

## A NEW APPROACH FOR BIDDING STRATEGY OF GENCOS USING PARTICLE SWARM OPTIMIZATION COMBINED WITH SIMULATED ANNEALING METHOD<sup>\*</sup>

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**Abstract**– This paper describes a procedure that uses particle swarm optimization (PSO) combined with the simulated annealing (SA) to analyze the bidding strategy of Generating Companies (Gencos) in an electricity market where they have incomplete information about their opponents.

In the proposed methodology, Gencos prepare their strategic bids according to the Supply Function Equilibrium (SFE) model and they change their bidding strategies until Nash equilibrium points are obtained. Nash equilibrium points constitute a central solution concept in the game theory and are computed with solving a global optimization problem. In this paper a new computational intelligence technique is introduced that can be used to solve the Nash optimization problem. This new procedure, namely PSO-SA is based on the PSO algorithm and SA method. SA method is used to avoid becoming trapped in local minima or maxima and improve the velocity's function of particles. The performance of the PSO-SA procedure is compared with the results of other computational intelligence techniques such as PSO, Genetic Algorithm (GA), and a mathematical method (GAMS/DICOPT).

**Keywords**– Energy market, deregulation, Nash equilibrium point, optimal bidding strategy, particle swarm, simulated annealing

### 1. INTRODUCTION

Recent changes in the electricity industry in several countries have led to a less regulated and more competitive energy market. In this condition, cost is replaced with the price and each Genco will try to maximize its own profit. For a Genco, it is critical to devise a good bidding strategy according to its opponents' bidding behavior, the model of demand and power system operating conditions. So Gencos should solve a game theory problem. Game theory is the study of multi-person or multi-firm decision-making problems. The most commonly encountered solution concept in the game theory is that of the Nash equilibrium [1]. Therefore the rational Gencos should bid at their Nash equilibrium strategies to obtain their optimum profit.

In [2], [3], and [4], a cooperative game was used to analyze the possible coalitions and collusions of participants in electricity markets. A non-cooperative incomplete game was employed in [2], [5] and [6] to choose a Genco's optimal bidding strategy among the sets of discrete bids. The bidding problem was modeled as a bi-level problem in [7] by assuming the complete information on a Genco's opponents. The Independent System Operator (ISO)'s market clearing problem was modeled as a non linear optimal power flow (OPF) problem and the Newton approach was employed to solve it. [8] describes a procedure

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that uses PSO combined with Lagrangian Relaxation framework to solve a power-generator scheduling problem known as the unit commitment problem.

This paper extends the proposed method in [7] for developing a more general approach to Genco's optimal bidding strategies with incomplete information in electricity markets. The problem of computing Nash equilibrium points can be formulated as a global optimization problem. This formulation allows us to consider computational intelligent techniques to detect Nash equilibria.

PSO is a stochastic optimization method capable of handling non differentiable, nonlinear, and multi module objective functions. The PSO approach is motivated from the social behavior of bird flocking and fish schooling. PSO has a population of individuals that move through the D-dimensional search space and each individual has a velocity that acts as an operator to obtain a new set of individuals. Individuals, called particles, adjust their movements depending on both their own experience and the population's experience. At each iteration, a particle moves towards a direction computed from the best visited position and the best visited position of all particles in its neighborhood. In this approach, except the particle that is the best experience of particles, the effect of other particles is ignored. So the probability of becoming trapped in the local points is increased. In this paper, to avoid this problem, the PSO algorithm is combined with the SA approach. SA employs a search which not only accept changes that decrease the objective function (assuming a minimization problem), but also some changes that increase it are accepted with a probability. The remaining sections of this paper are organized as follows:

The problem formulation is given in section 2. Sections 3, 4, 5, include a brief review of the PSO, SA algorithm, and game theory concepts. Section 6 describes a new algorithm named PSO-SA, and section 7 shows the solution method. Section 8 gives an illustrative example. Section 9 provides the conclusions.

## 2. PROBLEM FORMULATION

### a) Estimating opponent's unknown information

It is necessary for a Genco to model its opponents' unknown information. It is supposed that all Gencos own only the thermal units, so the most important parameters for Gencos are  $a$ ,  $b$  and  $c$  coefficients of second order generating cost function as  $aP^2 + bP + c$ , where  $P$  is the active power output of a generating unit. All Gencos try to hide this information from the others, so the opponents should estimate them based on the available information.

The available information of Gencos about their opponents is incomplete and it is supposed that they are only aware of the minimum and maximum generation levels of their opponents as well as their fuels type.

Reference [8] has presented a method to obtain the fuel cost of a generator as a quadratic function of active power generation. This function is expressed as:

$$F(P) = \alpha P^2 + \beta P + \gamma \quad (1)$$

In this function  $F(P)$  is measured in MJ/h or MBtu/h. By considering the High Heat Value (HHV) of fuels and the fuels price (in \$/m<sup>3</sup> or \$/lit),  $F(P)$  is obtained in \$. So the fuel price should be forecasted for a future time to obtain the fuel cost. We can define several scenarios with definite probability for the fuel price, so the different types for  $\alpha$ ,  $\beta$  and  $\gamma$  coefficients will result.

The total cost of operation includes the fuel cost, the cost of labor, supplies and maintenance. These costs, except for the fuel cost, are expressed as a fixed percentage of the fuel cost. So the total generation cost can be expressed by  $aP^2 + bP + c$  where  $a$ ,  $b$ , and  $c$  include  $\alpha$ ,  $\beta$ , and  $\gamma$  plus some percentage due to the cost of labor, maintenance and supplies.

### b) Genco's bids in energy market

In a power market, Gencos may prepare their strategic bids according to the 4 economic models in imperfect competition. These models are Bertrand, Cournot, Stackelberg and SFE where the Stackelberg model is similar to the Cournot model [1]. Fig.1 illustrates where the intensity of the competition predicted by the basic formulation of each of the models places them along the competitive spectrum.

In the Bertrand model, Gencos compete against each other using prices as strategy choices and Gencos bid at their marginal cost at the Nash equilibrium point.

In the classic model of Cournot, Gencos compete against each other using quantities as strategy choices. Genco's products are assumed to be homogenous, Demand is price-responsive, and Market Clearing Price (MCP) is the intersection of aggregated supply and market demand curves. The Stackelberg model is similar to the Cournot model. However, the competitors do not offer their output quantities simultaneously. The so-called "leader" will make the first move, which is followed by that of the followers who take into account the leader's action [1].

In the SFE model, Gencos compete with each other through the simultaneous choice of supply functions. Klemperer and Meyer developed SFE in order to model competition in the presence of demand uncertainty. The SFE model was used by Green and Newbery for analyzing the competitive strategic bidding in electricity markets [1].

Among these models, only SFE enables a Genco to link its bidding price with the bidding quantity of its product and only this model is the closest to the actual behavior of players in the actual power market. So in this paper, we suppose that Gencos use the SFE model to prepare the strategic bids of their units and the unit's bid function is expressed as follows:

$$\rho_j = \mu_j \cdot p_j + MC_j \quad (2)$$

where:

$\rho_j$  : The offered price of unit  $j$

$p_j$  : The quantity corresponding to  $\rho_j$

$\mu_j$  : Mark-up coefficient of unit  $j$

$MC_j$  : Marginal cost of unit  $j$

The generating cost function is a function of active power generation and is expressed as:

$$C_j = C(p_j) = a_j \cdot p_j^2 + b_j \cdot p_j + c_j \quad (j = 1, 2, \dots, n_i) \quad (3)$$

where  $a_j$ ,  $b_j$ , and  $c_j$  are the coefficients of the generating cost function.

So the marginal cost of each unit is expressed as:

$$MC_j = 2 a_j p_j + b_j \quad (4)$$

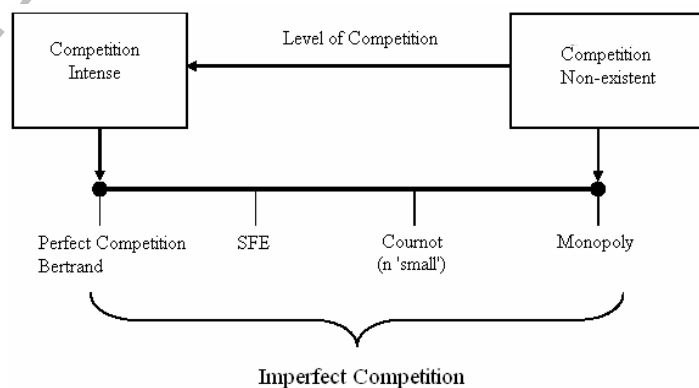


Fig. 1. Equilibrium models and predicted degree of competition [1]

The real power markets in the world are not characterized by perfect competition, but are rather an oligopoly market, i.e. a market in which there is not only one player (a monopoly) and not an infinite amount of players (perfect competition). In an oligopoly market, if all Gencos bid equal to their marginal cost, the market power will not produce. So the exercise of market power by each price maker player is used as an opportunity to add a mark-up to the player's own resultant supply function. A player will behave as a price taker in the market if the mark-up coefficient is 0.

### c) Market clearing model

We suppose the ISO uses a security-constrained economic dispatch to clear the market after collecting bids. The ISO minimizes consumer payments subject to the bids and line flow constraints. Accordingly, locational marginal prices (LMPs) are calculated as follows [6]:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n_G} \sum_{j=1}^{n_i} p_{ij} (\mu_{ij} p_{ij} + MC_{ij}) \\ \text{subject to:} \quad & B\theta = P_G - P_D \\ & F_l^{\min} \leq F_l \leq F_l^{\max} \quad (l=1,2,\dots,L) \\ & p_{ij}^{\min} \leq p_{ij} \leq p_{ij}^{\max} \end{aligned} \quad (5)$$

where:

$n_G$  : Number of Gencos

$n_i$  : Number of units of each Genco

$B$  : Network susceptance matrix

$\theta$  : Vector of bus voltage angles

$P_G$  : Vector of bus generation

$P_D$  : Constant vector of bus loads

$F_l$  : Power flow on line  $l$

$F_l^{\max}, F_l^{\min}$  : Upper and lower flow limits on line  $l$

$L$  : Number of lines in the system

$p_{ij}^{\min}, p_{ij}^{\max}$  : Upper and lower bounds of unit  $j$  of Genco  $i$

In (5), the first equality constraint is the DC power flow equation, the second constraint is the transmission line constraint, and the third is the generation capacity constraint for each unit. The LMP at each bus is the lagrangian multiplier of the corresponding power flow constraint.

Once the energy market is cleared, each unit will be paid according to its LMP times its awarded generation. So the payoff for Genco  $i$  is calculated as follows:

$$R_i = \sum_{j=1}^{n_i} (LMP_j) \cdot p_j - \sum_{j=1}^{n_i} (a_j p_j^2 + b_j p_j + c_j) \quad (6)$$

## 3. PARTICLE SWARM OPTIMIZATION (PSO) METHOD

PSO is a computation technique introduced by Kennedy and Eberhart in 1995, which was inspired by the social behavior of bird flocking or fish schooling (Reynolds, 1987). They theorize that the process of cultural adaptation can be summarized in terms of three principles: evaluate, compare, and imitate. An organism, a bird in PSO, evaluates its neighbors, compares itself to others in the population and then

imitates only those neighbors who are superior. So they behave with two kinds of information: their own experience and the knowledge of how the other individuals around them have performed. [9]

The PSO approach has some similarities to GA and evolutionary algorithms. PSO has a population of individuals that move through the D- dimensional search space and each individual has a velocity that acts as an operator to obtain a new set of individuals. Individuals, called particles, adjust their movements depending on both their own experience and the population's experience. At each iteration, a particle moves towards a direction computed from the best visited position and the best visited position of all particles in its neighborhood. [8,10,11,12]

In PSO, the p-th particle is represented as  $X_p = \{x_{p1}, x_{p2}, \dots, x_{pD}\}$ , where  $x_{pj}$  is the value of the j-th coordinate in the D dimensional space. The best experience of all particles is represented by the symbol  $G = \{g_1, g_2, \dots, g_D\}$  and the best visited position of the p-th particle is represented as  $P_p = \{p_{p1}, p_{p2}, \dots, p_{pD}\}$ . The rate of the position change, which is the velocity for particle p, is represented as  $V_p = \{v_{p1}, v_{p2}, \dots, v_{pD}\}$ .

The position of a particle changes according to its velocity, which is adjusted at each iteration. Particle p is repositioned according to the d- coordinate of its velocity, which is calculated as follows:

$$v_{pd}^{itr+1} = \omega v_{pd}^{itr} + c_1 rand(0,1)(p_{pd} - x_{pd}) + c_2 rand(0,1)(g_d - x_{pd}) \quad (7)$$

The factor  $\omega$  is the inertia weight and is similar to the effect of temperature in simulated annealing. If the inertia weight is large, the search becomes more general. The coefficients of  $c_1$  and  $c_2$  are learning factors, which help particles to accelerate towards better areas of the solution space. The function  $rand(0,1)$  generate a random number uniformly between 0 and 1.

The velocity of each dimension has upper and lower limits,  $V_{max}$  and  $V_{min}$ , which are defined by the user. The new position of a particle is updated as follows:

$$x_{pd}^{itr+1} = x_{pd}^{itr} + v_{pd}^{itr+1} \quad (8)$$

The PSO algorithm can start with a population of particles with random positions or with a population of particles created heuristically and can stop when its iteration reaches  $itr_{max}$ , which is the maximum number of iterations defined by the user.

#### 4. FRAMEWORK OF SIMULATED ANNEALING (SA)

Simulated annealing is motivated by an analogy to annealing in solids. The idea of SA comes from a paper published by Metropolis *et al.* in 1953. If we heat a solid past the melting point and then cool it, the structural properties of the solid depend on the rate of cooling. If the liquid is cooled slowly enough, large crystals will be formed. However, if the liquid is cooled quickly (quenched) the crystals will contain imperfections. Metropolis's algorithm simulated the material as a system of particles. The algorithm simulates the cooling process by gradually lowering the temperature of the system until it converges to a steady, frozen state [13].

SA's major advantage over other methods is an ability to avoid becoming trapped in local minima (assuming a minimization problem). The algorithm employs a random search which not only accepts changes that decrease the objective function  $f$ , but also some changes that increase it. The latter are accepted with a probability  $p = \exp(-\delta f / T)$  where  $\delta f$  is the increase in  $f$  and  $T$  is a control parameter, by which the analogy with the original application is known as the system "temperature" irrespective of the objective function involved. It can be appreciated that as the temperature of the system decreases the probability of accepting a worse move is decreased. This is the same as gradually moving to a frozen state

in physical annealing. The implementation of the basic SA algorithm is straightforward. Figure 2 shows its structure. As shown in this figure, the users should estimate suitable values for the initial temperature, and final temperature (Terminate search). Further, they should use appropriate rules for decreasing temperature [13].

A suitable initial temperature  $T_0$  is one that results in an average increase of acceptance probability  $p_0$  of about 0.8. In other words, there is an 80% chance that a change which increases the objective function will be accepted. The value of  $T_0$  will clearly depend on the scaling of  $f$  and, hence, be problem-specific. It can be estimated by conducting an initial search in which all increases are accepted and calculating the average objective increase of observed  $\delta f^+$ .  $T_0$  is then given by:  $T_0 = -\delta f^+ / \ln(p_0)$

The best final temperature for terminating the search is to let the temperature decrease until it reaches zero. However, this can make the algorithm run for a lot longer. In practice, it is not necessary to let the temperature reach zero because as it approaches zero the chances of accepting a worse move are almost the same as the temperature being equal to zero. Therefore, the stopping criteria can either be a suitably low temperature or when the system is "frozen" at the current temperature (i.e. no better or worse moves are being accepted).

Once we have our starting and stopping temperature we need to get from one to the other. That is, we need to decrement our temperature so that we eventually arrive at the stopping criterion. One way to decrement the temperature is a simple linear method. An alternative is a geometric decrement where  $t = t\alpha$  (where  $\alpha < 1$ ). Experience has shown that  $\alpha$  should be between 0.8 and 0.99, with better results being found in the higher end of the range. Of course the higher the value of  $\alpha$ , the longer it will take to decrement the temperature to the stopping criterion.

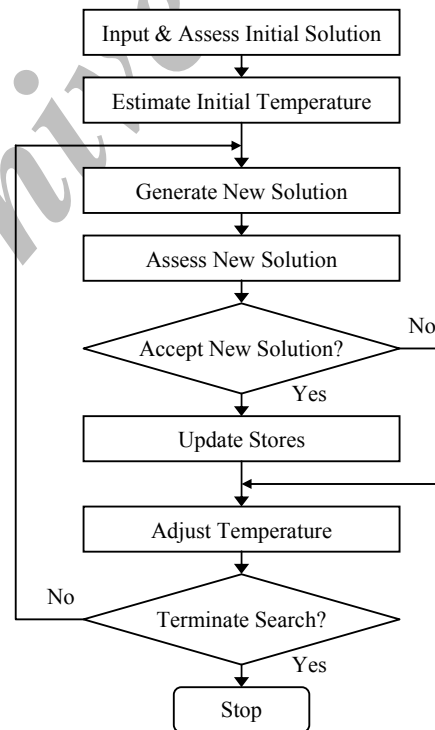


Fig. 2. The structure of the simulated annealing algorithm

## 5. BASIC CONCEPTS OF GAME THEORY AND NASH EQUILIBRIUM DEFINITION

Game theory is the study of multi-person or multi-firm decision-making problems. In the field of industrial organization in economics, the game theory is used extensively to study auction behavior, bargaining, principal-agent relationships, product differentiation, and strategic behavior by firms. There are three main mathematical models or forms used in the study of games, the strategic form, the extensive form, and the coalitional form [1]. These approaches differ in the amount of detail on the play of the game built into the model. The history of electricity markets shows that their behavior is near to the strategic form [1]. Therefore this section explains the basic concept of this form.

The strategic (or normal) form representation of a game includes three components:

- The set of players,  $i \in N, N = \{1, \dots, n\}$ , in the game, which is assumed finite;
- The pure strategy space,  $S_i, S_i = \{s_{i1}, \dots, s_{im_i}\}$  which contains the individual strategies available to player  $i$  ( $s_{ij}$ ), where  $s_{ij}$  is an arbitrary strategy; and
- The payoff function  $u_i : S \rightarrow R$  (real set) for each player  $i$  is also defined, where  $S = S_1 \times S_2 \times \dots \times S_n$  is the Cartesian set of all sets ( $S_i$ ).

In the game theory, the most commonly encountered solution concept is the Nash equilibrium. A strategy is a Nash equilibrium for a player if that player cannot increase its own payoff by undertaking any strategy other than its equilibrium strategy, given the strategy choice of its rivals. In a Nash equilibrium, each player will decrease its payoff if it deviates from its Nash equilibrium strategy, assuming all other players continue to play their existing strategies. As a result, a Nash equilibrium point is the "best response", in the sense that no player has an incentive to deviate from its strategy choice, given all other player's strategy choices. Definition 1 gives a formal definition of the Nash equilibrium.

**Definition 1:** In the  $n$  player strategic form game, the profile strategies  $(s_1^*, \dots, s_n^*)$  are a Nash equilibrium if, for each player  $i$ ,  $s_i^* \in S_i$  is player  $i$ 's best response to the strategies specified for the other  $(n-1)$  players,  $s_{-i}^* = (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_n^*)$ , such that  $u_i(s_i^*, s_{-i}^*) \geq u_i(s_{ij}, s_{-i}^*)$ , for every feasible strategy  $s_{ij} \in S_i$  [1].

As mentioned, the problem of finding a Nash equilibrium point can be formulated as a problem of detecting the global minimum of a real valued function. To this end, three functions,  $x$ ,  $z$ , and  $g$  are defined as follows [14]:

$$\begin{aligned} x_{ij}(s) &= u_i(s_{ij}, s_{-i}) \\ z_{ij}(s) &= x_{ij}(s) - u_i(s) \\ g_{ij}(s) &= \max(z_{ij}(s), 0) \end{aligned} \quad (10)$$

Now, we define the real valued function  $v$  by:  $v(s) = \sum_{i \in N} \sum_{1 \leq j \leq m_i} [g_{ij}(s)]^2$ . Function  $v$  is continuous, differentiable, and satisfies the inequality  $v(s) \geq 0$ . Furthermore,  $s^*$  is a Nash equilibrium, if and only if, it is a global minimum of  $v$ , i.e.  $v(s^*) = 0$ , (which means no player changes its strategy)

This formulation of finding Nash equilibrium points allows us to consider computational intelligence methods such as PSO and SA methods.

## 6. THE PROPOSED PSO-SA ALGORITHM

As previously mentioned, the PSO algorithm is a population of random solutions, in which each individual is referred to as a particle and presents a candidate solution to the optimization problem. A particle in PSO, like any living object, has a memory in which remains its best experience and the best experience of other particles. In this technique, each candidate solution is associated with a velocity vector, which is adjusted according to the particle's memory. This procedure is repeated until almost all particles converge to the best solution. So in each particle's point of view, its own experience and the best experience of other particles are considered and the experience of others is not regarded. Therefore the probability of

becoming trapped in the local minima or maxima is increased. To overcome this problem, it is good to modify the PSO method. Among the computational intelligent techniques, the SA method has an ability to avoid this problem. This algorithm employs a random search which even accept the bad experience with definite probability. So in the new proposed method, the PSO algorithm is combined with the SA method.

Because of the similarity between the operation of iteration in the PSO algorithm and temperature ( $T$ ) in the SA method (but in the opposite direct), in the new proposed algorithm, the iteration in the PSO algorithm is replaced with temperature. Then in each temperature, for calculating each particle's velocity in the PSO method, the experience effect of all particles is applied considering the corresponding probability defined in the SA method. Hence, the modified velocity of each particle is calculated regarding the personal initial velocity, distance from the personal best position, the distance from the global best position, and the distances from the other particle's position randomly. In the proposed method, the modified velocity function is expressed as follows:

$$v_{pd}^T = \omega v_{pd}^{T_{prev}} + c_1 rand(0,1)(p_{pd} - x_{pd}) + c_2 rand(0,1)(g_d - x_{pd}) + \sum_{j=1, j \neq d}^D \exp(-\delta f_{pj}/T) unit(\exp(-\delta f_{pj}/T) rand(o,1))(x_{pj} - x_{pd}) \quad (9)$$

In this equation, *unit* function randomly accept the bad experience of particles based on the calculated probability in the SA method. The function *unit* returns 1 if  $\exp(-\delta f_{pj}/T) > rand(0,1)$ , 0 otherwise.

After calculating the new velocity of particles in each temperature, their new position is updated.

The proposed PSO-SA algorithm can be described as follows for the minimization problem of function  $f(x)$  subject to some constraints:

Step 1) Generation of initial population  $(x_1, x_2, \dots, x_i, \dots, x_n)$

Step 2) The best experience of particle  $i$  is initialized ( $p_i$ )

Step 3) An initial velocity vector is assigned to each particle

Step 4) Determine the initial value of temperature  $T=T_0$  and the final Temperature  $T=T_f$

Step 5) Set the temperature decreasing rule

Step 6) Objective function evaluation

**for** each particle  $i = 1 \dots n$  **do**

**If** any security function is observed

Add a penalty term to the cost function ( $f(x)$ )

**Else**

Evaluate the cost function for that particle

**End.**

**End.**

Step 7) Particle's velocity modification

**for** particle  $i = 1 \dots n$  **do**

update its memory ( $p, g$ )

$$v_i^T = \omega v_i^{T_{prev}} + c_1 rand(0,1).(p_i - x_i) + c_2 rand(0,1).(g - x_i)$$

**for** (particle  $j = 1 \dots n$  &  $j \neq i$ ) **do**

**If** ( $\exp(-\delta f_j / T) > rand(0,1)$ )

$$v_i^{T_{next}} = v_i^{T_{next}} + \exp(-\delta f_j / T).(x_j - x_i)$$

**End.**

**End.**

**End.**

Step 8) Particles movement with using Eq.(8)

Step 9) If the Stopping temperature is not reached, decrease the temperature and return to Step 6



## 7. THE PROPOSED SOLUTION METHOD

As previously mentioned, Gencos try to choose the best bidding strategies in the energy market to maximize their profits. In order to reach this target they should consider their opponent's activities and the power system conditions. Each Genco has the knowledge of its own payoffs and generation costs, but could lack such information on the other Gencos. Hence they should model their opponents with approximate information. Section 2a shows an approach to reach this information with a good approximation. If it is supposed that all Gencos are intelligent, they can apply the proposed approach to be aware of their opponent's payoff function and try to maximize their own revenue (Nash equilibrium) by acknowledging the opponent's bidding strategies. (Gencos maximize their profits at Nash equilibrium points.)

The Genco's payoff function (Eq. (6)) is obtained after the market clearing procedure, which is formulated as a global minimization problem (section 2c). So the proposed solution method in providing optimal bidding strategies of Gencos includes two optimization problems. In this paper the PSO-SA algorithm is used to solve the Nash optimization problem. In reference [8], the PSO algorithm is used to solve the unit commitment problem (UCP). Because of the similarity between the UCP and the market clearing (MC) problem, the proposed method in [8] is used to solve the MC problem.

The proposed algorithm to obtain the optimum bidding strategy of Gencos is shown in Fig. 3. This algorithm contains the steps that should be taken into consideration when applying the PSO-SA algorithm to the bidding strategy problem. As shown in this algorithm, continuous sets are considered for the mark-up strategy of Gencos with upper and lower limits ( $\mu_i^{low} \leq \mu \leq \mu_i^{up}$ ).

A game problem may have only one Nash equilibrium, multiple Nash equilibria, or none at all. The nonexistence of the Nash equilibrium is contributed to system constraints that cause the discontinuity in a Genco's optimal behavior to other Genco's bids. Meanwhile, binding constraints in a power system could also be responsible for the existence of multiple Nash equilibria. If the problem has no Nash equilibria, the algorithm shown in Fig. 3 will not converge.

The computational requirement for the proposed algorithm will increase with the number of units or Gencos. However, a Genco can assign similar mark-up strategies to units located in one region. A Genco may speed up the convergence of the algorithm by providing a good estimate of the initial mark-up strategy, which could be based on historical bidding quantities.

## 8. SIMULATION RESULTS

The proposed methodology is implemented over the IEEE 39-bus system. There are ten units in the system and each unit is supposed to be a Genco. The information on load service entities (LSE) and the network is given in [15]. To have a more competitive market the total capacity of the units are increased to 1.66 times the total demand. So the modified information of the Gencos has been listed in Table I. It is supposed that Gencos can predict the exact fuel price, hence one generation cost structure is defined in Table 1, and in this table the units or Gencos are named according to their bus numbers.

In the following case studies, it is supposed that the mark-up strategy of Gencos varies between zero to 1 and unit 30 is considered as a slack bus. So the results of all units except for unit 30 are presented.

**Case 1.** In this case all Gencos are price takers and they bid at their marginal cost, making their mark-up strategy equal to zero. ISO clears the market using the security-constrained economic dispatch (5).

The MW dispatched from each Genco, and the ISO's objective function (the value of (Eq. (5)) in the optimum awarded generation of Gencos) are tabulated in Table 2, when applying three evolutionary methods (PSO-SA, PSO and GA), and the mathematical method (GAMS/DICOPT) [15]. As shown, the optimal solution of PSO-SA is near to the results of the GAMS approach, and better than the results of the GA and PSO method.

The dynamic of the convergence between the PSO-SA and the GA is illustrated in Fig. 4.

**Case 2.** In this case, Gencos use the method introduced in section 7 for calculating their optimal mark-up strategies. Tables 3 and 4 contain the optimal mark-up strategy and the MW dispatched when using PSO-SA (with  $itr_{max}=200$ ) and mathematical method (GAMS) to solve the game problem for finding Nash equilibrium strategies. Table 5 presents the obtained profit of Gencos. The results of the PSO-SA and the GAMS methodologies show that the optimal solution of PSO-SA method is better than GAMS approach (The obtained profit of Gencos in the PSO-SA method is greater than GAMS approach). But because of combining the SA method with the PSO algorithm, the speed of the PSO-SA algorithm is slower than when using just the PSO method.

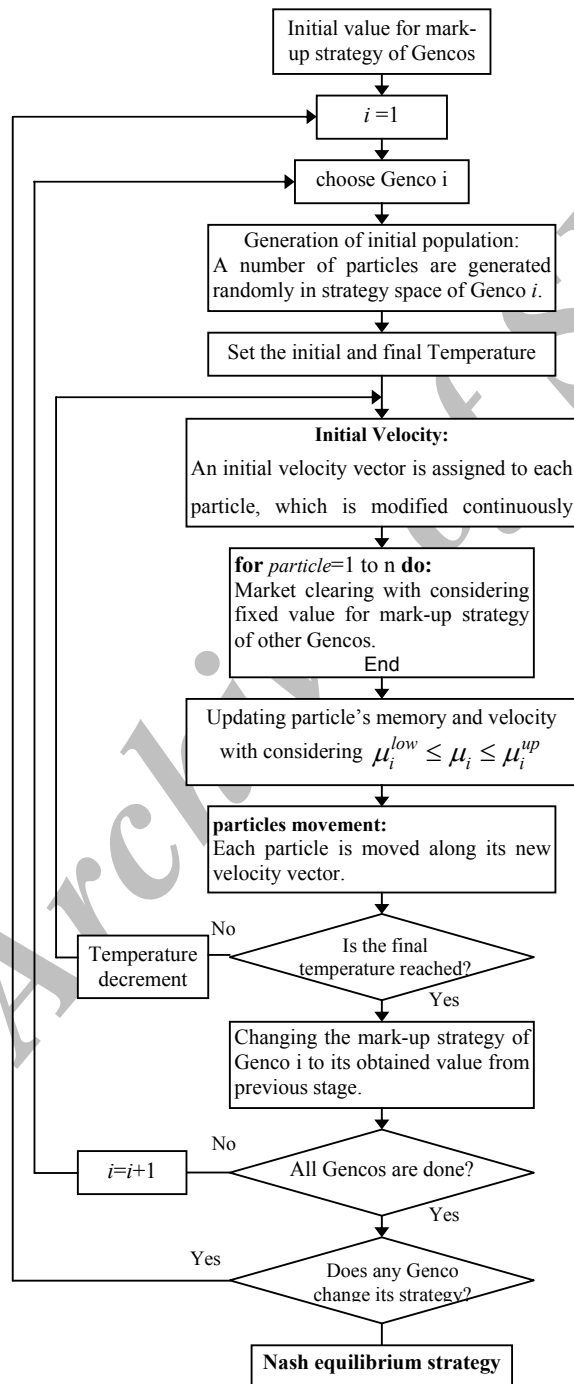


Fig. 3. Procedure of optimal bidding strategy calculation using PSO-SA algorithm

Table 1. Modified cost coefficients of units

UNITS	$a$	$b$	$c$	Pmin (MW)	Pmax (MW)
30	0.01	0.3	0.2	0	750
31	0.02	0.5	0.2	0	1245.5
32	0.03	0.4	0.2	0	1050
33	0.04	0.7	0.2	0	932
34	0.03	0.9	0.2	0	915
35	0.05	0.4	0.2	0	950
36	0.06	0.6	0.2	0	1060
37	0.07	0.3	0.2	0	940
38	0.006	0.8	0.2	0	1130
39	0.006	0.9	0.2	0	1300

Table 2. The results of GAMS, GA, and PSO for Case 1

	MW Dispatched from Gencos									ISO's Objective function
	31	32	33	34	35	36	37	38	39	
PSO-SA	1176	801.2	701.5	909	553.5	475	375.2	1130	1300	360273
PSO	1177	800.7	700.4	908	557.1	476.7	374.9	1125	1295	360287
GA	1170	806.5	700.5	912	551.3	472.5	370	1120	1293	360320
GAMS	1175	802	701.5	910	553	474.5	376	1130	1300	360275

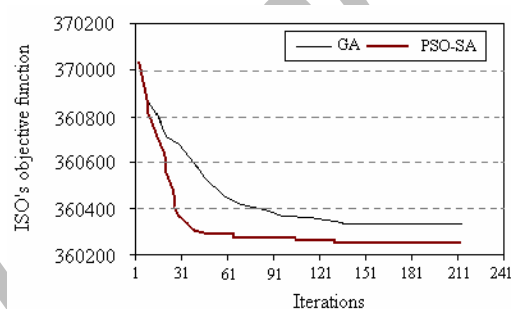


Fig. 4. Comparing the dynamic of convergence between PSO and GA approaches

Table 3. The mark-up strategy of Gencos with using PSO and GAMS

The Nash Mark-up strategy of Gencos									
	31	32	33	34	35	36	37	38	39
PSO-SA	0.012	0.008	0.0253	0.022	0.021	0.021	0	0	0
GAMS	0.01	0.006	0.02	0.02	0.02	0.02	0	0	0

Table 4. The MW dispatched from Gencos with using PSO and GAMS

The MW dispatched of Gencos									
	31	32	33	34	35	36	37	38	39
PSO-SA	1149.6	861.8	654.1	850.4	585.3	466	455	1130	1300
GAMS	1150	865	650	850	583.5	462	457	1130	1300

Table 5. The obtained profit of Gencos

	31	32	33	34	35	36	37	38	39
PSO-SA	53392.6	37680	28203.1	37046.6	23617.8	19705.4	17249.9	70651	79824
GAMS	53385	37672.3	28203	37040	23612.5	17695.1	17243.6	70645	79821

## 9. DISCUSSION ON THE PROPOSED METHOD

The proposed method in calculating the optimal mark up strategies of Gencos can be extended to a real large scale system. But there are more difficulties than the small case study like the one shown in the previous section. Some of them are explained below:

1- As previously mentioned, Gencos should estimate the opponents unknown information where one of the most important parameters in the accurate forecast of a, b, and c parameters is the fuel price. When the number of Gencos is increased, they may predict the fuel price differently. Therefore the accuracy of the forecasted parameters of each Genco about its opponents is low. In this condition the obtained Nash equilibrium strategies are probabilistic and not deterministic.

2- As shown in the market clearing model (See section 2c), the problem of finding optimal bidding strategies includes many variables such as mark up strategy of each Genco, the angle of each bus, the awarded quantity of each Genco, and the power flow of each line. Therefore the number of variables has a straight relationship with the number of Gencos and system buses. The larger system has a great number of buses and Gencos and this leads to an increase in the number of variables and consequently the required time for the convergence of the algorithm.

The authors of this paper propose that in order to examine the global solution for large systems, the bidding space of each Genco should be divided into several segments and search for the optimal response in each segment by applying the method introduced in section 7 and choose the best solution. It is proposed that for speeding the convergence of the algorithm for large systems, it is better to use the proposed method (PSO-SA algorithm) for calculating Nash equilibrium points and a mathematical method (such as GAMS) for solving the ISO's market clearing problem.

## 10. CONCLUSIONS

The history of electricity markets shows that these markets are not fully competitive. The finite number of power suppliers, the lack of enough transmission capacity, etc, are some reasons for the lack of achievement. So, each Genco or player in these markets should be able to choose a good bidding strategy in order to maximize its benefit.

In this paper, a new approach is proposed for presenting the bidding strategy of Gencos. The proposed method is based on the behavior of the participants, and while considering the ISO's objective function, combined with the PSO-SA approach to obtain the Nash equilibrium strategy of Gencos and market equilibrium points, as well as a method for market power monitoring that can be used by market operators. This method can be extended to more complicated networks and the simulation results show its high precision and capabilities.

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