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VARIED FLOW THROUGH SYSTEM OF GUTTERS AND GRATES*

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Abstract– An adaptation of the Newton Method is used to solve the combined system of ordinary differential and algebraic equations that describe the spatially varied flows that occur from lateral inflow with periodic outflow along open channels. The application selected is lateral inflow into gutters with periodically spaced grates in its bottom through which the accumulated flow is discharged. The occurrence of critical flow may allow the lateral inflow to be solved as a separate problem, and the outflow to be handled separately. Where subcritical flow exists, the composite system of differential and algebraic equations resulting from a series of gutter-grates must be solved simultaneously. Solutions to such example applications are given.

Keywords– Open channel flow, differential equations, gutters, grates, hydraulics, lateral inflow/outflow, numerical analysis, system of equations, varied flow, water flow

1. AN INTRODUCTION TO EQUATION FOR SPATIALLY OF VARIED FLOWS

Spatially varied flows occur when the flow rate in an open channel increases or decreases in magnitude because there is either lateral inflow or outflow from the channel. Such flows are governed by the following first order ordinary differential (ODE) that can be found in text books [1-4]. (This equation can be obtained from the energy principle for the outflow situations, but requires the momentum principle for lateral inflow.)

$$\frac{dY}{dx} = \frac{S_o - S_f + [Q^2/(gA^2)](\partial A/\partial x\,|_Y) - Qq^*/(gA^2) - F_q}{1 - F_r^2} \tag{1}$$

in which

$F_q = 0$ for bulk lateral outflow

$F_q = (Vq^*)/(2gA) = (Qq^*)/(2gA^2)$ for seepage flow, and

$F_q = (V-U_q)q^*/(gA) + (h_c/A)(MA/Mx)|_Y$ for bulk lateral inflow

The variables in Eq. 1 have the following meanings: Y = depth of flow (m or ft); x = position along the channel (m or ft); S_o= bottom slope of channel; S_f= slope of energy line, and in this paper will be computed using Mannings Equation; Q = the volumetric flow rate (m^3/s or ft^3/s); g=acceleration of gravity (9.81 m/s^2 when using SI units, 32.2 ft/s^2 when using ES units); A=cross-sectional area of flow (m^2 or ft^2); $MA/Mx|_Y$=the change in area with respect to x with the depth Y held constant (m or ft); q*=the lateral inflow or outflow, inflow is positive and outflow is negative (m^2/s or ft^2 /s); F_r^2= the Froude number squared and in general is evaluated from $Q^2T/(gA^3)$ (T=top width); V=velocity (Q/A) (m/s or ft/s); U_q=velocity component of lateral inflow in direction of the main flow (m/s or ft/s).

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It is important to understand what influence each term in Eq. (1) has. The third term in its numerator, along with the portion of F_q containing $MA/Mx|_Y$, will be referred to as the non-prismatic term. It is zero if the size and shape of the channel do not change. The fourth term in its numerator, along with the first part of F_q is referred to as the lateral inflow/outflow term. The depth increases in the downstream direction when dY/dx is positive, and decreases if it is negative. The denominator of Eq. (1) is positive for subcritical flows, i.e., when $Y>Y_c$ (Y_c=critical depth), and is negative for supercritical flows, i.e., when $Y<Y_c$. If the channel size reduces in the positive x direction the non-prismatic term adds to the negativeness of the numerator for subcritical flows, thus tending to decrease the depth and vice versa for supercritical flows. On the other hand, an expanding cross-section causes the depth to increase more in the downstream direction if the flow is subcritical and decrease if supercritical. The term $Qq^*/(gA^2)$ has the effect of increasing the depth for lateral outflow ($q^*<0$) and decreasing the depth for lateral inflow ($q^*>0$) for subcritical flow in the channel, and the opposite effects when the main channel flow is supercritical. Notice for a prismatic channel that $F_q=Qq^*/(gA^2)$ when $U_q=0$, and therefore for prismatic channels the effect of the lateral outflow term is twice as large for bulk lateral inflow as for bulk lateral outflow. However, the influence of U_q can reverse this effect if its magnitude is larger than V, but in the same direction as V, and if U_q's direction is opposite to V then the above effects on the depth are reversed. In brief, the effects of the terms in Eq. (1) are varied and can become confusing unless one carefully examines its sign in relationship to the sign of the denominator. Obviously the magnitudes of the non-prismatic, or the lateral inflow/outflow terms can easily exceed the difference between S_o and S_f, and consequently have the dominate influence.

In the remainder of this paper it will be assumed that $(MA/Mx)|_Y=0$ because its inclusion adds another unneeded dimension of complexity to attempts to summarize relationships between variables. Also, U_q will be taken as zero. With these assumptions the ODE for spatially varied <u>inflow</u> is

$$\frac{dY}{dx}=\frac{S_o-S_f-2Qq^*/(gA^2)}{1-F_r^{\,2}} \tag{2}$$

and for spatially varied <u>outflow</u> is

$$\frac{dY}{dx}=\frac{S_o-S_f-Qq_o^*/(gA^2)}{1-F_r^{\,2}} \tag{3}$$

in which $q_o^* = -q^*$.

2. INFLOW TO GUTTERS AND OUTFLOW THROUGH GRATES

The open channel flows resulting from the accumulations of lateral inflow into gutters from rainfall on roadways, and the outflow through grates (racks) that are periodically spaced along the gutters into storm drains is the problem that is dealt with in this paper. Under some circumstances the inflow to gutters and the outflow from grates can be handled as separate problems, whereas under other conditions the two flows must be handled as a single problem because each effects the other. Later in this paper the problem of a series of gutter-grates (the combined problem) will be handled. When the grates have more than ample capacity to receive all the gutter flow, it can be solved first because critical flow occurs where the grates begin, and thereafter the flow through the grates can be solved as a separate problem. To handle situations in which the grates easily accommodate the gutter flow, as well as to develop the necessary background for coping with a series of gutter-grates, the flow in a single gutter will be dealt with first. Next the flow from a single grate will be handled. Subsequently the two processes will be combined. Flow through grates in the bottom of channels is not limited to taking gutter flow into storm drains. Nor is the accumulation of lateral inflow to create a main channel flow limited to gutters. In other words, the solution techniques described below have other applications.

a) Gutter flow- handled separately

Assume that a length L of gutter with a mild bottom slope S_o is supplied by a lateral inflow q^* over its entire length, and that at both ends of this length of gutter there are drains with sufficient capacity to accept all of the flow. Under these assumptions the depths will be critical at both ends of the gutter, and the flow inbetween will be subcritical lateral inflow, as shown on Fig. 1.

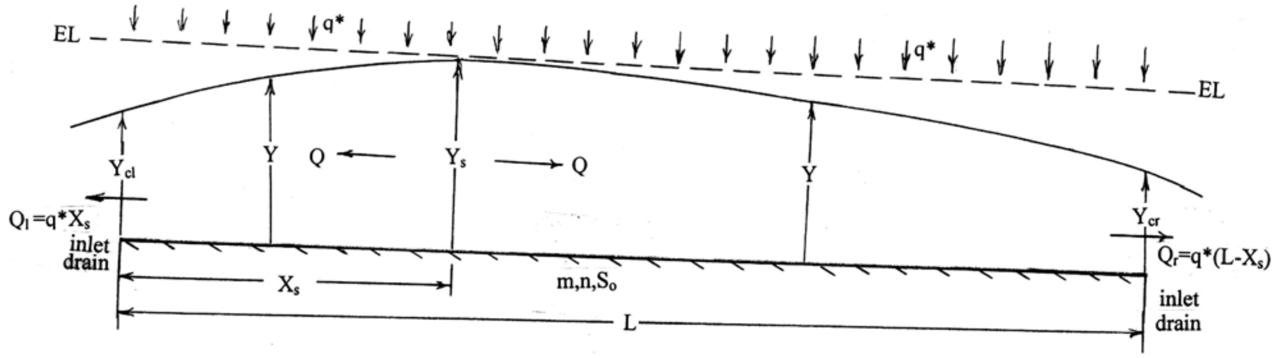

Fig. 1. Lateral Inflow across a gutter with complete flow removal at both its ends

The position X_s that separates the flow moving in the left direction from that moving in the right direction, and the depth Y_s at this position, are unknown, as well as the two critical depths at the left and right ends of the gutter . Thus there are four unknowns, Y_{cl}, Y_{cr}, Y_s and X_s. It will be assumed that the lateral inflow q^* is known, and to allow q* to vary in magnitude is given by $q^*=a_ox^2+a_1x+a_2$. The gutter's bottom slope S_o, its Mannings n, and geometric properties are also known. Four equations are needed to solve for the four unknown variables. The flow rates at the left and right ends of the gutter are given by integrating q* from x=0 to $x=X_s$ and from $x=X_s$ to x =L, respectively. If q* is constant ($a_1=a_o=0$), these integrations produce $Q_l = X_s q^*$ and $Q_r = (L-X_s)q^*$, in which the flow rate to the left Q_l moves up against the adverse bottom slope S_o, and Q_r is the flow rate passing the right end of the gutter. If a_o and a_1 are not zero then $Q_l= a_oX_s^3/3+a_1X_s^2/2+a_2 X_s$ and $Q_r = a_oL^3/3+a_1L^2/2+a_2 L- Q_l$. There are two methods for handling the flow on the left of the separate point; one is to use a positive x direction in the direction of flow, and then S_o is negative. The other is to use x as positive in the direction of the gutter slope, i.e. from left to right, and consider the flow in the left portion of the gutter negative, i.e. Q is negative. The latter approach will be used. The four equations needed are two critical flow equations and two ODE's, as given below.

$$F_1 = Q_l^2T_l - (gA_l^3) = 0 \tag{4}$$

$$F_2 = Q_r^2T_r - (gA_r^3) = 0 \tag{5}$$

$$F_3 = Y_s - Y_{sode}(Y_{cl}) = 0 \quad (Y_{sode} \text{ comes from Sol. of Eq. 2 with } x = 0 \text{ to } x=X_s) \tag{6}$$

$$F_4 = Y_s - Y_{sode}(Y_{cr}) = 0 \quad (Y_{sode} \text{ comes from Sol. of Eq. 2 with } x = L \text{ to } X_s) \tag{7}$$

The solution of the ODE (Eq. 2), which is denoted by $Y_{sode}(Y_{cl})$ in F_3 (Eq. (6)) starts with a depth slightly above the critical depth Y_{cl} at the x=0 end of the gutter associated with Q_l, and ends at X_s. For F_4 the ODE is solved starting with a depth slightly above the critical depth Y_{cr} at x=L end of the gutter associated with Q_r, and ending the solution at position X_s. The solution on the left side of X_s (e.g. F_3) uses Q as negative as mentioned above, and S_f will be computed from $S_f = |nQ(P/A)^{2/3}/(C_uA)|[nQ(P/A)^{2/3}/(C_uA)$, thus producing a negative value for S_f. On the left side of X_s the term $2Qq^*/(gA^2)$ in the numerator of the ODE, Eq. (2), adds to its positive amount causing the depth Y to increase in the positive x direction. When solving the ODE on the right of X_s, the computed values for both S_f and $2Qq^*/(gA^2)$ in the numerator of the ODE are positive and the negative sign in front of these terms will cause the depth Y to decrease in the positive x direction.

Often gutters have a cross-section that consists of one-half of a triangle as shown in Fig. 2.

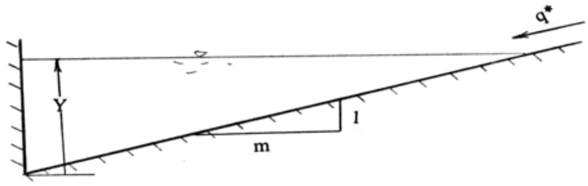

Fig. 2. Cross-Section of Gutter

For such triangular gutters, the area, perimeter and top width are given by, $A = 0.5mY^2$, $P = Y + Y(1+m^2)^{1/2} = Y\{1+(1+m^2)^{1/2}\}$ and $T = mY$, respectively. (Notice these are different than the special trapezoidal section with b = 0.) For triangular gutters the first two of the above four equations become (the critical flow equations):

$$F_1 = mY_{cl}Q_l^2 - .125gm^3Y_{cl}^6 = 0 \quad \text{or } Y_{cl} = [8Q_l^2/(gm^2)]^{0.2} \tag{8}$$

$$F_2 = mY_{cr}Q_r^2 - .125gm^3Y_{cr}^6 = 0 \quad \text{or } Y_{cr} = [8Q_r^2/(gm^2)]^{0.2} \tag{9}$$

b) Solution of combined systems of algebraic and ordinary differential equations

An adaptation of the Newton Method is well suited for solving combined systems of algebraic and ODE's such as Eqs. 4-7. The Newton iterative formula for solving a system of equations can be written as [5, 8],

$$\{x\}^{(m+1)} = \{x\}^{(m)} - [D]^{-1}\{F\}^{(m)} \tag{10}$$

in which {x} is the column vector of unknowns, {F} is a column vector of equations, [D] is called the Jacobian and is a square matrix of partial derivatives, i.e.

$$[D] = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & . & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & . & \frac{\partial F_2}{\partial x_n} \\ . & . & . & . \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & . & \frac{\partial F_n}{\partial x_n} \end{bmatrix}$$

These partial derivatives must be evaluated numerically for the equations coming from solutions of ODE's, and can be evaluated numerically for the algebraic equations. A numerical evaluation of these partial derivatives requires that the equation F_i be computed twice, or

$$\frac{\partial F_i}{\partial x_j} = \frac{F_i(x_1,x_2,..x_j+\Delta x_j, ..x_n\Delta - F_i(x_1,x_2,..x_n)}{\Delta x_j}$$

Equation (10) indicates that the Newton method solves a system of nonlinear equations by iteratively solving a system of linear equations because $[D]^{-1}\{F\}$ represents the solution of the linear system of equations

$$[D]\{z\} = \{F\} \tag{11}$$

That is that the amount subtracted from the current estimate of the unknown vector is the solution {z} to the above linear system of equations. Thus in practice the Newton method solves a system of equations by the iterative formula,

$$\{x\}^{(m+1)} = \{x\}^{(m)} - \{z\} \tag{12}$$

where the superscript (m) denotes the iteration number. Each ODE might be considered a function of unknowns as described in [5], Chapter 12. When the correct combination of these unknowns are used, then each ODE will produce the dependent value (for our ODE's the Y's) that satisfy the other equations. Thus Eqs. (6) and (7) are written as given above so that the Newton Method drives the solution of the ODE's, $Y_{sode}(Y_{cl})$ and $Y_{sode}(Y_{cr})$ toward Y_s. In writing a computer program to solve a system of equations such as Eq. (4) through 7, one wants to use subroutines (or functions) that use the Runge-Kutta [6], Bulirsch-Stoer [7], or some other method for solving ODE's, and also a linear algebra subroutine (or function) to solve the linear system, Eq. (11). Also this program should contain a subroutine (function) to evaluate the equations, i.e., Eqs. (4-7) (below this subroutine is named FUN.) This subroutine will call on the ODE-Solver to evaluate the ODE-equations, i.e. Eqs. (6) and (7). In this program the unknown vector will be stored in an array, say X(4), that holds the iterative values for Y_{cl}, Y_{cr}, Y_s and X_s for the above problem. Also two arrays, say F(4) and F1(4) for the above problem, are needed to store the values of the equations. Then the portion of the program that implements the Newton Method could be as given below (Fortran has been used and comments after the statements indicate what each statement does.) This portion of the main program is written in general so that if the parameter N is changed, a larger system of equations can be solved using the same code. (The CD in the back cover of [5] contains source code in Fortran and C to solve ODE's, ODESOL, and solve linear algebraic equations, SOLVEQ.)

```
      M=0                  // set Newton's iteration counter to 0
10    CALL FUN(F)          // supplies the values to the equations which when satisfied = 0
      DO 20 J=1,N          // This Do loop numerically evaluates the derivatives in the Jacobian
      XX=X(J)              // XX stores original value of unknown
      X(J)=1.005*X(J)      // increments unknown,i.e., gives xj+)xj
      CALL FUN(F1)         // Evaluates equations with incremented unknown
      DO 15 I=1,N          // This DO loop numerically evaluates the elements in the Jacobian.
15    D(I,J)=(F1(I)-F(I))/(X(J)-XX)  // index I denotes the row and J the column
20    X(J)=XX              //  Sets unknown back to non-incremented value
      CALL SOLVEQ(N,1,N,D,F,1,DD,INDX) // returns solution {z} of [D]{z}={F} in array F
      DIF=0.
      DO 30 I=1,N          //  This Do loop implements Newton iterative formula {x}={x}-{z}
      X(I)=X(I)-F(I)       //  substracts correction
30    DIF=DIF+ABS(F(I))    // accumulates absolute values of corrections
      M=M+1                // increments Newton's iteration counter
      IF(NCT.LT.MAX .AND. DIF.GT.ERR) GO TO 10 // repeat another Newton iteration until convergence
```

c) Gutter flow – handled separately (continued)

There are so many variables involved in the flow into a gutter that it is not practical to provide design charts that relate all these variables. Therefore, consider lateral inflow into a gutter that is 280 m long, and is triangular in shape with a side slope m = 4, a Mannings n = .013, and a bottom slope S_o=0.0009. The flow variables for the following 15 situations, in which the lateral inflow varies across the gutter as given below, have been solved and the solutions are given in Table 1. (The coefficient a_o=0 in the quadratic equation defining the inflow q* in all solutions.)

No.	Coefficients a_1	a_2	Explanation
1	$4.\times10^{-4}$	0	A constant inflow that result in a total Q=0.112 m^3/s
2	$6.\times10^{-4}$	0	A constant inflow that result in a total Q=0.168 m^3/s
3	$8.\times10^{-4}$	0	A constant inflow that result in a total Q=0.224 m^3/s
4	$2.\times10^{-4}$	1.42857×10^{-6}	q* at beg. 1/2 of No. 1 with increase with x so Q=0.112 m^3/s
5	0	2.85714×10^{-6}	q* at beg. 0, with increase with x so Q=0.112 m^3/s
6	$6.\times10^{-4}$	-1.42857×10^{-6}	q* at beg. .0006, with decrease with x so Q=0.112 m^3/s
7	$8.\times10^{-4}$	-2.85714×10^{-6}	q* at beg. .0008, with decrease with x so Q=0.112 m^3/s
8	$3.\times10^{-4}$	2.142857×10^{-6}	q* at beg. 1/2 of No. 2 with increase with x so Q=0.168 m^3/s
9	0	4.28571×10^{-6}	q* at beg. 0, with increase with x so Q=0.168 m^3/s
10	$9.\times10^{-4}$	-2.14286×10^{-6}	q* at beg. .0009, with decrease with x so Q=0.168 m^3/s
11	$12.\times10^{-4}$	-4.28571×10^{-6}	q* at beg. .0012, with decrease with x so Q=0.168 m^3/s
12	$4.\times10^{-4}$	2.857143×10^{-6}	q* at beg. 1/2 of No. 3 with increase with x so Q=0.224 m^3/s
13	0	5.714286×10^{-6}	q* at beg. 0, with increase with x so Q=0.224 m^3/s
14	$12.\times10^{-4}$	-2.85714×10^{-6}	q* at beg. .0012, with decrease with x so Q=0.224 m^3/s
15	$16.\times10^{-4}$	-5.71429×10^{-6}	q* at beg. .0016, with decrease with x so Q=0.224 m^3/s

Expanding the number of solutions in which the lateral inflow is constant, i.e., coefficients a_o and a_1 are zero and obtaining such a series of solutions for different bottom slopes with S_o varying from .0001 to .0018 gives the results displayed in Fig. 3. The general trend of lines on these graphs indicate that the separation point X_s changes very little with increasing lateral inflow if the bottom slope of the channel is small. This trend is evident by the flatness of the curves on the first graph for S_o=.0001 and .00025, etc. Another observation is that the depth at this separate point Y_s doesn't change much with the bottom slope. For example, if the lateral inflow q*=.00305 m^2/s then Y_s=0.4 m when S_o=.0018, and Y_s=0.57 when S_o=.0001. By comparing the 3rd and 4th graphs which provide the depths at the left and right sides, Y_l and Y_r, respectively, one notes that the variation in depth with S_o is slightly less on the right side than the left, but as one would expect, both of these depths increase with the decreasing bottom slope of the gutter, as well as increasing q*.

Table 1. Summary of solution results as related to pattern of q*(Triangular Gutter with m=4, L=280 m, n=.013, & So=.0009)

Prob. No.	Variation of Coef. of x	Inflow q* Const	Q(tot) (m^3/s)	Y (m)	X (m)	Y(left) (m)	Y(right) (m)
1	0.0000000	0.0004	0.112	0.196	42.0	0.108	0.215
2	0.0000000	0.0006	0.168	0.242	49.8	0.135	0.250
3	0.0000000	0.0008	0.224	0.280	55.5	0.159	0.277
4	0.142857E-05	0.0002	0.112	0.189	51.8	0.095	0.219
5	0.285714E-05	0.0000	0.112	0.196	75.8	0.081	0.223
6	-0.142857E-05	0.0006	0.112	0.207	37.9	0.119	0.211
7	-0.285614E-05	0.0008	0.112	0.220	36.0	0.130	0.206
8	0.214286E-05	0.0003	0.168	0.240	63.5	0.123	0.254
9	0.428571E-05	0.0000	0.168	0.257	91.5	0.110	0.258
10	-0.214286E-05	0.0009	0.168	0.252	43.5	0.148	0.244
11	-0.428571E-05	0.0012	0.168	0.265	40.3	0.159	0.239
12	0.285714E-05	0.0004	0.224	0.281	71.8	0.146	0.283
13	0.571429E-05	0.0000	0.224	0.296	100.1	0.133	0.287
14	-0.285714E-05	0.0012	0.224	0.289	47.4	0.171	0.272
15	-0.571429E-05	0.0016	0.224	0.301	43.2	0.183	0.265

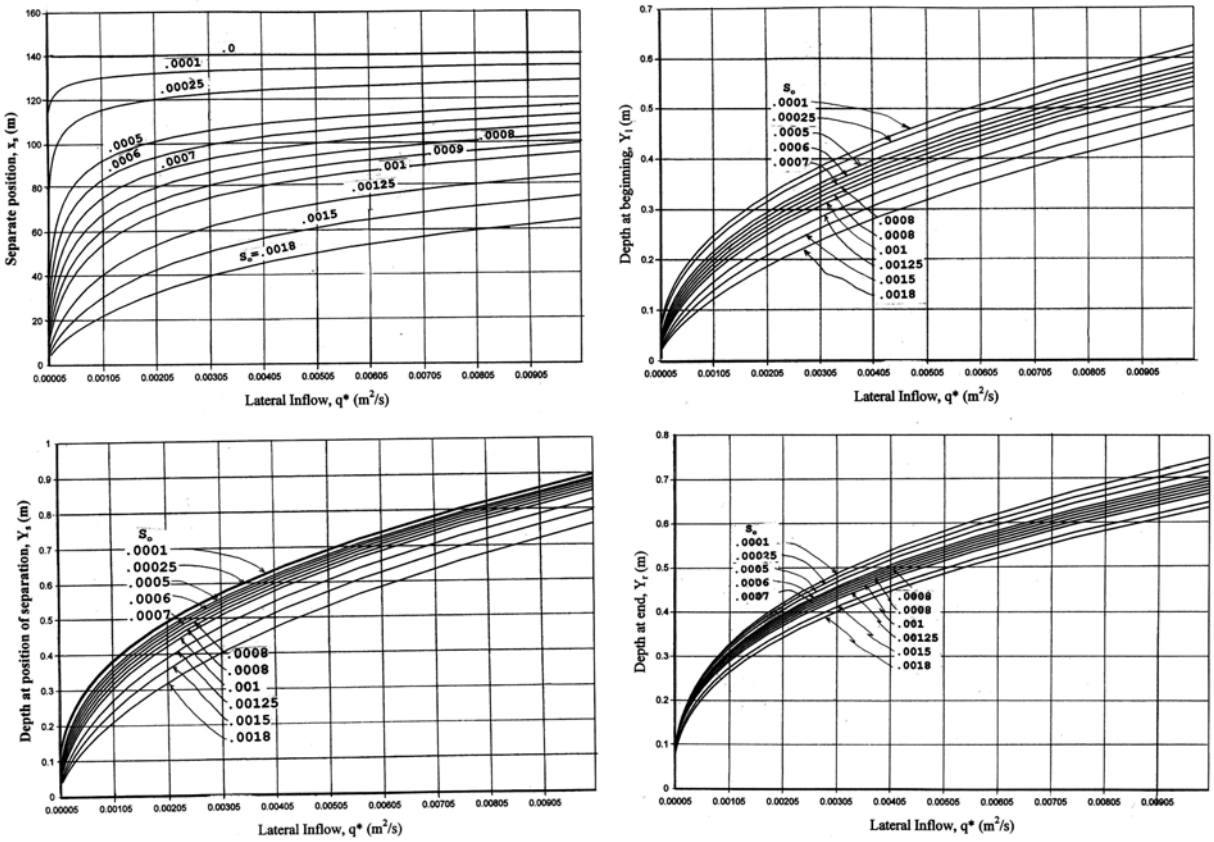

Fig. 3. Graphs showing the relationship of the four solved variables to q* and S_o for our gutter

d) Flow into grates on bottom of channel–again outflow handled separately

The flow from the bottom of a gutter through grates is a problem in which the lateral outflow depends upon the depth of the flow at any position. The orifice formula is generally used to define this lateral outflow

$$q_o^* = C_d(2g)^{1/2}(A_o/L)\ Y^{0.5} = C_d(2g)^{1/2}(fb)Y^{0.5} \tag{13}$$

in which (A_o/L) is the area of the opening per unit length, with f equal to the fraction of the bottom that is open. Assume that the storm drain that receives the flow from the grates has the capacity to carry off the flow. If this is not the case then the lateral outflow will be restricted by the hydraulics of the storm drain system of pipes, etc. The ODE that describes the change in depth across an outflow grate is Eq. (3). The flow rate Q will need to be obtained by numerically integrating the lateral outflow, i.e.,

$$Q = Q_o - Iq_o^*dx \tag{14}$$

in which Q_o is the flow rate at the beginning of the grate. An easy, not very precise, but generally adequate means for carrying out the numerical integration is to use the trapezoidal rule, or

$$Iq_o^*dx = \Gamma\{(x_{i+1}-x_i)[(q_o^*)_i + (q_o^*)_{i+1}]/2\} = \Gamma\{)x(q_o^*)_{av}\} \tag{15}$$

in which subscript i denotes the past position and subscript i+1 denotes the current position. For the solutions that follow, the grate will be given a triangular section (as the gutter) as shown in Fig. 2 when solving for area, perimeter, etc., but when using Eq. (13) the discharge will be assumed through a flat bottom with a width b.

If the flow in the gutter at both ends of the grate, is critical, as assumed in the gutter flow described previously, then the solution to the ODE will begin with a depth just slightly below critical depth, resulting in a negative denominator for the ODE, causing the depth to generally decrease in the x direction as shown in Fig. 4. Under these assumptions the solution of Eq. (3) can proceed until the depth becomes zero, or the flow rate becomes zero (both of which should occur essentially simultaneously.) The resulting distances provide the length of grate needed to discharge the flow from both the left and right sides of the gutter, and their sum is the length of grate needed. Table 2 provides results from solutions for which Fig. 3 applies in which the lateral inflow is varied from a $q^* = 0.01\ m^2/s$, near the largest abscissa value on the graphs, to $q^*=0.00001\ m^2/s$. In obtaining these solutions the grate was assumed to have a bottom width b = 0.3 m and a side slope beyond this of m = 4, a bottom slope S_o=.0009 and Mannings n=.013. One-half of the 0.3 m bottom was assumed open, i.e., f=0.5 and the discharge coefficient was assumed to be C_d=0.45.

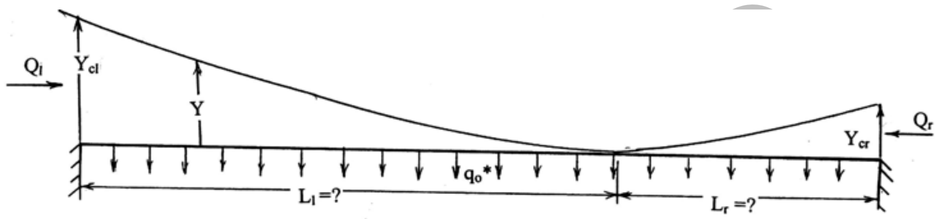

Fig. 4. Supercritical flow into grates

Table 2. Lengths of grates needed to discharge the lateral inflow from the previous gutter with a length of L = 280 m

q* (m^3/s)	Y_s (m)	X_s (m)	Y_l (m)	Y_r (m)	Q_{tot} (m)	L_r (m)	L_l (m)	L_{tot} (m)
0.010000	0.8610	99.4831	0.5502	0.6983	2.8000	4.00	10.35	14.35
0.007500	0.7642	95.4515	0.4824	0.6280	2.1000	3.00	8.30	11.30
0.005625	0.6774	91.0944	0.4220	0.5649	1.5750	2.40	6.65	9.05
0.004219	0.5994	86.4719	0.3684	0.5084	1.1812	1.80	5.30	7.10
0.003164	0.5293	81.6047	0.3208	0.4577	0.8859	1.40	4.20	5.60
0.002373	0.4663	76.4696	0.2786	0.4121	0.6645	1.00	3.30	4.30
0.001780	0.4096	71.1217	0.2412	0.3712	0.4983	0.80	2.60	3.40
0.001335	0.3586	65.5984	0.2082	0.3343	0.3738	0.60	1.95	2.55
0.001001	0.3127	59.9516	0.1790	0.3011	0.2803	0.40	1.45	1.85
0.000751	0.2714	54.1960	0.1532	0.2711	0.2102	0.40	0.95	1.35
0.000563	0.2345	48.5359	0.1307	0.2441	0.1577	0.20	0.55	0.75
0.000422	0.2015	43.0383	0.1110	0.2196	0.1183	0.20	0.15	0.35
0.000317	0.1722	37.6955	0.0938	0.1975	0.0887	0.20	0.05	0.25
0.000238	0.1465	32.7211	0.0790	0.1774	0.0665	0.20	0.05	0.25
0.000178	0.1240	28.1094	0.0663	0.1593	0.0499	0.08	0.01	0.09
0.000134	0.1046	24.0098	0.0555	0.1429	0.0374	0.04	0.01	0.05
0.000100	0.0880	20.3881	0.0463	0.1281	0.0281	0.04	0.01	0.05
0.000075	0.0739	17.2467	0.0386	0.1147	0.0210	0.04	0.01	0.05
0.000056	0.0620	14.5584	0.0321	0.1027	0.0158	0.04	0.01	0.05
0.000042	0.0520	12.2875	0.0268	0.0918	0.0118	0.04	0.01	0.05
0.000032	0.0436	10.3610	0.0223	0.0821	0.0089	0.04	0.01	0.05
0.000024	0.0366	8.7299	0.0186	0.0733	0.0067	0.04	0.01	0.05
0.000018	0.0307	7.3692	0.0155	0.0655	0.0050	0.04	0.01	0.05
0.000013	0.0257	6.2142	0.0129	0.0585	0.0037	0.04	0.01	0.05
0.000010	0.0216	5.2400	0.0107	0.0522	0.0028	0.04	0.01	0.05

Let's examine the case in which the grate is relatively short so only a portion of the flow is taken from the grate. Then two of the possible situations are shown in Fig. 5; the first contains subcritical flow through the entire grate length, and upstream and downstream therefrom the flows are also subcritical with the downstream depth as the control; the second contains supercritical flow throughout the grate, and the upstream depth Y_1 is the control. If the channel downstream is not steep, in the second case, then a supercritical flow over the first portion of the grate will result in a hydraulic jump either within the grate length, or in the downstream channel.

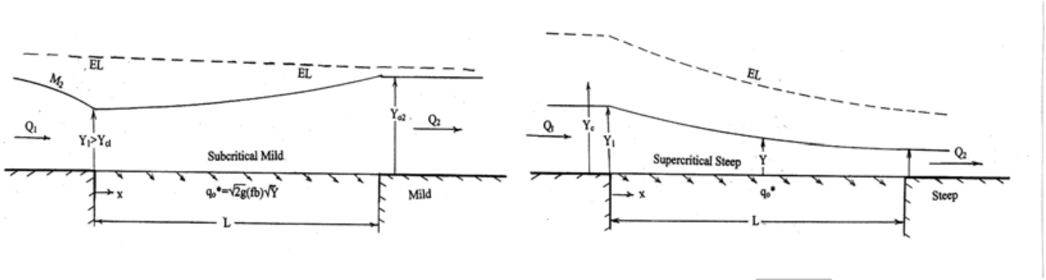

Fig. 5. Two cases of incomplete inflow through a grate

For the first case, if the channel is mild and long downstream from the grate, then normal depth Y_{o2}, based on the flow rate Q_2 remaining in the channel will exist at the end of the grate, and since the term $Qq_o^*/(gA^2)$ contributes to the positiveness of the numerator of the ODE and its denominator is positive since $F_r^2<1$, the depth increases across the grate as shown in the sketch. An M_2-GVF will occur upstream from the grate. The solution to this subcritical flow situation is to simultaneously satisfy Mannings Equation in the downstream channel and solve the ODE through the grate to give the variation in depth across the grate, as well as evaluate Iq_o^*dx so the continuity equation $Q_1 = Q_2 + Iq_o^*dx$ can be satisfied. In other words there are three unknowns involved, Y_1, $Y_2 = Y_{o2}$ and Q_2. The three available equations are: (1) Mannings Equation in the downstream channel, (2) the ODE across the grate, and (3) the continuity equation, or

$$F_1=nQ_2P^{2/3} - C_uA^{5/3}S_o^{1/2} =0 \tag{16}$$

$$F_2=Y_1-Y_{1ode}(Y_{o2})=0 \text{ (with } Y_{1ode} \text{ from the sol. of Eq. 3 from x=L to 0)} \tag{17}$$

$$F_3=Q_1-Q_2-Iq_o^*dx =0 \tag{18}$$

Table 3 provides solutions in which subcritical flow occurs across the grate. For these solutions the length of the grate is L = 0.5 m, its C_d=0.45, its bottom width b = 0.3 m, f=0.5, and m = 4. The channel is n=.013, and its bottom slope is S_o=0.0009. Note as the inflow decreases (or similarly if the length of grate increases), the upstream Froude Number gets closer to unity, and the smallest Q_1=0.5 m^3/s in Table 3 is close to the limit for having subcritical flow across the grate since F_{rl} = .883.

If the channel is steep (the second case in Fig. 5) so the upstream flow is supercritical, then the denominator of the ODE is negative and the depth will decrease across the grate. The upstream depth will

Table 3. Solutions with subcritical flow across grate and uniform flow downstream therefrom

Q_1 (m^3/s)	Y_1 (m)	Y_2 (m)	Iq_o*dx (m^3/s)	Q_2 (m^3/s)	F_{r1}	F_{r2}
1.1	0.660	.686	0.123	0.977	.703	0.566
1.0	0.632	0.659	0.120	0.880	0.712	0.563
0.9	0.602	0.631	0.118	0.782	0.724	0.558
0.8	0.601	0.569	0.113	0.686	0.740	0.554
0.7	0.532	0.532	0.111	0.589	0.751	0.549
0.6	0.491	0.491	0.107	0.493	0.781	0.543
0.5	0.439	0.439	0.103	0.397	0.883	0.535

not be affected by the outflow from the grate, so the solution can proceed by solving the ODE starting with depth Y_1 at the upstream end of the grate and let this solution provide the outflow Iq_o^*dx as well as the depth Y_2 at the end of the grate.

If the grate is followed by another gutter with lateral inflow, or something else that effects the flow so that normal depth does not exist downstream then we may want to solve the problem with subcritical flow over the grate, but be able to specify what the flow rate is that leaves the end of the grate as well as what flow rate enters. In other words, the depths at the beginning and end of the grate are sought that will cause a specified amount of outflow over the grate. For this problem two equations are available; the solution of the ODE across the grate and the continuity equation, namely Eqs. (18) and (1). The only difference is that the depth Y_2 will be used in place of the normal depth Y_{o2} in Eq. (18). These two equations solve for the depths at the beginning and end of the grate, Y_1 and Y_2, respectively, with the flow rates entering and leaving the grate $Q_{in} = Q_1$ and Q_{out},$= Q_2$, respectively, specified.

Table 4 contains a series of such solutions for the grate dealt with earlier, in which all specified outflow Q_{out}=0 and the inflow Q_{in} are as shown in the first column. Note how the upstream Froude Number, F_{r1}, increases with decreasing Q_{in}. The solution fails when Q_{in} is given as 0.071 m^3/s because for this and smaller values the flow is supercritical, at least over the first portion of the grate.

Table 4. Solution of the subcritical flow through grate with L=0.5 m, n=.013, S_o=.0009, f=.5 & C_d=.45 (Inflow as given in column 1 with zero outflow)

Q_{in} (m^3/s)	Y_1 (m)	Y_2 (m)	F_{r1}
0.200	1.790	1.790	0.011
0.140	0.877	0.877	0.044
0.100	0.449	0.445	0.171
0.080	0.291	0.277	0.449
0.078	0.278	0.260	0.509
0.076	0.265	0.244	0.586
0.074	0.253	0.225	0.693
0.072	0.241	0.200	0.905

3. COMBINED PROBLEM-GUTTER INFLOW AND GRATE OUTFLOW

If the length of the grate (or lateral outflow length) is less than that needed for all of the gutter flow at the beginning of the grate to enter, the depth feather down to zero, and it is necessary to combine the problems of lateral inflow along the length of the gutter with the lateral outflow along the length of the grate. First, consider the case where the flow throughout both the lateral inflow length and the lateral outflow length are subcritical. This means that the depth of water over the grate, or outflow length, is sufficient to cause the depths in the gutter both upstream and downstream from the grate to be above critical depth. For this case the depths throughout the lateral outflow length of the grate are subcritical, and this outflow is not separated from the lateral inflow by a control caused by the depth being critical at either the upstream or downstream end of the grate (or gutter).

Assume that there are a series of grates equally spaced along a gutter that receives a constant lateral inflow q^*, so that the problem can be defined as depicted in Fig. 6, in which the length being considered can start in the gutter at the end of the grate, proceed through the gutter, and then finally though the grate. If one wishes to find the point X_m in the grate where the flow rate Q goes to zero then the control section can begin here and then proceed to the gutter, then through the gutter length, and finally through the grate to the same point in the outflow length where the flow rate is zero.

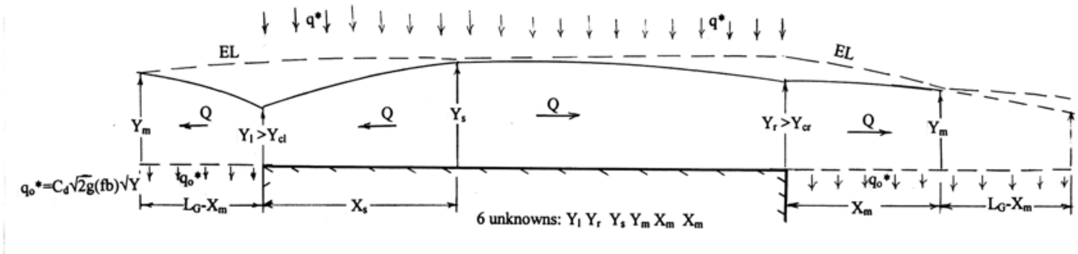

Fig. 6. Combined problem of lateral inflow into gutter which is discharged through grates

Assume the following variables are known and constant: b, m, S_o, n, L, L_G and q*. The unknown variables are: (1) the depth on the left side Y_l at the position between the lateral inflow and lateral outflow sections and $Y_l > Y_{cl}$ based on $Q_l = X_s q^*$; (2) the depth on the right side Y_r between the lateral inflow and lateral outflow sections and $Y_r > Y_{cr}$ based on $Q_r = (L\text{-}X_s)q^*$; (3) the depth Y_s in the gutter lateral inflow length where the flow rate is zero, i.e., the flow separates from moving left to right; (4) the position X_s where the flow in the gutter separates. In addition to these four variables one might add: (5) the depth Y_m within the grate outflow length where the flow separates in moving from upstream to downstream, i.e., where Q = 0 within the grate length; and (6), the position X_m where this Q = 0 occurs. These latter two variables shown on Fig. 6 can be determined by the solution of the spatially varied flow through out the entire grate length. In the description that follows consider only the first four variables as unknown by solving the ODE over the entire length of the grate, thus incorporating Y_m and X_m as part of this solution.

To solve these four unknown variables requires the simultaneous solution of the following four equations:

$$F_1 = Y_s - Y_{sode}(Y_l) = 0 \quad \text{with ODE solved from } x=0 \text{ to } x=X_s \tag{19}$$

$$F_2 = Y_s - Y_{sode}(Y_r) = 0 \quad \text{with ODE solved from } x=L \text{ to } x=X_s \tag{20}$$

$$F_3 = Y_l - Y_{lode}(Y_r) = 0 \quad \text{with ODE solved from } x'=0 \text{ to } x'=L_G \tag{21}$$

$$F_4 = Lq^* - \int_0^{L_G} q_o^* dx' = 0 \quad \text{Continuity Eq.} \tag{22}$$

Note that the first two equations are identical to Eq. 6 and 7 used to solve the gutter flow in which critical depths at both ends were assumed, but the starting depths Y_l and Y_r are now above the critical depths.

It has already been noted that because the lateral inflow term $2Qq^*/(gA^2)$ adds to the negativeness of the numerator of the ODE on the right side of X_s, and to the positiveness of the numerator of the ODE on the left side of X_s, that the depth Y_s will be larger than both Y_l and Y_r. Where Q is positive within the grate length, the term $Qq_o^*/(gA^2)$ in Eq. 2 adds to the numerator's positiveness thus making Y_m larger than Y_r, generally. Likewise where Q is negative from position L_G-X_m in the grate to position L_G, this term tends to make Y_m larger than Y_l. With a slope S_o greater than zero one would expect X_m to be larger than $L_G/2$, and therefore the length L_G-X_m will generally be quite small in consideration that the length of the grate will generally be much smaller than the length of the gutter. Therefore, generally there will not be as much difference between Y_l and Y_r (and Y_m) as there will be if the depths are critical at both ends of the gutter. The position X_s in the gutter that separates positive from negative Q's would be expected to be smaller than L/2, even for relatively small slopes S_o. In fact the effects of the increased depth, since larger depths are required for the accumulated inflow to exit through the grates, can easily result in no length of negative Q through the gutter, also resulting in no length of negative Q through the grate.

Table 5 contains such solutions for a gutter-grate combination using the example gutter and grate dealt with previously. For the first nine solutions in this table the bottom slope has been specified as S_o=0.0 (flat) so the position where the inflow and outflow separates from moving upstream and downstream are in the middle of the gutter and grate respectively, i.e. a 140 m and 0.25 m. Notice how rapidly the depths decrease with decreasing lateral inflows until when q*=0.000216 m^2/s, the Froude Number at the two ends of the gutter approach unity. For smaller values of q*, critical depths govern at the ends of the gutter so it can be solved separately as described earlier. For the larger values of q* the grate cannot discharge the flow until the depth becomes sufficiently large, and for the solutions using the larger q*'s there is essentially no change in depth over the entire gutter-grate system. For these cases the problem might be solved by finding this needed depth so the inflow that has been accumulated over the gutter length is discharged through the grate. For these larger q* (q*>. 0004 in Table 5)this near constant depth can be computed from Eq. (13). The total accumulated inflow over the gutter is Lq* and this amount must go into the grate, or $Lq^* = C_d(2g)^{.5}\{fbL_G\}Y^{.5}$, or upon substituting in the values used for this gutter-grate system $Y= (1872.98q^*)^2$, thus when q*=.0004 m^2/s, Y=0.561 m. Notice further from this example how rapidly the position of separation in the gutter moves to its upstream end as the bottom slope S_o of the system increases, with q*=.0003 m^2/s (the last three solutions in Table 5.) In this example a bottom slope of S_o=.00015 results in no reverse flow in the gutter, since X_s is zero or less.

Table 5. Solution to a combined Gutter-grate under subcritical flow conditions. (gutter:L=280 m,m=4, n=.013; grate:L_G=0.5 m,n=.013,b=0.3 m,f=0.5,C_d=0.45)

So	q* m^2/s)	Y_l (m)	Y_r (m)	Y_s (m)	X_s (m)	F_{rl}	F_{rr}	X_m (m)	Y_m (m)
0.0	0.001	3.508	3.508	3.508	140.0	0.001	0.001	0.250	3.508
0.0	0.0009	2.842	2.842	2.842	140.0	0.002	0.002	0.250	2.842
0.0	0.0008	1.068	1.068	1.068	140.0	0.021	0.022	0.250	1.068
0.0	0.0005	0.877	0.877	0.877	140.0	0.022	0.022	0.250	0.877
0.0	0.0004	0.561	0.561	0.562	140.0	0.054	0.054	0.250	0.561
0.0	0.0003	0.314	0.314	0.323	140.0	0.171	0.171	0.250	0.316
0.0	0.00025	0.214	0.214	0.245	140.0	0.372	0.372	0.250	0.222
0.0	0.00022	0.154	0.154	0.220	140.0	0.744	0.744	0.250	0.175
0.0	0.000216	0.142	0.141	0.218	140.0	0.903	0.907	0.250	0.170
0.00005	0.0003	0.316	0.312	0.323	87.0	0.104	0.240	0.356	0.317
0.0001	0.0003	0.318	0.311	0.322	39.6	0.047	0.303	0.431	0.318
0.00015	0.0003	0.319	0.309	0.319	-0.7	0.000	0.358	0.500	0.319

4. NO NEGATIVE FLOW RATES

When the combination of variables for a gutter-grate problem result in no reverse flow at the beginning of the gutter, the mathematical problem simplifies because the position X_s that separates positive from negative flows is known; X_s = 0 and Y_s does not exist. However, unless other variables are just the right magnitude there will be flow in the channel at the end of the grate, i.e. the grate will not discharge all of the flow that accumulates over the inflow length of gutter. While there are other ways of posing the problem, assume that the unknowns are: (1) the depth Y_l on the left side of the gutter, which is also let to be the depth at the end of the grate, (2) the depth Y_r on the right side of the gutter, which is also the depth at the beginning of the grate, and (3) the flow rate Q_{out} that leaves the end of the grate to add to the flow in the next series of gutter grates, as shown in Fig. 7.

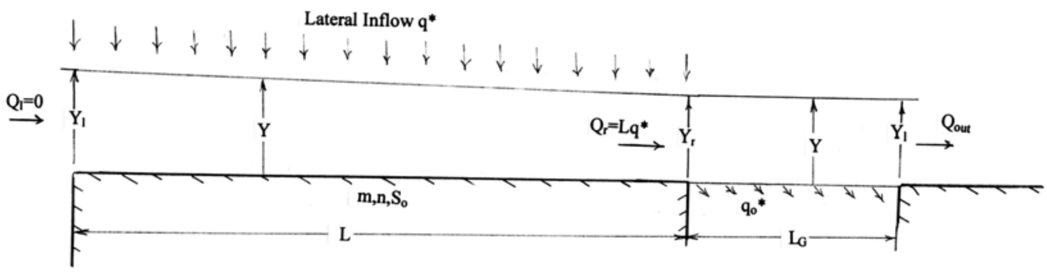

Fig. 7. Wall after gutter to prevent downstream flow

The three equations needed to solve for these three unknowns are:

$$F_1 = Y_r - Y_{rode}(Y_l) = 0 \quad \text{with the ODE solved over } x = 0 \text{ to } x = L \tag{23}$$

$$F_2 = Y_l - Y_{lode}(Y_r) = 0 \quad \text{with the ODE solved over } x' = 0 \text{ to } x' = L_G \tag{24}$$

$$F_3 = q^*L - \int_0^{L_G} q_o^* dx' - Q_{out} = 0 \tag{25}$$

a) Last grate with vertical wall at end

At the end of a gutter-grate system assume that the channel ends, as shown in Fig. 8, so that no flow passes beyond the last grate. If the lateral inflow is larger than can be discharged by the preceding grates then the depth will increase over this last grate so that all flow rates coming from previous gutter-grates will be discharged through this last grate. For this last gutter-grate the same three equations, 24-26 are available with F_2 modified so that rather than forcing the depth from the solution of the ODE to be the same as the depth at the beginning of the gutter Y_l it equals the depth at the end of the grate Y_e, or Eq. 25 becomes:

$$F_2 = Y_e - Y_{eode}(Y_r) = 0 \quad \text{with the ODE solved over } x' = 0 \text{ to } x' = L_G$$

Now the outflow $Q_{out} = 0$ is known, and it will be replaced by the depth Y_e as an unknown. The three unknowns for this end grate are: Y_l , Y_r, and Y_e.

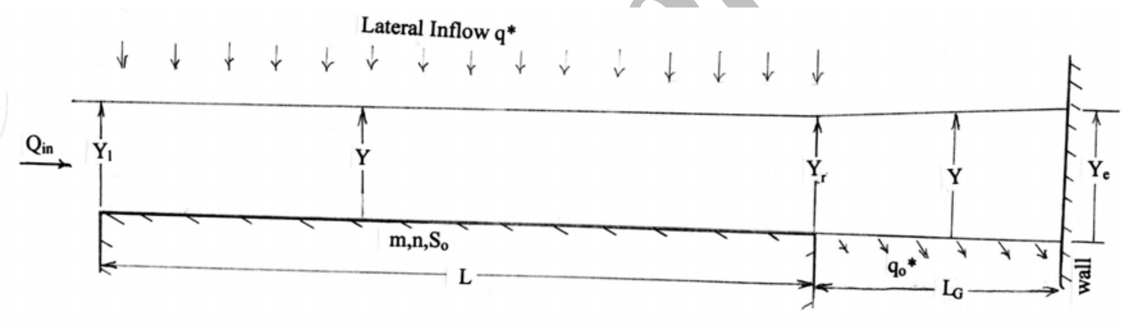

Fig. 8. Gutter-grate system with a vertical wall

5. SUBCRITICAL FLOW THROUGH n GUTTER-GRATES

As a final application consider the problem in which the lateral inflow into gutter 1 does not all discharge into grate 1 with its outflow Q_{out1} flowing into gutter 2, etc. This process of having the excess flow passed into the next gutter-grate continues to the last, which will be designated the nth gutter-grate, where the flow is terminated with a wall so that $Q_{outn} = 0$, as shown in Fig. 9.

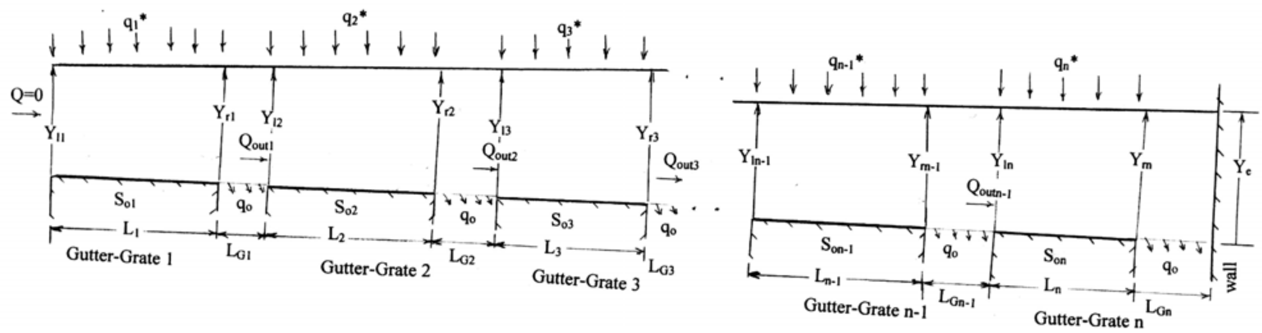

Fig. 9. A series of n gutter-grates containing subcritical flows

To describe the variables involved in these n gutter-grates an extra subscript will be added to denote the number of the gutter-grate as shown in Fig. 9. Thus the depth on the left side of Gutter 1 will be identified by Y_{l1}, and the depth on its right side, which is also the depth at the beginning of Grate 1, is Y_{r1}. The depth at the end of Grate 1 will be the same depth as the beginning of gutter 2 and is Y_{l2}, etc. The depth at the end of the n (and final grate) is Y_e. A grate does not exist upstream from Gutter 1, and therefore assume no reverse flow occurs in this gutter, i.e. the flow at its beginning is zero (or a vertical wall exists at the beginning.)

The unknown variables for this system of n gutter-grates are: Y_{l1}, Y_{r1}, Q_{out1}, Y_{l2}, Y_{r2}, Q_{out2}, . . . Y_{li}, Y_{ri}, Q_{outi}, . . Y_{ln}, Y_{rn}, Y_e. In other words the number of unknowns equals 3n. Therefore 3n simultaneous equations are needed, three from each gutter-grate. For each of these gutter-grates there are two ODE's available and one continuity equation, as has been used in the previous applications. This system of 3n equations consists of:

$$F_1 = Y_{r1} - Y_{r1ode}(Y_{l1}) = 0 \text{ with ODE solved over } x=0 \text{ to } x=L_1$$

$$F_2 = Y_{l2} - Y_{l2ode}(Y_{r1}) = 0 \text{ with ODE solved over } x'=0 \text{ to } x'=L_{G1}$$

$$F_3 = q_l{*}L_l - Iq_o{*}\, dx - Q_{out1} = 0$$

$$F_4 = Y_{r2} - Y_{r2ode}(Y_{l2}) = 0 \text{ with ODE solved over } x=0 \text{ to } x=L_2$$

$$F_5 = Y_{l3} - Y_{l3ode}(Y_{r2}) = 0 \text{ with ODE solved over } x'=0 \text{ to } x'=L_{G2}$$

$$F_6 = q_2{*}L2 - Iq_o{*}\, dx - Q_{out2} + Q_{out1} = 0 \tag{26}$$

$$F_{3i-2} = Y_{ri} - Y_{riode}(Y_{li}) = 0 \text{ with ODE solved over } x=0 \text{ to } x=L_i$$

$$F_{3i-1} = Y_{li+1} - Y_{liode}(Y_{r1}) = 0 \text{ with ODE solved over } x'=0 \text{ to } x'=L_{Gi}$$

$$F_{3i} = q_i{*}L_i - Iq_o{}^{*}dx - Q_{outi} + Q_{outi-1} = 0$$

$$F_{3n-2} = Y_{ln} - Y_{rnode}(Y_{ln}) = 0 \text{ with ODE solved over } x=0 \text{ to } x=L_n$$

$$F_{3n-1} = Y_e - Y_{lnode}(Y_{rn}) = 0 \text{ with ODE solved over } x'=0 \text{ to } x'=L_{Gn}$$

$$F_{3n} = q_n{*}L_i - Iq_o{*}\, dx = 0$$

The Jacobian matrix for use in the Newton Method from these equations forms a special banded matrix with one non zero element in the 1st column, i.e. $D_{1,1}$ =,/ 0, with three non zero elements in the 2nd column, i.e. $D_{1,2} = 1$, $D_{2,2}$ =,/ 0 and $D_{3,2}$ =,/ 0. In the 3rd, 6th, or overall 3i, columns that are associated with the variables Q_{outi}, there are four non zero elements. In these columns the diagonal elements will equal $D_{3i,3i} = -1.0$, and non zero elements will be in the next four rows below this diagonal position coming from Q_{outi-1} in the three equations for gutter-grate i. In general columns 3i-2 (those associated with Y_l) will have $D_{3i-4,3i-2} = 1$, and $D_{3i-2,3i-2}$ =,/ 0 (i.e. two elements in these columns will be non zero). Usually columns 3i-1 (those associated with Y_r) will have $D_{3i-2,3i-1} = 1$, and $D_{3i-1,3i-1}$ (the diagonal element) and $D_{3i,3i-1}$ (the element just below the diagonal) =,/ 0. Since for the last channel Y_e replaces Q_{outn} as the unknown variable, the last or (3n)th column of the Jacobian matrix will contain only one non zero element in the second from the last row, i.e. $D_{3n-1,3n} = 1$.

As an example problem, a series of six gutter-grates exists. Solutions to the nine cases given in Table 6 will be obtained. For all of these solutions the side slope of the triangular gutter is m = 4, the Mannings n= .013, and a bottom slope S_o=0.0009, for both gutter and grate. The grates discharge though a bottom width b = 0.3 m, have one-half this bottom open, or f = .5, and have a discharge coefficient C_d=0.45. Also for all cases the lateral inflow has been specified as q^*=0.0012 m^2/s. What vary from case to case are the length of the gutter and the length of the grate, as given in Table 6.

The solutions to these 9 cases are summarized in Table 7. In addition to these variables, the solution provides the profiles across each of the gutters and the grates, and the varying lateral outflow from each such profile across each grate. Furthermore, the flow rate at each position across the gutters and grates are given. It is not practical to give these profiles, etc. Note the following from these solutions: (1) All Froude Numbers are less than unity so the flow throughout the six gutter-grates is subcritical for all nine cases, (2) The depth increase from the beginning to the end of the series of gutter-grates. (This is due to the fact that the channel slopes downward and the grates discharge less flow than has accumulated in the upstream gutters, i.e. the Q_{out}'s are positive.) (3) As the lengths of grates have been increased in increase cases numbers, the discharge past upstream grates, Q_{out} is smaller, as expected, and this results in smaller depths, (4) The largest Froude Number of each case occurs at the right side of the first gutter where the depth is smallest. (As the length of the downstream grates is increased, or the lateral inflow is increased this Froude Number increases, and when its value becomes unity, or close thereto, the flow downstream from it will change from sub- to supercritical.), (5) For Case 9 the Froude Number at the right side of Gutter 1 is very close to unity, and this represents a limiting combination of grate lengths for the specified lateral inflow $q^* = 0.0012\ m^2/s$. If the length of the grate is increased from 1.35 to 1.4 m critical depths occur. Either smaller values of q*, or longer grate lengths than used in Case 9 will result in some supercritical depth and hydraulic jumps.

Table 6. Cases to be solved for a series of 6 Gutter-Grates (L=length of gutter, G=length of grate)

N	Case																	
	1		2		3		4		5		6		7		8		9	
	L	G	L	G	L	G	L	G	L	G	L	G	L	G	L	G	L	G
1	280	0.8	280	0.8	280	1.0	280	1.0	280	1.2	280	1.3	280	1.3	280	1.3	280	1.35
2	250	0.8	250	0.8	280	1.0	280	1.0	280	1.2	280	1.3	280	1.3	280	1.4	280	1.4
3	200	0.9	200	0.9	280	1.0	280	1.2	280	1.2	280	1.3	280	1.4	280	1.4	280	1.4
4	180	1.0	200	1.0	280	1.0	280	1.2	280	1.2	280	1.3	280	1.4	280	1.4	280	1.4
5	160	1.0	200	1.0	280	1.0	280	1.2	280	1.2	280	1.3	280	1.4	280	1.4	280	1.4
6	140	1.0	200	1.0	280	1.0	280	1.2	280	1.2	280	1.3	280	1.4	280	1.4	280	1.4

Table 7. Solutions of the principal variables for the 9 cases of 6 Gutter-Grates
(the 6th entry under Q_{out} is the depth Y_e at end)

No	Case 1					Case 2					Case 3				
	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}
1	0.395	0.565	0.175	0.000	0.317	0.482	0.702	0.157	0.000	0.184	0.751	0.997	0.070	0.000	0.076
2	0.576	0.749	0.291	0.157	0.221	0.707	0.915	0.254	0.084	0.129	0.999	1.248	0.109	0.016	0.053
3	0.756	0.916	0.302	0.132	0.149	0.918	1.091	0.272	0.071	0.090	1.250	1.500	0.120	0.014	0.036
4	0.920	1.075	0.242	0.084	0.098	1.093	1.252	0.214	0.049	0.058	1.502	1.753	0.104	0.010	0.025
5	1.078	1.219	0.140	0.045	0.060	1.254	1.397	0.122	0.027	0.037	1.754	2.006	0.063	0.006	0.017
6	1.221	1.346	1.347	0.019	0.033	1.398	1.523	1.524	0.012	0.021	2.007	2.258	2.259	0.003	0.012

No	Case 4					Case 5				
	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}
1	0.436	0.637	0.123	0.000	0.234	0.386	0.542	.099	0.000	0.350
2	0.646	0.873	0.210	0.083	0.146	0.558	0.770	.155	0.096	0.189
3	0.877	1.119	0.209	0.066	0.093	0.777	1.016	.169	0.066	0.107
4	1.122	1.370	0.171	0.035	0.056	1.019	1.266	.146	0.036	0.063
5	1.372	1.623	0.101	0.018	0.034	1.269	1.519	.089	0.018	0.038
6	1.624	1.876	1.877	0.007	00.020	1.520	1.772	1.773	0.007	0.023

Table 7. (Continued)

No	Case 6					Case 7					Case 8					Case 9				
	Y_l	Y_r	Q_{out}	G_{rl}	Frr	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}	Yl	Y_r	Q_{out}	F_{rl}	F_{rr}	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}
1	0.361	0.427	0.103	0.000	0.638	0.359	0.394	0.109	0.000	0.779	0.359	0.389	0.110	0.000	0.802	0.358	0.371	0.103	0.000	0.906
2	0.468	0.606	0.168	0.156	0.347	0.450	0.516	0.193	0.181	0.526	0.448	0.495	0.178	0.184	0.584	0.443	0.489	0.172	0.177	0.593
3	0.623	0.837	0.188	0.124	0.178	0.547	0.711	0.213	0.197	0.280	0.533	0.692	0.203	0.194	0.291	0.527	0.685	0.199	0.193	0.296
4	0.844	1.085	0.163	0.065	0.096	0.724	0.952	0.185	0.108	0.140	0.705	0.932	0.179	0.109	0.145	0.699	0.924	0.176	0.110	0.147
5	1.088	1.337	0.100	0.030	0.055	0.958	1.203	0.113	0.047	0.074	0.938	1.183	0.110	0.047	0.076	0.930	1.175	0.109	0.048	0.077
6	1.339	1.590	1.591	0.011	0.031	1.206	1.456	1.458	0.016	0.040	1.185	1.435	1.437	0.016	0.041	1.178	1.428	1.430	0.016	0.041

If the length of Grate 1 is increased from 1.35 m to 1.4 m, then the flow becomes critical at its beginning, and the flow over its first portion will be supercritical, with a hydraulic jump followed by subcritical flow over its remaining length. For this case, in which a hydraulic jump occurs within the Grate's length, the first three equations (or the 3 equations for the grate in which the jump occurs) in Eq. (27) are replaced by the following four equations.

$F_1 = Y_{1j} - Y_{1jode}(Y_c) = 0$ with ODE solved from x=0 to X_j

$F_2 = Y_{l2} - Y_{l2ode}(Y_{2j}) = 0$ with ODE solved from $x = X_j$ to L_{G1}

$$F_3 = Q_1 - \int_0^{X_j} q_o^* dx - \int_{X_j}^{L_{G1}} q_o^* dx - Q_{out2} = 0 \quad \text{(The Continuity Equation)}$$

$$F_4 = (m/6)[Y_{1j}^3 - Y_{2j}^3] + 2Q_j^2/(gm)[1/Y_{1j}^2 - 1/Y_{2j}^2] = 0 \quad \text{(The Momentum Equation)}$$

in which Y_{1j} is the depth upstream of the hydraulic jump, Y_{2j} is the depth downstream of the hydraulic jump, X_j is the position of the hydraulic jump, Q_1 is the flow rate at the end of gutter 1 and equals $L_1(q_1^*)$, Y_c is the critical depth associated with Q_1 and for the triangular gutter equals $[8Q_1^2/(gm^2)]^{1/5}$, Q_j is the flow rate at the hydraulic jump and is obtained by subtracting the integrated outflow from the beginning of the grate to the position of the hydraulic jump from Q_1. The first two variables solved previously, namely Y_{l1} and Y_{r1}, are replaced by the three unknowns: Y_{1j} , Y_{2j} and X_j.

Solutions to three additional cases are given below in which the length of Grate 1 has been specified as 1.4 m, 1.6 m and 1.7 m, respectively in Table 8. The length of all gutters is 280 m, with an inflow of $q^*=0.0012$ m^2/s, and the length of Grates 2 through 6 are 1.4 m as in Case 9. For these solutions the depth at the beginning of Gutter 1 is obtained by solving Eq. (2) starting with a depth just slightly above Y_c and proceeding upstream to its beginning. This solution does not depend upon any of the variables downstream, and therefore does not need to be part of the simultaneous solution. Notice from these solutions that as the length of Grate 1 is increased, the position of the jump also increases. Eventually, with longer grate lengths the hydraulic jump will move into the next gutter. This same effect will occur by specifying longer lengths to other downstream grates, or smaller lateral inflows to gutters. By specifying different bottom slopes or side slopes, etc. for individual gutters and/or grates the hydraulic jump might take place in a different grate, or gutter. The position where critical depth occurs will separate the problem into two problems; the one upstream from this position, and the one downstream from where critical flow occurs.

Table 8. Solutions of the principal variables for 3 cases of Gutter-Grates in which a hydraulic jump occurs in Grate 1

	Case 10 (L_1=1.4 m) Y_1=.3107 m,Y_2=.3860 m,X_j=.1441 m					Case 11 (L_1=1.6 m) Y_{1j}=.2642 m,Y_{2j}=.3881 m,X_j=.5190 m					Case 12 (L_1=1.7 m) Y_{1j}=.2491 m,Y_{2j}=.3843 m,X_j=.671 m				
No	Y_1	Y_r	Q_{out}	F_{rl}	F_{rr}	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}	Y_l	Y_r	Q_{out}	F_{rl}	F_{rr}
1	0.359	0.367	0.097	0.000	0.916	0.359	0.367	0.077	0.000	0.916	0.359	0.367	0.066	0.000	0.916
2	0.440	0.484	0.168	0.172	0.600	0.424	0.466	0.152	0.147	0.629	0.417	0.457	0.143	0.134	0.643
3	0.523	0.679	0.196	0.192	0.300	0.508	0.658	0.184	0.186	0.314	0.501	0.647	0.178	0.183	0.321
4	0.693	0.919	0.174	0.110	0.148	0.673	0.896	0.167	0.112	0.154	0.663	0.885	0.163	0.112	0.157
5	0.924	1.169	0.108	0.048	0.078	0.902	1.146	0.104	0.049	0.081	0.891	1.135	0.103	0.049	0.082
6	1.172	1.422	1.424	0.016	0.042	1.149	1.399	1.401	0.017	0.043	1.138	1.388	1.390	0.017	0.044

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