# OPTIMUM ROUTE LOCATION MODEL CONSIDERING THE TRIP GENERATION CENTERS AND PROTECTED ZONES\*

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**Abstract**— The purpose of this research is to present an optimum model for route location considering access to all the main points of the roads, passing compulsory points and having the minimum total cost. It is assumed in this research that the involved areas are level or rolling. The major capability of such models is predicting protected areas in an optimal corridor route location. In this paper, after describing the fulfilled studies in the field of road alignment, two mathematical models for alignment in level areas are presented. In the first model the forbidden zones are not considered, but in the second model the forbidden zones are prevented from being passed through by modeling these zones in a circular shape. In this research the mathematical non-linear programming has been used for modeling.

**Keywords**– Route location, forbidden areas, optimization, cost function

# 1. INTRODUCTION

Route location is usually a complicated and tedious part of roads design projects and in this regard many factors should be taken into account by designers. Designers are usually confronted with a wide area in order to select the best corridor, so these areas are divided and reduced to some possible routes by means of engineering judgment. Recommended routes are then compared with each other from an economic point of view, and finally the route with the minimum cost is chosen. After this stage the engineering details for the chosen route is recommended. Using mathematical and computerized models, design engineers are able to significantly speed up the design procedure.

# 2. EFFECTIVE PARAMETERS OF CORRIDOR SELECTION

The accurate location and details of roads can not be specified all at once in one stage. Thus alignment is specified in various stages, using different-scale aerial maps and photographs [1]. The choice of a route means to achieve a balanced status which will be able to develop the maximum number of accessions by minimum costs, and keep the undesirable environmental impact at the minimum rate, some of the factors that should be taken into account at the time of designing the best alignment are:

accessibility to natural topography, geometric design criteria, geological aspects, road basement and road bedding materials, the availability of suitable materials, road repair and maintenance, and aesthetic and environmental factors [2].

## 3. ALIGNMENT METHODS

Several research studies have been carried out in the area of road alignment. Turner considers the optimization problem to be a network issue where the links of the network are in accordance with the road

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blocks, and then by using network optimization models such as the shortest path method, solve the problem [3].

Athanassoulis and Calogero divided the route into subdivisions in a way that in each stage local decision making would be possible [4]. Dinardo modeled the alignment three dimensionally and used dynamic programming for finding the route with the minimum cost [5]. In another model Nassiri and Ghaffari converted the alignment into an issue of selecting the optimal point in continuous transversal plates in the route corridor [6]. In this method, the maximum permitted longitudinal gradient and the minimum vertical curve radius are considered the constraints of the optimum point selection at each plate. Dynamic Programming was used to overcome this problem. Hogon modeled the alignment three dimensionally by dynamic programming. He started by a large network first, and then for better accuracy of the alignment around the resulting solution, used networking and then repeated this process to reach a suitable alignment [7]. Nicholson, in a similar method, used a large network for alignment and then by discontinuous calculus of variation determined the final route [8].

One of the methods likely to optimize a mild sloped route on a three dimensional basis is the Numerical Search Method. An heuristic method which solves the problem by using a genetic algorithm has been represented by Jong [9]. In a similar method, Jha used GIS for the input of the data in his algorithm; therefore his model has more accuracy in finding costs that are related to the geographical conditions of the area, compared to Jong's model.

The existing models, although seeming to be appropriate, have major deficiencies and are not applicable in actual cases. None of the present models have been able to consider all the effective factors for the provision of a comprehensive model for alignment. Therefore, none of these models present specific optimized models for alignment projects, taking into consideration some of the effective parameters.

# 4. EVALUATION OF ROUTE FINDING MODELS

Road designers generally predict some options as the preliminary routes from past experiences as well as employer requirements and general topography. These planned routes are then modified by designers after considerable investigations, and trial and error stages. These modified plans are then finalized as the main route, based on the engineering judgment, limitations of the design, compulsory points and some other issues. The quality of these routes depend considerably on the experimental background of the designer, therefore, this method could not possibly be considered as scientific. In case the preliminary routes are not appropriate, costs will be imposed on the projects and extra costs in other stages cannot be prevented or compensated.

#### 5. OPTIMIZATION BY MATHEMATICAL PROGRAMMING METHOD

In this method mathematical programming has been used for modeling. These methods convert the problem to a mathematical model and then solve it. Converting a problem to a mathematical model increases the ability to investigate it, and therefore, provides better opportunities to benefit from a variety of mathematical programs. Therefore, the mathematical programming is first defined and then a basic theorem which has been used in solving problems has been represented:

Definition: The Mathematical programming model is shown in Eq. (1),

$$Min \quad f(x), \quad x \in S, \quad x \in E^n \tag{1}$$

Where

 $f: E^n \to R$  is an optional function and S is the subset of the  $E^n$  points of the vector space. If S is the total  $E^n$  vector space, then the model could be called Unlimited Mathematical Programming. If not, the model is called Limited Mathematical Programming. The unlimited mathematical programming may be defined in Eq. (2).

Min 
$$f(x)$$
  
 $g_i(x) \ge 0$   $i = 1,2,...,m$  (2)  
 $h_j(x) = 0$   $i = 1,2,...,p$ .

The conditions for the presence of answers for these models and the basic theorem for solutions are represented as follows:

(The Kuhn Tucker Theorem): In the case that f,  $g_i$  and  $h_j$  are the first grade derivatives, and the first grade condition for the limited quality is present,  $x^*$  the general minimum of the convex programming, based on Eq. (2), is obtained by [10]:

The practicability conditions of X\*:

$$g_i(x^*) \ge 0 \quad i = 1, 2, ..., m$$
 (3)

$$h_{i}(x^{*}) = 0 \quad j = 1, 2, ..., r$$
 (4)

Complementary condition:

$$u_i^* g_i(x^*) = 0 \quad i = 1, 2, ..., m$$
 (5)

Lagrangian condition:

$$\nabla f(x^*) - \sum_{i=1}^{m} u_i \nabla g_i(x^*) - \sum_{j=1}^{r} v_j \nabla h_j(x^*) = 0$$
 (6)

where:

 $u_i$  = Lagrange multipliers,

 $v_i$  = Lagrange multipliers.

The  $\nabla$  operator which has been used in the above equations is defined as follows:

$$\nabla f(x) = \left(\frac{\partial f(x)}{\partial x_1}, \frac{\partial f(x)}{\partial x_2}, \dots, \frac{\partial f(x)}{\partial x_n}\right)^T \tag{7}$$

That is called the delta operator or the f(x) gradient.

Solving the equations that result from Eq. (6) is a difficult task. (For studying the solution methods for these equations readers may refer to reference [10, 11]).

## 6. MATHEMATICAL MODEL OF OPTIMAL ALIGNMENT

Finding an optimum alignment will practically result in an alignment that has the minimum cost. So the major factor in alignment design in mountainous areas is the minimization of earthwork operations, since the topographical conditions of these areas impose some restrictions on the projects, among which route alignment is a function of those restrictions. This means that in following the standards in the permitted minimums and maximums in the geometric design of roads, the topography of the area plays the major role. But in level areas which generally consist of mild gradients, topography is not the main factor for road alignment. In such areas, the length of the route and the conditions of the subgrade which influence construction and utilization costs, are the determinant factor in the road cost function. Therefore, minimizing the cost function in these areas means finding the shortest possible route that, while having access to the points on different parts of the route, passes through areas where construction costs are much

less than in other areas. Usually the natural situation of the ground in these areas has some local rough places like mountains and valleys. Due to the considerable cost of road construction in such areas, it is prudent not to pass the route from these points.

Therefore, the optimum mathematical alignment procedure should not only have the minimum costs, but it should also fulfill some other restrictions. One of these circumstances is the compulsory points that the path should pass through. On the contrary, there are some areas in the design corridor that the path should not pass through. These are briefly pointed out as follows:

- The areas where passing through requires high costs such as valleys, lakes and mountains
- Protected environmental areas
- Protected military areas
- Some lands with agricultural applications
- Those areas that are not appropriate to pass through due to their geological conditions and
- Some other forbidden zones that the path should not pass through for other reasons.

## 7. SELECTION OF THE MODEL

The general form of the connection of the points to each other and the grade of communication between the routes are among the most important factors in the optimization of intercity transportation networks. The defined problem in this paper is finding an optimal route for the purpose of connecting two main cities and providing accessibility to the important points along the road. These are connected by the model shown in Fig. 1.

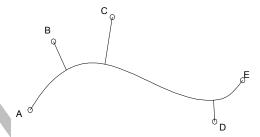


Fig. 1. The general shape of the optimal route

As shown in Fig. 1, many possible routes are able to connect the points A to E, and at the same time have access to points B, C and D. The optimal route in this case is the route that has the minimum cost for the construction and utilization of the main and access roads.

### 8. OPTIMUM ALIGNMENT MODEL IN UNCONSTRAINED AREAS

It is assumed, in the first stage, that there are no restrictions and/or forbidden zones in the area. In these cases the problem will be solved by a mathematical model with no constraints. As mentioned before, in rolling and level areas, the costs are mostly dependant.

#### a) Unit cost of the route length

The most important factor for choosing a route among some other options is the ratio of benefits to the cost within the lifetime of the road. The cost of a road or railroad in their service life are categorized into three classes; design, construction and operation. Any one of these classes include various expenditures. OECD has classified the costs of a route as presented in Table 1.

Classification Example Consulting and supervision Design and administrative costs Construction costs Land supply, earthwork and pavement Maintenance costs Lighting, mowing, pavement Vehicles operation costs Fuel, tire wear, depreciation of vehicles User costs Travel time costs Vehicle hours times unit value of time Estimated accident rates times unit Accidents costs accident costs Social, environmental Noise, air pollution, wetland loss

Table 1. Various costs of a route in its service life [1]

The major part of roads and railroads cost, despite their sums, is directly dependant on the length of the route. The know-how of such relevance and its cost function can be found in some research works [12-15].

The total cost function in the analysis of this research is equal to the sum of the above mentioned costs; therefore, the cost of construction and the utilization of the unit length of roads and railroads is calculated by Eq. (8):

$$C_{Total} = C_c + C_O \tag{8}$$

Where:

C<sub>C</sub>=The average cost of road construction per unit of the road length

C<sub>0</sub>=The average operation cost of the route length within the road service life.

The topographical situation of the area, whether it is level, rolling or mountainous, has a great effect on the cost of the road. On the other hand, the road grade (freeway, expressway, major highway or arterial) is effective on its construction cost. Therefore, in an intercity road or railroad network, the minimum construction cost is a form of the network in which in addition to the appropriate connection of the urban areas, natural resources and industrial centers, the length of the roads are optimal.

In this model, the total cost of construction and the utilization of the unit length of the base road which connect the starting and terminating points of the route is shown by p, and the total construction and operation costs of the per unit of the road length for the  $i^{th}$  section has been considered as  $q_i$ .

In practice, for some projects, a few of the road points are defined as compulsory points due to specific reasons, the points that the base route should pass through. In this model, for the definition of these points, it is possible to define the construction and operation cost of the access road to the base route as infinite. In this case the optimization of the model reduces the access route length to the minimum in order to reduce the total cost.

## b) Length of road sections

The route length functions are calculated assuming the intersection of the access roads with the major road as  $J_i$  and the coordinates of the point as  $x_i$  and  $y_i$  as follows (as shown in Fig. 2):

$$L_{i} = \sqrt{(x_{i-1} - x_{i})^{2} + (y_{i-1} - y_{i})^{2}}$$
(9)

$$d_i = \sqrt{(x_i - a_i)^2 + (y_i - b_i)^2}$$
(10)

In which:

 $L_i$  =Length of major road section

 $x_i, y_i$ =Coordinates of the intersection point

 $d_i$ =Length of minor road section, and

 $a_i, b_i$  =Coordinates of the access point.

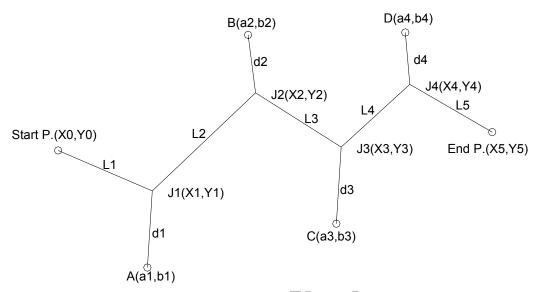


Fig. 2. The coordinates of the road points in the local coordinate system

#### c) The road cost function

By multiplying the cost of the unit length in each section of the road by its length, the cost of each section is found. Now by adding these costs, the total cost is obtained as follows:

$$f = p \sum_{i=1}^{n+1} L_i + \sum_{i=1}^{n} q_i d_i$$
 (11)

Where

f = Total cost

p = Unit cost of the main route length

 $q_i$  = Unit cost of the i<sup>th</sup> section, and

n =the number of access points.

It is assumed that the unit cost of the major road length will remain constant within the length of the road, but the unit cost of access roads, considering their grade and based on their traffic volume, is varied.

## d) The cost function minimization

The coordinates of the intersections between access roads and major roads  $(x_i, y_i)$ , are unknown variations of the equation. By using the function minimization methods,  $x_i$  and  $y_i$  quantities are found in a way that the cost function (f) will have its minimum quantity. As mentioned before in this research, mathematical programming models have been used for the optimization of the cost function. The Lagrange method is used as one of the minimization methods.

In the Lagrange method, in order to find the minimum of a function, the detailed derivative of the function, based on free variables, is first calculated. Then by considering the obtained derivative functions as zero and their simultaneous solution in a derivative equations system, the quantities of variables are obtained as shown in Eq. (12).

$$\begin{cases} \frac{\partial f}{\partial x_{i}} = \frac{q_{i}(x_{i} - a_{i})}{\sqrt{(x_{i} - a_{i})^{2} + (y_{i} - b_{i})^{2}}} \\ + p(\frac{x_{i} - x_{i-1}}{\sqrt{(x_{i} - x_{i-1})^{2} + (y_{i} - y_{i-1})^{2}}} + \frac{x_{i} - x_{i+1}}{\sqrt{(x_{i} - x_{i+1})^{2} + (y_{i} - y_{i+1})^{2}}}) = 0 \quad (i = 1, ..., n) \end{cases}$$

$$\begin{cases} \frac{\partial f}{\partial y_{i}} = \frac{q_{i}(y_{i} - b_{i})}{\sqrt{(x_{i} - a_{i}) + (y_{i} - b_{i})}} \\ + p(\frac{y_{i} - y_{i-1}}{\sqrt{(x_{i} - x_{i-1})^{2} + (y_{i} - y_{i-1})^{2}}} + \frac{y_{i} - y_{i+1}}{\sqrt{(x_{i} - x_{i+1})^{2} + (y_{i} - y_{i+1})^{2}}} = 0 \quad (i = 1, ..., n) \end{cases}$$

After solving the above mentioned equations, the values of  $(x_1, x_2, x_3, ..., y_1, y_2, y_3, ...)$  are found. By determination of the  $x_i$  and  $y_i$ , the basic shape of the major road and the length of access roads are specified.

By upgrading the degrees freedom of the cost function (increasing the road access points), finding the derivative functions and their simultaneous solution will be more difficult and require computer programs. Therefore, it would be possible to solve these equations more accurately with appropriate computer programs. Software such as Lingo, Mathematica, and Solver Program (installable on Excel), are appropriate to solve the model functions numerically, after appropriate programming.

# e) Modification of the designed route

After designing the route by the computer program, the design engineer can modify the designed route on the various geographical maps. If the route has passed through inappropriate points, by definition of the Compulsory point for the program, the designer can finalize the optimal route. For instance, a preliminary route may pass through a lake, a river or a marsh. In such cases, the design engineer can define the best intersection of route passing through the river (best intersection for construction of a bridge), as the compulsory point for the program, based on the river engineering data and economic and technical factors. There are sometimes mountains in level areas that the route should inevitably pass through. In such cases too, the engineer may define the best intersection as the compulsory point for the construction of a tunnel, based on the type of area and its specific circumstances, like topographical, geological conditions, materials of the mountains, performance possibilities, and technical limitations. Therefore, the more complete the data entered in the model, the closer the output of the model will be to the optimum route.

#### f) Sensitivity analysis of the cost function

The cost function of the model includes two main sections; one is illustrative of the cost of the main road, and the other includes the cost of access roads. Taking into account the fact that usually the quality of the main road is better than that of the access roads, therefore, the cost of construction of the unit length is considerably more than that of the access roads. On the other hand, the utilization cost of the unit length of the main road is more than the service cost of the unit length of the access roads. Therefore, the higher the total cost of construction and utilization of the main road compared with the access roads is, the more direct the general shape of the main road becomes. On the contrary, in the case of the cost of the main road being less than the cost of access roads, the main road will be closer to these access points. Figure 3 illustrates this fact.

As seen in Fig. 3, the model's input data are managed so that by constant construction and the utilization cost of the unit length of access roads, at one stage, the cost of the main road is assumed as 1/2 of the unit cost and at another time it is assumed to be 10 unit costs. These are compared with each other in Fig. 3.

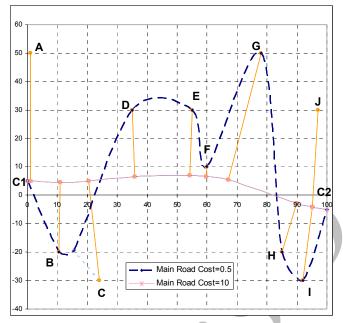


Fig. 3. The output data of the unconstrained alignment model for two types of construction and utilization cost of the main road [16]

In some projects there are areas around the road where access to them, either from industrial, commercial or a social point of view is vital. In such cases the total cost of the construction and utilization of these areas is comparatively higher than other access roads. In such situations it seems natural that the main route should be adjacent to these points. The example in Fig. 4 shows the output of the optimization program for two routes that are only different in construction and utilization costs of the access road to point C.

As shown in Fig. 4, the construction and utilization cost of Point C of the access road is assumed as 0.5 of the cost unit, and in another place it is assumed 3 times the unit cost. In Fig. 4 the output of the unconstrained alignment model is seen.

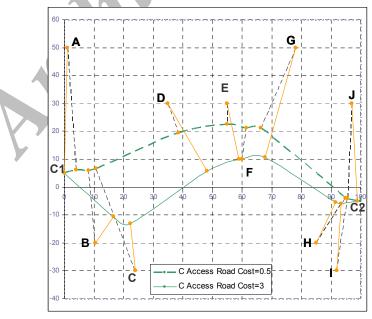


Fig. 4. The output of unconstrained model for two types of construction and utilization cost [16]

#### 9. CONSTRAINED ALIGNMENT MATHEMATICAL MODEL

In road projects, there are areas that the route should not pass through. Therefore, by modeling and defining these limited areas the model should be complete. As a definition of these areas in a mathematical model, considering their irregular shape, these areas should be defined on the basis of regular geometric shapes. In this research, a circle has been taken as the regular shape for the definition of forbidden zones, in a way that the user leads the alignment program by specifying the center of the circle and their radius as appropriate for the shape of the forbidden zones.

In this model, by applying a coefficient in the cost function, the quantity of the cost that has been caused by passing through various areas of the design corridor is controlled. In this method the situation of all sections of the route against the forbidden zones are controlled. In case a section of the route passes through this limited area, the cost of that specific section, or the coefficient of that segment in the objective function will become infinite. Therefore, by such an approach the program for the minimization of the goal function will make all the route sections away from these areas.

## a) The minimum space between the main route and the forbidden zones

It is assumed that the point  $Z_0$  is going to be connected to the point  $Z_n$  by a main route, in a way that this route can have access to inter road points with the coordinates  $(a_i, b_i)$ . The points  $F_1$  to  $F_m$  with the coordinates  $(p_i, q_i)$ , and the radius  $R_i$  are defined in Fig. 5.

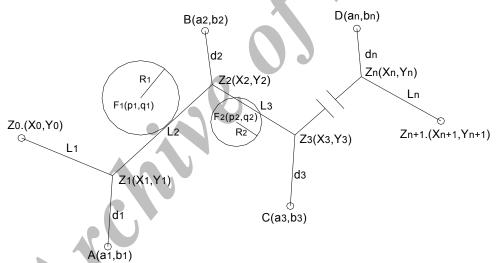


Fig. 5. The coordinates of the points of the main route and forbidden zones in the local coordinates system

For the calculation of distance from i<sup>th</sup> sections of the road to the j<sup>th</sup> forbidden area, firstly the equation of that road section should be specified and then the function of the space between the point and the line should be specified.

The equation of the line passing through the point  $Z_i(X_i, Y_i)$  and the point  $Z_{i+1}(X_{i+1}, Y_{i+1})$  is as follows:

$$\frac{y - y_i}{x - x_i} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \tag{13}$$

If the line gradient is called S<sub>i</sub>, the line equation may be rewritten as follows:

$$y - s_i x + (s_i x_i - y_i) = 0 (14)$$

Where:

$$s_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \tag{15}$$

In this case, the distance between the Fi forbidden area center with coordinates  $(p_i,q_i)$  and the line with Eq. (14) is as follows:

$$T_{ij} = \frac{\left| q_i - s_i . p_i + s_i . x_i - y_i \right|}{\sqrt{1 + s_i^2}} \tag{16}$$

#### b) Route direction coefficient

In this method, the coefficient Cii, in which i is the number of the main route sections and j is the forbidden area, is determined according to the following conditions (Fig. 6):

$$C_{ij} = \begin{cases} 1 & T_{ij} \ge R_j \\ M & T_{ij} < R_j \end{cases}$$
 (17)

Where

 $C_{ij}$  =The control coefficient of the passing of i<sup>th</sup> section of the main route from the j<sup>th</sup> forbidden area  $T_{ij}$  =The minimum distance between the i<sup>th</sup> section of the main route and the center of the j<sup>th</sup> forbidden area  $R_{j}$  =The radius of the j<sup>th</sup> forbidden area

M =a assumed large number.

Using Eq. (17), when a section of the main road passes through one of the forbidden zones, the control coefficient of Cij, will be a large number, e.g. M. Using this method it would be possible to prevent passing all sections of the major road from any of the forbidden zones. The cost coefficient of each section of the major road taking all the forbidden zones into account is calculated as follows:

$$C_i = \prod_{j=1}^m C_{ij} \tag{18}$$

in which:

m=the number of forbidden zones

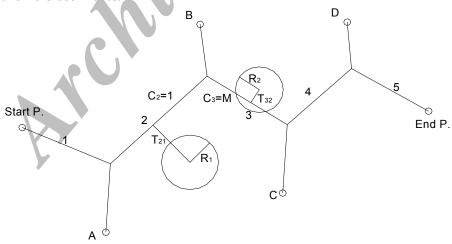


Fig. 6. The Ci quantities in different situations as compared to the forbidden zones

#### c) The road cost function

By putting these coefficients into the cost function, the cost of any section that has passed through the forbidden area will become infinite. In the minimization process of the cost function it would be possible to make all points of the major road away from the forbidden zones.

$$f = p \sum_{i=1}^{n} C_i L_i + \sum_{i=1}^{n} q_i d_i$$
 (19)

To clarify the output of the model, an example is solved in two different cases. Assume that a major road is designed to connect two cities and there are also ten different areas which should have access to the Major road and should pass through a rolling area. From some of the variables such as; the coordinates of cities, access areas, and forbidden zones, the construction and operation cost of major and access roads should be identified [13].

Figure 7 compares the output of the model for two different cases, i.e., with and without forbidden zones.

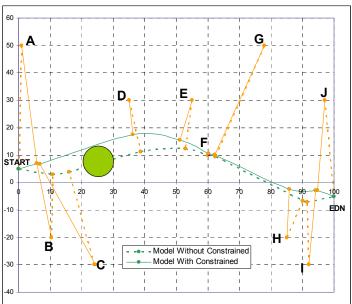


Fig. 7. Comparison between the output of the constrained and unconstrained alignment model [16]

However the minor modifications to the model can be formed by engineering judgment.

As illustrated in Fig. 7, the final output of the model depends on the correct assignment of the compulsory points as well as forbidden zones.

#### 10. CONCLUSION

The model presented in this paper yields the final route corridor in a way that economic, social and political requirements of the project are fulfilled. Many of the effective parameters on the construction and utilization cost function of the road and railroad are directly dependant on the length of the route. The main objective of this paper was to design the primary route corridor for a road or railway. At this stage, recognition of general natural specifications such as mountains, lakes, valleys, lagoons, etc. will satisfy the project requirements. The presented models are not, however, suitable for mountainous or rolling areas with high gradients, because in such areas topography is one of the major factors of the route design.

All the cost parameters of the route length can enter the model. The natural and geographical phenomena like valleys, mountains, lakes, lagoons, soft soils, and environmentally protected areas can be included in this model. Recognition of the accurate boundaries and specifications of the areas however, requires some systems like the Geographical Information System.

The findings of this paper can be summarized as follows:

- -Provision of an efficient comprehensive plan requires appropriate mathematical models so that the cost minimization is possible.
- -The significance of computerized and mathematical programs for designing the preliminary routes is due to the fact that in road and railroad projects, the additional length of the route will significantly increase the total costs of the project.
- -The present model in this paper is a computerized aid that experienced engineers can benefit from in planning roads. For the accurate design of the final route the designers need to make use of accurate topographical and geographical maps. This paper is the starting point in the field of computerized and mathematical models for route design that should be developed by other researchers.

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