

STABILITY AND VIBRATION OF ELASTICALLY SUPPORTED, AXIALLY MOVING ORTHOTROPIC PLATES *

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Abstract– In this study, the vibration and stability of orthotropic plates, moving on elastic foundation or elastic supports and subjected to in-plane forces, are investigated by classical plate theory. Firstly, a solution based on the exact finite strip method is developed for multi-span moving plates on an elastic foundation. Then a formulation which is an extension of the approximate finite strip method is presented for axially moving plates on discrete or distributed elastic supports. By some examples, the reliability of both proposed methods is investigated for the stability and the vibration analysis.

Keywords– Stability, free vibration, axially moving, orthotropic plate, elastic foundation, elastic supports, exact solution, finite strip method (FSM)

1. INTRODUCTION

From the point of view of mechanics, the out of plane vibration or instability is a typical phenomenon of axially moving materials, encompassing systems such as moving strings and belts, high speed magnetic tapes, paper and plastic sheets in process, band saw blades, conveyor belts, etc. Recent developments in research into axially moving materials are reviewed by Wickert & Mote [1] and Pellicano & Vestroni [2].

In axially traveling systems, the transverse vibration of the moving material often becomes a serious problem in achieving good quality. The axial speed of a structure may significantly affect dynamic characteristics of the system even at low velocity, giving rise to variations of natural frequencies and complex modes. In a certain *critical speed*, first, natural frequency vanishes and the axially moving structure may experience severe vibrations and bifurcation instability. Thus, accurate prediction of stability and vibration characteristics of such structures is a requisite for the analysis and optimal design of a broad class of technological devices.

Avoiding complication, a lot of earlier works for modeling two-dimensional axially moving continua used the one-dimensional string or beam theory instead of the plate theory. Although this simplification leads to reasonable results for a narrow strip, two-dimensional analysis is required for the modeling of many problems such as wide width plates, various forces across the width, no free lateral boundaries, intermediate supports, catching higher modes of vibration or buckling etc.

Following the early work of Ulsoy and Mote [3] for two-dimensional plate modeling of a wide band saw blade, some research efforts have been directed toward studying the dynamic behavior of axially moving plates. In 1995, Lengoc and Mccalion [4] considered cutting conditions on the dynamic response of bandsaw blades. Lin [5] investigated the stability and the vibration characteristics of a single-span moving isotropic plate. A mixed finite element formulation was developed by Wang [6] for a traveling orthotropic plate. Lou and Hutton [7] presented the formulation of a moving triangular isotropic plate

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element and compared the results with the Rayleigh-Ritz method. A spectral element method was formulated by Kim *et al.* [8], for thin plates moving with constant speed under a uniform axial tension and more recently, equilibrium, membrane force and buckling stability of full simply supported traveling plates were obtained analytically by Luo and Hamidzade [9].

The objective of this paper is to analyze stability and free vibration of orthotropic plates moving on distributed or discrete elastic supports by the classical plate theory, which can be used to represent two-dimensional moving media supported by air bearings, recording heads, tensioner arms and the like. Also, by considering elastic supports with unlimited stiffness, rigid intermediate rollers and local supports can be modeled. Previous works on elastically supported moving materials are limited to one-dimensional string models [10-13].

In this study, two types of the finite strip method (FSM) are developed for elastically supported, axially moving plates. For the prediction of natural frequencies and buckling stresses of flat-plate structures, what may be termed an exact finite strip method (E-FSM) may be used in some circumstances, wherein the strip properties are based on the direct solution of the governing differential equations of classical plate theory. It happens that it was also in 1968 that pioneering works in this area were published by Wittrick [14, 15] and by Smith [16]. As well as the E-FSM, approximate energy-based or work-based finite strip method (A-FSM), based on the use of an assumed displacement field, has been considerably developed and has been proven effective in a large number of engineering applications particularly in the analysis of plate and plate assemblies [17-22]. The reader is referred to the early text of Cheung [23] and recent text of Cheung and Tham [24] for general descriptions.

By the E-FSM developed in this study, an exact stiffness matrix for dynamic stability and vibration of a multi-span orthotropic plate guided by an elastic foundation is derived. The plate is assumed to be subjected to a state of plane stress which is laterally invariant, and it is further assumed that the mode of vibration varies sinusoidally in the lateral direction. The exact solutions obtained for such plates are indeed valuable, as they serve as important benchmark solutions for checking the convergence, validity and accuracy of numerical methods for the analysis of axially moving plates.

Based on Hamilton's principal, the approximate finite strip (A-FSM) is extended for orthotropic plates moving on discrete or distributed elastic supports. The displacement functions of each finite strip are assumed to be Hermitian polynomials in the direction of strip width, while in the longitudinal direction sinusoidal trigonometric functions or bubble functions corresponding to the pre-set end boundaries of the plate have been used.

2. EQUATION OF MOTION

An orthotropic plate moving on an elastic foundation and some local elastic supports is shown in Fig. 1. The plate has a constant velocity v along x -direction which is mentioned as the axial direction. By using Hamilton's principle, the equation of motion for transverse displacement w in the fixed coordinate (x,y,z) can be derived. Assuming that the boundaries are fixed both at the inlet and the outlet, the fluxes in and out are equal and the net energy flux through the boundaries is zero. Therefore Hamilton's principle takes the familiar form

$$\delta \int_{t_i}^{t_f} (U - T) dt = 0 \quad (1)$$

where U is the total potential energy of the particles which fill domain A and T is the kinetic energy of the structure mass. U includes the strain energy due to bending U_b , the effect of in-plane forces on the transverse deflection U_g , and the potential energy of elastic foundation and elastic supports U_s .

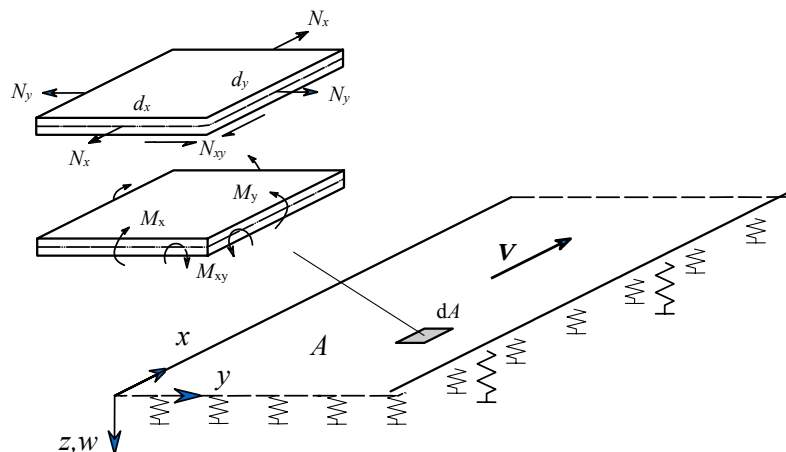


Fig. 1. An elastically supported, axially moving orthotropic plate

$$U = U_b + U_g + U_s \quad (2)$$

The strain energy for bending of the orthotropic thin plate with median surface A is

$$U_b = \frac{1}{2} \int_A [D_{11} \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + D_{22} \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2D_{12} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + D_{66} \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2] dA \quad (3)$$

where D_{ij} are stiffness coefficients of the plate. The potential energy due to in-plane forces is

$$U_g = \frac{1}{2} \int_A [N_x \left(\frac{\partial w}{\partial x}\right)^2 + N_y \left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy} \left(\frac{\partial w}{\partial x}\right) \left(\frac{\partial w}{\partial y}\right)] dA \quad (4)$$

in which N_x and N_y are in-plane forces per unit length in the x and y -directions respectively (positive when tensile) and N_{xy} is shear force per unit length. The potential energy of the elastic foundation and local discrete or distributed elastic supports is

$$U_s = \frac{1}{2} \int_A \kappa_f w^2 dA + \sum_{i=1}^{nes} \frac{1}{2} \int_A \kappa_{s_i}(x, y) w^2 dA \quad (5)$$

in which κ_f is elastic foundation stiffness, κ_{s_i} are stiffness functions for the local elastic supports and nes is the number of these supports in the area A . The local supports may be discrete line or point supports or a uniformly distributed support in a rectangle area. κ_{s_i} is defined for a frictionless elastic line support parallel to the y -direction at $x = x_p$, as:

$$\kappa_{s_i} = \kappa_l \delta(x - x_p) \quad (6)$$

For an elastic point support located at (x_p, y_p) , κ_{s_i} is expressed as

$$\kappa_{s_i} = \kappa_p \delta(x - x_p) \delta(y - y_p) \quad (7)$$

and for a uniformly distributed elastic support with a rectangle area in $x_p < x < x_q$, $y_p < y < y_q$, κ_{s_i} is defined by

$$\kappa_{s_i} = \kappa_d H[(x_q - x)(x - x_p)] \times H[(y_q - y)(y - y_p)] \quad (8)$$

where $\delta(x)$ and $H(x)$ are Dirac delta function and Heaviside unit function, respectively. κ_l , κ_p and κ_d are the spring stiffness for line, point and uniformly distributed supports, respectively.

The kinetic energy T for the domain A can be written as:

$$T = \frac{1}{2} \int_A \rho h [v^2 + \left(\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial x}\right)^2] dA \quad (9)$$

where ρ is mass per volume of the plate. By making use of Hamilton's principle in equation (1) and replacing the expressions of potential and kinetic energies from Eqs. (2-5) and (9), the differential equation of motion for the elastically supported moving orthotropic plate is derived as:

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2} - 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \kappa_f w + \sum_{i=1}^{nes} \kappa_{s_i} w + \rho h \left(\frac{\partial^2 w}{\partial t^2} + 2v \frac{\partial^2 w}{\partial x \partial t} + v^2 \frac{\partial^2 w}{\partial x^2} \right) = 0 \quad (10)$$

where

$$D_3 = D_{12} + 2D_{66}$$

3. THE EXACT FINITE STRIP METHOD (E-FSM)

Figure 2a shows an orthotropic plate moving on an elastic foundation with intermediate rigid rollers. The plate is assumed to be subjected to a basic state of plane stress which is invariant in the direction of plate width and simply supported along the transport direction (axial direction). It is further assumed that the traveling speed v is constant throughout and the rollers are frictionless, bilateral and perpendicular to the transport direction.

In the free vibration analysis, it can be postulated that, whatever type of vibration mode occurs, it is sinusoidal in the y -direction. This means that along any line in the plate structure parallel to y -axis, transverse displacement w vary sinusoidally, so w can be written in the form

$$w(x, y, t) = W_n(x) [\exp(ik_n y) - \exp(-ik_n y)] e^{\lambda t} \quad (11)$$

where $k_n = n\pi/b$ is the wave number of the n th mode in y -direction ($n = 1, 2, \dots$), b is the plate width, $W_n(x)$ is the shape function along the axial direction obtained by solving the equation of motion for n th mode, and values of λ extracted from an eigenvalue problem are, in general, complex numbers as:

$$\lambda = \sigma + i\omega$$

where σ and ω are the real and imaginary parts of eigenvalues λ , respectively. While all eigenvalues λ are purely imaginary ($\lambda = i\omega$), the traveling plate is stable and the values of ω are the natural frequencies of the plate. By increasing transport speed, the natural frequencies of the plate decrease and at a certain speed, the first natural frequency of vibration vanishes ($\omega = 0$) and the plate becomes unstable. This certain speed may be mentioned as *critical speed* v_{cr} . For axial speeds higher than the critical speed, the plate can experience divergence or flutter instability that, in unstable conditions, the real part of at least one of the eigenvalues λ is non-zero ($\sigma \neq 0$).

The critical speed can also be obtained by a static stability analysis based on the fact that the axially moving plate becomes unstable if multiple equilibrium positions exist at any problem specification. The critical speed is the lowest speed at which multiple equilibrium positions exist [5]. From Eq. (10), the equilibrium position in static stability analysis satisfies the following equation

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2} - 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \kappa_f w + \sum_{i=1}^{nes} \kappa_{s_i} w + \rho h v^2 \frac{\partial^2 w}{\partial x^2} = 0 \quad (12)$$

Also, the displacement function for buckling modes in the exact method can be written as:

$$w(x, y) = W_n(x)[\exp(ik_n y) - \exp(-ik_n y)] \quad (13)$$

In the present method, the plate structure is divided into a small number of components in which each component fills the domain between two rollers or a roller and an end boundary. For example, the structure shown in Fig. 2a can be divided into three components. Fig. 2b shows one of these components. The assumption of sinusoidal modes implies that the displacements (w_1 , θ_1 , w_2 , and θ_2) and the additional forces (Q_1 , M_1 , Q_2 , and M_2), that appear on the edge of every plate component due to vibration or buckling, vary sinusoidally in the y -direction (Fig. 2b). The exact stiffness matrix is extracted for each component and by assembling stiffness matrices of all components, overall stiffness matrix of the structure will be obtained.

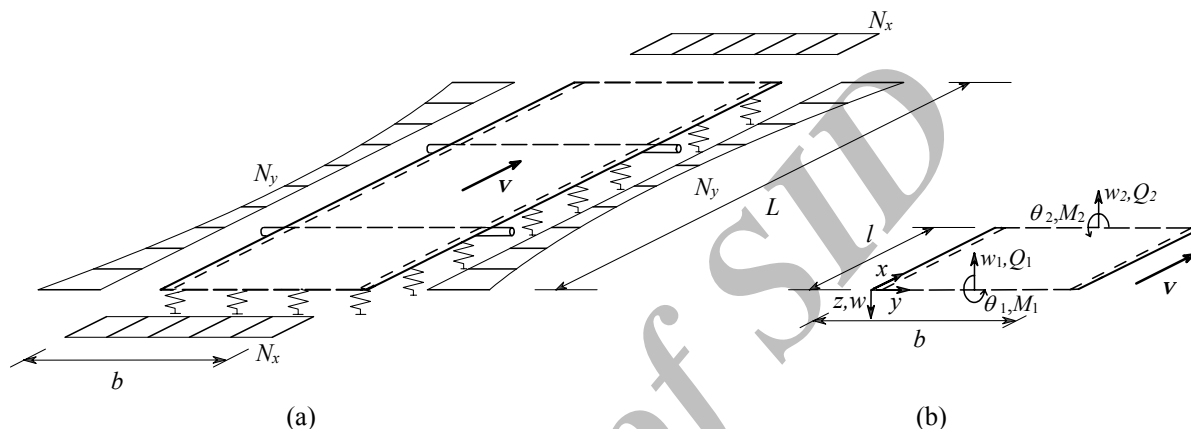


Fig. 2. a) A multi-span orthotropic plate moving over an elastic foundation and subjected to in-plane loads. b) Displacements and forces on nodal lines due to buckling or vibration for one of the plate components.

In the free vibration study, the determination of the exact stiffness matrix for the component shown in Fig. 2b requires solving the differential equation of motion presented in Eq. (10) as satisfy the boundary conditions. Substituting w from Eq. (11) into equation of motion (10) in the absence of shear in-plane force ($N_{xy}=0$) and local supports ($\kappa_{si} = 0$), gives an ordinary differential equation with complex terms for n th mode as

$$D_{11} \frac{\partial^4 W_n}{\partial x^4} + [\rho h v^2 - N_x - 2D_3 k_n^2] \frac{\partial^2 W_n}{\partial x^2} + 2\rho h \lambda v \frac{\partial W_n}{\partial x} + (\rho h \lambda^2 + k_n^2 N_y + D_{22} k_n^4 + \kappa_f) W_n = 0 \quad (14)$$

When N_y is a function of x , because of the variable coefficient of W_n in the equation, a solution in the general case can only be obtained by a numerical integration method. If, however, N_y is constant, the differential equation has constant coefficients and it becomes possible to obtain an exact general solution, leading to explicit expressions for the elements of the stiffness matrix. For uniform N_y , the solution of this fourth order differential equation can be written in the general form of

$$W_n(x) = \sum_{m=1}^4 A_{mn} \exp(r_{mn} x) \quad (15)$$

where A_{mn} are coefficients derived from boundary conditions at $x=0$ and $x=l$ and r_{mn} , the wave numbers in x -direction can be obtained by substituting Eq. (15) into Eq. (14), which yields a polynomial equation for n th mode as

$$D_{11} r_{mn}^4 + (\rho h v^2 - N_x - 2D_3 k_n^2) r_{mn}^2 + 2\rho h \lambda v r_{mn} + (\rho h \lambda^2 + k_n^2 N_y + D_{22} k_n^4 + \kappa_f) = 0 \quad (16)$$

Equation (16) has four roots ($m=1$ to 4) corresponding to each mode ($n=1, 2, 3, \dots$), which, in general, are complex.

In the stability study, by using Eqs. (12), (13) and (15), Eq. (16) is eliminated as the following form

$$r_{mn}^4 + \alpha_1 r_{mn}^2 + \alpha_2 = 0 \quad (17)$$

where

$$\alpha_1 = (\rho h v^2 - N_x - 2D_3 k_n^2) / D_{11}, \quad \alpha_2 = (k_n^2 N_y + D_{22} k_n^4 + \kappa_f) / D_{11}$$

and four series of roots are

$$r_{mn} = \pm \frac{1}{\sqrt{2}} \sqrt{\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_2}} \quad (18)$$

For a component of the moving plate shown in Fig. 2b, the displacement function of the element assumed by Eqs. (11) or (13) satisfies the boundary conditions at $y=0$ and $y=b$ as at these two sides the transverse displacement w and resultant bending moment per unit length of x -direction M_y vanish. By satisfying the boundary conditions on two other sides, the stiffness matrix of the element can be obtained. These edge conditions, as shown in Fig. 2b, are defined by

$$\begin{aligned} \text{at } x=0: \quad Q_x &= Q_1, \quad w = -w_1 \\ M_x &= -M_1, \quad \frac{\partial w}{\partial x} = -\theta_1 \end{aligned} \quad (19)$$

$$\begin{aligned} \text{at } x=l: \quad Q_x &= -Q_2, \quad w = -w_2 \\ M_x &= M_2, \quad \frac{\partial w}{\partial x} = -\theta_2 \end{aligned} \quad (20)$$

where, the resultant shear force and bending moment per unit length of y -direction (Q_x and M_x respectively) are related to w by the following equations.

$$Q_x = -[D_{11} \frac{\partial^3 w}{\partial x^3} + (D_{12} + 4D_{66}) \frac{\partial^3 w}{\partial x \partial y^2}] + (N_x - \rho h v^2) \frac{\partial w}{\partial x} + N_{xy} \frac{\partial w}{\partial y} - \rho h v \frac{\partial w}{\partial t} \quad (21)$$

$$M_x = -[D_{11} \frac{\partial^2 w}{\partial x^2} + D_{12} \frac{\partial^2 w}{\partial y^2}] \quad (22)$$

In the equation of shearing force Q_x , allowance has been made for the Kirchhoff edge effect and for the components of the in-plane loads arising from distortion of the plate. The effect of the axial velocity on the edge shear has also been considered [4].

Edge displacements and edge forces vectors can be defined by

$$\{d\} = \{\theta_1, w_1, \theta_2, w_2\}^T \quad (23)$$

$$\{p\} = \{M_1, Q_1, M_2, Q_2\}^T \quad (24)$$

Using expressions (11), (15) and (19) through (24) edge displacements and edge forces vectors in free vibration may be written as:

$$\{d\} = \{\bar{d}\} [\exp(ik_n y) - \exp(-ik_n y)] e^{\lambda t} \quad (25)$$

$$\{p\} = \{\bar{p}\} [\exp(ik_n y) - \exp(-ik_n y)] e^{\lambda t} \quad (26)$$

where

$$\{\bar{d}_n\} = -\sum_{m=1}^4 \left\{ \begin{array}{c} r_{mn} \\ 1 \\ r_{mn} \exp(r_{mn}l) \\ \exp(r_{mn}l) \end{array} \right\} A_{mn}$$

$$\Rightarrow \{\bar{d}_n\}_{4 \times 1} = [X_n]_{4 \times 4} \{A_n\}_{4 \times 1} \quad (27)$$

$$\{\bar{p}_n\} = \sum_{m=1}^4 \left\{ \begin{array}{c} D_{11}r_{mn}^2 - D_{12}k_n^2 \\ -D_{11}r_{mn}^3 + (D_{12}k_n^2 + 4D_{66}k_n^2 + N_x - \rho h v^2)r_{mn} - \lambda \rho h v \\ -(D_{11}r_{mn}^2 - D_{12}k_n^2) \exp(r_{mn}l) \\ [D_{11}r_{mn}^3 - (D_{12}k_n^2 + 4D_{66}k_n^2 + N_x - \rho h v^2)r_{mn} + \lambda \rho h v] \exp(r_{mn}l) \end{array} \right\} A_{mn}$$

$$\Rightarrow \{\bar{p}_n\}_{4 \times 1} = [Y_n]_{4 \times 4} \{A_n\}_{4 \times 1} \quad (28)$$

Combining Eqs. (27) and (28) by elimination $\{A_n\}$ yields

$$\{\bar{p}_n\} = [s_n] \{\bar{d}_n\} \quad (29)$$

where

$$[s_n] = [Y_n][X_n]^{-1} \quad (30)$$

Here, $[s_n]$ is the exact stiffness matrix of a component of the plate that in general contains complex elements associated with the transport speed v ; however, it is Hermitian in form. In the free vibration analysis, the elements of the stiffness matrix are transcendental functions of the natural frequencies, axial velocity, in-plane forces, and of the wave numbers in x and y -directions. By assembling the stiffness matrix of all elements of the moving plate and eliminating constrained degrees of freedom, the overall exact stiffness matrix of the multi-span orthotropic plate moving over an elastic foundation $[S_n]$ is obtained. By the present method, precise results can be obtained by only a few components, leading to a small order of overall stiffness matrix. The natural frequencies can be determined by vanishing the determinant of $[S_n]$, i.e.

$$\det[S_n(\lambda)] = 0 \quad (31)$$

Equations (27) through (30) can be used for static stability analysis too, by replacing λ with zero. Here, an eigenvalue problem is also produced, but its eigenvalue is the axial speed. Thus the critical speed v_{cr} is obtained by the following expression.

$$\det[S_n(v_{cr})] = 0 \quad (32)$$

Although the E-FSM method leads to the precise results for vibration and dynamic stability of axially moving orthotropic plates on an elastic foundation by only a few components, the method has some restrictions concerning boundary conditions, various plane stresses and elastic supports. Moreover, in the exact method, the coefficients of the overall stiffness matrix are not linear functions of the eigenvalues, and standard eigenvalue routines can not be used to extract natural frequencies or critical speed. In the next section, an energy-based finite strip method is developed for an orthotropic plate moving on an elastic foundation and local elastic supports to remove restrictions of the exact method.

4. THE APPROXIMATE FINITE STRIP METHOD (A-FSM)

Consider an axially moving orthotropic plate with arbitrary boundary conditions and in-plane forces which is supported by distributed or discrete elastic springs (Fig. 3a). The plate has the length L , width b , and

thickness h in the x , y and z -directions, respectively. For each finite strip of the plate, shown in Fig. 3b, the transverse displacement function can be expressed in the natural coordinates (ξ, η) as

$$w = \sum_{n=1}^r S_n(\xi) \langle f(\eta) \rangle \{d_n(t)\} \tag{33}$$

where $S_n(\xi)$ is the n th term of the basic function series in the axial direction, $f(\eta)$ is the interpolation function in the lateral direction and $d_n(t)$ is the vector of degrees of freedom corresponding to the n th harmonic which is given by

$$\{d_n(t)\} = \langle \theta_{1n}, w_{1n}, \theta_{2n}, w_{2n} \rangle^T \tag{34}$$

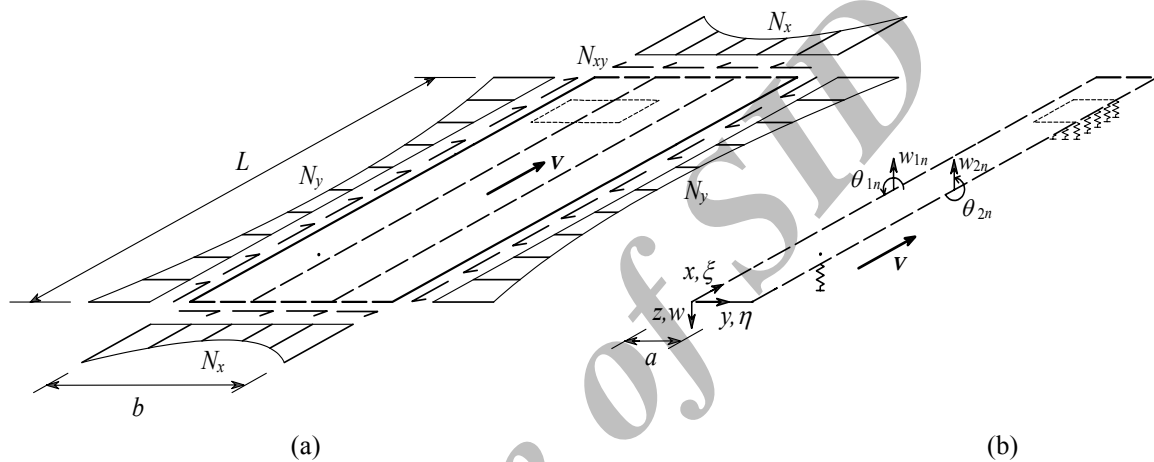


Fig. 3. a) An elastically supported, axially moving orthotropic plate which is divided to some finite strips. b) nodal line displacements for one of the finite strips of the plate

A Hermitian polynomial function has been chosen for the interpolation vector $f(\eta)$ for a strip with the width a as:

$$\langle f(\eta) \rangle = \langle a\eta(1-2\eta+\eta^2), (1-3\eta^2+2\eta^3), a(-\eta^2+\eta^3), (3\eta^2-2\eta^3) \rangle \tag{35}$$

where $\eta=y/a$. Also, trigonometric or bubble functions are considered as basic functions $S_n(\xi)$ corresponding to particular end conditions. $S_n(\xi)$ corresponding to different types of end conditions at $x=0$ and $x=L$ are given by

Trigonometric functions:

$$\begin{aligned} S_n(\xi) &= \sin n\pi\xi && \text{for both simply supported} \\ S_n(\xi) &= \sin n\pi\xi \times \sin \pi\xi && \text{for both clamped} \end{aligned}$$

Bubble functions:

$$S_n(\xi) = \frac{1}{2^{2n}} \xi^n (1-\xi)^n \quad \text{for both simply supported}$$

in which $\xi=x/L$ and $n=1,2,\dots$. Other basic functions can certainly be used here, such as the mode shapes of free vibration of a beam [18], but the basic functions introduced above have a faster convergence rate.

The energy method based on Hamilton's principle of Eq. (1) is adopted to develop the A-FSM for the axially moving orthotropic plate on elastic foundation and elastic supports. By using Eq. (3) for the bending strain energy of the plate U_b and replacing the displacement field of a strip from Eq. (33), it can be written as:

$$U_b = \frac{1}{2} \sum_{m=1}^r \sum_{n=1}^r \{d_m\}^T [k_b]_{mn} \{d_n\} \quad (36)$$

where $[k_b]$ is the bending stiffness matrix of the strip and may be obtained by

$$[k_b]_{mn} = \int_{st} [B_m]^T [D] [B_n] dA \quad (37)$$

in which st denotes the area of the strip and $[B_n]$ and $[D]$ are defined as

$$[B_n] = \left\{ -\frac{1}{L^2} \frac{\partial^2}{\partial \xi^2}, -\frac{1}{a^2} \frac{\partial^2}{\partial \eta^2}, -\frac{1}{aL} \frac{2\partial^2}{\partial \xi \partial \eta} \right\}^T \langle f(\eta) \rangle S_n(\xi) \quad (38)$$

$$[D] = \begin{bmatrix} D_{11} & & \text{Symm.} \\ D_{12} & D_{22} & \\ 0 & 0 & D_{66} \end{bmatrix} \quad (39)$$

Also, from Eqs. (4) and (33), the potential energy due to in-plane forces U_g can be obtained by

$$U_g = \frac{1}{2} \sum_{m=1}^r \sum_{n=1}^r \{d_m\}^T [k_g^s]_{mn} \{d_n\} \quad (40)$$

in which $[k_g^s]$ is the stability matrix for a stationary plate, as:

$$[k_g^s]_{mn} = \int_{st} \left[\frac{1}{L^2} N_x \langle f(\eta) \rangle^T \langle f(\eta) \rangle S'_m S'_n + \frac{1}{a^2} N_y \langle f(\eta) \rangle_{,\eta}^T \langle f(\eta) \rangle_{,\eta} S_m S_n + \frac{1}{aL} 2N_{xy} \langle f(\eta) \rangle^T \langle f(\eta) \rangle_{,\eta} S'_m S_n \right] dA \quad (41)$$

where a prime denotes differentiation with respect to ξ .

The potential energy of the spring systems U_s for each strip which is guided by an elastic foundation and nes elastic supports is obtained by Eqs. (5) and (33) as:

$$U_s = \frac{1}{2} \sum_{m=1}^r \sum_{n=1}^r \{d_m\}^T [k_s]_{mn} \{d_n\} \quad (42)$$

where $[k_s]$ denotes the spring stiffness matrix including the stiffness matrix of the elastic foundation $[k_s^f]$ and the stiffness matrices of the local elastic supports $[k_{s_i}]$ as:

$$[k_s]_{mn} = [k_s^f]_{mn} + \sum_{i=1}^{nes} [k_{s_i}]_{mn} \quad (43)$$

in which

$$[k_s^f]_{mn} = \int_{st} \frac{1}{L} \kappa_s^f \langle f \rangle^T \langle f \rangle S_m S_n dA \quad (44)$$

$$[k_{s_i}]_{mn} = \int_{st} \frac{1}{L} \kappa_{s_i} \langle f \rangle^T \langle f \rangle S_m S_n dA \quad (45)$$

Thus, the variation of the total potential energy in Eq. (1) is written as

$$\delta \int_{t_i}^{t_f} U dt = \int_{t_i}^{t_f} \sum_{m=1}^r \sum_{n=1}^r \{\delta d_m\}^T ([k_b]_{mn} + [k_g^s]_{mn} + [k_s]_{mn}) \{d_n\} dt \quad (46)$$

The kinetic energy for every strip of the moving plate T of Eq. (9) can be extended in the following form

$$T = \frac{1}{2} \int_{st} \rho h \left(\frac{\partial w}{\partial t} \right)^2 dA + \frac{1}{2} \int_{st} \rho h v^2 \left(\frac{\partial w}{\partial x} \right)^2 dA + \frac{1}{2} \int_{st} 2 \rho h v \frac{\partial w}{\partial x} \frac{\partial w}{\partial t} dA + \frac{1}{2} \rho h v^2 A \quad (47)$$

or

$$T = T_0 + T_g + T_v + T_c \quad (48)$$

The first term of Eq. (47), T_0 , is the kinetic energy for a non-moving plate and the variation of T_0 together with displacement function of Eq. (33) leads to

$$\delta \int_{t_i}^{t_f} T_0 dt = - \int_{t_i}^{t_f} \sum_{m=1}^r \sum_{n=1}^r \{\delta d_m\}^T [m]_{mn} \{\ddot{d}_n\} dt \quad (49)$$

in which a dot means differentiation with respect to t and $[m]$ is the mass matrix expressed by

$$[m]_{mn} = \int_{st} \rho h \langle f \rangle^T \langle f \rangle S_m S_n dA \quad (50)$$

The second term of Eq. (37), T_g , is related to the effect of axial speed on the transverse deflection. By using Eq. (33), we have

$$\delta \int_{t_i}^{t_f} T_g dt = \int_{t_i}^{t_f} \sum_{m=1}^r \sum_{n=1}^r \{\delta d_m\}^T [k_g^m]_{mn} \{d_n\} dt \quad (51)$$

where $[k_g^m]$, which can be called the stability matrix of an axially moving plate is obtained by

$$[k_g^m]_{mn} = \int_{st} \frac{1}{L^2} \rho h v^2 \langle f \rangle^T \langle f \rangle S'_m S'_n dA \quad (52)$$

The third term T_v has the form of

$$T_v = \frac{1}{2} \int_{st} 2 \rho h v \frac{\partial w}{\partial x} \frac{\partial w}{\partial t} dA \quad (53)$$

It can be found that

$$\delta \int_{t_i}^{t_f} T_v dt = \int_{t_i}^{t_f} \int_{st} \rho h v \left(\frac{\partial \delta w}{\partial x} \frac{\partial w}{\partial t} \right) dA dt + \int_{st} \rho h v (\delta w \frac{\partial w}{\partial x}) dA \Big|_{t_i}^{t_f} - \int_{t_i}^{t_f} \int_{st} \rho h v (\delta w \frac{\partial^2 w}{\partial x \partial t}) dA dt \quad (54)$$

and by substituting the displacement field of Eq. (33) into Eq. (54), it may be written

$$\delta \int_{t_i}^{t_f} T_v dt = - \int_{t_i}^{t_f} \sum_{m=1}^r \sum_{n=1}^r \{\delta d_m\}^T [g]_{mn} \{\dot{d}_n\} dt \quad (55)$$

where the gyroscopic $[g]$ matrix is

$$[g]_{mn} = \int_{st} \frac{1}{L} \rho h v \langle f \rangle^T \langle f \rangle (S_m S'_n - S'_m S_n) dA \quad (56)$$

Finally, the forth term T_c is a constant value for a moving plate which has a constant velocity (this term is omitted by variation). By using Hamilton's principle in Eq. (1) for free vibration, the governing equation of motion for the elastically supported, axially moving orthotropic plate can be formulated as

$$[m]\{\ddot{d}\} + [g]\{\dot{d}\} + ([k] - [k_g])\{d\} = \{0\} \quad (57)$$

in which total stiffness matrix $[k]$ and total stability matrix $[k_g]$ are

$$[k] = [k_b] + [k_s] \quad (58)$$

$$[k_g] = [k_g^m] - [k_g^s] \quad (59)$$

By assembling $[m]$, $[g]$ and $[k]$ matrices and also $\{d\}$, $\{\dot{d}\}$ and $\{\ddot{d}\}$ vectors of all strips of the moving plate and eliminating constrained degrees of freedom, Eq. (57) for the whole plate is obtained as

$$[M]\{\ddot{\delta}\} + [G]\{\dot{\delta}\} + ([K] - [K_g])\{\delta\} = \{0\} \quad (60)$$

For free vibration analysis, the degrees of freedom vector can be represented as

$$\{\delta\} = \{\hat{\delta}\} e^{\lambda t} \quad (61)$$

where λ is, in general, a complex number. Substituting Eq. (61) into Eq. (60) gives

$$\lambda^2 [M]\{\hat{\delta}\} + \lambda [G]\{\hat{\delta}\} + ([K] - [K_g])\{\hat{\delta}\} = \{0\} \quad (62)$$

and the Eq. (62) can be rewritten in the form of

$$\lambda [\Gamma]\{\bar{\delta}\} + [A]\{\bar{\delta}\} = \{0\} \quad (63)$$

where

$$\{\bar{\delta}\} = \begin{Bmatrix} \lambda \{\hat{\delta}\} \\ \{\hat{\delta}\} \end{Bmatrix}, \quad [\Gamma] = \begin{bmatrix} [M] & [0] \\ [0] & [K] - [K_g] \end{bmatrix} \text{ and } [A] = \begin{bmatrix} [G] & [K] - [K_g] \\ [K_g] - [K] & [0] \end{bmatrix} \quad (64)$$

In the eigenvalue problem (63), $[\Gamma]$ is a symmetric matrix and $[A]$ is a skew-symmetric one. The eigenvalues λ which are generally complex, can be obtained from a non-symmetric eigenvalue solver. The interpretation of λ in the A-FSM is the same as the E-FSM.

For buckling analysis, equation of motion (61) is eliminated in the form of

$$([K] - [K_g])\{\delta\} = \{0\} \quad (65)$$

where $[K_g]$ is a linear function of the square of the axial speed v^2 , so Eq. (65) is an eigenvalue problem, by which the critical speed of the plate v_{cr} can be derived.

5. NUMERICAL INVESTIGATIONS

a) General

The orthotropic plates treated in the numerical results have dimensionless orthotropic material properties of $E_y/E_x=0.5$, $G_{xy}/E_x=0.25$, and $\nu_{xy}=0.2$, where E_x and E_y are major and minor elastic modulus of the plate,

respectively and G_{xy} is the shear modulus. Also, ν_{xy} and ν_{yx} are the plate major and minor Poisson's ratios, respectively which have the relation of $\nu_{xy}/\nu_{yx}=E_x/E_y$. Non-dimensional variables used in the results are introduced as

$$\Omega = \omega \frac{b^2}{2\pi^2} \sqrt{\frac{\rho h}{D_{11}}}, \quad (c, c_{cr}) = (v, v_{cr}) \frac{b}{\pi} \sqrt{\frac{\rho h}{D_{11}}}, \quad (k_x, k_y, k_{xy}) = (N_x, N_y, N_{xy}) \frac{b^2}{\pi^2 D_{11}},$$

$$r = \frac{L}{b}, \quad (K_f, K_d) = (\kappa_f, \kappa_d) \frac{b^4}{D_{11}}, \quad K_p = \kappa_p \frac{b^2}{D_{11}} \quad (66)$$

where Ω is dimensionless frequency of vibration, c and c_{cr} denote non-dimensional axial speed and critical axial speed of the plate, respectively, k_x and k_y are in-plane load parameters along x and y -directions, respectively (positive when tensile) and k_{xy} is shear load parameter. r is aspect ratio of the plate, K_f and K_d are stiffness parameters of the elastic foundation and local distributed elastic support, respectively, and K_p is the non-dimensional stiffness of elastic point support. D_{11} in the relations (66) is defined as:

$$D_{11} = \frac{E_x}{12(1 - \nu_{xy}\nu_{yx})} \quad (67)$$

For the E-FSM method, the parameter n implies the number of half-waves in the direction perpendicular to the simple edges. Also, for the A-FSM, m and NS are the number of terms of basic functions and the number of finite strips, respectively. For convenience, four capital letters are used to describe the edges of plates. The symbolism CFSF, for example, identifies a plate with edges clamped, free, simply supported and free; start counting counter-clockwise from the left edge of the plate.

b) Comparison study

To illustrate the ability of the A-FSM in the modeling of elastically supported plates, a comparison study has been made with the results obtained by Cheung *et al.* [20] for a stationary isotropic plate on a non-homogeneous elastic foundation shown in Fig. 4. The plate is simply supported at all edges. In Table 1, the first frequency parameter of the plate is given for different values of stiffness of local distributed elastic supports.

Two types of basic functions have been used to extract the results; the trigonometric and the bubble functions. As shown in the table, the results have been obtained with $m=1, 3$ and 5 , and $NS=10$. Since the nature of free vibration modes of a simply supported plate is sinusoidal, the trigonometric series leads to better results for $m=1$, but the rate of convergence is faster for the bubble function series. Anyway, the results obtained by both basic functions with $m=3$ have a complete agreement with the results of reference [20].

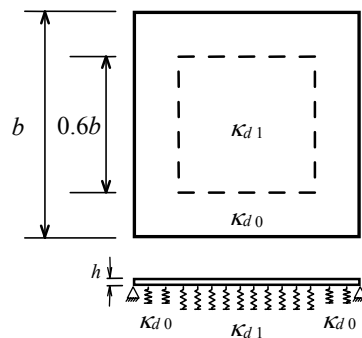


Fig. 4. A SSSS square isotropic plate on symmetrically distributed elastic supports

Table 1. Fundamental frequency parameter Ω_1 for a stationary isotropic square plate on non-homogeneous elastic foundation. ($\nu=0.3$)

K_{d0}	K_{d1}	A-FSM*			Ref [20]
		$m=1$	$m=3$	$m=5$	
0	320	1.2917 <u>1.3112</u> ⁺	1.2913 <u>1.2912</u>	1.2912 <u>1.2912</u>	1.291
0	800	1.6329 <u>1.6423</u>	1.6310 <u>1.6308</u>	1.6308 <u>1.6308</u>	1.631
0	1600	2.0273 <u>2.0773</u>	2.0716 <u>2.0711</u>	2.0711 <u>2.0710</u>	2.071
320	800	1.6802 <u>1.6932</u>	1.6795 <u>1.6794</u>	1.6794 <u>1.6794</u>	1.679
320	1600	2.1168 <u>2.1192</u>	2.1128 <u>2.1125</u>	2.1125 <u>2.1124</u>	2.113

*NS=10

+ underlined numbers are associated with bubble functions and others obtained by trigonometric functions.

To demonstrate the reliability of the A-FSM in the analysis of moving plates, natural frequencies of a single-span SFSF isotropic plate subjected to a uniform axial tension (Fig.5), are compared with those obtained by Lou and Hutton using Rayleigh-Ritz method [7]. The dimensionless natural frequencies of the first ten modes are shown in Table 2 for a stationary plate and a moving one with $c=1.839$. The other dimensionless parameters of the problem are: $r=2.92$, $\nu=0.3$ and $k_x=9.157$. The FSM results have been obtained by $m=7$ and $NS=16$ for the highest mode. A very close agreement is observed.

For comparing the E-FSM with the A-FSM, a double-span moving orthotropic plate, shown in Fig. 6, is analyzed by both methods. The plate is subjected to biaxial in-plane forces (N_x and N_y) and supported by an elastic foundation with stiffness κ_f in one of the spans. As shown in the figure, the plate has clamped boundaries at $X=0$ and $X=L$, and simple supports at $Y=0$, $Y=b$ (CSCS). The parameters of this example are: $K_f=4000$, $k_x=10$, $k_y=-2$ and $r=2$.

In the finite strip formulation developed in Section 4, longitudinal direction of the strips has been considered along the axial speed direction (Fig. 3). Nevertheless, an alternative finite strip formulation can also be extended in which the strips longitudinal direction be perpendicular to the speed direction (Fig. 6). The present example has been solved by means of this alternative formulation in which the finite strip displacement field of Eq. (33) is replaced with

$$w = \sum_{n=1}^r S_n(\eta) \langle f(\xi) \rangle \{d_n(t)\} \quad (68)$$

in which $\eta=y/b$ and $\xi=x/l$ as shown in Fig. 6b. Here, $S_n(\eta)$ are the basic functions which develop transverse displacement w along strip length (y -direction) and $f(\xi)$ are Hermitian polynomials for interpolating w along the width of the strip (x -direction). The other relationships corresponding to this formulation can be derived based on the displacement field defined in Eq. (68).

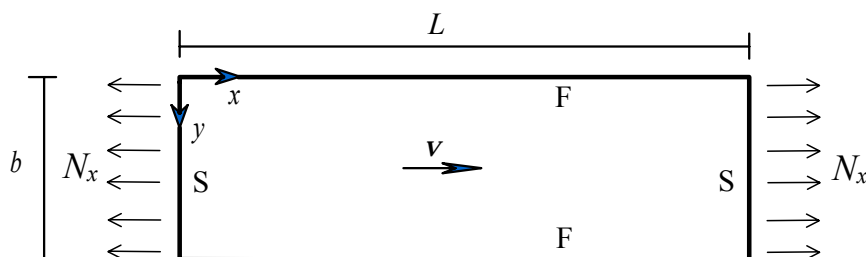


Fig. 5. An axially moving isotropic plate subjected to uniform axial tension

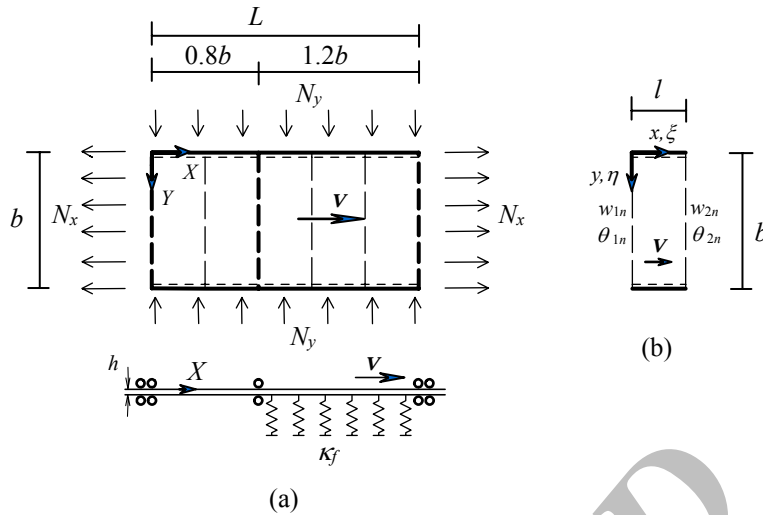


Fig. 6. a) A double-span axially moving orthotropic plate supported by an elastic foundation in one of its spans. b) one of finite strips of the plate

Table 2. Normalized natural frequencies Ω of the SFSF isotropic plate for stationary and moving situations. ($\nu=0.3$)

Mode	Stationary plate		Moving plate	
	A-FSM	Ref. [7]	A-FSM	Ref. [7]
1	0.0826	0.0827	0.0533	0.0534
2	0.0899	0.0902	0.0629	0.0629
3	0.1682	0.1683	0.1150	0.1151
4	0.1827	0.1827	0.1330	0.1331
5	0.2118	0.2118	0.1850	0.1850
6	0.2597	0.2599	0.1907	0.1909
7	0.2808	0.2811	0.2152	0.2155
8	0.2890	0.2890	0.2470	0.2470
9	0.3596	0.3597	0.2824	0.2832
10	0.3868	0.3874	0.3122	0.3130

For this example, the critical speed and the first five frequencies of vibration associated with $c=0.4c_{cr}$ have been listed in Table 3. The A-FSM results have been obtained by the alternative formulation with a different number of finite strips NS . By the formulation, modeling of the roller located at $X= 0.8b$, is simply done by setting the deflection of the corresponding nodal line to zero. With 15 strips in the length L , the results are very close to the exact values.

Also, validity of the results obtained from the E-FSM has been examined by comparing them with the results of A-FSM. The E-FSM results have been derived only by two plate components corresponding to two spans of the plate and only by one degree of freedom associated with the rotation of the nodal line at $X= 0.8b$. As shown in Table 3, the results of A-FSM and E-FSM are agreeable.

The exact stiffness matrix components are transcendental functions of the eigenvalues and a trial and error procedure is required to derive natural frequencies or critical speeds from the eigenfunction of the structure presented in Eq. (31) or (32). Fig. 7 shows the variation of $\det[S_n(\lambda)]$ in a logarithmic scale versus the vibration frequency parameter Ω for the double-span moving orthotropic plate. In the natural

frequencies Ω_i , $\det[S_n(\lambda)]$ vanishes. As shown in the figure, the first and the third natural frequencies correspond to the mode of vibration which has a one half wave in y -direction ($n=1$), the second and the fourth ones are associated with $n=2$ and the fifth natural frequency concerns $n=3$.

Table 3. Non-dimensional critical speed c_{cr} and natural frequency parameters Ω_i corresponding to $c/c_{cr}=0.4$ for the double-span moving plate

	<i>A-FSM</i>				<i>E-FSM</i>
	<i>NS =5</i>	<i>NS =10</i>	<i>NS =15</i>	<i>NS =20</i>	
c_{cr}	3.932	3.923	3.921	3.921	3.921 (1) ⁺
Ω_1	2.371	2.290	2.285	2.284	2.284 (1)
Ω_2	2.749	2.663	2.657	2.655	2.655 (2)
Ω_3	3.227	3.208	3.207	3.207	3.207 (1)
Ω_4	3.419	3.399	3.397	3.397	3.397 (2)
Ω_5	3.930	3.845	3.838	3.836	3.835 (3)

⁺The number of half waves along y -direction n is shown in the parentheses

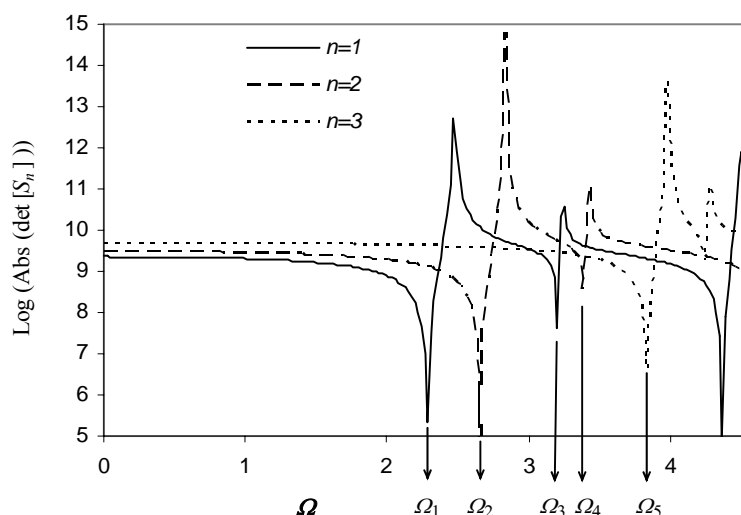


Fig. 7. Absolute of overall stiffness matrix determinant with respect to frequency parameter in the exact method

c) Numerical examples

In this subsection, some examples concerning the stability and vibration of axially moving orthotropic plates on stationary elastic supports are solved by the E-FSM or the A-FSM.

Figure 8 shows an axially traveling orthotropic plate which is supported by a partial elastic foundation with the length of $\beta \times L$ in the middle of the plate length L ($r=3$ and $K_f=4000$). The plate is under uniform axial tension ($k_x=10$) and has CSCS boundaries. Dynamic stability and vibration of the plate have been evaluated by the E-FSM with three components and four degrees of freedom. The critical speed of the plate and also fundamental natural frequencies concerning different axial speed ratios (c/c_{cr}) are shown in Table 4 for $\beta=0, 1/3, 1/2, 2/3$ and 1.0 . As the table shows, when the axial speed increases, the fundamental frequency decreases. For $c/c_{cr}=1$, all fundamental frequencies become zero, according to the definition of critical speed.

A study is carried out to demonstrate the ability of the A-FSM for modeling axially moving plates with elastic point supports. Figure 9 shows an orthotropic plate under a various tension which moves over two elastic point supports. The axial tension vary linearly along the plate width with a maximum normalized value of $k_x=2.0$. The lateral edges of the plate are free and two other edges can be assumed

simple by connivance (SFSF). The first three natural frequencies of the plate in stationary and moving situations have been listed in Table 5, for different values of stiffness of elastic point supports K_p . The results have been obtained by trigonometric basic functions.

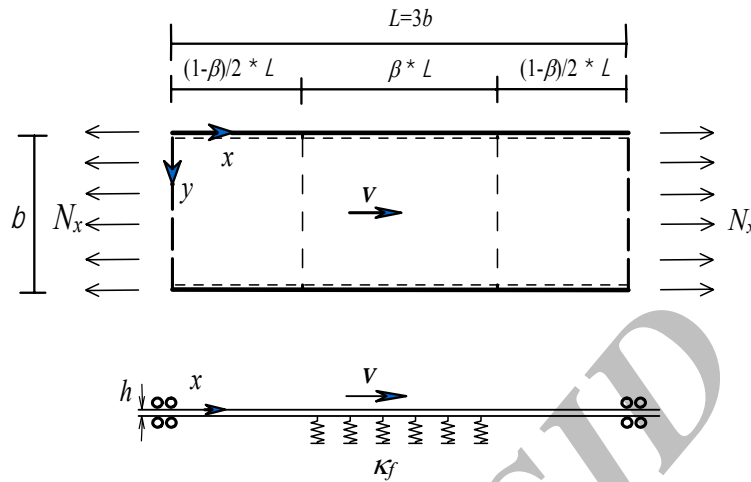


Fig. 8. An orthotropic plate moving on a locally distributed elastic support

Table 4. Exact values of dimensionless critical speed c_{cr} and fundamental frequency Ω_1 for an orthotropic plate moving on partial elastic foundation

β	χ_{cr}	Ω_1			
		$c/c_{cr} = 0$	$c/c_{cr} = 0.25$	$c/c_{cr} = 0.5$	$c/c_{cr} = 0.75$
0	3.60008	.694714	.653787	.530606	.322704
1/3	3.81198	1.64582	1.55288	1.27314	.800973
1/2	4.00268	2.12375	2.00676	1.65477	1.06048
2/3	4.40317	2.88380	2.72277	2.24145	1.43726
1	4.94861	3.27851	3.06203	2.42851	1.40430

Another example is associated with a SFSF orthotropic plate moving on an elastic foundation, shown in Fig. 10. The plate is under a uniform axial tension and a linearly various shear force with maximum dimensionless value of k_{xy0} . This form of in-plane loading may occur when an eccentricity exists between the end rollers. The characteristics of the example are $k_x = 4$, $r = 3$ and $c = 1.0$. The fundamental frequencies Ω_1 for the plate with $K_f = 0, 50$ and 100 and $k_{xy0} = 0, 1.0$ and 2.0 , obtained by the A-FSM with trigonometric basic functions, have been recorded in Table 6. Also, in Fig. 11, the variation of Ω_1 versus aspect ratio r is shown. The results in the figure obtained by bubble basic functions correspond to $k_x = 0.2$, $k_{xy0} = 0$ and $c = 0.4$.

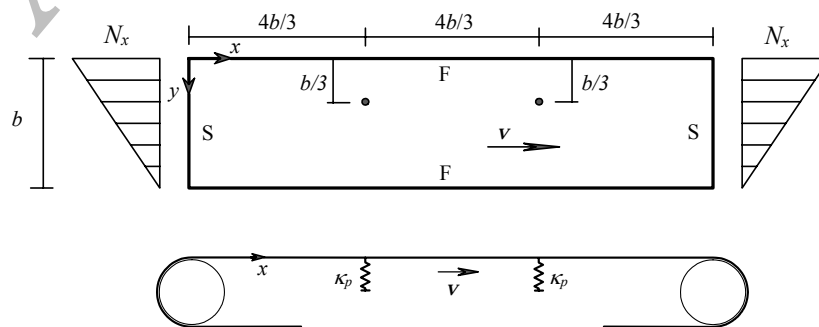


Fig. 9. An orthotropic plate moving on an elastic foundation subjected to axial tension and various shear force

Table 5. Natural frequency parameters Ω_i for SFSF plate with two elastic point supports in stationary and moving conditions obtained by A-FSM*

K_p	Stationary plate; $c=0$			Moving plate; $c=0.5$		
	Ω_1	Ω_2	Ω_3	Ω_1	Ω_2	Ω_3
0	0.1135	0.1969	0.2506	0.0832	0.1780	0.2088
1e1	0.1155	0.1990	0.2515	0.8532	0.1801	0.2097
1e2	0.1276	0.2190	0.2584	0.0980	0.1986	0.2182
1e3	0.1448	0.2853	0.3629	0.1179	0.2397	0.3399
1e4	0.1481	0.3004	0.4291	0.1225	0.2553	0.3869

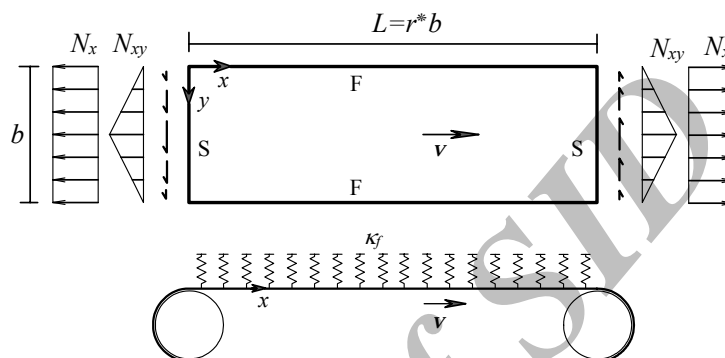


Fig. 10. An orthotropic plate moving on an elastic foundation subjected to axial tension and various shear force

The dynamic stability of the plate shown in Fig. 10 can be analyzed by the E-FSM, if the shear force N_{xy} vanishes. Here, the sinusoidal modes of buckling occur along x -direction that means the transverse displacement due to buckling, which was earlier defined by Eq. (13), must be changed as

$$w(x, y, t) = W_n(y)[\exp(ik_n x) - \exp(-ik_n x)] \tag{69}$$

The other steps of the E-FSM expressed in Section 3, can be simply modified based on the displacement field of Eq. (69). This modified formulation is used to extract exact values of critical speed c_{cr} for the plate shown in Fig. 10 with $N_{xy}=0$. Table 7 shows c_{cr} at two values of axial tension, for different values of the aspect ratio and the stiffness of the elastic foundation. Also, the variation of c_{cr} versus aspect ratio of the plate r has been drawn in Fig. 12 at $k_x=0.2$ for $K_f=0, 10$ and 100 . As shown in the figure, for the plate without foundation ($K_f=0$), c_{cr} continuously reduces when r increases and the minimum value of c_{cr} is 0.441 which is the critical speed parameter of a very slender plate ($r \rightarrow \infty$). For $K_f=0$, the buckling of the plate occurs by only one half a sinusoidal wave in x -direction ($n=1$). The buckling behavior of the plates with the elastic foundation ($K_f=10$ and 100) differs from the plate without an elastic foundation ($K_f=0$). For the plates with an elastic foundation, the critical speed depends on the number of half wavelengths. The minimum values of c_{cr} for $K_f=10$ and 100 , are 0.914 and 1.49 , respectively.

Table 6. First frequency parameters Ω_1 of an orthotropic plate moving on an elastic foundation derived by the A-FSM*

k_{xy0}	$K_f = 0$	$K_f = 50$	$K_f = 100$
0	0.260	0.411	0.519
1.0	0.244	0.394	0.503
2.0	0.197	0.353	0.465

* $m=3, NS=6$

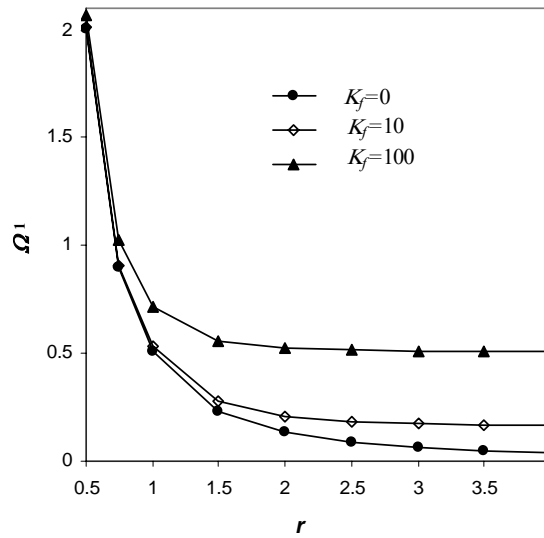


Fig. 11. The variation of fundamental frequency of an SFSF orthotropic plate moving on an elastic foundation derived by SA-FSM ($m=3, NS=6$)

Table 7. Exact non-dimensional critical speed c_{cr} for a SFSF orthotropic plate moving on an elastic foundation

r	$k_x = 0.2$			$k_x = 0.4$		
	$K_f=0$	$K_f=10$	$K_f=100$	$K_f=0$	$K_f=10$	$K_f=100$
0.5	2.0447	2.0510	2.1065	2.0930	2.0991	2.1535
1	1.0913	1.1374	1.4891	1.1794	1.2221	1.5548
1.5	0.7993	0.9327	1.5946	0.9159	1.0344	1.6561
2	0.6681	1.1374	1.4891	0.8040	1.2221	1.5548
2.5	0.5979	0.9967	1.5611	0.7466	1.0925	1.6239
3.0	0.5561	0.9327	1.4891	0.7136	1.0344	1.5548

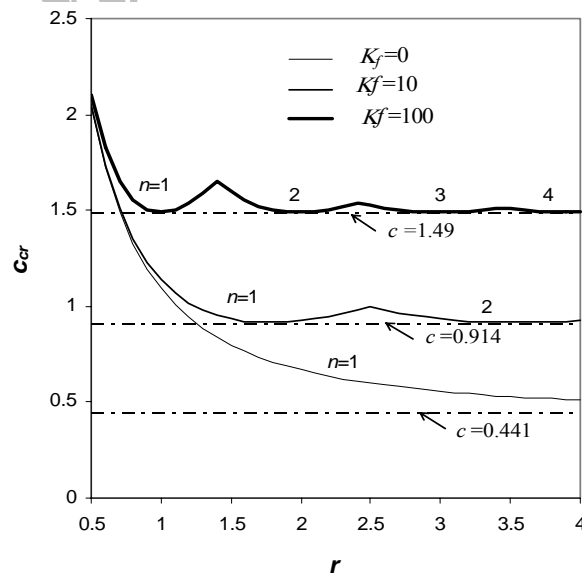


Fig. 12. The variation of dimensionless critical speed of an SFSF orthotropic plate moving on an elastic foundation obtained by exact FSM ($k_x=0.2$)

6. CONCLUDING REMARKS

In order to analyze elastically supported, axially moving orthotropic plates and to consider the effects of gyroscopic and in-plane forces on dynamic stability and vibration characteristics of such plates, two kinds of FSM have been presented in this study; one exact and one approximate.

By the exact method, the exact stiffness matrix of an axially moving orthotropic plate on an elastic foundation has been obtained, so precise results can be derived for stability and vibration of a multi-span moving plate, by only a few numbers of elements. The natural frequencies and critical speed of such plates obtained by the exact method can serve as a benchmark for checking the accuracy of numerical methods. Nevertheless the exact method has some restrictions about boundaries, loads and supports. However, the approximate method is able to model different boundary conditions, various plane stresses and discrete and distributed elastic supports.

Validity and accuracy of the methods and the rate of convergence of the A-FSM have been shown by some comparative studies. Furthermore, some examples have been presented to examine the ability of the methods for modeling different problems. The effects of aspect ratio, stiffness of elastic supports and in-plane forces on critical speed and natural frequencies of moving plates have been studied in these examples.

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