

“Technical Note”

**NON-LINEAR ELASTIC STABILITY OF RECTANGULAR
FRAMES UNDER VARIOUS LOADING***

M. Z. KABIR** AND A. MOSLEHITABAR

Dept. of Civil Engineering, Amirkabir University of Technology, Tehran, I. R. of Iran
Email: mzkabir@cic.aku.ac.ir

Abstract– The post-buckling behavior of rectangular frames in an elastic domain is studied in depth. In analysis, unsymmetrical geometry, sway possibility and support conditions are considered in order to find their influences on load-deflection paths and non-linear deformations. The static perturbation technique is used for analysis and discussion. The first, a second order perturbation problem as an accurate measurement for the frame, is solved and the solutions compared with previously published papers. The results reveal that symmetric frames with a sway movement, due to lack of an axial force in the beam in the first order perturbation analysis, have a symmetric bifurcation point. However, the post-buckling behavior of un-symmetric frames with or without sway is bifurcated in an asymmetric manner.

Keywords– Non-linear, post-buckling, stability, frames, perturbation

1. INTRODUCTION

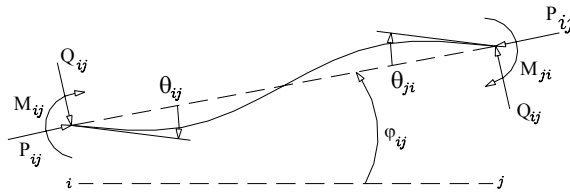
The analysis of elastic post-buckling behavior of frames is rather complicated since it necessitates a geometrically non-linear bending. The post-critical analysis of elastic structures invariably requires the solution of a set of non-linear differential equations and is based solely on equilibrium equations. Roorda and Chilver [2] employed the perturbation method with power series expansions for the analysis of non-linear equations. A nonlinear finite element analysis for frames is proposed by Care *et al.* [3]. The post-buckling behavior of perfect framed structures, as well as the nonlinear geometric analysis of imperfect frames, can be handled with this method. Bazant and Cedolin [4] adopted the stiffness matrix method and stability functions from the linear theory, enhancing it using additional terms and used it to deal with the second order deformation in non-linear behavior. The present work employs the suggested static perturbation method of Roorda and Chilver [2], based on the equilibrium and flexibility approach and by simple manipulations for the higher order non-linear stability. The solutions for utilizing the behavior of rectangular frames with symmetric and non-symmetric geometry and applied loading is obtained. The main assumptions of the coming analysis are based on frame continuity and the straightness of slender members. The degree of instabilities at the onset of buckling in a frame is generally not high, and the purpose of this paper is not to suggest that this instability is important for practical purposes, but rather to show that the application of non-linearity is extensively dependent on the geometry of the frame and the applied load configurations.

2. ANALYSIS

Consider a straight member ij of length L_{ij} and flexural stiffness EI_{ij} of a continuous, rigidly jointed plane frame.

*Received by the editors January 8, 2002 and in final revised form January 25, 2004

**Corresponding author

Fig. 1. End forces and deflections of member ij

The unbuckled states of the member, which is supposed to be axially rigid but flexible in bending, are sketched in Fig.1 along with the positive directions of the rotations (θ_{ij}, θ_{ji}), chord rotation (φ_{ij}), end moments (M_{ij}, M_{ji}), end shears (Q_{ij}) and axial force (P_{ij}). In the following, for simplicity, dimensionless parameters, m_{ij}, m_{ji}, p_{ij} and q_{ij} , are utilized instead of the end forces and are, respectively, defined as: $M_{ij}(\frac{L}{EI})_{ij}, M_{ji}(\frac{L}{EI})_{ji}, P_{ij}(\frac{L^2}{EI})_{ij}$ and $Q_{ij}(\frac{L^2}{EI})_{ij}$.

A nonlinear differential equation of the deformed member ij can be written as follow:

$$\theta''(x) + p_{ij} \sin \theta(x) - q_{ij} \cos \theta(x) = 0 \quad (1)$$

with boundary conditions

$$\theta'(0) = -m_{ij}, \quad \theta'(1) = m_{ji} \quad (2)$$

where $\theta(x)$ is the member rotation at a distance of x from node i . In this study, the static perturbation technique is applied for the solution of nonlinear differential Eq. (1). For this reason, $\theta, p_{ij}, q_{ij}, m_{ij}$ and m_{ji} are expressed in power series form as functions of some small parameter, ε , which increases from zero along an equilibrium path, leaving a known equilibrium point. Utilizing equilibrium conditions of member ij results in a power series with respect to ε . Equating coefficients of the same power of ε , the following infinite system of perturbation equations are obtained:

$$\dot{\theta}'' + p_{ij} \dot{\theta} = \dot{m}_{ij} + \dot{m}_{ji} \quad (3a)$$

$$\ddot{\theta}'' + p_{ij} \ddot{\theta} = \ddot{m}_{ij} + \ddot{m}_{ji} - 2\dot{p}_{ij} \dot{\theta} \quad (3b)$$

Equation (3a) and (3b) are ordinary differential equations and can be readily solved by making use of the boundary conditions, Eq. (2). Following the solution of Eqs. (3a) and (3b), the first and second order end rotations, $\dot{\theta}_{ij}, \dot{\theta}_{ji}, \ddot{\theta}_{ij}$ and $\ddot{\theta}_{ji}$, are as follows [5]:

$$\dot{\theta}_{ij} = f_{ij} \dot{m}_{ij} + g_{ij} \dot{m}_{ji}, \quad \dot{\theta}_{ji} = g_{ij} \dot{m}_{ij} + f_{ij} \dot{m}_{ji} \quad (4.1)$$

$$\ddot{\theta}_{ij} = f_{ij} \ddot{m}_{ij} + g_{ij} \ddot{m}_{ji} + 2\dot{p}_{ij} (F_{ij} \dot{m}_{ij} + G_{ij} \dot{m}_{ji}), \quad \ddot{\theta}_{ji} = g_{ij} \ddot{m}_{ij} + f_{ij} \ddot{m}_{ji} + 2\dot{p}_{ij} (G_{ij} \dot{m}_{ij} + F_{ij} \dot{m}_{ji}) \quad (4.2)$$

in which

$$f_{ij} = \frac{1}{p_{ij}} \left[1 - \sqrt{p_{ij}} \cot \sqrt{p_{ij}} \right], \quad g_{ij} = \frac{1}{p_{ij}} \left[1 - \sqrt{p_{ij}} \csc \sqrt{p_{ij}} \right] \quad (5.1)$$

$$F_{ij} = -\frac{1}{2p_{ij}} \left[\frac{2}{p_{ij}} - \cot \sqrt{p_{ij}} \left(\cot \sqrt{p_{ij}} + \frac{1}{\sqrt{p_{ij}}} \right) - 1 \right], \quad G_{ij} = -\frac{1}{2p_{ij}} \left[\frac{2}{p_{ij}} - \csc \sqrt{p_{ij}} \left(\cot \sqrt{p_{ij}} + \frac{1}{\sqrt{p_{ij}}} \right) \right] \quad (5.2)$$

In the following, the post-buckling behavior of three frames shown in Fig.2, with typical specifications including: sway possibility, boundary conditions, frame symmetry or asymmetry and relative stiffness of beams and columns, is investigated.

The frame shown in Fig. 2a is an asymmetric side-sway permitted frame whose post-buckling paths are depicted in Fig. 3a for different values of α . In the case of $\alpha=0$, the results coincide with those obtained by

Roorda, [1]. It should be noted that in the case of $\alpha=1$, since the first order axial force in the beam becomes zero, the post-buckling path is symmetric.

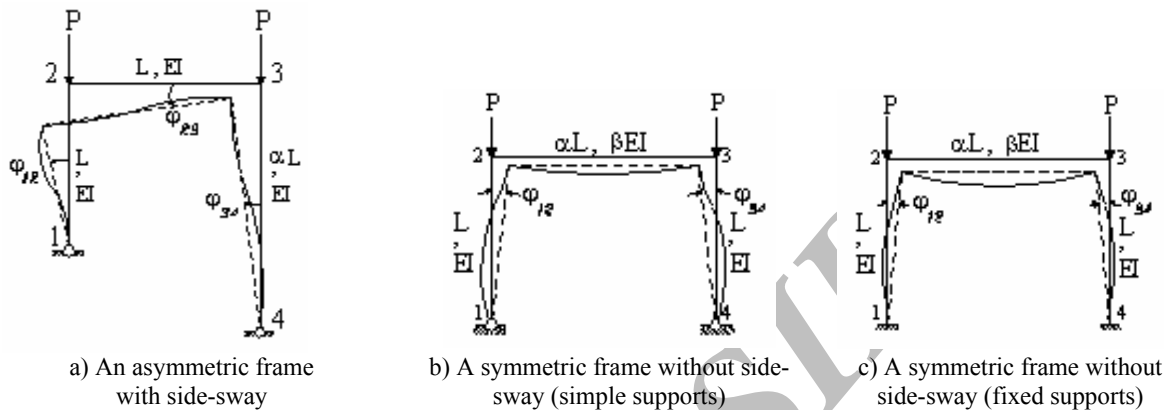


Fig. 2. Frame configuration

The post-buckling behavior of symmetric side-sway prevented frames, shown in Fig.2b and Fig.2c, can be observed in Fig. 3b and Fig. 3c, respectively.

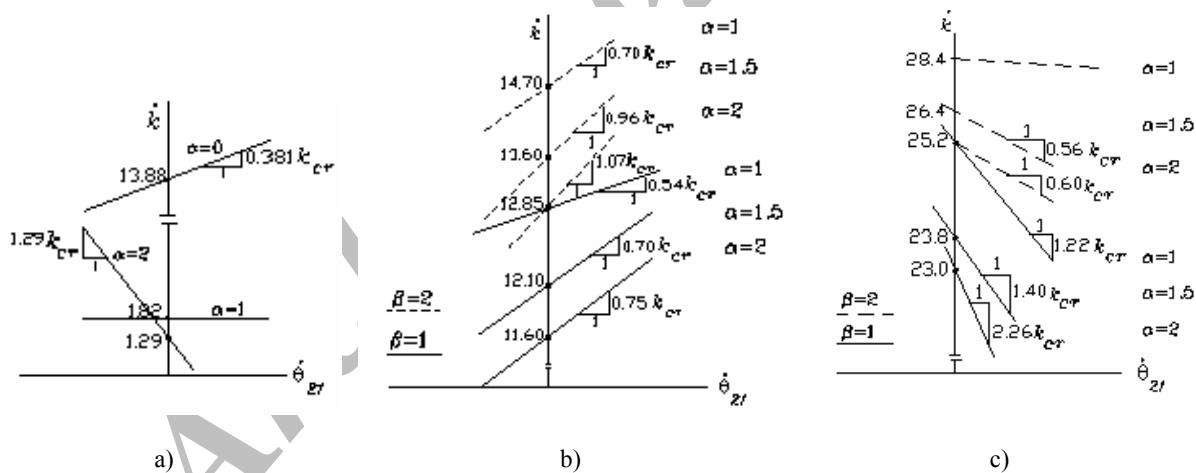


Fig. 3. Post-buckling path of frames shown in Fig.2

A comparison between Fig. 3b and Fig. 3c indicates that simple support reactions improve post-buckling stiffness rather than fixed supports. The initial post-buckling paths of frames shown in Fig. 2b and Fig. 2c are, respectively

$$\frac{\dot{k}}{\dot{\theta}_{21}} = \frac{-\alpha^3}{16\beta^2 f_{12} F_{12}}, \quad \frac{\dot{k}}{\dot{\theta}_{21}} = \frac{1}{24} \frac{\alpha^3}{\beta^2} \frac{f_{12}}{f_{12}^2 - g_{12}^2} \frac{\frac{3}{2} \frac{g_{12}}{f_{12}}}{\frac{g_{12}}{f_{12}^2} (G_{12} f_{12} - F_{12} g_{12}) - \frac{1}{f_{12}} (F_{12} f_{12} - G_{12} g_{12})}$$

3. CONCLUSIONS

In this paper, the non-linear post-buckling behavior of rectangular plane frames is analyzed using the standard procedure of the static perturbation technique. The solution of a highly non-linear problem is reduced to the recursive solution of an infinite set of linear problems. Based on parametric studies made in this study, the following remarkable conclusions are pointed out:

1. In the stability of symmetric portal frames with permitted sway, due to zeroes of axial force in the beam resulting from the first order non-linear analysis, the post buckling of the frame becomes unstable. But, at the second order analysis of the beam, axial force exists and stabilizes the post-buckling of the frame.
2. The post-buckling behavior of non-symmetrical frames is stable due to a tensile force in the beam.
3. In portal frames, when sway is not permitted, the potential loading of frames beyond buckling is reduced due to the compressive force in the beam and the decreasing of total frame stiffness.
4. Reducing the ratio of S_b/S_c , S_b and S_c become beam and column stiffnesses, respectively, improving the post-buckling behavior in sway permitted frames and weakening this capacity in non-permitted sway frames.

The influence of support conditions on the post buckling stiffness of frames is revealed in sway-prevented frames, the simple support reactions improve post-buckling stiffness rather than fixed supports.

REFERENCES

1. Roorda, J. (1965). Stability of structures with small imperfections. *J. Engng. Mech. Div. ASCE 91 EMD*.
2. Roorda, J. & Chilver, A. H. (1970). Frame buckling: An illustration of the perturbation technique. *Int. J. Non-Linear Mechanics*, 5, 235-246.
3. Care, R. F., Lawther, R. E. & Kabaila, A. P. (1977). Finite element post-buckling analysis for frames. *International Journal for Numerical Methods in Engineering*, 11, 833-849.
4. Bazant, Z. P. & Cedolin, L. (1989). Initial postcritical analysis of asymmetric bifurcation in frames. *J. Struct. Engng.* 113(7) 1501-1517.
5. Moslehitabar, A. (1999). Elastic post-buckling stability of rectangular frames, M. Sc. Thesis, Amirkabir University of Technology, Tehran, Iran (in Persian).