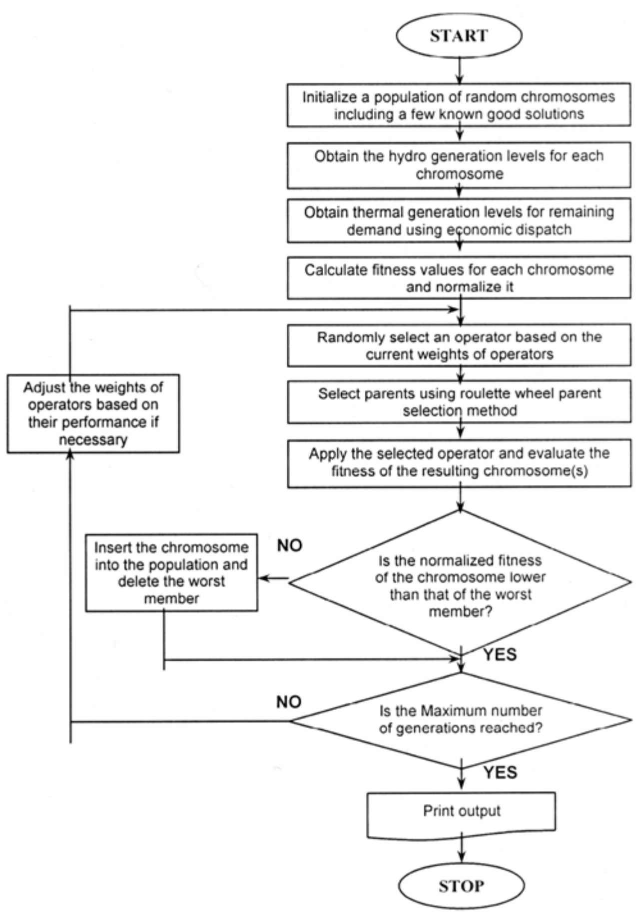

Fig. 3. Flow diagram of the proposed method

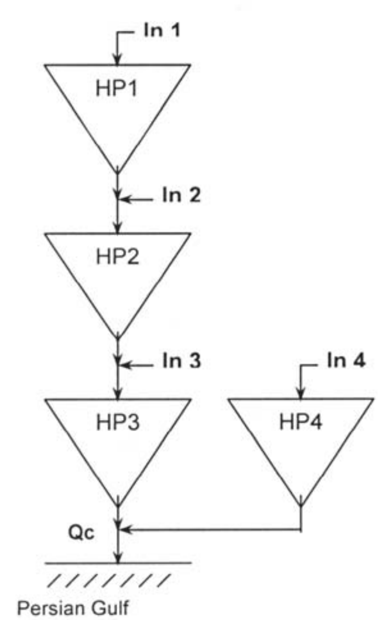

Fig. 4. Schematic diagram of the test system

Table 1. Various combinations of GA characteristics

R	R	R	R	R	R	R	R	B	B	B	B	B	B	B	B
D	D	D	D	F	F	F	F	D	D	D	D	F	F	F	F
P	P	M	M	P	P	M	M	P	P	M	M	P	P	M	M
E	I	E	I	E	I	E	I	E	I	E	I	E	I	E	I

R= Real, B= Binary, D= Dynamic tuning, F= Fix weights, P= Pure initialization,
M= Mix initialization, E= Exponential normalization, I= Inverse normalization

4. CASE STUDY AND RESULTS

In order to examine the performance of the proposed hybrid GA with real number encoding and dynamic tuning of control parameters, two types of tests are performed. In the first step, the solution of the HTC problem is obtained using the LR technique in which the hydro subproblem is solved using the method of stochastic dynamic programming with successive approximations presented in [4]. Since actual inflows are available for the last 37 years, they are used as scenarios for solving the hydro subproblem in the LR technique. The convergence criterion is the difference between upper and lower bounds of the dual objective function. The maximum allowable tolerance for this difference is adopted to be 0.5 percent of the calculated lower bound for the dual objective function. Then the results are compared with those obtained using binary and real encoded hybrid GAs. In the second step, the performance of binary (discrete) and real (continuous) hybrid GAs are compared with each other under equal circumstances. All tests are performed on Khuzestan hydro-thermal power system in Iran, which is described in detail in [5]. A brief description of this system and its characteristics are given in appendix A. The schematic diagram of the hydro network for this system is illustrated in Fig. 4.

In order to compare the performance of binary and real GAs, 16 different combinations of GA characteristics are considered. These include different normalization schemes, different initialization techniques and dynamic tuning or fixed weights for GA operators. The various combinations are summarized in Table 1.

Note that in the pure initialization technique, the initial population is composed of purely random chromosomes, while in mix mode two chromosomes, which are known to be good solutions, are used together with 28 randomly generated chromosomes to form the initial population. To implement the resulting 16 different algorithms, we developed software within MATLAB environment. Each one of 16 different algorithms is run for 10 different initial populations which are randomly generated. However, in order to provide equal circumstances for the 16 different algorithms, the same starting points of random number generator are used for each one of the 16 different algorithms. Therefore, 160 different runs are performed, each one allowed to proceed up to 1500 generations. The results are summarized in Tables 2 and 3 for binary and real GAs, respectively.

In Tables 2 and 3 the abbreviations given in the first column indicate the characteristics of the specific GA for which the results of different runs are illustrated. For example RDME is the abbreviation for Real GA with Dynamic tuning of control parameters, Mixed initial population and Exponential normalization, as shown in Table 1. Also the CPU times given in the last column are the average values for 10 different runs.

Table 2. The fitness values of the best chromosome–Binary (values in $ A)

	RUN No. 1	RUN No. 2	RUN No. 3	RUN No. 4	RUN No. 5	RUN No. 6	RUN No. 7	RUN No. 8	RUN No. 9	RUN No. 10	Average fitness	CPU TIME
BFMI	172535	175756	176533	178368	175919	166777	180504	177219	182388	180729	176673	6819 s
BFME	171157	171460	184108	178377	164767	172510	170475	175812	175534	170755	173496	6825 s
BFPI	170155	167275	178798	172395	199915	180451	179098	176790	166328	198248	178945	6819 s
BFPE	172836	172738	167243	169038	167342	172937	171717	168362	173942	175462	171162	6825 s
BDMI	177927	175315	165585	174343	181775	168248	177190	185727	193256	173283	177265	7186 s
BDME	170735	167915	165270	169879	194777	176516	168076	172111	175059	168683	172902	7192 s
BDPI	172780	225885	173987	170834	168730	175838	173927	169930	190335	184478	180672	7186 s
BDPE	175210	182564	170966	176866	177075	180193	169847	181062	182716	175139	177164	7192 s
Lagrangian relaxation using dynamic programming with successive approximations										159483	15492 s	

Considering the results given in Tables 2 and 3, the following general conclusions can be drawn:
The first point is concerned with the advantage of real GA over binary GA. The average fitness over 80 runs of real GA is \$163,055 with an average CPU time of 6502 seconds, i. e. 4.33 seconds per iteration. These values for binary GA are \$176,035 and 7006 seconds or 4.67 seconds per iteration, respectively. Hence, there is an average of about 8% improvement in the quality of solutions with about 7.7% lower CPU time. Next, consider the behavior of GAs with and without dynamic tuning of operator weights. The average fitness over 40 runs of real GA with dynamic tuning is \$161,579 with an average CPU time of 6686 seconds while those for real GA with fixed operator weights are \$164,531 and 6318 seconds, respectively. Thus, the average improvement of \$2952 in the solutions is obtained at the expense of increasing the average CPU time by 368 seconds. Considering the low costs of computation on PCs as compared to system costs, it is concluded that dynamic tuning of operator weights is an efficient technique for real GAs. This efficiency is also notable for binary GA with mixed initial population and exponential normalization with an average fitness of \$172902 and average CPU time of 7192 seconds.

Table 3. The fitness values of the best chromosome -Real GA values in \$

	RUN No. 1	RUN No. 2	RUN No. 3	RUN No. 4	RUN No. 5	RUN No. 6	RUN No. 7	RUN No. 8	RUN No. 9	RUN No. 10	Average fitness	CPU Time
RFMI	166676	164361	161551	170910	167421	163926	167667	163530	167662	163578	165728	6314^{s}
RFME	161439	162520	167657	162590	167262	163937	163167	166639	166458	160519	164219	6321^{s}
RFPI	164124	163472	166391	165668	160704	164881	165365	167243	165862	165574	164928	6314^{s}
RFPE	159828	162973	165155	162275	168929	164082	162889	160945	161985	163431	163249	6321^{s}
RDMI	161711	157388	157430	163549	160823	167247	165416	157447	158206	159144	160836	6682^{s}
RDME	164206	159744	164351	163116	161632	162010	165082	160241	159117	162953	162245	6691^{s}
RDPI	161394	162942	157525	159591	161807	162614	162996	165459	166168	157326	161782	6682^{s}
RDPE	163520	159668	158277	162396	163062	162152	163699	161849	159154	160739	161452	6691^{s}
Lagrangian relaxation using dynamic programming with successive approximations											159483\$	15492^{s}

Furthermore, note that the effect of including a few non-random chromosomes in the initial population is not significant. The average fitness over 40 runs of real GA with and without completely random initial populations have been \$162,853 and \$163,257, respectively, while those corresponding to binary GA are \$176986 and \$175,084, respectively. Finally consider the effect of fitness normalization technique on the quality of solutions. The average fitness over 40 runs of binary GA with exponential and inverse normalizations have been \$173,681 and \$178,389, i. e., about an \$4708 improvement with an average of about 6 seconds increase in CPU time. This improvement was prominent for real GA with pure random initial population and dynamic tuning of operator weights, with an average fitness of \$161,452. The general behavior of real and binary GAs with various characteristics is shown in Fig. 5. Figure 6 shows the typical competition among operators for real and binary GAs with dynamic tuning of operator weights.

Regarding the variance of the solutions, we note that it generally decreases at first, as the algorithm tends to converge. However the rate of decrease is different for each one of 16 GAs proposed in the paper. The variances of the fitness values for each one of the 16 GAs when they are terminated are shown in Table 4.

Table 4. Variance of the fitness values for each one of the 16 GAs when they are terminated

GA Type	BFMI	BFME	BFPI	BFPE	BDMI	BDME	BDPI	BDPE
Variance	159.91	188.25	112.67	160.49	151.53	140.65	126.06	124.55
GA Type	RFMI	RFME	RFPI	RFPE	RDMI	RDME	RDPI	RDPE
Variance	202.48	180.05	173.61	145.21	172.54	231.78	175.82	156.86

As can be seen, the variances of solutions for binary GAs are relatively low since the probability of premature convergence is relatively high for this type of GA, mainly due to their inherent discretization. The dispersion of chromosomes at the end of optimization is greater for RDME type of GA, which uses real number encoding and dynamic tuning of control parameters. However, this is a preliminary investigation and this subject needs an independent and more careful study.

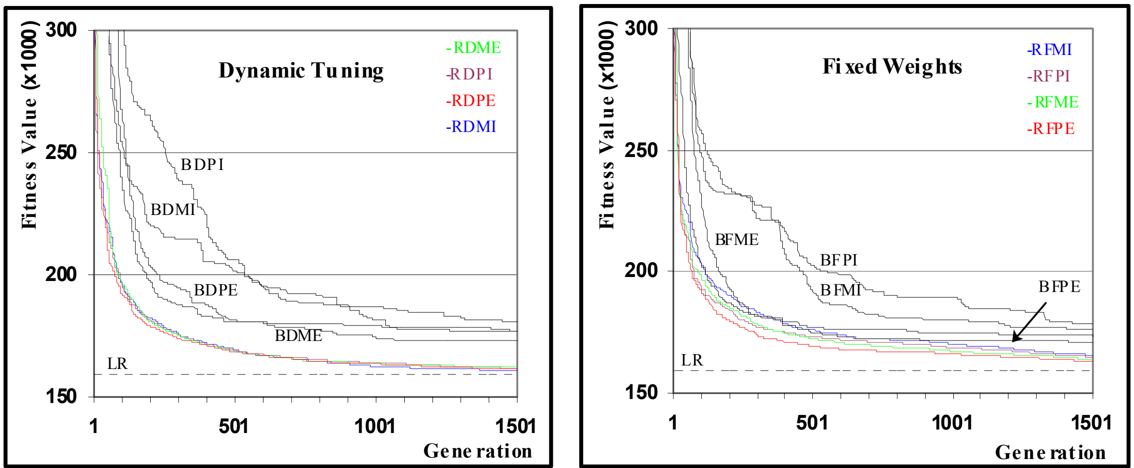

Fig. 5. General behavior of real and binary GAs with various characteristics

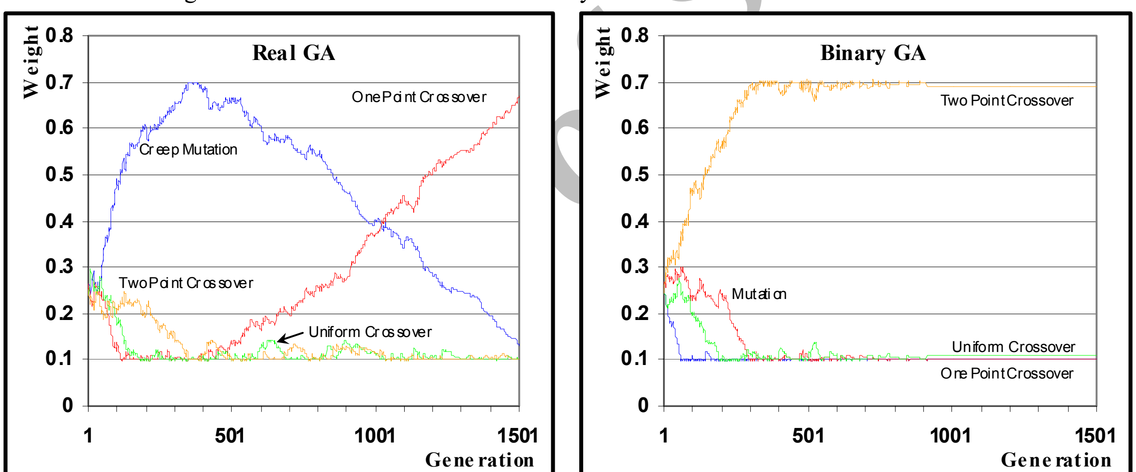

Fig. 6. Typical competition among operators for real and binary GAs

Considering the merits of real GA with dynamic tuning of operator weights, it is concluded that this type of GA is an efficient solution method for a long-term HTC problem with cascaded reservoirs and stochastic inflows. This fact is particularly revealed when the results obtained by RDMI and RDPE GAs are compared with that obtained by application of the classical LR technique using dynamic programming with successive approximations. As is illustrated in Table 3, there are several cases for which the costs obtained by real GA are lower than those obtained by the LR technique, for instance, see the run numbers 2,3,8,9 and 10 of RDMI and 3 and 9 of RDPE. The fact that the costs obtained by LR are greater than those obtained by real GAs are partly due to the use of heuristic methods during the feasibility phase of the LR technique. In addition, the LR technique generally obtains locally optimum solutions, while real GAs a much greater probability of finding global optimum [28]. As another advantage of GAs over the classic LR technique, one can mention the dimensionality problem associated with this method, which prohibits its practical application for systems with more than four cascaded reservoirs when used together with stochastic dynamic programming [4]. However, not only is this problem not limiting real GAs as seriously as the LR technique, but also other practical features of the system such as stochastic energy demand and forced outages of generating units can be easily implemented by real GAs, which if not impossible, are very difficult to be considered by LR and other classical techniques.

5. CONCLUSIONS

As more practical aspects of hydro-thermal power systems, such as stochasticity of reservoir inflows are incorporated into mathematical models for long-term HTC, the effectiveness of conventional optimization techniques for solving them decreases. The flexibility of GAs makes them an attractive alternative, particularly when they are customized with the special structure of the problem. The hybrid GA proposed in this paper which was developed on the basis of real number encoding, has shown its effectiveness over the conventional binary GA and LR technique. The accuracy of this method is very close to that of the analytical method while it obtains the optimal solution much faster. In this special GA, appropriate crossover and mutation operators, as well as some conventional optimization techniques are adopted. The method gives promising results for the case of cascaded hydro systems and can handle situations which, if not impossible, are very difficult to analyze using conventional methods.

The proposed method can also be used for generation expansion planning studies by the inclusion of investment costs of alternative plans in the objective function. Furthermore, the proposed method can easily handle the long-term HTC when, in addition to reservoir inflows, system demand and generating unit failures are stochastic. Devising more effective operators for real number encoding based on the concepts of a conventional optimization theory is the author's current research interest.

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NOMENCLATURE

T	the planning horizon, number of periods
M	number of thermal power plants
N	number of hydro power plants
x_{mt}	output of thermal power plant m in period t
y_{nt}	water released from reservoir n in period t
u_{nt}	output of hydro power plant n in period t
v_{nt}	water content of reservoir n in the beginning of period t
$GC_m(x)$	generating cost of thermal power plant m as a function of its output x
$TC_n(v)$	terminal cost of reservoir n as a function of its water content v
Q_{nt}	inflow of water to reservoir n in period t
$\underline{x_{mt}}, \overline{x_{mt}}$	minimum and maximum allowable values for x_{mt}
$\underline{y_{nt}}, \overline{y_{nt}}$	minimum and maximum allowable values for y_{nt}
$\underline{u_{nt}}, \overline{u_{nt}}$	minimum and maximum allowable values for u_{nt}
$\underline{v_{nt}}, \overline{v_{nt}}$	minimum and maximum allowable values for v_{nt}
s_{nt}	water spilled from reservoir n in period t
D_t	energy demand in period t
UP_n	set of reservoirs upstream to hydro plant n

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Appendix A- Characteristics of Khuzestan power system [5]

The Khuzestan power system is the largest hydro- thermal generating system in Iran. Currently it has two hydro power plants in operation and up to six plants under construction, of which two power plants are planned to begin operation in the near future. Hence, we have considered 4 hydro power plants for this system as shown in Figure 4. These hydro power plants have a total number of 20 units with a total installed capacity of 3520 MW. Also, there are two thermal power plants with 12 units which together produce 2258 MW. Therefore, the total installed capacity of this system is 5778 MW where there is a local power demand of 3086 MW at peak period with a load factor of 82%. The extra supply of this system should be transmitted to the neighboring power systems, which should at least be 1000 MW for 4 hours (daily peak) every day. Due to the fact that the reservoirs of this system are multi-purpose, there is a minimum demand for water supply of 150 CM/S at downstream which must be satisfied (i.e. Qc in Fig. 4). Table A1 gives the characteristics of generating units and system demand of this system. The demand figures are the algebraic sum of local demand, and import and export of energy with neighboring regions.

Table A1. Characteristics of Khuzestan power system

	Plant	No. of units	Unit capacity (MW)	Reservoir volume (MCM)	Max. discharge rate (CM/S)	Min. discharge rate (CM/S)	Water head losses (M)	Max. eff. Head (M)
Hydro plants	HP1	4	250	228	710	0	2.9	170
	HP2	4	250	2750	850	85	3.1	168
	HP3	4	250	3134	850	85	3.2	165
	HP4	8	65	3345	420	100	1.6	180

				Coefficients of quadratic cost function	a	b	c
Thermal plants	RAMIN	6	310		0.005	6.8	500
	MODHEJ1	2	145		0.003	7.5	200
	MODHEJ2	4	27		0.010	50	150

Period	1	2	3	4	5	6	7	8	9	10	11	12
Load Factor (%)	57	60	73	77	82	79	68	63	61	63	65	62
Demand (GWH)	2138	2231	2537	2735	2983	2710	2158	2107	2093	2151	2205	2167

A non-convex relation between head and volume for reservoirs of Khuzestan power system is suggested in [5] and is also used in our case studies. These are as follows:

For HP3 (Abbaspur) Dam: $$h_{ABS} = 134.178510 \cdot (V_{ABS})^{0.13512489} + 133.80723$$

For HP4 (Dez) Dam: $$h_{DEZ} = 15.763685 \cdot (V_{DEZ})^{0.29909223} + 171.54242$$

The efficiency of power generation is assumed to be 0.87 for all hydro power plants. The head-volume relations for HP1 and HP2 (Karun4 and Karun3) are assumed to be similar to those of HP3. The terminal cost functions for HP3 and HP4 are as follows

For HP3 (Abbaspur Dam) $\begin{cases} -0.5875441*(x-1432) + 500 & \text{if } (x>1432 \text{ and } x<2283) \\ -0.5875441*(x-2283) + 500 & \text{if } (x>2283 \text{ and } x<3134) \\ 500 & \text{if } (x<1432 \text{ or } x>3134) \end{cases}$

For HP4 (Dez Dam) $\begin{cases} -0.3723008*(x-779) + 500 & \text{if } (x>779 \text{ and } x<2122) \\ -0.3723008*(x-2122) + 500 & \text{if } (x>2122 \text{ and } x<3465) \\ 500 & \text{if } (x<779 \text{ or } x>3465) \end{cases}$

For intermediate reservoirs, based on the current operation policy of Khuzestan Power Company established for future hydro power plants, no terminal cost is considered. Finally, in this study the value of k of section 3b3 is taken to be equal to 2.

Appendix B- Lagrangian relaxation method [8]

The Lagrangian relaxation (LR) technique has been reported in the literature as the most effective technique for solving the HTC problem with deterministic inflows [10, 12]. Its effectiveness essentially comes from its ability to decompose the original problem into a number of subproblems for thermal and hydro plants. More specifically, in the LR technique, the dual of the original problem as described in section 2a is formed as follows:

$$\underset{\lambda}{Max} \ \underset{x,y}{Min} \ E\left[\sum_{m=1}^{M}\sum_{t=1}^{T} GC_m(\mathrm{x}_{mt}) + \sum_{n=1}^{N} TC_n(\mathrm{v}_{n,T+1})\right] + \sum_{t=1}^{T}\lambda_t \cdot [D_t - (\sum_{m=1}^{M}\mathrm{x}_{mt} + \sum_{n=1}^{N}\mathrm{u}_{nt})]$$

subject to: *constraints (2)-(6)*

In the above problem, λ_t is the Lagrange multiplier corresponding to period **t**. It can be observed that the objective of the dual problem is separable in terms of hydro and thermal subproblems. The thermal subproblem is, for m=1,…,M,

$$Minimize \ \sum_{t=1}^{T}[GC_m(\mathrm{x}_{mt}) - \lambda_t \mathrm{x}_{mt}]$$

subject to: $\underline{x_{mt}} \le x_{mt} \le \overline{x_{mt}}$, t = 1,...,T,

Since thermal plant generations in each period are independent of their generations in the other periods, thermal subproblems can also be decomposed in time. Therefore, the solution of thermal subproblem m for period t, say x^*_{mt}, is found by equating the derivative of $[GC_m(\mathrm{x}_{mt}) - \lambda_t \mathrm{x}_{mt}]$ to zero and comparing the result, $\tilde{x}_{mt}$**,** with the lower and upper bounds. Hence, the optimal solution is

$$x^*_{mt} = \min\{ \overline{x_{mt}}, \ \max\{ \underline{x_{mt}}, \tilde{x}_{mt} \}\}$$

The hydro subproblem is, for n=1,…,N

$$Minimize \ E\left[TC_n(v_{n,T+1}) - \sum_{t=1}^{T}\lambda_t u_{nt}(v_{nt}, y_{nt})\right]$$

subject to: *constraints (2), (4), (5) and (6)*

Dynamic programming is applied to solve the hydro subproblem, starting from terminal cost function and moving backward to period t=1. As can be seen, when reservoirs are cascaded, the hydro subproblem cannot be decomposed into separate subproblems for each hydro plant. Therefore, when reservoirs are cascaded, obtaining the solution of hydro subproblem with stochastic inflows becomes very time consuming even by application of techniques such as dynamic programming with successive approximations [4]. This is mainly due to the explosive increase of the number of states, as already mentioned.

After obtaining hydro and thermal subproblem optimal solutions, they are coordinated by updating Lagrange multipliers using any one of the existing methods. While updating Lagrange multipliers is the critical part of the LR technique, most of the existing methods for this purpose are either oscillating (i.e. subgradient method, [12]) or have a slow rate of convergence (i.e. cutting plane method [21]). The problem of finding an effective method for updating Lagrange multipliers is not fully solved and is still under investigation, see [12, 36, 37] for example. When Lagrange multipliers are updated, they are returned back to the hydro and thermal subproblems to determine new generation levels corresponding to the updated dual variables. The procedure is repeated until most convergence criterion is satisfied. Then an additional procedure should be followed in order to obtain a primal feasible solution, since dual solutions are not necessarily feasible in the primal problem. There are efficient methods for this last procedure, see [38] for example.