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ADAPTIVE RADAR DETECTION OF FLUCTUATING TARGETS IN AUTOREGRESSIVE INTERFERENCE*

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Abstract– In this paper, an adaptive detection scheme for fluctuating targets with a swerling I model in AR interference is presented. Since the proposed detector uses more information from the target signal in its structure, it has better performance compared with those detectors which do not use the target amplitude model. Performance improvement of this detector compared with the previously AR model based adaptive detector (ARGLR) is shown by simulation results. Besides, another detector is proposed for the known amplitude situation whose performance can be used as an upper bound for all similar detectors.

Keywords– Detector, radar, adaptive detector, fluctuation, ARGLR, detection theory

1. INTRODUCTION

The detection theory has been used in radar signal detection since the 1950's. The first research in this case was made by Marcum [1]. He considered the detection of a completely known signal in white Gaussian noise using multiple received samples. This research was continued by Swerling [2-4], and since then, radar detection has been consistently developing.

The design of adaptive radar detectors is a very important application of the detection theory in radar signal processing. Hence, several detectors have been proposed in this field which can be divided into three main categories: SMI based detectors [5-10], GLR based detectors [11-19] and AR model based detectors [20-26]. Whereas AR model based detectors consider some special interference modeling, their performance is more desirable compared with others [27-29]. In these detectors the target is assumed to be nonfluctuating. But this is not the case in many real scenarios, so in this paper the AR model based adaptive detector for fluctuating targets (SW1) was considered.

This paper is organized as follows. In section II the problem formulation is presented. In section III the proposed detector will be introduced. Section IV is devoted to performance evaluation by computer simulation, and finally section V gives a summary and conclusions.

2. PROBLEM FORMULATION

We have considered coherent detection in a pulsed radar system for detecting targets in a primary data vector. The system also uses L secondary data vectors which are assumed to be signal free for adaptation. So, the following detection problem is considered:

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$$\begin{aligned} H_0: \quad & \underline{y}(0) = \underline{n}(0) \\ & \underline{y}(k) = \underline{n}(k) \qquad k = 1,2,\ldots,L \\ H_1: \quad & \underline{y}(0) = \underline{n}(0) + b\underline{s} \\ & \underline{y}(k) = \underline{n}(k) \qquad k = 1,2,\ldots,L \end{aligned} \tag{1}$$

Where $\underline{y}(k)$'s, $k = 0,1,\ldots,L$ are complex N-tuple vectors denoting the received primary and secondary signal. Also, $\underline{s}$ is a complex N-tuple vector which denotes the target signal and is given by:

$$\underline{s} = \begin{bmatrix} 1 & e^{j\Omega} & \ldots & e^{j(N-1)\Omega} \end{bmatrix}^T \tag{2}$$

This vector corresponds to a target whose normalized Doppler is a known constant Ω. b is the unknown complex amplitude of the reflected signal from the target. $\underline{n}(k)$'s are also complex N-tuple interference vectors which are assumed to be an AR process of order M given by [29]:

$$n_{k,i} = \sum_{r=1}^{M} a_r n_{k,i-r} + w_{k,i} \qquad \begin{matrix} k = 1,2,\ldots,L \\ i = 1,2,\ldots,N \end{matrix} \tag{3}$$

$$\underline{a} = \begin{bmatrix} a_1 & \ldots & a_M \end{bmatrix}^T \tag{4}$$

Where $w_{k,i}$'s are zero-mean discrete complex white Gaussian noise with variance σ^2, and $\underline{a}$ is the AR parameters vector. We assume that $\underline{a}$ and σ^2 are also unknown, but fixed constants which are the same under the two hypotheses.

Sheikhi assumed b to be unknown and derived a GLRT for the hypothesis-test [24-26]. In this paper, we assume a different assumption regarding b in two different cases.

3. DESIGN OF NEW ADAPTIVE DETECTORS

CASE 1: We assume the target has fluctuation with the Swerling I model. Therefore, the target amplitude b to be given by:

$$b = ue^{j\varphi} \tag{5}$$

u has Rayleigh distribution with unknown parameter m, as follows:

$$f_u(u) = \frac{u}{m}\exp(-\frac{u^2}{2m}), \qquad u \geq 0 \tag{6}$$

and φ has uniform distribution in the interval $[0,2\pi)$.
For deriving the decision rule, we first calculate the likelihood ratio as follows:

$$LR = \frac{f_Y[\underline{y}(0),\underline{y}(1),\ldots,\underline{y}(L) \mid H_1]}{f_Y[\underline{y}(0),\underline{y}(1),\ldots,\underline{y}(L) \mid H_0]} = L\left(\underline{y} \mid \underline{a}',\underline{a}'',\sigma_0^2,\sigma_1^2,\varphi,u\right) \tag{7}$$

For the sake of brevity, we use $L\left(\underline{y} \mid \underline{a}',\underline{a}'',\sigma_0^2,\sigma_1^2,\varphi,u\right)$ instead of $L\left(\underline{y} \mid \underline{a},\sigma^2,\varphi,u\right)$, albeit the latter representation is more usual. Since $w_{k,i}$'s are Gaussian and independent identically distributed (i.i.d):

$$f_{w_i}(w_i) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{|w_i|^2}{2\sigma^2}\right) \tag{8}$$

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$$f_{\underline{w}}[\underline{w}_k]=\prod_{i=1}^{N}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}\left|w_{k,i}\right|^2\right) \tag{9}$$

Considering the AR model of interference yields:

$$w_{k,i}=n_{k,i}-\sum_{r=1}^{M}a_r n_{k,i-r} \tag{10}$$

and so:

$$\begin{aligned} & f_Y[\underline{y}(0),...,\underline{y}(L)\mid H_1]= \\ & =f_n[\underline{y}(0)-ue^{j\varphi}\underline{s},\underline{y}(1),...,\underline{y}(L)\mid H_1] \\ & =f_W[\underline{w}_0,\underline{w}_1,...,\underline{w}_L\mid H_1] \end{aligned} \tag{11}$$

therefore:

$$\begin{aligned} & f_Y[\underline{y}(0),...,\underline{y}(L)\mid H_1]=f_{\underline{w}}(\underline{w}_0\mid H_1)\ \cdots\ f_{\underline{w}}(\underline{w}_L\mid H_1) \\ & =\left(\frac{1}{2\pi\sigma_1^2}\right)^{N(L+1)}\prod_{k=0}^{L}\exp\left(-\frac{1}{2\sigma_1^2}\sum_{n=1}^{N}\left|x_{k,n}-\sum_{r=1}^{M}a''_r x_{k,n-r}\right|^2\right) \end{aligned} \tag{12}$$

$$\begin{aligned} & x_{0,n}=y_{0,n}-ue^{j\varphi}s_n \\ & x_{k,n}=y_{k,n} \qquad \begin{matrix} n=1,2,...,N \\ k=1,2,...,L \end{matrix} \\ & s_n=e^{j(n-1)\Omega} \end{aligned} \tag{13}$$

If the poles of the AR process are not too close to the unit circle [23], we have:

$$\begin{aligned} & f_Y[\underline{y}(0),...,\underline{y}(L)\mid H_1]\cong \\ & \left(\frac{1}{\pi\sigma_1^2}\right)^{N(L+1)}\prod_{k=0}^{L}\exp\left(-\frac{1}{\sigma_1^2}\sum_{n=M+1}^{N}\left|x_{k,n}-\sum_{r=1}^{M}a''_r x_{k,n-r}\right|^2\right) \end{aligned} \tag{14}$$

In a similar way, the following equation can be written for H_0 condition:

$$\begin{aligned} & f_Y[\underline{y}(0),...,\underline{y}(L)\mid H_0]\cong \\ & \left(\frac{1}{\pi\sigma_0^2}\right)^{N(L+1)}\prod_{k=0}^{L}\exp\left(-\frac{1}{\sigma_0^2}\sum_{n=M+1}^{N}\left|y_{k,n}-\sum_{r=1}^{M}a'_r y_{k,n-r}\right|^2\right) \end{aligned} \tag{15}$$

Based on the detection theory, the ALR can be used because the distribution of parameters u and φ are known. Hence:

$$\begin{aligned} & L\left(\underline{y}\mid\underline{a}',\underline{a}'',\sigma_0^2,\sigma_1^2\right)= \\ & \int_u\int_\varphi L\left(\underline{y}\mid\underline{a}',\underline{a}'',\sigma_0^2,\sigma_1^2,\varphi,u\right)f_\varphi(\varphi)f_u(u)\,d\varphi du \\ & =\frac{\left(\frac{1}{\pi\sigma_1^2}\right)^{N(L+1)}\prod_{k=1}^{L}\exp\left(-\frac{1}{\sigma_1^2}\sum_{n=M+1}^{N}\left|y_{k,n}-\sum_{r=1}^{M}a''_r y_{k,n-r}\right|^2\right)}{\left(\frac{1}{\pi\sigma_0^2}\right)^{N(L+1)}\prod_{k=0}^{L}\exp\left(-\frac{1}{\sigma_0^2}\sum_{n=M+1}^{N}\left|y_{k,n}-\sum_{r=1}^{M}a'_r y_{k,n-r}\right|^2\right)}\times B \end{aligned} \tag{16}$$

$$\begin{aligned} B=\frac{1}{2\pi}\int_u\int_\varphi & \exp\left(-\frac{1}{\sigma_1^2}\sum_{n=M+1}^{N}\left|\begin{matrix} y_{0,n}-ue^{j\varphi}s_n \\ -\sum_{r=1}^{M}a''_r\left(y_{0,n-r}-ue^{j\varphi}s_{n-r}\right)\end{matrix}\right|^2\right) \\ & \times\frac{u}{m}\exp\left(-\frac{u^2}{2m}\right)d\varphi\,du \end{aligned} \tag{17}$$

Now, using definitions:

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$$\underline{e} = \begin{bmatrix} e_{M+1} & \dots & e_N \end{bmatrix}^T, \qquad e_n = y_{0,n} - \sum_{r=1}^{M} a''_r y_{0,n-r} \tag{18}$$

$$\underline{\psi} = \begin{bmatrix} \psi_{M+1} & \dots & \psi_N \end{bmatrix}^T, \qquad \psi_n = s_n - \sum_{r=1}^{M} a''_r s_{n-r} \tag{19}$$

$$\underline{\psi} = \psi_{M+1}\underline{\theta}, \qquad \underline{\theta} = \begin{bmatrix} 1 & e^{j\Omega} & \dots & e^{j(N-M-1)\Omega} \end{bmatrix}^T \tag{20}$$

We can write:

$$\sum_{n=M+1}^{N} \left| y_{0,n} - ue^{j\varphi} s_n - \sum_{r=1}^{M} a''_r \left(y_{0,n-r} - ue^{j\varphi} s_{n-r} \right) \right|^2 = \left| \underline{e} - ue^{j\varphi} \underline{\psi} \right|^2 \tag{21}$$

and so the term B can be rewritten as:

$$B = \frac{1}{2\pi m} \exp\left(-\frac{|\underline{e}|^2}{\sigma_1^2} \right) \times C \tag{22}$$

$$C = \int_u u \exp\left[-\frac{1}{\sigma_1^2}\left(u^2 |\underline{\psi}|^2 \right) - \frac{u^2}{2m} \right] \times D du \tag{23}$$

$$D = \int_\varphi \exp\left[\frac{1}{\sigma_1^2}\left(ue^{j\varphi} \underline{e}^H \underline{\psi} + ue^{-j\varphi} \underline{\psi}^H \underline{e} \right) \right] d\varphi \tag{24}$$

By the following definition:

$$t = \underline{e}^H \underline{\psi} \quad \rightarrow \quad t^* = \underline{\psi}^H \underline{e} \tag{25}$$

The integral argument of D can be simplified as:

$$te^{j\varphi} + t^* e^{-j\varphi} = 2\,\mathrm{Re}\left\{ te^{j\varphi} \right\} = 2|t|\cos(\varphi + \angle t) \tag{26}$$

and so the term D becomes:

$$D = \int_0^{2\pi} \exp\left[\frac{2u}{\sigma_1^2} |t| \cos(\varphi + \angle t) \right] d\varphi \tag{27}$$

Considering the modified Bessel function of the first kind:

$$I_0(\eta) = \frac{1}{2\pi} \int_0^{2\pi} e^{\eta \cos(\theta - \phi)} d\theta = \frac{1}{\pi} \int_0^{\pi} e^{\eta \cos\theta} d\theta \tag{28}$$

D can be written as:

$$D = 2\pi I_0\left(\frac{2u}{\sigma_1^2} \left| \underline{e}^H \underline{\psi} \right| \right) \tag{29}$$

Thus:

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$$B = \frac{1}{m}\exp\left(-\frac{|\underline{e}|^2}{\sigma_1^2}\right)\times G \tag{30}$$

$$G = \int_u u I_0(\alpha u)\exp(\beta u^2)du \tag{31}$$

$$\alpha = \frac{2\left|\underline{e}^H \underline{\psi}\right|}{\sigma_1^2} \tag{32}$$

$$\beta = -\left(\frac{|\underline{\psi}|^2}{\sigma_1^2} + \frac{1}{2m}\right) \tag{33}$$

By solving the G integral we have:

$$G = -\frac{1}{2\beta}\exp\left(-\frac{\alpha^2}{4\beta}\right) \tag{34}$$

And finally:

$$B = \frac{\sigma_1^2}{2m|\underline{\psi}|^2 + \sigma_1^2}\exp\left[\frac{1}{\sigma_1^2}\left(\frac{2m\left|\underline{e}^H \underline{\psi}\right|^2}{2m|\underline{\psi}|^2 + \sigma_1^2} - |\underline{e}|^2\right)\right] \tag{35}$$

Therefore:

$$L(\underline{y} \mid \underline{a}', \underline{a}'', \sigma_0^2, \sigma_1^2) = \frac{\left(\frac{1}{\pi\sigma_1^2}\right)^{N(L+1)} \prod_{k=0}^{L}\exp\left(-\frac{1}{\sigma_1^2}\sum_{n=M+1}^{N}\left|y_{k,n} - \sum_{r=1}^{M} a_r'' y_{k,n-r}\right|^2\right)}{\left(\frac{1}{\pi\sigma_0^2}\right)^{N(L+1)} \prod_{k=0}^{L}\exp\left(-\frac{1}{\sigma_0^2}\sum_{n=M+1}^{N}\left|y_{k,n} - \sum_{r=1}^{M} a_r' y_{k,n-r}\right|^2\right)} \times$$
$$\times \frac{\sigma_1^2}{2m|\underline{\psi}|^2 + \sigma_1^2}\exp\left[\frac{1}{\sigma_1^2}\left(\frac{2m\left|\underline{e}^H \underline{\psi}\right|^2}{2m|\underline{\psi}|^2 + \sigma_1^2}\right)\right] \tag{36}$$

Consequently, considering the definitions of Y and $\underline{P}$ matrices:

$$\prod_{k=0}^{L}\exp\left(-\frac{1}{\sigma_1^2}\sum_{n=M+1}^{N}\left|y_{k,n} - \sum_{r=1}^{M} a_r'' y_{k,n-r}\right|^2\right) = \exp\left[-\frac{1}{\sigma_1^2}(\underline{P} - Y\underline{a}'')^H(\underline{P} - Y\underline{a}'')\right] \tag{37}$$

$$\prod_{k=0}^{L}\exp\left(-\frac{1}{\sigma_0^2}\sum_{n=M+1}^{N}\left|y_{k,n} - \sum_{r=1}^{M} a_r' y_{k,n-r}\right|^2\right) = \exp\left[-\frac{1}{\sigma_0^2}(\underline{P} - Y\underline{a}')^H(\underline{P} - Y\underline{a}')\right] \tag{38}$$

Where:

$$\underline{a}' = [a_1' \quad \dots \quad a_M']^T \qquad , \quad \underline{a}'' = [a_1'' \quad \dots \quad a_M'']^T \tag{39}$$

$$\underline{e} = \underline{P}_0 - Y_0 \underline{a}'' \tag{40}$$

Therefore, the likelihood ratio will become as follows:

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$$L\left(\underline{y} \mid \underline{a}', \underline{a}'', \sigma_0^2, \sigma_1^2\right) = \frac{F_1}{F_0}$$
$$F_1 = \left(\frac{1}{\pi\sigma_1^2}\right)^{N(L+1)} \times \exp\left[-\frac{1}{\sigma_1^2}\left(\underline{P} - Y\underline{a}''\right)^H \left(\underline{P} - Y\underline{a}''\right)\right]$$
$$\times \frac{\sigma_1^2}{\sigma_1^2 + 2m\left|\underline{\psi}\right|^2} \times \exp\left[\frac{1}{\sigma_1^2}\left(\frac{2m\left|\left(\underline{P}_0 - Y_0\underline{a}''\right)^H \underline{\psi}\right|^2}{\sigma_1^2 + 2m\left|\underline{\psi}\right|^2}\right)\right] \tag{41}$$
$$F_0 = \left(\frac{1}{\pi\sigma_0^2}\right)^{N(L+1)} \times \exp\left[-\frac{1}{\sigma_0^2}\left(\underline{P} - Y\underline{a}'\right)^H \left(\underline{P} - Y\underline{a}'\right)\right]$$

Now, because the parameters m and σ_1^2 in the F_1 statement are unknown, and the distribution of these two parameters are also unknown, we apply the GLR test on the likelihood ratio. By some computations, we can obtain the ML estimation of these parameters as follows:

$$\hat{m}_{ML} = \frac{\left|\left(\underline{P}_0 - Y_0\underline{a}''\right)^H \underline{\psi}\right|^2 - \sigma_1^2\left|\underline{\psi}\right|^2}{2\left|\underline{\psi}\right|^4} \tag{42}$$

$$\hat{\sigma}_{1\,ML}^2 = \frac{1}{N(L+1)-1}\left[\left(\underline{P} - Y\underline{a}''\right)^H \left(\underline{P} - Y\underline{a}''\right) - \frac{\left|\left(\underline{P}_0 - Y_0\underline{a}''\right)^H \underline{\psi}\right|^2}{\left|\underline{\psi}\right|^2}\right] \tag{43}$$

Substituting Eqs. (42) and (43) in F_1 and applying the log operator, we obtain:

$$\max_{m,\sigma_1^2} \ln F_1 = -N(L+1)\ln\pi - \left[N(L+1)-1\right]\ln\hat{\sigma}_{1\,ML}^2 - \ln\frac{\left|\left(\underline{P}_0 - Y_0\underline{a}''\right)^H \underline{\psi}\right|^2}{\left|\underline{\psi}\right|^2} - N(L+1) \tag{44}$$

Also, with simplifying Eq. (43):

$$\hat{\sigma}_{1\,ML}^2 = \frac{1}{N(L+1)-1}\left(\underline{P}' - Y'\underline{a}''\right)^H \left(\underline{P}' - Y'\underline{a}''\right) \tag{45}$$

Where $\underline{P}'$ and Y' are:

$$Y = \begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_L \end{bmatrix}, \qquad Y_i = \begin{bmatrix} y_{i,M} & \cdots & y_{i,1} \\ \vdots & \ddots & \vdots \\ y_{i,N-1} & \cdots & y_{i,N-M} \end{bmatrix}_{(N-M)\times M}$$
$$\underline{P} = \left[\underline{P}_0^T \quad \underline{P}_1^T \quad . \quad . \quad . \quad \underline{P}_L^T\right]^T \qquad \underline{P}_i = \left[y_{i,M+1} \quad . \quad . \quad . \quad y_{i,N}\right]^T \tag{46}$$
$$Y' = \begin{bmatrix} Y_0' \\ Y_1 \\ \vdots \\ Y_L \end{bmatrix}, \qquad Y_0' = HY_0, \qquad \underline{P}' = \left[\underline{P'}_0^T \quad \underline{P}_1^T \quad . \quad . \quad . \quad \underline{P}_L^T\right]^T, \qquad \underline{P}'_0 = H\underline{P}_0$$
$$H = I - \frac{\underline{\theta}\,\underline{\theta}^H}{\underline{\theta}^H\underline{\theta}}, \qquad \underline{\theta} = \left[1 \quad e^{j\Omega} \quad \cdots \quad e^{j(N-M-1)\Omega}\right]^T$$

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Also, since the AR parameter vector, $\underline{a}''$ is unknown, if we substitute its ML estimation in Eq. (45), σ_1^2 becomes an estimation of its real value. Unfortunately, there is no closed form for F_1 to compute the ML estimation of the $\underline{a}''$ parameter vector. In other words, it is impossible to convert it to a least squares problem. Then, we used the covariance method for the estimation of parameter $\underline{a}''$. This method is not necessarily optimal, but is similar to what is used in ARGLR. We have:

$$\begin{aligned}\underline{\hat{a}}'' &= \left(Y'^H Y'\right)^{-1}\left(Y'^H \underline{P}'\right)\\ \hat{\sigma}_1^2 &= \frac{1}{N(L+1)-1}\left(\underline{P}' - Y'\underline{\hat{a}}''\right)^H\left(\underline{P}' - Y'\underline{\hat{a}}''\right)\end{aligned} \tag{47}$$

It has to be mentioned that the process has two steps. First, it projects the primary data in the perpendicular subspace of target signal space using the H transform. Then, in the second step it estimates the variance of the AR process. Substituting Eq. (47) in F_1 yields:

$$\ln F_1 = -N(L+1)\ln\pi - \left[N(L+1)-1\right]\ln\hat{\sigma}_1^2 - \ln\hat{\sigma}_t^2 - N(L+1) \tag{48}$$

$$\hat{\sigma}_t^2 = \frac{\left|\left(\underline{P}_0 - Y_0\underline{\hat{a}}''\right)^H \underline{\psi}\right|^2}{\left|\underline{\psi}\right|^2} = \left(\underline{P}''_0 - Y''_0\underline{\hat{a}}''\right)^H\left(\underline{P}''_0 - Y''_0\underline{\hat{a}}''\right) \tag{49}$$

$$\underline{P}''_0 = H'\underline{P}_0, \qquad Y''_0 = H'Y_0 \tag{50}$$

$$H' = I - H \tag{51}$$

Similar to the F_1, for term F_0 we have:

$$\begin{aligned}\hat{\sigma}_0^2 &= \frac{1}{N(L+1)}\left(\underline{P} - Y\underline{\hat{a}}'\right)^H\left(\underline{P} - Y\underline{\hat{a}}'\right)\\ \underline{\hat{a}}' &= \left(Y^H H\right)^{-1}\left(Y^H \underline{P}\right)\end{aligned} \tag{52}$$

Substituting Eq. (52) in F_0 yields:

$$\ln F_0 = -N(L+1)\ln\pi - N(L+1)\ln\hat{\sigma}_0^2 - N(L+1) \tag{53}$$

And finally, the decision rule will become:

$$\ln L(\underline{y}) = \ln F_1 - \ln F_0 = -\left[N(L+1)-1\right]\ln\hat{\sigma}_1^2 - \ln\hat{\sigma}_t^2 + N(L+1)\ln\hat{\sigma}_0^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{54}$$

Since we used the ALR and GLR tests to derive the structure of this detector, and considered the AR model for interference, we call it ARAGLR. Figure 1 shows the block diagram of the ARAGLR detector:

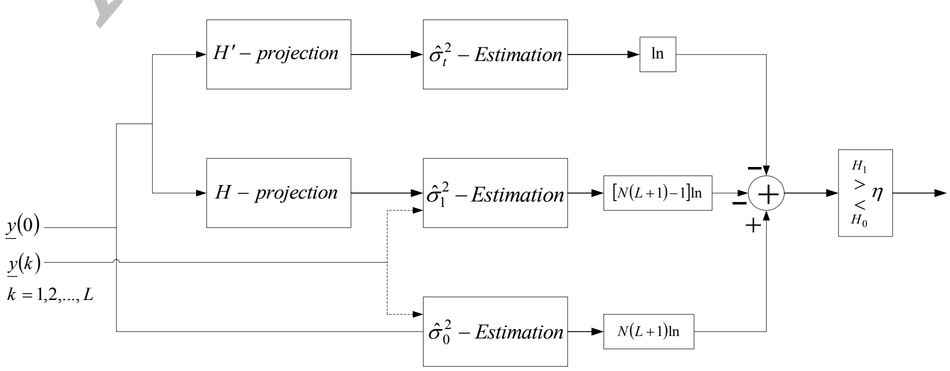

Fig. 1. Block diagram of ARAGLR detector

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Since this detector uses more information in comparison with the ARGLR, we expect the ARAGLR to have better performance than the ARGLR.

CASE 2: We assume b to be completely known. In this case, the amplitude and the phase of the target signal are constant and have known values. So the likelihood ratio is given by:

$$L\left(\underline{y} \mid \underline{a}', \underline{a}'', \sigma_0^2, \sigma_1^2\right) = \frac{f_Y[\underline{y}(0), \underline{y}(1), \ldots, \underline{y}(L) \mid H_1]}{f_Y[\underline{y}(0), \underline{y}(1), \ldots, \underline{y}(L) \mid H_0]} \tag{55}$$

$$f_Y[\underline{y}(0), \ldots, \underline{y}(L) \mid H_1] \cong \left(\frac{1}{\pi\sigma_1^2}\right)^{N(L+1)} \times \prod_{k=0}^{L} \exp\left(-\frac{1}{\sigma_1^2}\sum_{n=M+1}^{N}\left|x_{k,n} - \sum_{r=1}^{M} a_r'' x_{k,n-r}\right|^2\right) \tag{56}$$

$$x_{0,n} = y_{0,n} - b.s_n \quad n = 1,2,\ldots,N$$
$$x_{k,n} = y_{k,n} \quad k = 1,2,\ldots,L$$

$$f_Y[\underline{y}(0), \ldots, \underline{y}(L) \mid H_0] \cong \left(\frac{1}{\pi\sigma_0^2}\right)^{N(L+1)} \times \prod_{k=0}^{L} \exp\left(-\frac{1}{\sigma_0^2}\sum_{n=M+1}^{N}\left|y_{k,n} - \sum_{r=1}^{M} a_r' y_{k,n-r}\right|^2\right) \tag{57}$$

Since:

$$\sum_{n=M+1}^{N}\left|x_{0,n} - \sum_{r=1}^{M} a_r'' x_{0,n-r}\right|^2 = \left|\underline{e} - b\underline{\psi}\right|^2 = \left|\underline{e}\right|^2 + \left|b\underline{\psi}\right|^2 - b\underline{e}^H\underline{\psi} - b^*\underline{\psi}^H\underline{e} \tag{58}$$

Using Eqs. (5) to (20), we obtain:

$$\ln f_Y[\underline{y}(0), \ldots, \underline{y}(L) \mid H_1] = -N(L+1)\ln\pi - N(L+1)\ln\sigma_1^2 - \frac{1}{\sigma_1^2}\left(\underline{P} - Y\underline{a}''\right)^H\left(\underline{P} - Y\underline{a}''\right)$$
$$-\frac{1}{\sigma_1^2}\left(\left|b\underline{\psi}\right|^2 - b\underline{e}^H\underline{\psi} - b^*\underline{\psi}^H\underline{e}\right) \tag{59}$$

And so:

$$\ln f_Y[\underline{y}(0), \ldots, \underline{y}(L) \mid H_0] = -N(L+1)\ln\pi - N(L+1)\ln\sigma_0^2 - \frac{1}{\sigma_0^2}\left(\underline{P} - Y\underline{a}'\right)^H\left(\underline{P} - Y\underline{a}'\right) \tag{60}$$

Since parameters σ_0^2 and σ_1^2 are unknown, we should now compute the ML estimations of these parameters. Therefore, we obtain:

$$\hat{\sigma}_{1\,ML}^2 = \frac{1}{N(L+1)}\begin{bmatrix}\left(\underline{P} - Y\underline{a}''\right)^H\left(\underline{P} - Y\underline{a}''\right) \\ + \left|b\underline{\psi}\right|^2 - b^*\underline{\psi}^H\underline{e} - b\underline{e}^H\underline{\psi}\end{bmatrix} \tag{61}$$

Substituting $\hat{\sigma}_{1\,ML}^2$ in $f_Y[\cdots \mid H_1]$:

$$\ln f_Y[\underline{y}(0), \ldots, \underline{y}(L) \mid H_1] = -N(L+1)\ln\pi - N(L+1)\ln\hat{\sigma}_{1\,ML}^2 - N(L+1) \tag{62}$$

And also:

$$\hat{\sigma}_{0\,ML}^2 = \frac{1}{N(L+1)}\left(\underline{P} - Y\underline{a}'\right)^H\left(\underline{P} - Y\underline{a}'\right) \tag{63}$$

$$\ln f_Y[\underline{y}(0), \ldots, \underline{y}(L) \mid H_0] = -N(L+1)\ln\pi - N(L+1)\ln\hat{\sigma}_{0\,ML}^2 - N(L+1) \tag{64}$$

And so ML estimations of $\underline{a}'$ and $\underline{a}''$ are:

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$$\hat{\sigma}_{1\,ML}^{2}=\frac{1}{N(L+1)}\left(\underline{P'}-Y'\underline{a''}\right)^{H}\left(\underline{P'}-Y'\underline{a''}\right) \tag{65}$$

$$\underline{P'}=\left[\underline{P'}_{0}^{T}\quad \underline{P}_{1}^{T}\quad \ldots \quad \underline{P}_{L}^{T}\right]^{T} \tag{66}$$

$$\underline{P'}_{0}=\underline{P}_{0}-b\underline{P}_{t}\,\underline{P}_{t}=s_{M+1}\underline{\theta} \tag{67}$$

$$Y'=\begin{bmatrix} Y_0' \\ Y_1 \\ \vdots \\ Y_L \end{bmatrix} \tag{68}$$

$$Y_0'=Y_0+Y_t,\quad Y_t=\underline{\theta}\times\left[s_M\quad \ldots \quad s_1\right] \tag{69}$$

Consequently, the following statements will be obtained:

$$\hat{\sigma}_{1}^{2}=\frac{1}{N(L+1)}\left(\underline{P'}-Y'\underline{\hat{a}''}\right)^{H}\left(\underline{P'}-Y'\underline{\hat{a}''}\right) \tag{70}$$

$$\underline{\hat{a}''}=\left(Y'^{H}Y'\right)^{-1}\left(Y'^{H}\underline{P'}\right) \tag{71}$$

And so:

$$\hat{\sigma}_{0}^{2}=\frac{1}{N(L+1)}\left(\underline{P}-Y\underline{\hat{a}'}\right)^{H}\left(\underline{P}-Y\underline{\hat{a}'}\right) \tag{72}$$

$$\underline{\hat{a}'}=\left(Y^{H}Y\right)^{-1}\left(Y^{H}\underline{P}\right) \tag{73}$$

Finally, the decision rule is:

$$\begin{aligned}\ln L(\underline{y}) &= \ln f_Y[\underline{y}(0),\ldots,\underline{y}(L)\mid H_1]-\ln f_Y[\underline{y}(0),\ldots,\underline{y}(L)\mid H_0] \\ &= -N(L+1)\ln\hat{\sigma}_1^2+N(L+1)\ln\hat{\sigma}_0^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta\end{aligned} \tag{74}$$

Since this detector has been derived under the special conditions in which target amplitude is known, it is point optimum, hence we call it ARGLRPOD (ARGLR Point Optimum Detector). Since this detector uses the most information of the target amplitude, we use its performance as an upper bound for the achievable performance by target amplitude modeling.

4. PERFORMANCE EVALUATIONS

The behaviors of ARGLR and ARAGLR detectors have been simulated for comparison. The Monte-Carlo method was used for this purpose. In this procedure, we produce AR interference of order $M=4$ with parameters:

$$\underline{a}=[0.3-j0.7-0.4-j0.3-0.2+j0.1\quad -0.1-j0.2]^{T}$$

Since these simulations are performed for specific Signal to Noise Ratio (SNR), the SNR is defined as:

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$$SNR = \underline{S}_c^H R_N^{-1} \underline{S}_c \tag{75}$$

Where $\underline{S}_c$ is the signal vector of the target and R_N is the covariance matrix of interference (Note that "noise" used in SNR refers to the interference, which is the clutter with the AR model here). Based on the [30], for the AR process, R_N^{-1} can be calculated from:

$$R_N^{-1} = \frac{1}{\sigma_e^2}\left(AA^H - BB^H\right)$$

$$A = \begin{bmatrix} 1 & 0 & & \cdots & \cdots & & 0 \\ a_1 & 1 & & & & & \\ \vdots & a_1 & & \ddots & & & \vdots \\ a_M & \vdots & & \ddots & \ddots & & \vdots \\ 0 & a_M & & \ddots & \ddots & & \\ \vdots & \ddots & \ddots & & & & 0 \\ 0 & \cdots & 0 & a_M & \cdots & a_1 & 1 \end{bmatrix}_{N \times N}$$

$$B = \begin{bmatrix} 0 & & \cdots & \cdots & & 0 \\ \vdots & & & & & \\ 0 & & & & & \vdots \\ a_M & \ddots & & & & \vdots \\ \vdots & \ddots & \ddots & & & \\ a_1 & \cdots & a_M & 0 & \cdots & 0 \end{bmatrix}_{N \times N} \tag{76}$$

Where σ_e^2 is the variance of the input white Gaussian process to the AR filter. Thus, the SNR will be:

$$\underline{S}_c = b\underline{s} \rightarrow SNR = E\left\{\underline{S}_c^H R_N^{-1} \underline{S}_c\right\} = E\left(b^2\right)\underline{s}^H R_N^{-1} \underline{s} \tag{77}$$

For Rayleigh distribution:

$$SNR = 2m\underline{s}^H R_N^{-1} \underline{s} \tag{78}$$

When the amplitude of the target signal is known, (77) becomes:

$$SNR = |b|^2 \underline{s}^H R_N^{-1} \underline{s} \tag{79}$$

It is clear that the target signal phase has a uniform distribution in the interval $[0,2\pi)$.

Based on the simulation results, Figs. 2 to 4 have been obtained. Figure 2 shows the performance of ARAGLR and ARGLR against SNR at $P_{fa} = 0.001$. Figures 3 and 4 show the ROC of ARAGLR and ARGLR for N=10, L=15, and $\Omega = 4(rad/s)$ at $SNR = -10dB$ and $SNR = -15dB$, respectively. As the figures show, the performance of ARAGLR in low SNR and the low probability of a false alarm is better than ARGLR. Since the SNR and probability of a false alarm are relatively low in the practical applications, this is a very desirable result. Besides, the performance of both detectors depends on the assumed value for the Doppler shift (Ω) compared with the clutter spectrum. For instance, if in the above situations Ω is changed to π (rad/s), the superiority of the ARAGLR will be more apparent for higher SNR. Figure 5 shows the ROC of ARAGLR and ARGLR in these conditions at $SNR = 10dB$. The superiority of the ARAGLR compared with ARGLR has been clearly illustrated in this figure.

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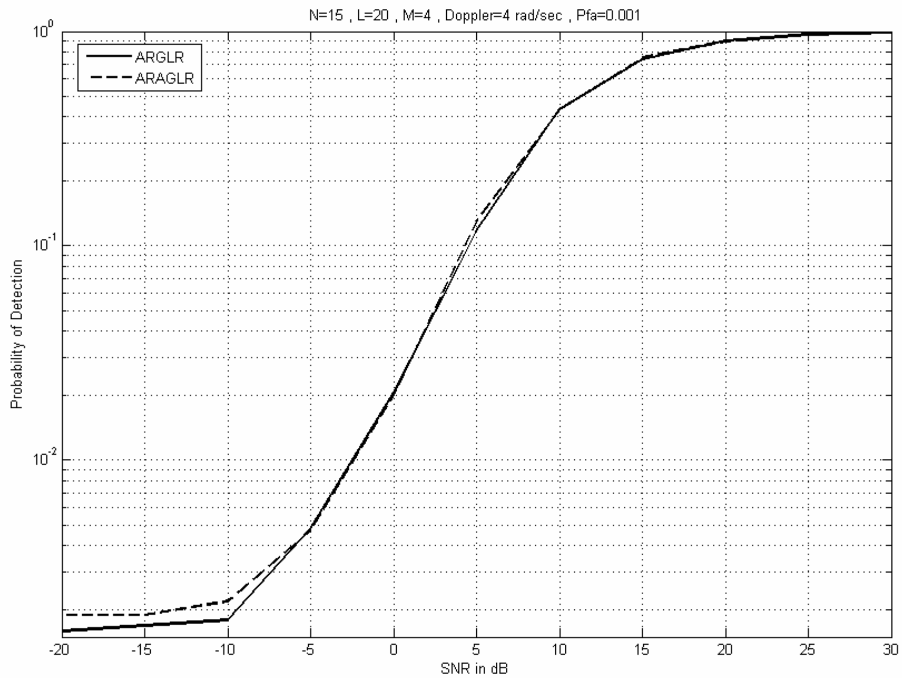

Fig. 2. P_d against SNR for ARAGLR and ARGLR

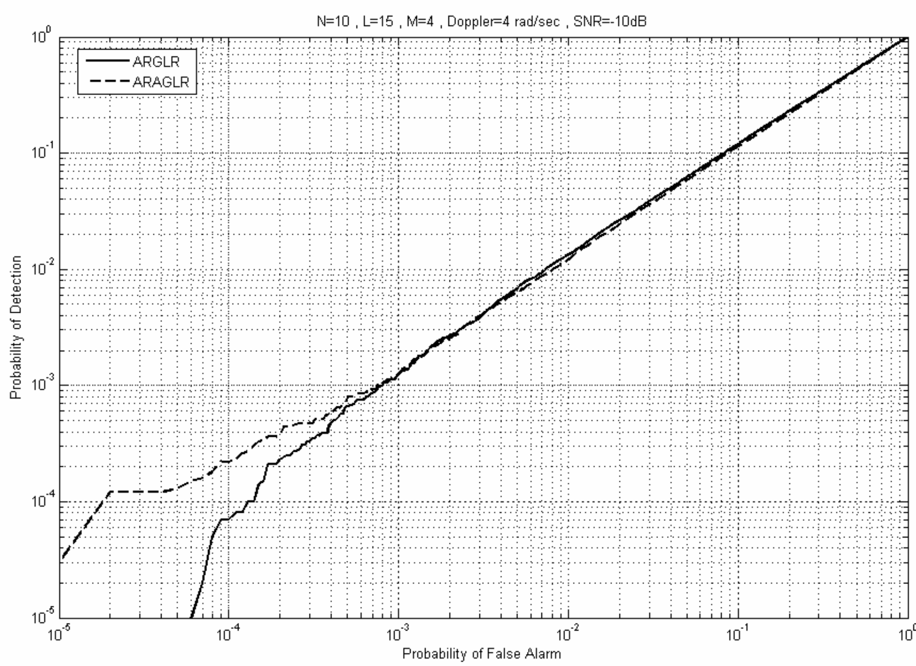

Fig. 3. ROC for ARAGLR and ARGLR ($\Omega = 4\, rad\ / s$, SNR=-10dB)

As we know, one of the most important features of any detector is its CFAR property. In other words, to compare the two detectors, we should compare their CFAR property in addition to their ROC's. It is proven that ARGLR has CFAR property for enough large pulse numbers [25]. So, because of the

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similarity of the deriving method for ARAGLR and ARGLR, it can be concluded that ARAGLR has a similar CFAR property. But, for more confidence, its CFAR property has been investigated and Fig. 6 is obtained. This figure shows P_{fa} against noise power for the fixed threshold as well as some various pulse numbers. As is seen, the ARAGLR detector has a very good CFAR property, even for the small values of pulse numbers.

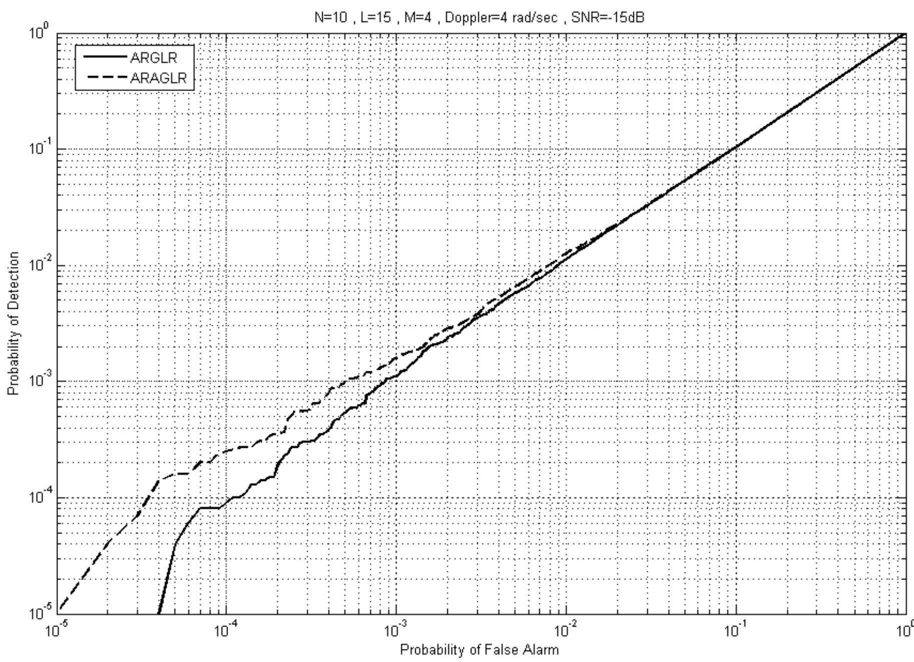

Fig. 4. ROC for ARAGLR and ARGLR ($\Omega = 4\, rad / s$, SNR=-15dB)

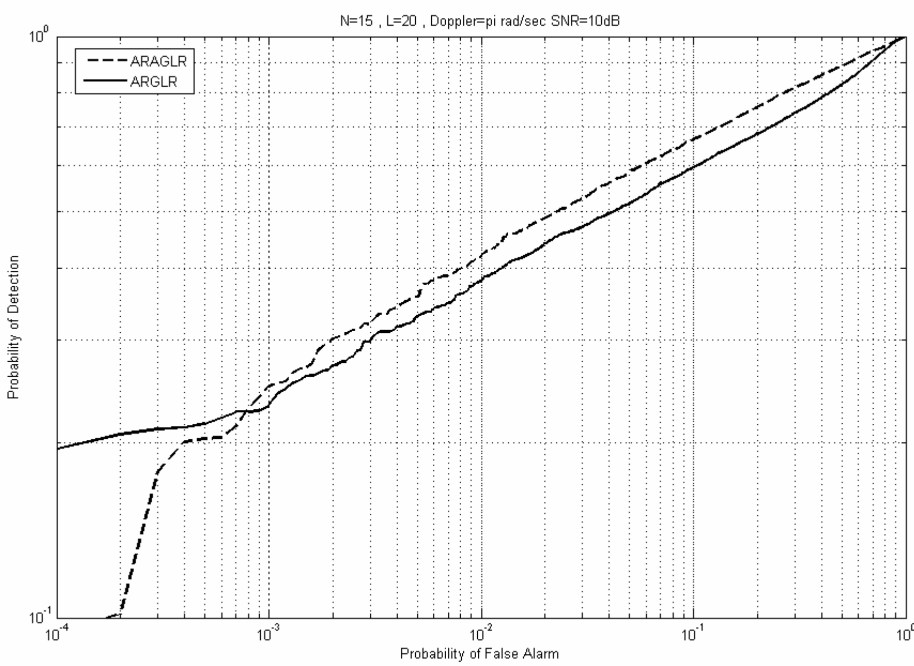

Fig. 5. ROC for ARAGLR and ARGLR ($\Omega = \pi\, rad / s$, SNR=10dB)

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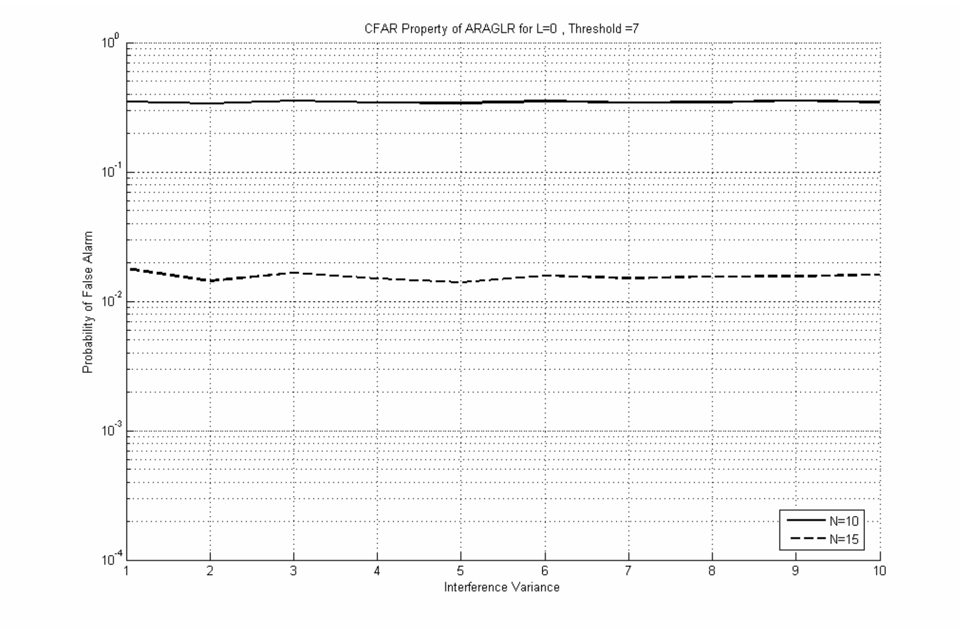

Fig. 6. P_{fa} against interference variance (CFAR property for ARAGLR)

Performance comparison of ARGLR and ARGLRPOD detectors against SNR at $P_{fa} = 0.01$ and $P_{fa} = 0.001$, have been shown in Figs. 7 and 8 respectively. As it is seen, there is 3-5dB improvement in the performance of the ARGLRPOD in comparison with ARGLR. Figure 9 shows the performance of ARGLRPOD in different amplitudes such that the dash-dotted line and solid line have maximum and minimum amplitudes in these simulations, respectively. Notice that in Fig. 9, the solid line shows the performance in the case that the real amplitude of the target signal matches that of the assumed amplitude. Also, the dashed line and dash-dotted line show the performance in the cases that real amplitude is three times and five times that of the assumed amplitude, respectively. As is seen, when the real amplitude increases, the performance of ARGLRPOD is improved. Therefore, we can design our interested detector for the smallest expected amplitude and be sure that we will not have inferior performance in all real cases.

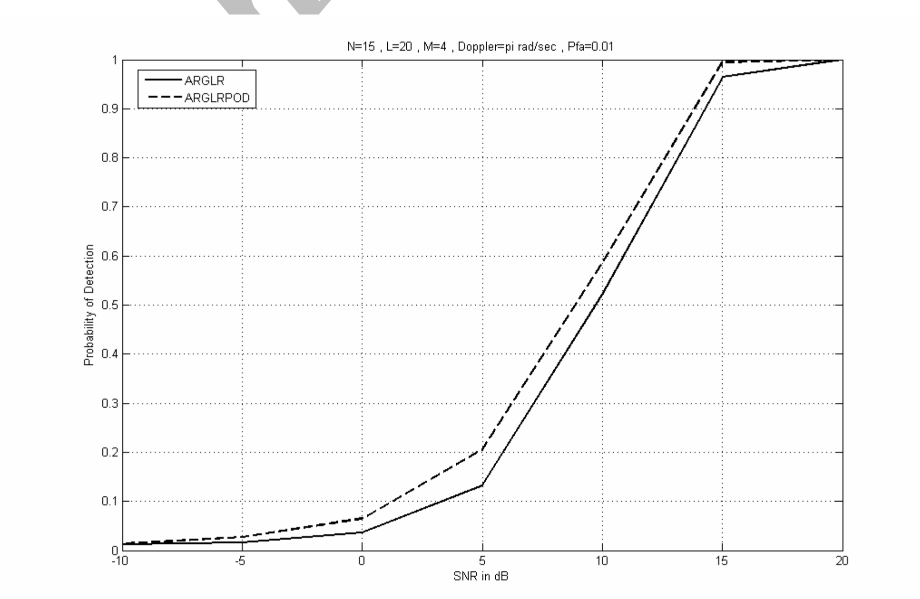

Fig. 7. P_d against SNR for ARGLRPOD and ARGLR (P_{fa}=0.01)

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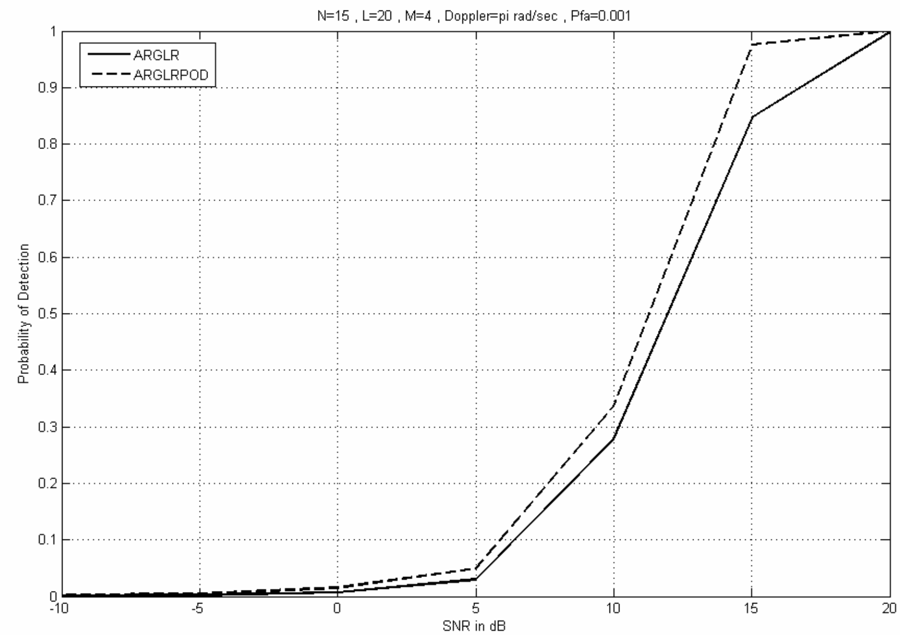

Fig. 8. P_d against SNR for ARGLRPOD and ARGLR (P_{fa}=0.001)

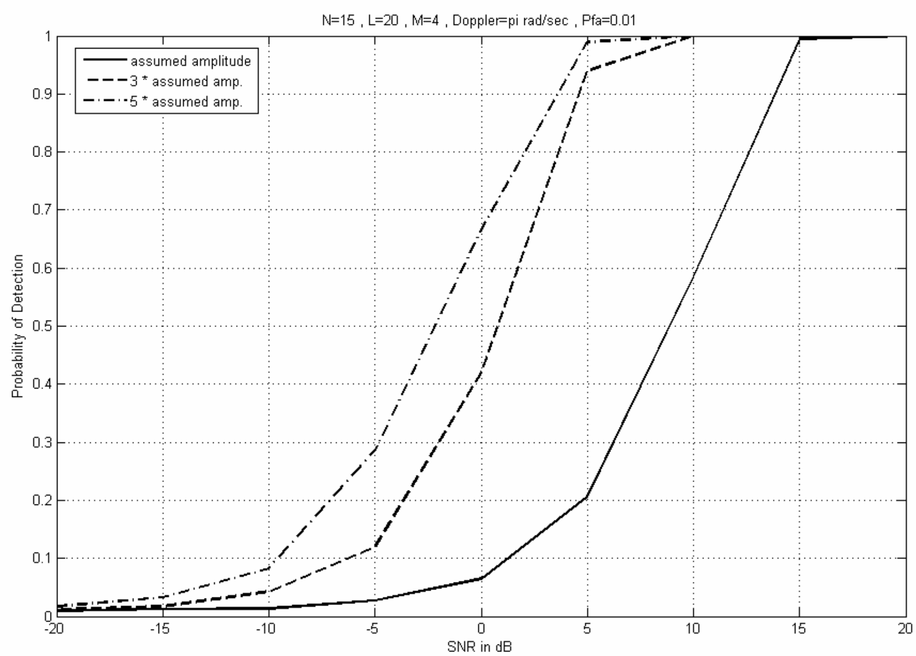

Fig. 9. P_d against SNR for ARGLRPOD and non-matched amplitude

5. SUMMARY AND CONCLUSIONS

In this paper we have designed ARAGLR and ARGLRPOD detectors in which the former uses some assumptions closer to the practical cases and real conditions for fluctuating targets, while the latter can be used for conservative designs. Simulation results show that the ARAGLR detector, in which the target amplitude has been assumed to have Rayleigh distribution, has better performance than the ARGLR,

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especially for low SNR and low probability of false alarms. Moreover, this new detector has CFAR property, even for small values of pulse numbers.

Also, for the ARGLRPOD detector in which the target amplitude is known, we see that its performance is improved 3-5dB compared with the ARGLR. These results show the importance of the ARAGLR detector (much consistency with the real conditions) and also the ARGLRPOD detector (design in the worst case).

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