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STRESS CONCENTRATIONS OF SYMMETRICALLY LAMINATED COMPOSITE PLATES CONTAINING CIRCULAR HOLES*

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Abstract– In this study, an analysis of fiber reinforced, symmetrically laminated composite plates containing circular holes has been carried out. First, the stress state of a layer in a laminated plate is studied. After obtaining the stress state for each layer due to the uniaxial loading of a plate, the stress concentrations around a circular hole are studied. A number of diagrams are drawn to show the stress concentrations around a hole for layers having different oriented fibers using different material pairs with different E_1/E_2 ratios (ratio of elasticity modulus of fiber direction to that of transverse direction). Graphs are given for various E_1/E_2 values for the circumferential stress values around the hole versus angular location of points for two different fiber orientation angles. Second, the failure of the laminated composite plate is studied. To determine the "*first–ply failure"* of a laminated plate, Tsai-Hill failure criterion is employed to find minimum bearing circumferential stresses and where they occur as a function of the fiber orientation angle.

Keywords– Laminated composite plates, fibrous composites, stress concentrations, circular holes

1. INTRODUCTION

Composite materials constitute a group of materials formed by putting together at least two different materials. A reinforced concrete beam and a car tire are examples for materials of this type. The aim of this three dimensional composition is to obtain a property which none of the constituents possesses. In other words, the target is to produce a material that possesses higher performance properties than its constituent parts for a particular purpose. Some of these properties are mechanical strength, corrosion resistance, high temperature resistance, heat conductibility, stiffness, lightness and appearance. In accordance with this definition, the following conditions must be satisfied by the composite material. It must be man-made and not natural. It must be comprised of at least two different materials with different chemical components separated by distinct interfaces. Different materials must be put together in a three dimensional unity. It must possess properties which none of the constituents possesses alone, and that must be the aim of its production. The material must behave as a whole, i.e. the fiber and the matrix material (material surrounding the fibers) must be perfectly bonded (Classical Lamination Theory-CLT).

Lamination is used to combine the best aspect of the constituent layers and bonding material in order to achieve a more useful material. The properties that can be emphasized by lamination are strength, stiffness, low weight, corrosion resistance, thermal insulation, etc. Laminates, as many other structures, could have holes to serve various purposes. An obvious purpose is to accommodate a bolt (Fig. 1).

Considering two different materials, reinforcing bars (fibers) and surrounding materials (matrix material), mechanical properties of each layer are given in two directions. In the literature, several books

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have been published about mechanics of composite plates [5-10]. Stress concentration for orthotropic plates having circular holes was firstly introduced by Green and Zerna [1]. Later, Broockman and Sierakowski [2] carried out an investigation on boron-aluminium composites with circular cut-outs subjected to loads parallel to the fibers of the composites. The stress distribution of glass-epoxy composite plates was given by Hyer and Liu [3] using photo-elastic methods. Kaltakci [4], pointed out the effect of fiber orientation on the stress concentration around the holes of single layered anisotropic plates and their failures by comparing the layer without a hole. Chen and Liu [11] studied on multiple-cell modeling by applying the boundary element method. Chen and Zheng [12] studied universal connections between the overall moduli of elastic fibrous composites. Temiz, Ozel and Aydin [13] carried out the stress analysis of isotropic and orthotropic laminae, both with a hole and without a hole, and laminated both with a hole and without a hole composite plates using the finite element method.

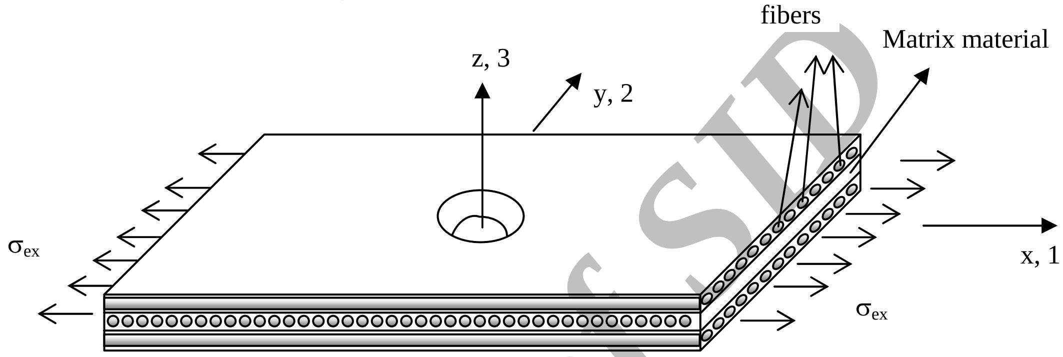

Fig. 1. Fiber reinforced cross- ply composite plate with a circular hole

In this paper, to avoid incorrect modeling of an orthotropic plate behavior by the finite element method, the analytical method is preferred. Using an equation given by Green and Zerna [1], the stress concentration equation is derived for a composite layer with an arbitrary fiber orientation angle and under general stress state. The effect of the fiber orientation angle, E_1/E_2 ratio (ratio of elasticity modulus of fiber direction to that of transverse direction), external loading and type of loading on the stress concentrations, as well as the failure of the symmetrically laminated plates which contain a circular hole, are studied in this paper.

2. ANALYSIS

a) Laminated composite plates

The stress-strain relation for a three dimensional linear elastic, anisotropic material is given as,

$$\{\sigma\} = [C]\{\varepsilon\} \tag{1}$$

Which is also known as Hooke's law. $\{\sigma\}$ and $\{\varepsilon\}$ are stress and strain vectors respectively. The $[C]$ matrix is called the material stiffness matrix, which has 21 independent material constants. For plane stress problems where the external stresses are in the plane of the plate, Hooke's law could be simplified to

$$\underbrace{\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}}_{\{\sigma\}} = \underbrace{\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}}_{[Q]} \underbrace{\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}}_{\{\varepsilon\}} \tag{2}$$

In which $[Q]$ is the reduced material stiffness matrix having elements as,

$$
\begin{aligned}
Q_{11} &= \frac{E_1}{1-\nu_{12}\nu_{21}} \\
Q_{12} &= Q_{21} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \\
Q_{22} &= \frac{E_2}{1-\nu_{12}\nu_{21}} \\
Q_{66} &= G_{12}
\end{aligned}
\tag{3}
$$

Here, E_1 is the elasticity modulus in the fiber direction, E_2 is the elasticity modulus in the transverse direction, ν_{12} and ν_{21} are the Poisson's ratio and G_{12} is the shear modulus. For a unidirectional fiber reinforced layer there are two principal material directions. One corresponds to fiber direction and the other corresponds to the matrix material direction denoted by subscripts 1 and 2 respectively. When these material directions are oriented by the angle α from the plate direction (Fig. 2), the stress strain relation is given as

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}
\tag{4}
$$

Where

$$
[\bar{Q}] = [T]^{-1}\,[Q]\,[R]\,[T]\,[R]^{-1}
$$

$$
[T] = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & 2\sin\alpha\cos\alpha \\ \sin^2\alpha & \cos^2\alpha & -2\sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha & \sin\alpha\cos\alpha & \cos^2\alpha-\sin^2\alpha \end{bmatrix}
\tag{5}
$$

$$
[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}
$$

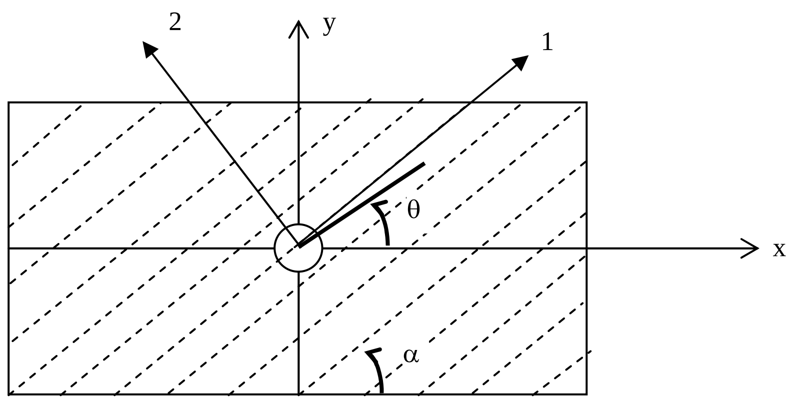

Fig. 2. Orthotropic layer with a circular hole

Classical Lamination Theory (CLT) assumes that all layers are perfectly bonded together in a plate and the in-plane deformations are continuous. In the case of bending, loading strain distribution could be rewritten as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{6}$$

The first vector on the right hand side is mid-plane strains, the second is curvatures and z is the depth from mid-plane. For a laminated composite plate (Fig. 3) the relation between applied forces and plate strains is given as

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \end{Bmatrix} \tag{7}$$

Where elements of [A], [B] and [D] matrices are defined as

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} (\overline{Q}_{ij})_k \, (z_k - z_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q}_{ij})_k \, (z_k^2 - z_{k-1}^2) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q}_{ij})_k \, (z_k^3 - z_{k-1}^3) \end{aligned} \tag{8}$$

in which n is the number of layers. For a given loading value of the laminated plate, Eq. (7) is solved for plate strains and curvatures. To obtain a layer stress state, mid-plane strains and curvatures, which are the same for all layers, are put in Eq. (4) and solved for stresses. It is obvious that due to symmetric lamination no moments are calculated.

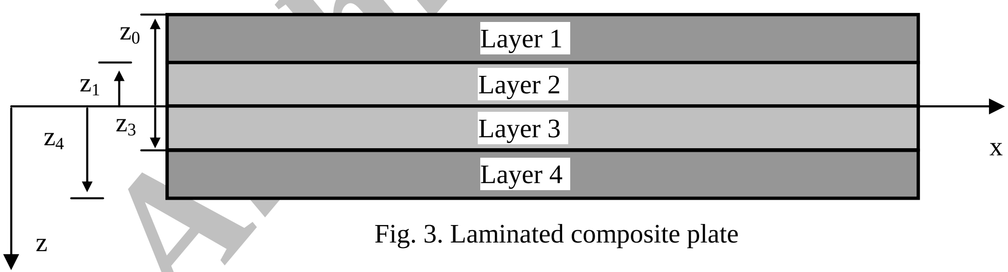

Fig. 3. Laminated composite plate

b) Stress concentrations around holes

A circular hole in a plate can be required for various purposes. If a small circular hole is made in the middle of the plate, the stress distribution in the neighborhood of the hole will be changed. The stress distribution around a hole is studied by several researchers. As described in [1], Green studied the problem using

$$F = F_0 + A_0 \log r_1 + B_0 \log r_2 + \sum_{n=1}^{\infty} \left(\frac{A_{2n} \cos 2n\theta_1}{(1+\gamma_1)^{2n} \, r_1^{2n}} + \frac{B_{2n} \cos 2n\theta_2}{(1+\gamma_2)^{2n} \, r_2^{2n}} \right) \tag{9}$$

where F is the Airy stress function, A_0, B_0, A_{2n} and B_{2n} are the constants related to boundary conditions. For the infinite orthotropic plate (ratio of the plate dimensions to the diameter of the circular hole is greater than 4) with a hole, as shown in Fig. 4, loaded in x direction (tension or compression) the circumferential stress around the hole is given as [1]

$$\sigma_\theta^i = \frac{(1+\gamma_1)(1+\gamma_2)(1+\gamma_1+\gamma_2-\gamma_1\gamma_2-2\cos 2\theta)}{(1+\gamma_1^2-2\gamma_1\cos 2\theta)(1+\gamma_2^2-2\gamma_2\cos 2\theta)}\sigma_x^i \tag{10}$$

where, σ_x^i is the stress applied in x-direction, θ, measured from the x axis in a counterclockwise direction, is the angle showing the direction in which the stress value is calculated, and γ_1, γ_2 are defined as

$$\gamma_1 = \frac{\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)+\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)^2-\frac{E_2}{E_1}\right]}\right]}-1}{\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)+\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)^2-\frac{E_2}{E_1}\right]}\right]}+1}$$

$$\gamma_2 = \frac{\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)-\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)^2-\frac{E_2}{E_1}\right]}\right]}-1}{\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)-\sqrt{\left[\left(\frac{E_2}{2G_{12}}-\nu_{21}\right)^2-\frac{E_2}{E_1}\right]}\right]}+1} \tag{11}$$

Using the circumferential stress, the stresses in x-direction, y-direction and shear stress around the hole are calculated by

$$\begin{aligned} \sigma_{\theta x}^i &= \sigma_\theta^i \sin^2\theta \\ \sigma_{\theta y}^i &= \sigma_\theta^i \cos^2\theta \\ \sigma_{\theta xy}^i &= -\sigma_\theta^i \sin\theta\ \cos\theta \end{aligned} \tag{12}$$

respectively. A special case of this problem is transversally loading the plate to the fibers (Fig. 5a). The circumferential stress around the hole could be calculated using Equation 10. However in this case, the material properties in fibers and matrix directions must be interchanged. This process is simplified by Kaltakci [4] as the interchanging of γ_1 and γ_2 in their position and sign. Thus, the circumferential stress around the hole is

$$\sigma_\theta^i = \frac{(1-\gamma_1)(1-\gamma_2)(1-\gamma_1-\gamma_2-\gamma_1\gamma_2-2\cos 2\theta)}{(1+\gamma_1^2+2\gamma_1\cos 2\theta)(1+\gamma_2^2+2\gamma_2\cos 2\theta)}\sigma_x^i \tag{13}$$

Under shear loading, fiber reinforced composite plates show different stress concentration characteristics than when they are axially loaded. In the case of α=0° (i.e. fibers are parallel to x-axis), the circumferential normal stress component around the circular cut-out for a plate subjected to uniform shear stress (Fig. 5b), is given as a function of θ [1]

$$\sigma_\theta^i = \frac{4\ (\gamma_1\gamma_2-1)\ \sin 2\theta}{(1+\gamma_1^2-2\gamma_1\cos 2\theta)(1+\gamma_2^2-2\gamma_2\cos 2\theta)} S^i \tag{14}$$

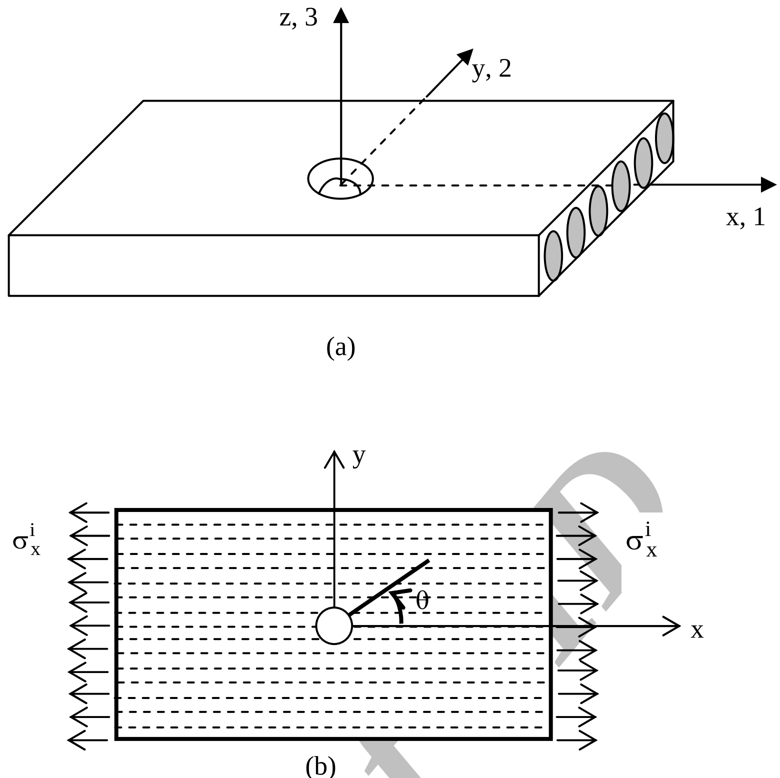

Fig. 4. Unidirectional composite plates

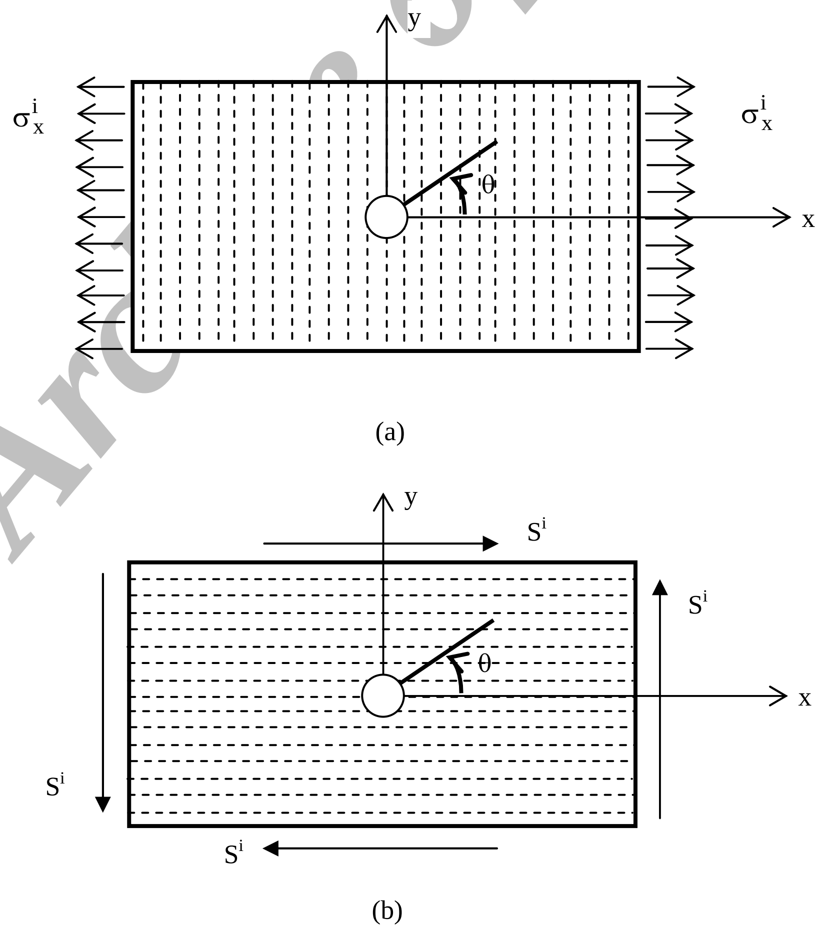

Fig. 5. Anisotropic plate subjected to transverse loading and shear

When the fibers in a composite plate are oriented at an angle α with the x-axis (Fig. 6), the solution of the problem is more complicated than the previous ones. The stresses, occupied by a layer due to the loading of the laminated composite plate, must be resolved into components associated with the axes of symmetry of the material by static equilibrium rules. Then, the problem in hand could be solved using Eqs. (10), (13) and (14) by superposition. Special care must be taken at this point. Locations of the calculated circumferential stress values are the same for the cases where fibers are parallel to the x-axis (i.e. Eq. (10) and (14)). However, when the fibers are transverse to the loading, circumferential stress values must be calculated in accordance with the case in which loading and fibers are parallel. The circumferential stress for the composite plate with the fibers oriented an angle α, subjected to general loading (Fig. 6), could be calculated as

$$\sigma_\theta^i = \frac{1}{(1+\gamma_1^2-2\gamma_1\cos 2(\theta-\alpha))(1+\gamma_2^2-2\gamma_2\cos 2(\theta-\alpha))}\times$$

$$\left[\begin{array}{l} (1+\gamma_1)(1+\gamma_2)(1+\gamma_1+\gamma_2-\gamma_1\gamma_2-2\cos 2(\theta-\alpha))\begin{bmatrix} 0.5\sigma_x^i(1+\cos 2(\theta-\alpha)) \\ +0.5\sigma_y^i(1-\cos 2(\theta-\alpha)) \\ +\sigma_{xy}^i \sin 2(\theta-\alpha) \end{bmatrix} \\ \\ +(1-\gamma_1)(1-\gamma_2)(1-\gamma_1-\gamma_2-\gamma_1\gamma_2+2\cos 2(\theta-\alpha))\begin{bmatrix} 0.5\sigma_x^i(1-\cos 2(\theta-\alpha)) \\ +0.5\sigma_y^i(1+\cos 2(\theta-\alpha)) \\ +-\sigma_{xy}^i \sin 2(\theta-\alpha) \end{bmatrix} \\ \\ +4(\gamma_1\gamma_2-1)\sin 2(\theta-\alpha)\left[0.5(\sigma_x^i-\sigma_y^i)\sin 2(\theta-\alpha)-\sigma_{xy}^i\cos 2(\theta-\alpha)\right] \end{array}\right] \tag{15}$$

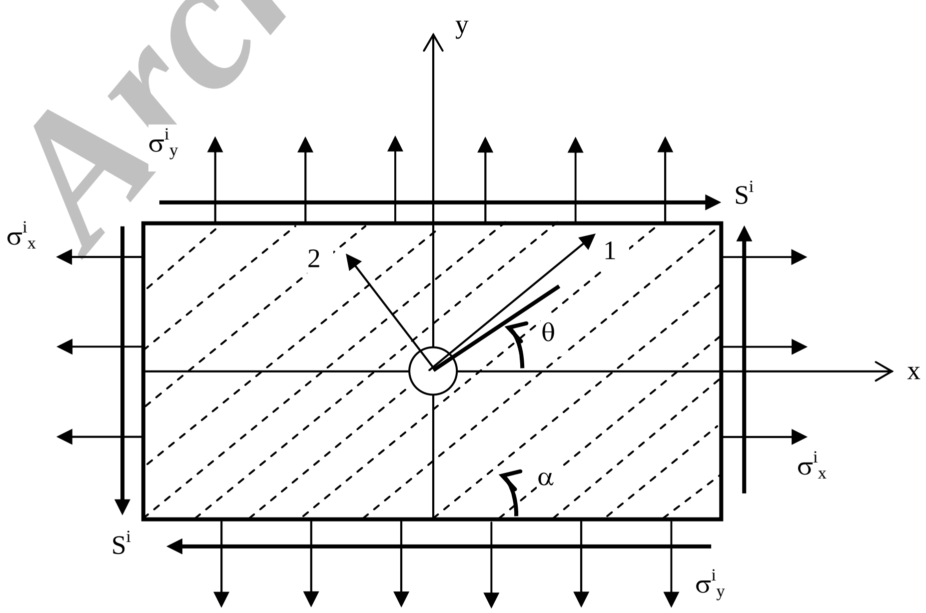

Fig. 6. Layer with arbitrarily oriented fibers subjected to general loading

c) Failure of fiber reinforced anisotropic layers

The prediction of the failure of structural components is usually accomplished by comparing the stresses or strains to the materials ultimate limits. Given a two-dimensional stress state, the calculation of the magnitudes of principal stresses is sufficient to predict the failure of isotropic materials. The determination of the failure of a laminated composite material presents several difficulties. Delamination, variation of stresses in the thickness direction and stress concentrations are some of those difficulties.

The failure of a laminated composite is more complicated than that of a single plate, due to the stacking together of several layers of material with different orientations and properties. Since stresses and strengths are different in different layers, it is possible that, as the loading increases, the stresses in one or more of the layers of a laminate would reach their ultimate strength earlier than others. It is, therefore, likely that the laminate would suffer damage in the form of local failures in those layers before it fails completely. For some applications the first failure of any layer is not acceptable because it degrades the strength and stiffness of the laminate. The failure prediction based on the first failure is commonly referred to as the first-ply failure criterion.

There are various quadratic failure criteria for composite plates. The most frequently used ones are Tsai-Hill, Hoffman, Hencky-Von Mises and Tsai-Wu criteria. Hill proposed an extension of the von Mises yield criterion to anisotropic materials with equal strengths in tension and compression. For a three-dimensional stress state Hill's criterion is given by

$$F(\sigma_x - \sigma_y)^2 + G(\sigma_x - \sigma_z)^2 + H(\sigma_z - \sigma_x)^2 + L(\sigma_{yz})^2 + M(\sigma_{xz})^2 + N(\sigma_{xy})^2 < 1 \quad (16)$$

Under plane stress assumption, Eq. (16) reduces to what is generally referred to as the Tsai-Hill criterion

$$\frac{1}{X_t^2}(\sigma_{\theta f}^i)^2 + \frac{1}{Y_t^2}(\sigma_{\theta m}^i)^2 - \frac{1}{X_t^2}\sigma_{\theta f}^i\,\sigma_{\theta m}^i + \frac{1}{G^2}(\sigma_{\theta fm}^i)^2 < 1 \quad (17)$$

Where G is the shear modulus, X_t and Y_t are the tensile strengths of the material in the fiber and transverse directions, respectively. $\sigma_{\theta fm}^i$ is the shear stress, $\sigma_{\theta f}^i$ and $\sigma_{\theta m}^i$ are the stresses in the fiber and transverse directions around the hole, respectively. This equation is known as Tsai-Hill failure criterion. The circumferential stresses, calculated by Eq. (15), can be resolved into components parallel to the fiber and transverse directions as

$$\begin{aligned} \sigma_{\theta f}^i &= \sigma_\theta^i \sin^2(\theta - \alpha) \\ \sigma_{\theta m}^i &= \sigma_\theta^i \cos^2(\theta - \alpha) \\ \sigma_{\theta fm}^i &= -\sigma_\theta^i \sin(\theta - \alpha)\cos(\theta - \alpha) \end{aligned} \quad (18)$$

Replacing this relation in Eq. (17), Tsai-Hill failure criterion are obtained as

$$\frac{(\sigma_\theta^i \sin^2(\theta-\alpha))^2}{X_t^2} + \frac{(\sigma_\theta^i \cos^2(\theta-\alpha))^2}{Y_t^2} - \frac{(\sigma_\theta^i)^2 \sin^2(\theta-\alpha)\cos^2(\theta-\alpha)}{X_t^2} + \frac{(-\sigma_\theta^i \sin(\theta-\alpha)\cos(\theta-\alpha))^2}{G^2} < 1 \quad (19)$$

3. NUMERICAL RESULTS AND DISCUSSION

A computer program has been prepared to study the effect of circular holes on stress distributions. Two different kinds of composite plates are studied as examples. In the first part, the effect of fiber orientation on the stress concentrations around the hole is studied for two different composite plates, namely graphite-epoxy and glass-epoxy. The material properties of those composites are given in Table 1.

Table 1. Material properties

Material properties	Graphite-epoxy	Glass-epoxy
E_1 (kg/cm^2)	2111000	549000
E_2 (kg/cm^2)	52700	183000
G_{12} (kg/cm^2)	26300	91400
$(X_t)_1$ (kg/cm^2)	10555	10555
$(X_t)_2$ (kg/cm^2)	422	281
$(X_c)_1$ (kg/cm^2)	7037	10555
$(X_c)_2$ (kg/cm^2)	1196	1407
G (kg/cm^2)	703	422
ν_{12}	0.25	0.25

In this table, subscripts 1 and 2 refer to the fiber and transverse directions, respectively. Moreover, subscribe c is used for compressive, while subscribe t is used for tensile strengths. For both composites, stress values in x and y directions around the hole are presented graphically for different fiber orientation angles. The stress concentration values given in the graphs (Figs. 9-12) are obtained by dividing the circumferential stress values by the uniaxial loading value of a layer considered, assuming that layer is loaded in x-direction. The variation of the stress concentration values for the glass-epoxy and graphite-epoxy composites is drawn depending on the circumferential location (the angle θ varies from 0^0 to 180^0), as shown in the figures.

An 8-layered graphite-epoxy composite plate is considered first. Stacking sequence is selected as $(0°/30°/60°/90°)_s$ to see the effect of the fiber orientation angle on stress distribution. In Fig. 7 and 8, the variation of the circumferential angle is shown on the horizontal axis and dimensionless stress values are shown on the vertical axis. The values shown on the vertical axis are calculated by dividing stress values around the hole in the x direction to the stress value occupied by the layer due to plate loading (Fig. 7).

For the same example, the second graph is drawn by the same way. The values shown on the vertical axis are calculated by dividing stress values around the hole in the y direction to the stress value occupied by the layer due to plate loading (Fig. 8).

As shown in Figs. 7 and 8, as the fiber orientation changes, stress values change. When examining those figures, the combined effect of material properties, the number of layers, and the stacking sequence must be considered. For example, in both figures, the first layer in which α=0° takes most of the x-direction load, whereas the fourth layer, in which α=90°, takes most of the y-direction load. In Fig. 7, maximum x-direction stress is obtained when α=0°, as fiber orientation changes from 0° to 90°, the maximum stress point moves to the end of the horizontal axis, while the value of that stress decreases. When fibers are oriented as 90°, maximum x-directional stress changes its sign due to Poisson's effect. When the stress states of the first and fourth layer are examined, it could be observed that the first layer takes most of the plate loading, and due to Poisson's ratio, there is small y-directional stress. The situation is vise-versa for the fourth layer, which causes negative stresses in x-direction.

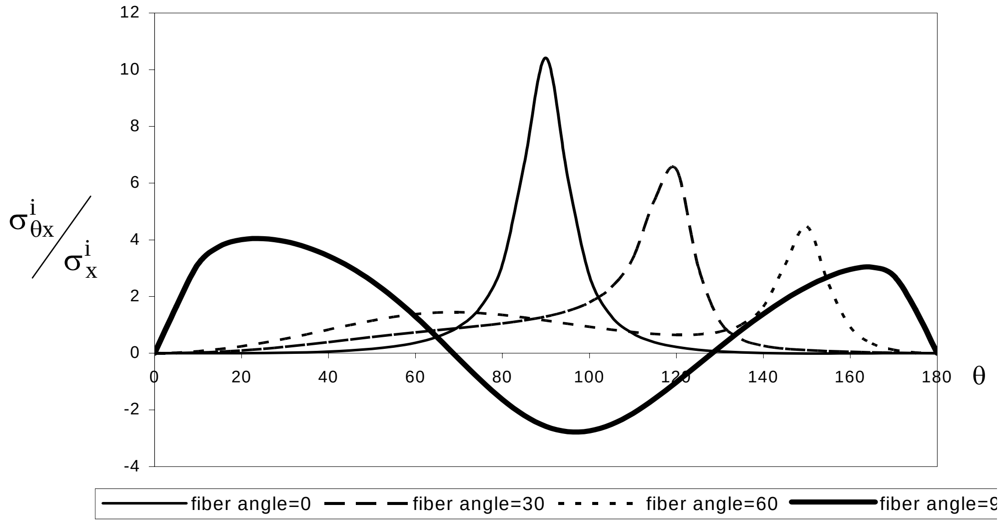

Fig. 7. Dimensionless x-direction stress values versus circumferential angle

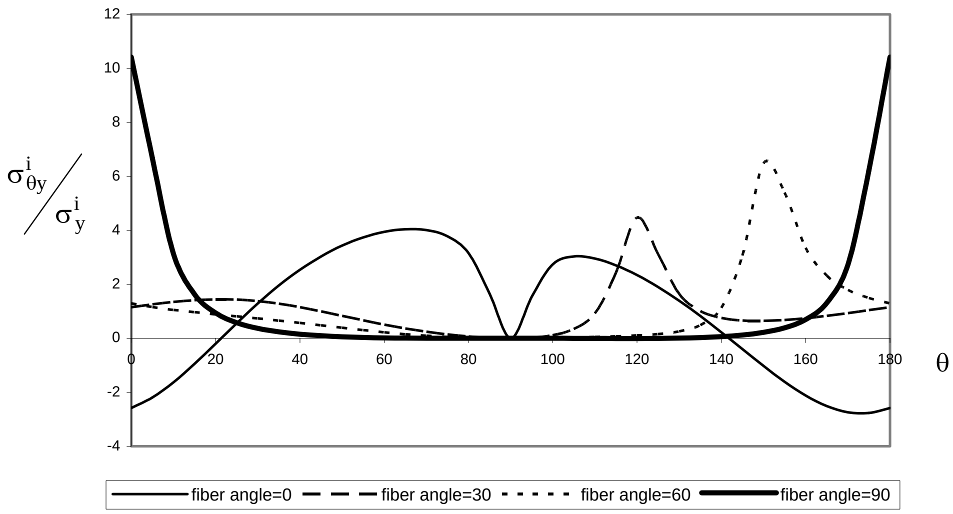

Fig. 8. Dimensionless y-direction stress values versus circumferential angle

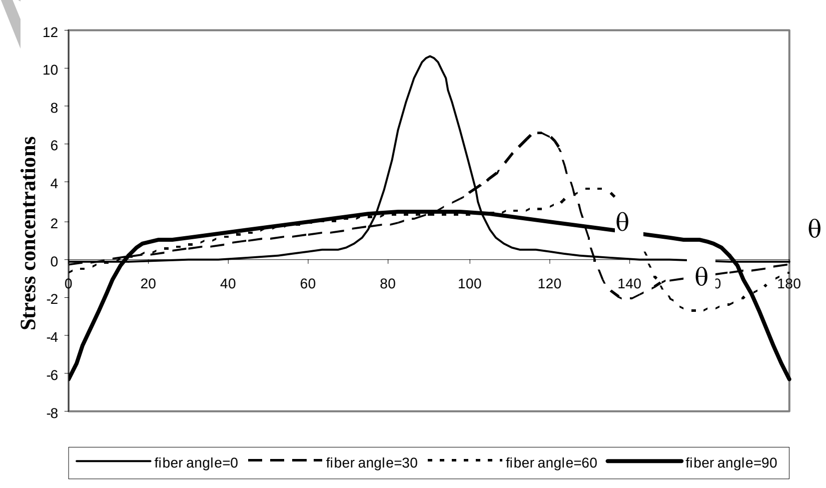

Fig. 9. Stress concentrations ($\sigma^i_\theta / \sigma^i_x$) versus θ for graphite-epoxy composite

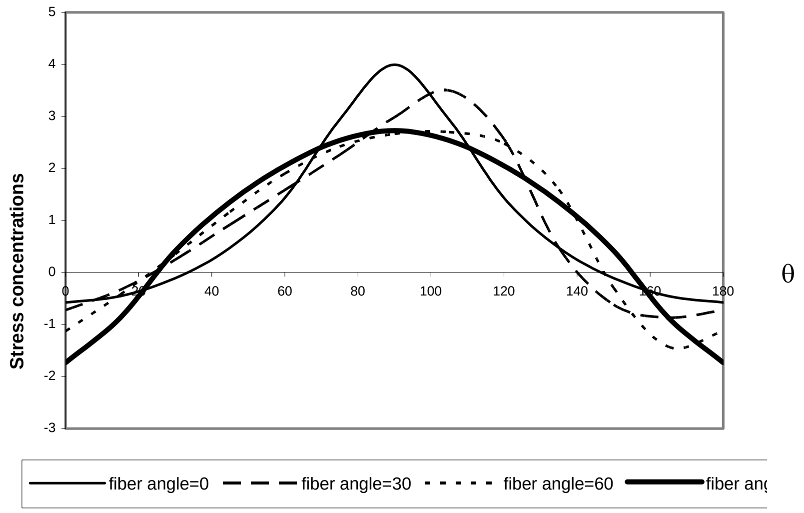

Fig. 10. Stress concentrations ($\sigma_\theta^i / \sigma_x^i$) versus θ for glass-epoxy composite

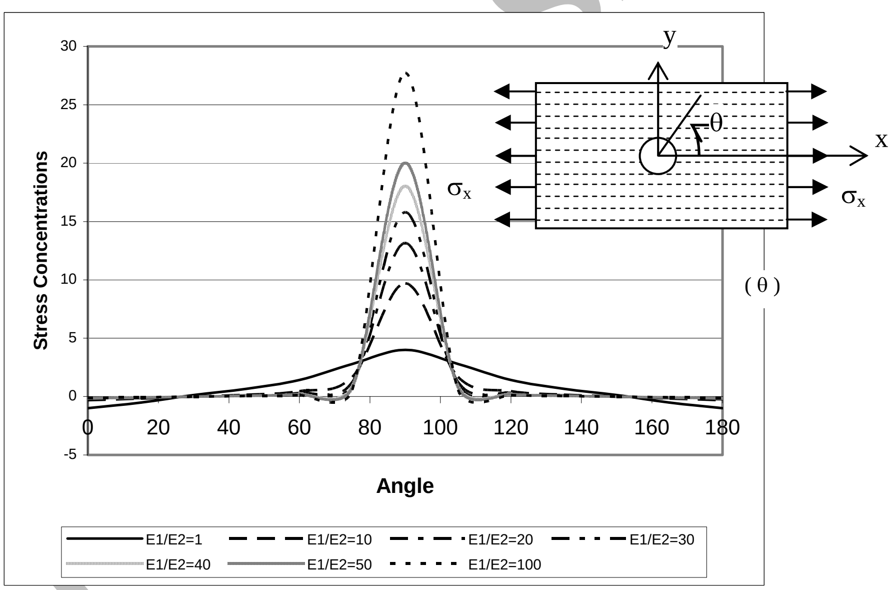

Fig. 11. Stress concentrations ($\sigma_\theta^i / \sigma_x^i$) versus θ for different E_1/E_2 ratio

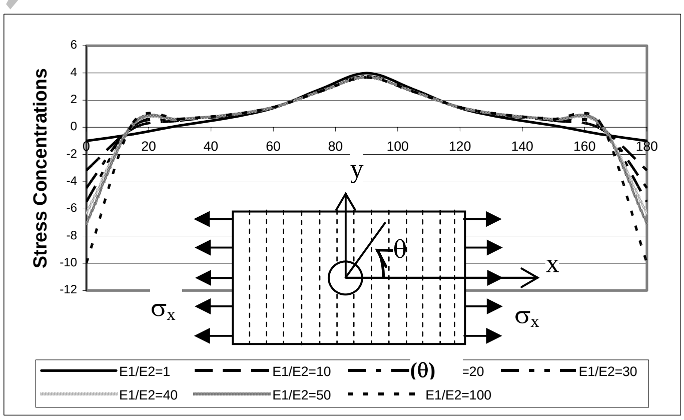

Fig. 12. Stress concentrations ($\sigma_\theta^i / \sigma_x^i$) versus θ for different E_1/E_2 ratio

In order to see the direct effect of the fiber orientation angle, and to compare the results of the computer program with the literature, a uniaxially loaded single layer is considered for graphite-epoxy composite (Fig. 9) and glass-epoxy composite (Fig. 10). Those results, obtained from the computer program, are compared with Kaltakci [4].

In Figs. 9 and 10 different circumferential stress concentration curves are given as a function of circumferential angel for two different material properties. A good agreement is observed between curves given by Kaltakci[4] and obtained in this study. If Fig. 9 is examined, one can note that in the laminated composite plate, the stress concentration value reaches a maximum at θ=90° for a layer in which the fiber orientation angle is 0°. This value is a minimum in the layer in which the fiber orientation angle is 90°. In laminated composite plates, each layer can have different concentration values depending on the fiber orientation angle, as shown in Fig. 9 and 10. The maximum concentration value and the location where it occurs, differ as the fiber orientation changes. Furthermore, the stress concentration values also change, depending on the ratio of the elasticity modulus of the fiber direction to the transverse one (E_1/E_2). In Figs. 9 and 10, it is observed that the maximum stress concentration occurs when the fibers are oriented parallel to the x-axis. The effect of the E_1/E_2 ratio on stress concentrations will be studied considering this layer as the critical one. For the same fiber orientations, different concentration factors are obtained for two different composites with different E_1/E_2 ratios. To see the effect of this ratio on the stress concentration value, a layer is considered which has a 0° fiber orientation angle. The same calculations are performed for a 90^0 fiber orientation angle, also.

In Figs. 11 and 12, the effect of E_1/E_2 ratio is shown for 0^0 and 90^0 fiber orientation angles. Using glass-epoxy material properties and increasing volumetric ratio of fibers, higher values of E_1/E_2 could be obtained. When the fibers are parallel to x axis, i.e. $\alpha=0^0$, as the E_1/E_2 ratio increases, the stress concentrations increase also. When $\theta=90^0$ and $E_1/E_2=100$, stress concentration value is calculated as 27.73 in the case of $\alpha=0^0$. But when values $\theta=0^0$ and $\theta=180^0$, the stress concentrations were not affected too much by the change in E_1/E_2 ratio. The situation is vice-versa for the second case (see Fig. 12). The positive stress concentration values show no significant changes with the E_1/E_2 ratio, whereas the negative values change from 1 to nearly 10.

In a symmetrically laminated composite plate, each layer is affected biaxial and shear stress. Depending on material properties, fiber orientation angles and the number of layers, the ratio of x-direction stress to y-direction stress could be changed. To see the effect of the stress ratio on a single layer in which the fiber angle is 0°, Fig. 13 is drawn. Four different loading values are considered for a graphite-epoxy layer. x-directional stress values around the hole are given as a function of circumferential angle θ.

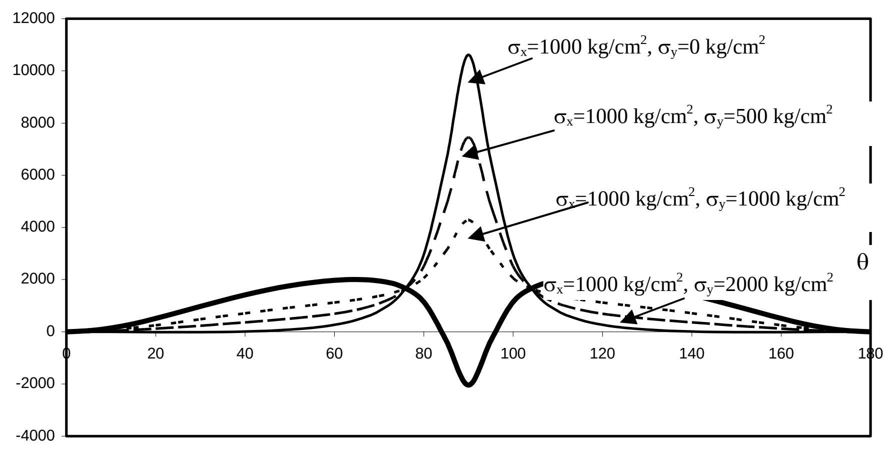

Fig. 13. x-directional stresses around the hole versus circumferential angle for different loading pairs

In Fig. 13, stresses around the circular hole are drawn in x-direction. When a layer is under the effect of uniaxial loading, the maximum stress obtained at the circumferential angle equals 90° (θ=90°). An increase in applied y-direction stress causes a decrease in the maximum stress value. Further increase leads maximum x-direction stress to a negative value, causing stress to turn from tension to comparison.

Table 2. Plate minimum circumferential stresses of graphite epoxy composite. (kg/cm^2)

	Compressive strength		Tensile strength	
Fiber angle	Minimum stress	Angular location (θ)	Minimum stress	Angular location (θ)
0	1196.00	29.0	422.00	0.0
5	1192.15	34.0	424.64	5.0
10	1181.65	39.0	432.71	10.0
15	1167.32	44.0	446.58	15.0
20	1152.93	49.0	466.93	20.0
25	1142.42	54.0	494.80	25.0
30	1139.52	1.0	531.72	30.0
35	1147.69	6.0	579.86	35.0
40	1170.49	11.0	642.33	40.0
45	1212.11	16.0	723.63	45.0
50	1278.26	21.0	830.41	50.0
55	1377.52	26.0	972.88	55.0
60	1523.81	31.0	1167.44	60.0
65	1741.26	36.0	1442.29	65.0
70	2075.08	41.0	1850.83	70.0
75	2617.84	46.0	2508.51	75.0
80	3575.67	51.0	3714.92	80.0
85	5342.92	56.0	6408.41	85.0
90	7037.00	61.0	10555.00	90.0

In the present work, Tsai-Hill failure criteria are employed for the failure of laminated composite plates. In laminated composites, due to layers having different fiber orientation angles, the stress state of layers is different from each other. Hence it is difficult to give failure load in x or in y direction individually. For example, a layer could have low x-directional stress, which may not cause failure, but due to higher y-directional stress and the effect of Poisson's ratio, the layer could fail. So, it is preferred to give minimum circumferential stresses as a function of the fiber angle.

4. CONCLUSIONS

In this study stress concentrations around the hole and critical circumferential stresses are investigated for laminated composite plates and a single orthotropic layer. Stress concentration values around the hole are obtained for a laminated composite plate having a different fiber orientation angle in each layer. It is observed that as the fiber orientation angle changes, maximum stress value around the hole and angular location of that stress value changes in a laminated composite plate. The same calculations are performed for the same fiber orientation angles for a single orthotropic layer affected by uniaxial loading. It is concluded that behavior is different for two cases in which one is single layer and the other is a layer in a laminated composite plate, both having the same material properties and fiber orientation angles due to the coupling effect in laminated composite plates. The ratio of biaxial loading (ratio of x-direction loads to y-direction loads) is also important to calculate design characteristics of a layer. As shown in Figure 13 positive stress concentration value is obtained for uniaxial loading of the plate, whereas the value is negative for the case in which ratio of x-direction loads to y-direction loads is 0.5 at the same angular location. As mentioned before, due to coupling effects in a laminated composite plate, critical circumferential stress values are given in Table 2 as a function of fiber angles for two cases. As seen from that table, critical values differ depending on the material strength considered in calculations.

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