

A MATHEMATICAL MODEL FOR A FIXED DESICCANT BED DEHUMIDIFIER CONCERNING ACKERMANN CORRECTION FACTOR*

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Abstract– Solid desiccants have long been used in dehumidification and cooling systems. Many investigators have presented mathematical work on it, but there is a considerable discrepancy between published values and experimental values. A mathematical model has been derived to show the dehumidification trend of a desiccant dehumidifier with no rotation. In this model, variation of several parameters such as air humidity ratio, air temperature, water content of desiccant bed and temperature of desiccant as a function of the length of the bed and time were investigated. The mathematical model was validated by experimental results on a fixed solid desiccant bed filled with silica gel.

The results indicated that the dehumidification rate along the length of the desiccant bed depends mostly on input humidity ratio, air stream velocity, heat and mass transfer from the air stream to the bed and the Ackermann heat transfer correction factor. By increasing input air relative humidity and temperature more than 50% and 95°C, respectively, the Ackermann factor corrected the heat transfer coefficient up to 4%. The results also showed that air stream velocity was one of the most effective parameters on the dehumidification rate of the wheel.

Keywords– Desiccant bed, dehumidification, heat transfer, mathematical model, mass transfer, Ackermann correction factor

1. INTRODUCTION

In the desiccant cooling process, which is a new kind of refrigeration method, fresh air is dehumidified and then sensibly and evaporatively cooled before being sent to the conditioned space [1]. Desiccant-enhanced air conditioning equipment has exhibited both the capability to improve humidity control and the potential to save energy costs by lowering the latent energy requirement of the supply air stream. Controlling temperature and humidity within a conditioned space is important for a wide variety of applications. A desiccant dehumidifier, running in an open cycle, can be driven by low-grade heat sources, e.g. solar energy, waste heat and natural gas.

A procedure was developed for the energy and exergy analysis of open-cycle desiccant cooling systems and applied to an experimental unit operating in ventilation mode with natural zeolite as the desiccant [2].

A simple model has been presented to evaluate the performance of rotary desiccant wheels based on different kinds of solid desiccants (e.g. silica gel). The 'Model 54' has been derived from the interpolation of experimental data obtained from the industry and the correlations developed for predicting outlet temperature and absolute humidity [3]. This model consists of 54 coefficients corresponding to each correlation for outlet absolute humidity and temperature, and it is found that the model predicts the performance of the silica gel desiccant rotor very well. A psychometric model is then presented to obtain

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relatively simple correlations for outlet temperature and absolute humidity. The developed psychrometric model is based on the correlations between the relative humidity and enthalpy of supply and regeneration air streams. The update on the desiccant wheel models is then developed. It is mentioned that the psychrometric model is valid only for the desiccant wheel running with identical volume air flows in the supply and regeneration side. When the system runs with volume air flow ratio between the supply and regeneration side, the model needs further modification. Correction factors have been developed and incorporated in order to update the model for correctly predicting the temperature and humidity of processed air at the outlet of the desiccant wheel [4].

Many mathematical models on the rotary desiccant dehumidifier have been proposed. A finite difference computer program was developed based on a detailed numerical analysis [5]. Detailed computer simulations, as well as a simplified psychrometric analysis have been applied to determine the increase in cooling system performance that could be realized through the use of nonhomogeneous or staged desiccant beds [6]. A mathematical model by the finite difference method has been proposed to predict the performance and optimize the operation parameters of rotary desiccant wheels. The effect of the rotational speed on the performance of an adiabatic rotary dehumidifier has been parametrically studied, and the optimal rotational speed determined by examining the outlet adsorption side humidity profiles and humidity wave front inside the desiccant dehumidifier [7-8]. The mathematical model of a rotary desiccant wheel has been applied to calculate the performance of a stationary or rotary bed and transient or steady state operation by considering some of the key components. Predicting the performance and evaluating the benefits of rotary desiccant wheels concerning the complicated heat and mass transfer in the rotary desiccant matrix has been also suggested [9]. Recently, the continuity and energy conservation equations for the transient coupled heat and mass transfer were established and solved using a finite differential model [10]. A simple mathematical model for the explanation of the rotary desiccant wheel has been presented, in which the optimum rotational speed for achieving the maximum performance offered [11].

The ultimate goal of this study is to develop a new mathematical model, in which the factor of Ackermann for heat transfer coefficient and also the variable density for air stream were taken into account and used to analyze desiccant performance. With the implementation of a control volume technique, difference equations have been developed which were, in turn, used in the derivation of the governing differential equations and also for heat and mass transfer both within the desiccant and in the air stream. These finite difference equations were solved using an explicit finite difference scheme. Boundary and initial conditions, air stream velocity and density are examples of the parameters that have been changed throughout the analysis of total mass and enthalpy (energy) changes.

2. MATHEMATICAL MODEL OF DESICCANT DEHUMIDIFIER

The desiccant wheel consisted of micro channels in which the air stream and dehumidification occurred as a heat exchanger. We assume the velocity of air stream and the diameter of micro channels affect desiccant performance.

The physics of this mechanism dictates a close look into the surface phenomena taking place as contrasting conditions transferring heat and moisture. With effects of this kind, a lumped capacitance assumption can be used. This assumption allows for the fixed wheel total enthalpy exchanger model to be described by a two-dimensional model solution. With lumped capacitance effects occurring at the desiccant material surface, a two-dimensional solution can be formulated taking into consideration the temperature and moisture effects along the length of the wheel channels. Figure 1 shows a schematic coordination diagram of a desiccant wheel.

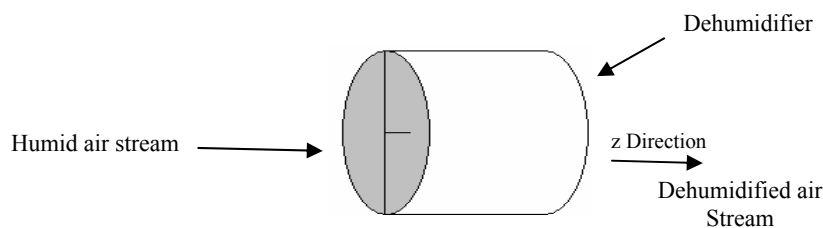


Fig. 1. Coordination diagram of desiccant matrix

Four equations concerning water content balance and energy conservation are used to describe the complicated heat and mass transfer occurring in moisture adsorption. Assumptions to obtain Eqs. (1-4) are as follows: 1. Effect of centrifugal force is neglected due to no rotation; 2. The shell of the rotary dehumidifier satisfies the insulated condition; 3. Heat and mass transfer in the radius direction is not taken into consideration; 4. Desiccant is uniformly distributed in the matrix, f_v , f_s are constant; 5. Thermal conductivity and diffusivity are isotropic.

Conservation of moisture for the processed air

$$\frac{K_Y \cdot f_v}{f_s} (Y_w - Y) = \frac{u}{f_s} \frac{\partial(Y \cdot \rho)}{\partial z} + \frac{\partial(Y \cdot \rho)}{\partial \tau} \quad (1)$$

Conservation of energy for the processed air

$$\frac{\partial(t \cdot \rho)}{\partial \tau} + \frac{u}{f_s} \cdot \frac{\partial(t \cdot \rho)}{\partial z} = \frac{A_f \cdot \alpha \cdot f_v}{f_s (c_{p_a} + Y \cdot c_{p_v})} (t_w - t) \quad (2)$$

Conservation of water content for the adsorbent

$$\frac{\partial W}{\partial \tau} - \text{Deff} \left[\frac{\partial^2 W}{\partial z^2} \right] = \frac{K_Y \cdot f_v}{M_w} (Y - Y_w) \quad (3)$$

In Eq. (3) the diffusion of liquid along the bed axes was taken into account.

Conservation of energy for the adsorbent

$$\frac{\partial t_w}{\partial \tau} - \frac{\lambda}{M_w (1 - f_s) (C_{pw} + WC_{pl})} \cdot \left[\frac{\partial^2 t_w}{\partial z^2} \right] = \frac{1}{[M_w (C_{pw} + WC_{pl})]} [A_f \cdot \alpha \cdot f_v (t - t_w) + K_Y f_v (Y - Y_w) Q] \quad (4)$$

The density of the air stream is calculated by

$$\rho = \frac{1}{0.00283(t + 273)} \quad (5)$$

In the above 5 equations, some auxiliary relations are needed. The Ackerman heat transfer correction factor was used as below [12]

$$A_f = \frac{C_f}{e^{C_f} - 1} \quad (6)$$

$$C_f = \frac{\sum_i J_{i,j} C_{p_{i,j}}}{h_j} \quad (j = \text{gas or liquid}) \quad (7)$$

For air stream process

$$C_f = \frac{J.C_{pe}}{\alpha} \quad (8)$$

Where C_{pe} and J are equivalent specific heat for humid air stream and the evaporation flux respectively as shown below

$$C_{pe} = C_{pa} + Y.C_{pv} \quad (9)$$

$$J = K_Y (Y - Y_w) \quad (10)$$

Mass transfer coefficient is given below [10]

$$K_Y = 0.704 m_i \text{Re}^{-0.51} \quad (11)$$

Heat transfer coefficient is given below [10]

$$\alpha = 0.671 m_i \text{Re}^{-0.51} C_{pe} \quad (12)$$

In this study, silica gel is chosen as the desiccant. The air humidity ratio near the wall of the adsorbent can be calculated from sorption isotherm presented by ASHRAE below, as Equilibrium isotherm

$$RH / 100 = (2.112W)^{Q/h_{fg}} (29.91P_{ws})^{(Q/h_{fg}-1)} \quad (13)$$

Saturation pressure (atm)

$$\log_{10} \frac{P_{ws}}{218.167} = -\left(\frac{z}{t + 273.15}\right) \left(\frac{a + bz + cz^3}{1 + dz}\right) \quad (14)$$

Where

$$z = 374.12 - t$$

$$a = 3.2437814$$

$$b = 5.86826(10)^{-3}$$

$$c = 1.1702379(10)^{-8}$$

$$d = 2.1878462(10)^{-3}$$

ASHRAE data

$$Y_w = \left(\frac{0.622RH}{\frac{P_a}{P_{ws}} - RH} \right) \quad (15)$$

The surface diffusivity for adsorption of silica gel is given as [10]

$$D_{eff} = D_0 \left[-0.974 \times 10^{-3} \frac{Q}{t_w + 273.15} \right] \quad (16)$$

Where Q is the heat of sorption [13]

$$Q = h_{fg} (1.0 + 0.2843 \exp(-10.28W)) \quad (17)$$

There are several kinds of adsorption isotherm which depend on the particular solid –liquid system. Figure 2 shows predicted isotherms at 33 °C from the best fit correlations for silica gel [14]. In this work ASHRAE used several presented isotherms for silica gel in this figure.

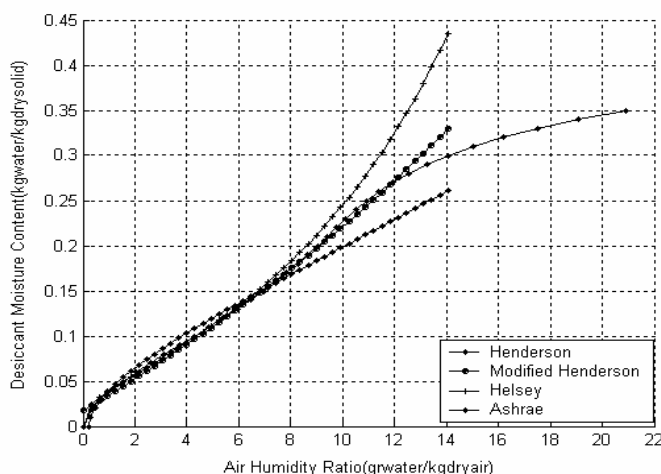


Fig. 2. Desiccant moisture content of silica gel versus air humidity ratio at $t=33\text{ }^{\circ}\text{C}$

Boundary conditions are

$$Y(0, \tau) = Y_{in} \quad (18)$$

$$t(0, \tau) = t_{in} \quad (19)$$

If the transient problem is taken into consideration, the initial conditions will also be necessary.
For desiccant

$$W(z, 0) = W_{eq|ta} \quad (20)$$

$$t_w(z, 0) = t_a \quad (21)$$

For air stream:

$$Y(z, 0) = Y_{eq|ta} \quad (22)$$

$$t(z, 0) = t_a \quad (23)$$

$$\frac{\partial W}{\partial z}(L, \tau) = 0 \quad (24)$$

$$\frac{\partial t_w}{\partial z}(L, \tau) = 0 \quad (25)$$

$$\frac{\partial W}{\partial z}(0, \tau) = 0 \quad (26)$$

$$\frac{\partial t_w}{\partial z}(0, \tau) = 0 \quad (27)$$

3. RESULTS AND DISCUSSION

Equations (1-4), in which time and space variation terms are included, can be discretized into finite difference equations groups. For space varying terms, two kinds of difference schemes, order one upwind difference scheme for the convection terms and a central difference scheme for the diffusion terms are adopted. To investigate the dynamic behavior of the dehumidification device and explicit scheme, a typical time stepping method is employed.

This mathematical model was validated by experimental results for a fixed desiccant bed dehumidifier filled with silica gel material. The apparatus includes a silica gel desiccant wheel with a thickness of 0.35 m, as shown in Fig. 3, and air with certain conditions pass through it. The wheel specification and also the operation condition are presented in Table 1. Initial water content of the bed was obtained in ambient humidity and temperature by using the equilibrium moisture content of solid desiccant (Silica gel). Then, the wet air stream was passed along the bed. Outlet relative humidity and temperature of the bed were measured at these conditions. The experimental tests and all measurements were repeated three times.

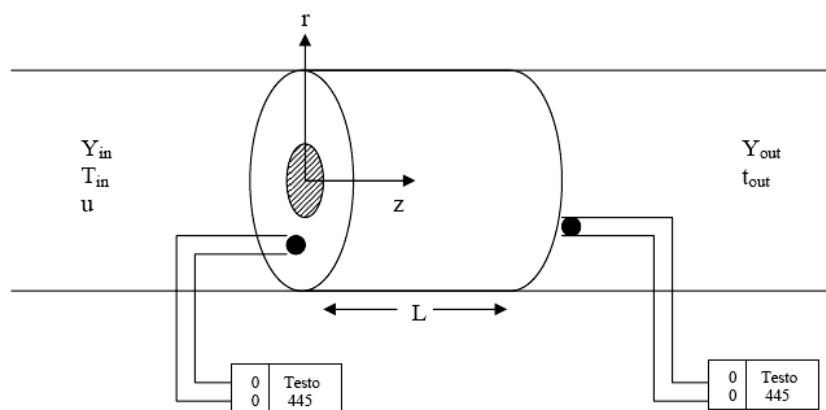


Fig. 3. Schematic of experimental apparatus

Table 1. Input data and calculation condition of the base case

Parameter	Value	Parameter	Value
RH_0 (%)	23.077	D_0 (m ² /s)	0.8×10^{-6}
RH_{in} (%)	24.5	u_0 (m/s)	3.5
RH_a (%)	18.2	L (m)	0.35
t_0 (°C)	33.041	λ (W/m·°K)	0.14417
t_{in} (°C)	32.8	f_s	0.149
t_a (°C)	33.4	f_v (m ² /m ³)	1468.85
w_0 (kg/kg)	0.15	M_w (kg/m ³)	720
		C_{pw} (J/kg·°K)	921

The temperature and the humidity of air at the outlet of the wheel were measured using a hygrometer device (Testo 445) with high accuracy, about 0.0001 and 0.01 for humidity and temperature, respectively.

Figure 4 shows the experimental results and model results both with and without the Ackermann factor. This figure shows that the model with the Ackermann factor is close to the experimental data. The difference between the model and the experimental points was slightly increased in the initial times. Therefore the mathematical model was confirmed and can be applied to predict the system behavior. Figure 4 also shows that the Ackermann factor corrects air humidity ratio about 4% and 20% at the beginning time and after 4 minutes, respectively in comparison with the results without considering the Ackermann factor.

The results of the changes of the air humidity ratio, bed water content, air temperature and bed temperature along the length of the bed versus time obtained from the presented model have been shown in Figs. 5-8.

Figure 5 shows the variation of the air stream humidity ratio versus time at different bed lengths of the desiccant. This figure shows that the air humidity at a point in the wheel increasing with time. This is due to the fact that the bed is going to the saturation state with time and it picks up less water, so the air

humidity increases with time. Also, this figure exhibits that the outlet humidity ratio (at $z=30$ cm) is less than that in the early points of the bed, which is obvious because of air dehumidification along the bed.

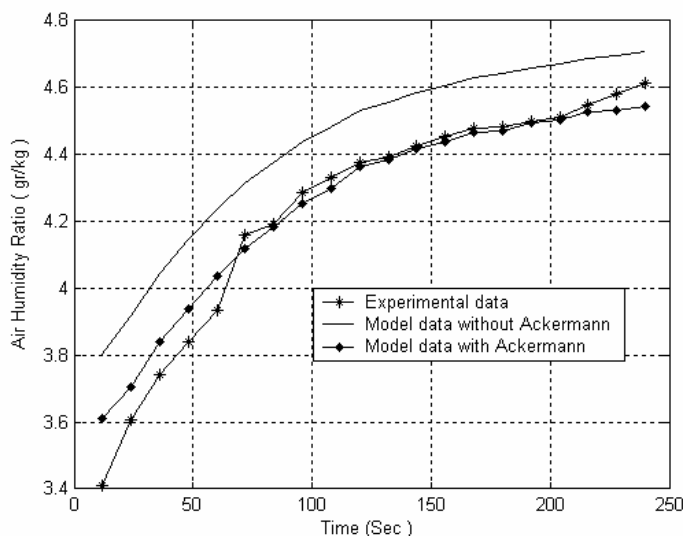


Fig. 4. Model and experimental air humidity ratio versus time

Variation of bed water content with time for a number of difference lengths of the bed is shown in Fig. 6. Water content at each point in the desiccant bed increases with time.

Also, at the end of the desiccant bed (35 cm), the bed water content was 0.1 and 0.11 at the time of 10 sec and 4 min, respectively. This figure also indicates that the dehumidification capacity of the desiccant bed is decreased along the bed. This is because more dehumidification takes place at the beginning of the bed compared with that at the end of the bed. Therefore, the slopes of the above curves were decreased at the end of the bed at different times of processing.

Variation of air temperature with time along the desiccant bed is shown in Fig. 7. Air temperature is increased with the length of the bed. This is due to the air dehumidification by the adsorber in which the latent heat of sorption is related. By the passing of time, it was decreased to air temperature at the initial condition.

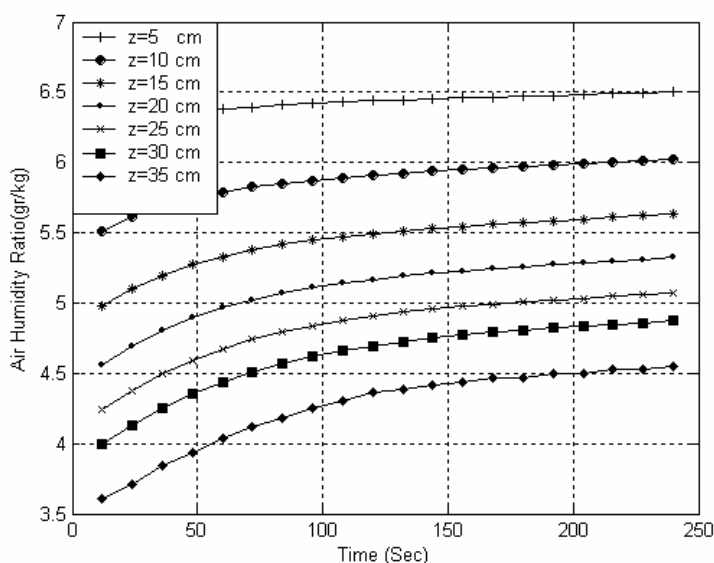


Fig. 5. Air humidity ratio versus time

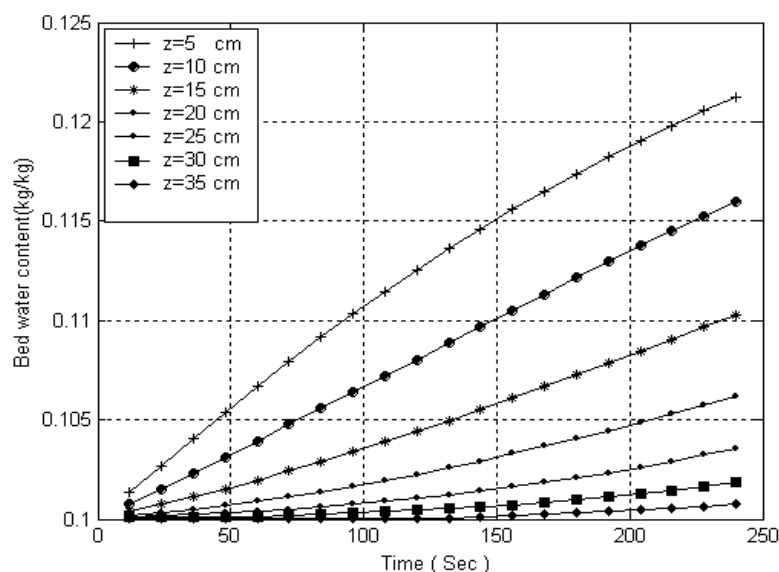


Fig. 6. Bed water content versus time

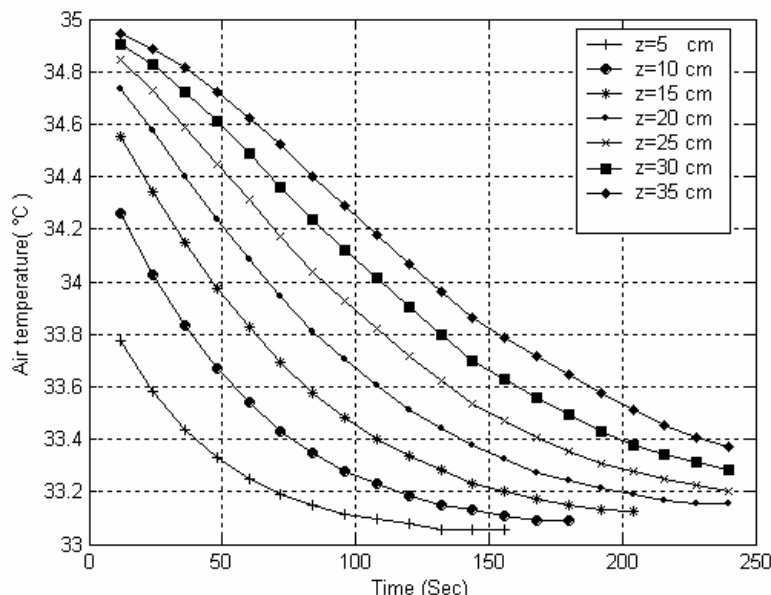


Fig. 7. Air Temperature versus time

In Fig. 8 the bed temperature profiles corresponding to time at each point of the bed were exhibited. This figure shows that temperature at each point of the bed decreases with time to the initial air temperature.

The effect of Ackermann heat transfer correction factor, A_f , on input air relative humidity according to different input air temperatures were presented in Fig. 9. This figure indicated that by increasing input air temperature or inlet air relative humidity, the Ackermann heat transfer correction factor decreased. This factor corrects heat transfer coefficient up to 4% at air relative humidity and input temperature more than 50 percent and 95 °C, respectively as shown in Fig. 9. Therefore it seems that Ackermann heat transfer correction factor should be considered during dehumidification and also periods for correction that regenerate heat transfer coefficient. The direction of mass transfer (vapor) and heat transfer between solid desiccant and gas phases is reversed, so the flux of the dehumidification affects the heat transfer coefficient, this effect is presented by the Ackermann factor.

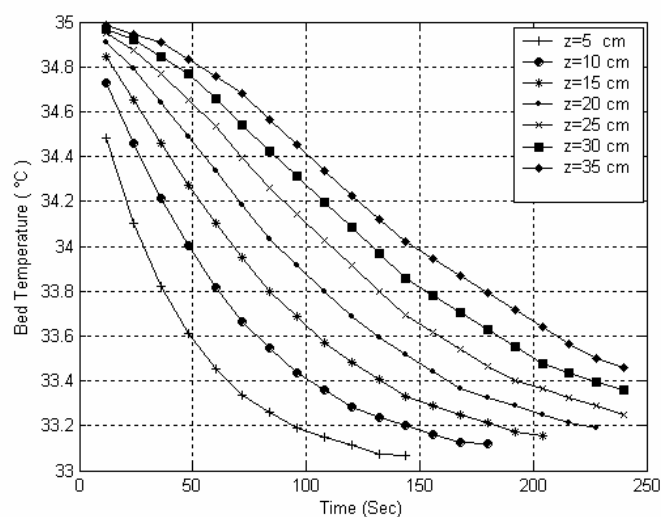


Fig. 8. Bed Temperature versus time

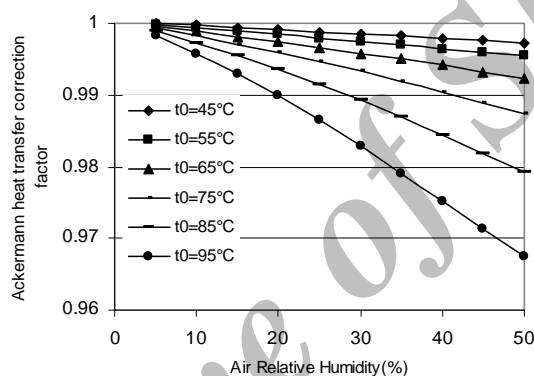


Fig. 9. Ackermann heat transfer correction factor versus air relative humidity

Figure 10 depicts the effect of air stream velocity on the average air humidity ratio in the outlet of the bed after 4 minutes. By increasing air stream velocity, the exit air humidity ratio increased, but in air stream velocities of less than 1 m.s^{-1} and more than 10 m.s^{-1} , humidity ratio have different behaviors. Air stream velocity also has an effect on pressure drop and mass transfer coefficient inside the bed, so real velocity profile should be considered correct for the model.

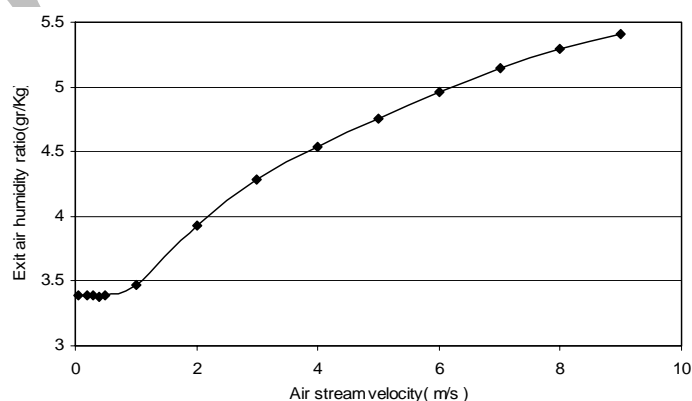


Fig. 10. Air humidity ratio versus air stream velocity

We are going to do further studies while the wheel is rotating. Momentum, heat and mass transfer occurring in moisture adsorption and regeneration will be investigated during bed rotation and experimental data and the mathematical model compared.

4. CONCLUSIONS

In this paper, we have presented a two dimensional mathematical model to predict the dehumidification trend of a fixed solid desiccant bed filled with silica gel and no rotation. Four differential equations concerning water content balance and energy conservation for flowing air and solid desiccant were used to describe the complicated heat and mass transfer occurring in moisture adsorption. The Ackerman heat transfer correction factor was applied to correct the heat transfer coefficient.

In this research, the effect of the Ackermann factor and the velocity of air on the performance of the dehumidification of air were also investigated. The results showed that the input air velocity between 1-10 m.s⁻¹ has a significant effect on the exit air humidity ratio. The heat transfer coefficient correction factor, A_f , changed the value of heat transfer up to 4%. Therefore, it was concluded that the Ackermann factor for heat transfer coefficient in modeling of the dehumidification system should be taken into account.

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