Iranian Journal of Science & Technology, Transaction B, Engineering, Vol. 31, No. B5, pp 487-500
Printed in The Islamic Republic of Iran, 2007

APPLICATION OF ANT ALGORITHM TO PIPE NETWORK OPTIMIZATION*

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Abstract– The application of ant algorithms, as any other evolutionary optimisation method, requires a number of controlling parameters to be known *a priori*. These parameters are often determined by sensitivity analysis as their values dramatically affect the performance of the methods. In addition to these parameters, a penalty parameter is usually to be defined for constrained optimisation problems. An ant algorithm with a minimum number of controlling parameters is introduced in this paper for pipe network optimisation problems. This method uses the interrelation between pheromone change and initial pheromone strength to initialize the pheromone trail strength at the start of the computation. Ant algorithms with an elitist strategy of pheromone updating are known for premature convergence leading to suboptimal solutions. Such suboptimal solutions are avoided by using the concept of pheromone strength limiter introduced in the literature for TSP. The introduction of this concept, however, requires the introduction of a new parameter adding to the number of controlling parameters of ant algorithms. A sensitivity analysis was carried out to find the proper value of the newly introduced parameter. The results suggest that a value in the range of 0.15-0.3 is the best value for the examples considered. The efficiency of the proposed ant algorithm is tested against two benchmark examples in the literature and the results are presented. This method is shown to be capable of locating the best ever solutions obtained for these problems.

Keywords– Ant algorithm, pipe networks, optimal design

1. INTRODUCTION

Optimization of pipe networks, when defined in a mathematical form, leads to a non-convex nonlinear constrained optimization problem with discrete search space. Conventional search methods are not capable of handling the difficulties encountered when attempting the solution of these problems. Any successful application of search methods to the optimal sizing of pipe networks requires a method capable of handling noisy objective function, discrete decision variables, while enjoying computational efficiency if an industrial use of the method is in mind. Evolutionary search methods, in particular genetic algorithms, have been successfully used for pipe network optimization in recent years [1-5]. Recently, Dorigo *et al*. [6] proposed a new evolutionary optimization method, namely the ant algorithm, based on the collective behaviour of the ants in their search for food. Ant algorithms were first proposed for the solution of difficult combinatorial optimization problems like the traveling salesman problem (TSP) and the quadratic assignment problem (QAP). This method has been shown to outperform other evolutionary optimization methods including Gas [6, 7]. More recently Dorigo and Di Caro [8] introduced a general framework for developing Ant Colony Optimization Algorithms (ACOAs), namely ant colony meta-heuristic. This enables the ACOAs to be applied to other engineering problems provided that the problem

*Received by the editors August 1, 2004; final revised form October 8, 2006.

can be properly formulated. The application of ant algorithms to water resource problems is of recent origin. Abbaspour *et al*. [9] used the ant algorithm for estimating the unsaturated soil hydraulic parameters. More recently Maier *et al*. [10] compared the performance of the ant algorithm with that of Genetic Algorithms (GAs) for the optimization of water distribution networks. The results are in line with the early experiences of Dorigo *et al*. [6] regarding the superiority of ant algorithms to other general purpose heuristics including genetic algorithms. This paper describes a variant of the ant colony optimization algorithm for the optimal design of pipe networks. The method differs from that of Maier *et al*. [10] in the number of parameters required to be defined *a priori* before the underlying problem can be solved. This is important since the proper values of these controlling parameters is usually determined in a trial and error fashion adding to the computational effort required by this method to solve any engineering problem. This is achieved using the relation between the reward factor and the initial value of the pheromone trail strength. The performance of the method is improved using the concept of trail limiting introduced by Stutzle and Hoos [11] as a remedy for premature convergence, which is often encountered in ant algorithms. The efficiency of this method is illustrated by its application to two pipe network design benchmark problems, with the results compared with the existing results in the literature.

2. ANT COLONY OPTIMIZATION

Ant algorithms were initially inspired by the observation that ants can find the shortest paths between food sources and their nest even though they are almost blind. Individual ants choose their paths from the nest to the food source in an essentially random fashion [6]. While walking from food sources to the nest and vice versa, however, ants deposit a substance called pheromone on the ground, forming, in this way, a pheromone trail. Ants can smell pheromone and, when choosing their way, tend to choose, in probability, paths marked by strong pheromone concentrations. The pheromone trail acts as a form of indirect communication called stigmergy [12], helping the ants to find their way back to the food source or to the nest. Also, it can be used by other ants to find the location of the food sources found by their nestmates. It has been shown experimentally [13] that this pheromone trail following behavior can give rise, once employed by a colony of ants, to the emergence of the shortest paths.

In the ant colony optimization (ACO) meta-heuristic, a colony of artificial ants cooperate in finding good solutions to discrete optimization problems. Artificial ants are similar to real ants in some aspects, while they could behave differently as required by the nature of the problem in hand. Cooperation, pheromone reinforcement and stochastic decision-making policy using pheromone trails are amongst the most important similarities between artificial and real ant colonies. While the real ants are almost blind, the artificial ants are usually made to use local information in choosing their path, thereby providing artificial ants with sight. Real ants update pheromone trails on the paths as they walk, while artificial ants may be required to update pheromone only when they have finished their tour. While all ants contribute to pheromone trail reinforcement in a real colony, only an ant that has created the best path may be allowed to lay pheromone.

The application of the ant algorithm to the arbitrary combinatorial optimization problem requires that the problem can be projected on a graph [7]. Consider a graph $\boldsymbol{G} = (\boldsymbol{D},\boldsymbol{L},\boldsymbol{C})$ in which $\mathbf{D}=\{d_1, d_2, \ldots, d_n\}$ is the set of decision points at which some decisions are to be made, $\mathbf{L}=\{l_{ij}\}$ is the set of options $j=1, 2, \ldots, J$ at each of the decision points $i=1,2,\ldots,n$ and finally, $\mathbf{C}=\{c_{ij}\}$ is the set of costs associated with options $\mathbf{L}=\{l_{ij}\}$. The components of sets $\boldsymbol{D}$ and $\boldsymbol{L}$ may be constrained if required. A feasible path on the graph is called a solution (φ), and the minimum cost path on the graph is called the optimal solution (φ^*). The cost of a solution is denoted by $f(\varphi)$, and the cost of the optimal solution by $f(\varphi^*)$. Once the problem has defined these terms, the ant algorithm can be applied. In TSP, for which the ant algorithm was originally applied, the graph ***G*** is defined with a fully connected graph connecting n cities to each other. Each city represents a decision point at which to decide the next city to go to. At each decision point i, the list of available cities j to go or equivalently the list of available arcs *(i,j)* to choose to go to city j, defines the set ***L***, and finally the distance between cities i and j represent the cost c_{ij} associated with the option *(i,j)*. A solution of the TSP is defined as a tour visiting each and every city once and only once with the tour having the same starting and finishing point. The feasibility of a trial solution in TSP is usually guaranteed by providing each ant with a list of allowable destinations called a tabu list which is continuously updated as the ant travels from city to another. The cost of the TSP solution is considered as the sum of the arcs lengths from which the solution is constructed. The TSP, therefore, is defined as finding the minimum length solution of the problem.

3. ANT ALGORITHM FOR PIPE NETWORK OPTIMISATION

A typical water distribution network is a collection of pipes, reservoirs, pumps, and different types of valves connected to each other in order to meet a specified demand at the nodes. Basically, the optimal design of pipe networks is a multi-objective task involving hydraulics, reliability, and water quality. The multi-objective design of water distribution networks is too difficult a problem when considering newly introduced optimization methods. Most of the investigation, therefore, considers the simpler problem of component design and, in particular the optimal pipe sizing problem.

The application of the ant algorithm to any combinatorial optimisation problem such as the pipe network optimisation problem, as pointed out earlier, requires that the problem is defined in terms of a graph. For this, consider the a typical network, shown in Fig. 1, consisting of a fixed head reservoir, a source node, as an energy creating device and a set of demand nodes and connecting pipes so that the network can deliver water at the demand nodes under allowable conditions. These conditions are often considered as pipe velocities and nodal pressures remaining in a pre-specified range defined by the maximum and minimum velocity and pressure values. The pipe diameters are considered the design parameters that can be chosen from a set of commercially available diameters. The pipe network optimisation problem is then defined as selecting the diameter of each pipe of the network so that the resulting network has a minimum cost, while meeting the required conditions defined above. The proper representation of a pipe network optimisation problem requires the definition of the graph ***G*** = ***(D,L,C)*** and its component. Here, each pipe is a decision point at which the diameter of the pipe is to be determined. The component of the decision set $\boldsymbol{D} = \{d_1, d_2, ..d_i, .., d_n\}$ is, therefore, the existing pipes of the network, where d_i represents the diameter of the *i*th pipe to be selected from a set of commercially available diameters $\boldsymbol{\varphi} = \{\varphi_{ij}\}$ which may or may not be the same for all the pipes. Assuming that these diameters are the same for all the pipes, then $\boldsymbol{\varphi}=(\varphi_1, \varphi_2, ..., \varphi_{nd})$ would represent the list of available options at each and every decision point of the problem. If uc_j is defined as the per unit length cost of the pipe with a diameter φ_j, the cost c_{ij} associated to the option φ_j at decision point d_i can now be calculated as the product of the per unit cost uc_j and the length le_i of the link under consideration. The cost of a trial solution f(φ), which may or may not be a feasible solution, is now calculated as the sum of the links cost given by

$$f(\varphi) = \sum_{i=1}^{n} uc_j \times le_i \tag{1}$$

Here a difficulty not present in other combinatorial optimization problems such as TSP arises. In TSP for example, a tabu list representing the list of available options at each decision point is so constructed that only feasible solutions are created. This is not possible in pipe network optimization problems, where the feasibility of the solution can only be determined after all the components of the solution are known. This means that some of the solutions created by the algorithm may be infeasible. This is the situation which arises in other evolutionary methods such as the genetic algorithm. A method for coping with this problem is to check the feasibility of the solution once they are totally constructed and reject the infeasible solutions. Rejecting infeasible solutions, however, is costly and may lead to the loss of useful information within some of the infeasible solutions. The usual remedy, which is adopted here, is to penalize the infeasible solutions so that the total cost of these solutions is much larger than that of the feasible solutions. This technique discourages the algorithm from creating more infeasible solutions. The total penalized cost of the network is defined as follows:

$$f(\varphi) = \sum_{i=1}^{n} uc_j \times le_i + \alpha_p CSV \qquad (2)$$

$$CSV = \{\sum_{i=1}^{n} (1 - V_i / V_{min}) + \sum_{i=1}^{n} (V_i / V_{max} - 1) + \sum_{in=1}^{nn} (1 - H_{in} / H_{min}) + \sum_{in=1}^{nn} (H_{in} / H_{max} - 1)\}$$

in which n and nn is the number of existing pipes and nodes, respectively; H_{in} is the nodal head; H_{min} and H_{max} are the minimum and maximum allowable hydraulic head; V_i is the pipe velocity; V_{min} and V_{max} are the minimum and maximum allowable flow velocity; *CSV* is the total constraint violation of the trial solution; and α_p is the penalty parameter with a large value when the constraints are violated, ie; the term in parenthesis is positive, and zero value otherwise. The second term, therefore, represents the penalty cost due to constraint violations.

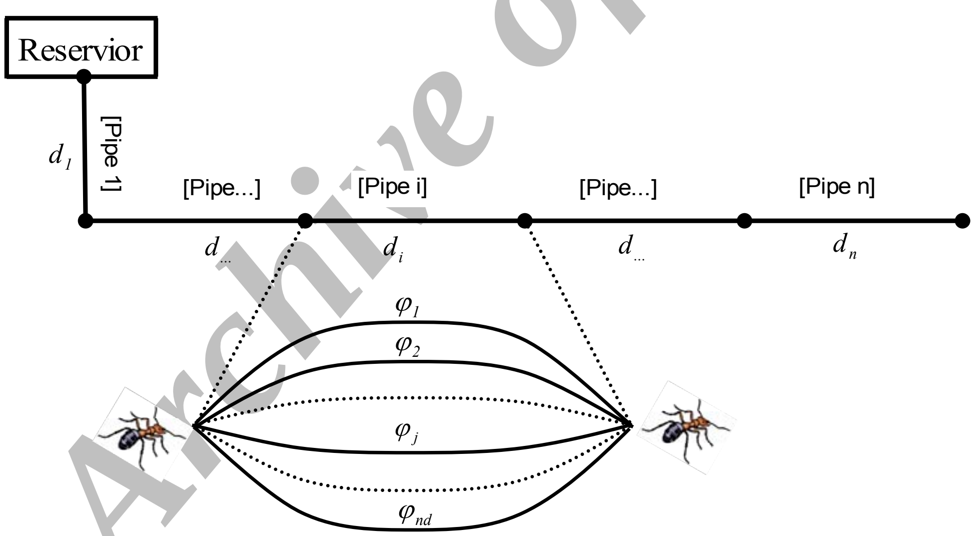

Fig.1. Representation of pipe network optimization problem in terms of graph

The problem under consideration is now formulated in a proper format required for the application of the ant algorithm. The method used in this paper is based on that suggested by Stuzle and Hoos [11] which may be defined by the following steps:

1- m ants are randomly placed on the n pipes of the network and the amount of pheromone trail on all arcs are initialized to some proper value at the start of the computation.

2- A transition rule is used at each pipe to decide which diameter is selected for the pipe under consideration. Once the diameter of the current pipe is selected, the ants move to the next pipe and the solutions are incrementally created by ants as they move from one pipe to the next. This procedure is repeated until all pipes of the network are covered. The transition rule used here is defined as follows [6]:

$$p_{ij}(k,t)=\frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{j=1}^{nd}[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}} \tag{3}$$

Where $p_{ij}(k,t)$ is the probability that the ant k selects diameter φ_j for the ith pipe at iteration t; $\tau_{ij}(t)$ is the concentration of pheromone on arc *(i,j)* at iteration t; $\eta_{ij} = 1/(uc_j \times le_i)$ is the heuristic value representing the cost of choosing option *j* for pipe *i*, and α and β are two parameters that control the relative weight of the pheromone trail and heuristic value referred to as the pheromone and heuristic sensitivity parameter, respectively. The heuristic value η_{ij} is analogous to providing the ants with sight and is sometimes called "visibility". This value is calculated once at the start of the algorithm and is not changed during the computation. The role of the parameters α and β can be best described as follows. If $\alpha = 0$, the cheapest diameters are more likely to be selected leading to a classical stochastic greedy algorithm. If, on the contrary, $\beta = 0$, only pheromone amplification is at work, leading to the pre-mature convergence of the method to a strongly sub-optimal solution [6]. A trade-off between the heuristic value and trail intensity therefore appears to be necessary.

3- Eq. (2) is used to calculate the total cost of the trial solutions generated. The generation of a complete trial solution and the calculation of the corresponding cost is called a cycle (k). The calculation of the total cost of a trial solution requires the determination of the constraint violation by the solution. This in turn requires that the distribution of nodal pressures and pipe velocities in the network are known, and can be readily obtained by a steady state analysis of the network using any available pipe network simulation code [14]. This, however, requires the definition of some parameters in the Hazen-Williams equation which states the relation between head loss and flow in each link. Here, a Hazen-Williams formula of the type

$$h_f = \mu L(\frac{Q}{C})^{\eta} D^{-\gamma} \tag{4}$$

is used, in which L = length of a pipe; Q = flow rate of a pipe; C = Hazen-Williams coefficient, D = internal diameter of a pipe and η, γ, and μ are empirical constants. The value of these parameters will be discussed later when presenting the results of the model for the benchmark problem.

4- The pheromone is updated after Steps 2 and 3 are repeated for all ants and, therefore, generation of the m trial solution and the calculation of their corresponding cost, referred to as an iteration (t). The general form of the pheromone updating used here is as follows [6]:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \Delta\tau_{ij} \tag{5}$$

where $\tau_{ij}(t+1)$ is the amount of the pheromone trail on option j of the ith decision point , i.e. arc *(i,j)*, at iteration *t+1*; $\tau_{ij}(t)$ concentration of pheromone on arc *(i,j)* at iteration t; $0 \le \rho \le 1$ is the coefficient representing the pheromone evaporation and $\Delta\tau_{ij}$ is the change in the pheromone concentration associated with arc *(i,j)*. The amount of pheromone trail $\tau_{ij}(t)$ associated to arc (*i,j*) is intended to represent the learned desirability of choosing option *j* when in decision point *i*. The pheromone trail information is changed during the problem solution to reflect the experience acquired by ants during problem solving. The main role of pheromone evaporation is to avoid stagnation, that is, the situation in which all ants end up doing the same tour. In addition, evaporation reduces the likelihood that high cost solutions will be selected in future cycles.

Different methods are suggested for calculating the pheromone change. The method used here is originally suggested by Stutzle and Hoos [11] in which only the ant which has produced the best solution of the iteration is allowed to contribute to pheromone change.

$$\Delta\tau_{ij} = \Delta\tau_{ij}^{best} \tag{6}$$

The amount of pheromone change is usually defined as [6]:

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{R}{f\,(\varphi)^{k}} & \text{if arc (i, j) is chosen by ant k} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $f\,(\varphi)^{k}$ is the cost of the solution produced by the ant k, and *R* is a quantity related to the pheromone trail called the pheromone reward factor. The amount of pheromone added to each of the options during a cycle is a function of the cost of the trial solution generated. The better the trial solution, and hence the lower the cost, the larger the amount of pheromone added to the option. Consequently, the solution components (diameter options) that are used by the best ant and form a part of the lower cost solution receive more pheromone and are more likely to be selected by future ants. This choice clearly helps to direct the search towards good solutions.

At the end of each iteration, each ant has generated a trial solution. The pheromone is updated before the next iteration starts. This process is continued until the iteration counter reaches its maximum value defined by the user.

The application of ACO as defined earlier requires *a priori* defining a number of parameters such as evaporation coefficient, pheromone and heuristic sensitivity parameter, pheromone reward factor and the initial value of the pheromone trail. The performance of the ACO algorithms is very sensitive to the values of these parameters which are problem dependant and are usually chosen in a trial and error fashion for each problem. This is one of the main shortcomings of the ACO algorithms compared to other evolutionary search methods such as GAs, where at least some common rules exist for defining the proper value of the parameters required. This is further complicated when attempting ACOAs for the optimisation of constrained problems such as pipe network optimisation problems where a penalty parameter has to be defined.

The pheromone reward factor *R* and the initial value of the pheromone concentration are interrelated. The pheromone reward factor has to be defined in such a way that the pheromone change defined by Eq. (7) is of the same order as the pheromone concentration during the computation process and in particular at the early stages of the computation. A very high value of the reward factor *R* leading to a higher value of the pheromone change compared to the pheromone concentration would lead to a rapid increase in the pheromone concentration of the options which have been chosen by ants in the early stages of the process. These paths will then dominate the ant decision table leading to the so-called premature convergence of the method. On the other hand, a high value of the pheromone trail compared to the pheromone change results in a very small change in the pheromone concentration of the options chosen by ants, hindering the convergence of the algorithm if it occurs at all. The values of pheromone concentration and pheromone change will now be of the same order in the early stages of the computation, giving ants enough chance to explore the search space. Here a value of unity is selected for the reward factor *R* to reduce the cost of sensitivity analysis. This, however, requires the careful initialisation of the pheromone trail to balance between convergence characteristics of the method and the quality of the final solution obtained. It can be shown that the pheromone trail update rule defined by Eq. (7) will lead to an analytically maximal trail strength calculated by a geometric series as [11]:

$$\tau_{max} = \frac{1}{f\,(\varphi)^{opt}} \tag{8}$$

Since the value of the reward factor is set to unity as noted earlier, here $f(\varphi)^{opt}$ is the cost of the optimal solution of the underlying problem. It can be seen that setting the initial pheromone trail to the pheromone change calculated at the first iteration $\frac{1}{f(\varphi)^{best}}$ will lead to a logical balance between current pheromone trail strength and the pheromone change, in particular at the beginning of the search preventing premature convergence to suboptimal solutions.

When an elitist strategy is used for pheromone updating, early stagnation of the search may occur. In this situation the trail strength on one of the options available at each decision point is so high compared to the other options that the same option is nearly always chosen by the ants. No better solutions are, therefore, found by the ants and usually the best found solution is constructed by most of the ants. Stutzle and Hoos [11] introduced an *MMAS* method to have a more direct control over the influence of the pheromone strength. They introduced two limiters for the pheromone trail to prevent early stagnation of the search at suboptimal solutions. These limiters act as the upper and lower bounds of the pheromone trail strength. The upper bound as defined in Eq. (8) was found to be of lesser importance, while the lower limit played a more decisive role. Stutzle and Hoos [11] introduced the following formula for the calculation of the lower trail strength limit based on some analytical arguments:

$$\tau_{\min} = \frac{\tau_{\max}(1 - p^{dec})}{np^{dec}}$$
$$p^{dec} = (p^{best})^{1/n} \tag{9}$$

where τ_{min} represents the lower limit for the pheromone trail strength; p^{dec} is the probability that an ant constructs each component of the best solution again, and p^{best} is the probability that the best solution is constructed again. This argument is based on the strong assumption that around good solutions other good or even better solutions are located. This is definitely the case for TSP, the problem for which the MMAS is proposed as it is shown that reasonably good tours are located in a small region of the search space. This is not necessarily true for the pipe network optimization problem in which good solutions may be surrounded by costly infeasible solutions. In the next section the application of the method on some benchmark problems in the literature is addressed.

3. TEST PROBLEMS

The first problem to be considered is a two-loop network with 8 pipes, 7 nodes, and one reservoir shown in Fig. 2 [2]. All the pipes are 1,000-m long and the Hazen-Williams coefficient is assumed to be 130 for all the pipes. The minimum nodal head requirement for all demand nodes is 30 m. There are 14 commercially available pipe diameters as listed in Table 1, while the nodal demands and elevations are shown in Table 2. Figure 3 shows the variation of the global best solution cost against the number of network analysis required for different values of p^{best}. All methods have been capable of locating the best ever obtained solution of 41900 within 5100 evaluations. The best performance was shown with p^{best} = 1.0, leading to faster convergence to the optimal solution in only 4700 evaluations. This compares favorably with the ~250,000 evaluations required by the method of Savic and Walters [2], ~53,000 evaluations required by the method of Cuncha and Sousa [15], and 9,201 evaluations required by the Fast Messy Genetic Algorithm of Boulos *et al*. [4] to get the least cost solution of 419,000 units. Figure 4 shows the effect of different values of the p^{best} on the number of ants constructing the iteration best solution at each iteration. The number of ants following the foot steps of the best iteration ant is very low at the beginning of the computation where the pheromone trail strength is more or less the same for all the options available at different decision points. As the pheromone trail built up on the options constructing the best solution, more ants follow the same path creating the same solution. This explains the sudden increase in the number of best solutions generated as the computation proceeds. For $p^{best}=1$, representing no trail limit, the number of ants creating the best solution at each iteration rises quickly to 100 percent, as expected, while this number decreases with decreasing values of p^{best}. For smaller values of p^{best}, the lower trail limit defined by Eq. (9) has a higher value and, therefore, comes into play sooner than that for higher values. For this test problem, however, the use of a lower trail limit leads to slower convergence since the method is able to locate the optimal solution before stagnation occur. It should be noted that these experiments are carried out with hydraulic parameter values η = 1.85, γ = -4.87, μ = 10.5088 for D in meters and Q in cubic meters per second [2, 4] and ACO parameter values m=100, ρ=0.9, α=1, β=0.1 and R=1 as explained before. A sensitivity analysis was only carried out to find the value of β=0.1 assuming fixed values of ρ=0.9 and α=1.

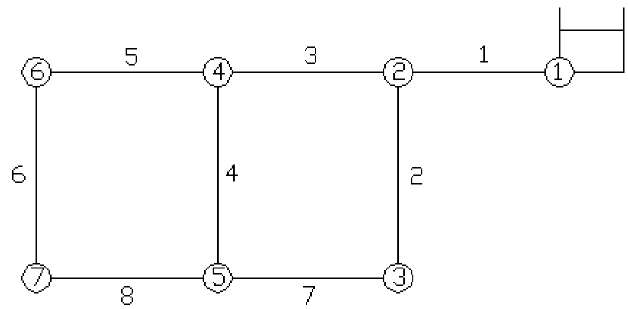

Fig. 2. Two loop network

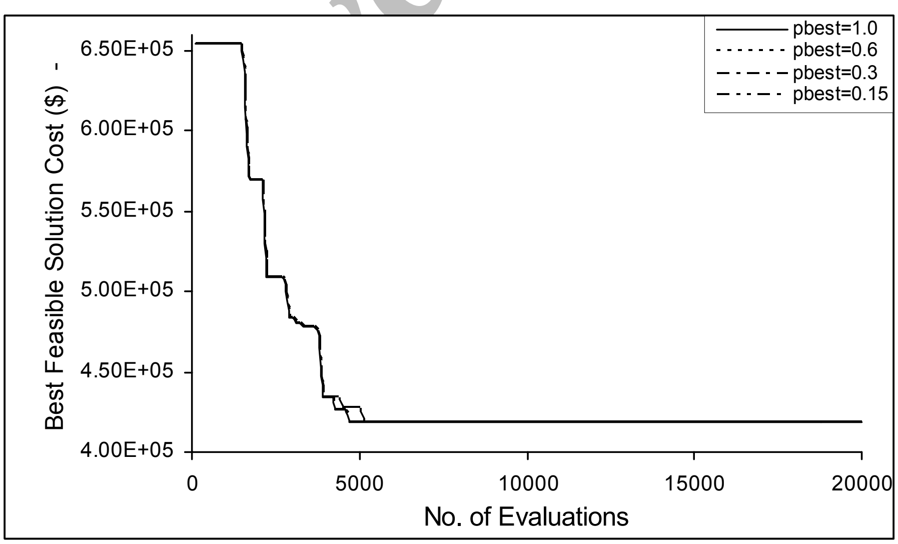

Fig. 3. Variation of the global best solution cost against the number of network analysis for different values of p^{best} (Test 1)

The second test problem concerns the rehabilitation of the New York City water supply network with 21 pipes, 20 demand nodes, and one reservoir as shown in Fig. 5 [1]. The commercially available pipe diameters and their respective costs are listed in Table 3, while the pipe and nodal data of the existing network are shown in Table 4. The Hazen-Williams coefficient is assumed to be 100 for all the pipes [1].

Figure 6 shows the variation of the global best solution cost against the number of network analysis required for different values of p^{best} using the same values of hydraulic and ACO parameters used before except for the new value of β =0.3. This example clearly shows the effect of the lower trail limit on the convergence characteristics and the quality of the final solution obtained. For the highest possible value of p^{best} =1, the method converges very quickly to a sub-optimal solution of 38.8M$ in just 5200 evaluations. For smaller values of p^{best} =0.6 and p^{best} = 0.3 the quality of the solution is improved to 37.6M$ and 37.1M$ with the latter representing the best ever solution obtained for this problem. These solutions are, however, obtained at the expense of slower convergence and within 8500 and 7000 evaluations, respectively. Decreasing the value of p^{best} to a smaller value of 0.15 shows an adverse effect on the quality of the solution obtained in the 9100 evaluation with a cost of 37.6M$. These solutions compare favorably with the ~1,000,000 evaluations required by the method of Savic and Walters [2] and 37,186 evaluations required by the Fast Messy genetic algorithm of Boulos *et al.* [4] to get the same solution of 37.1M$. The variation of the number of best solutions created at each iteration shows the same behavior as that of the previous example as shown in Fig. 7.

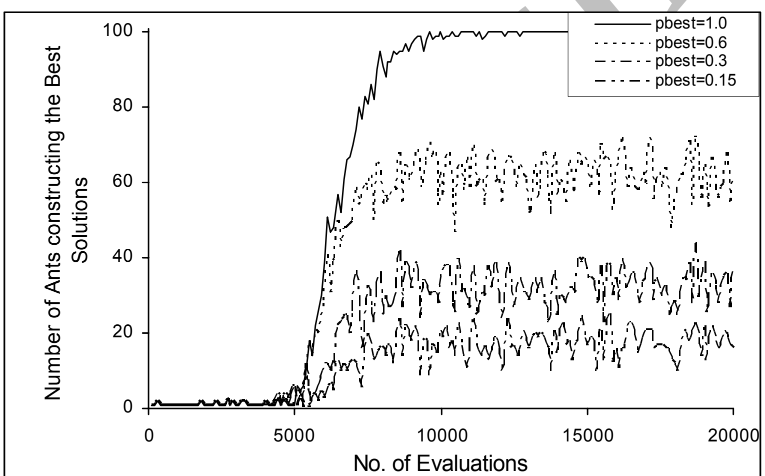

Fig. 4. The ratio of the best solution generated at each iteration for different values of p^{best} (Test 1)

Table 1. Cost data for the two-loop network

Diameter (cm)	2.54	5.08	7.62	10.16	15.24	20.32	25.4	30.48	35.56	40.64	45.72	50.8	55.88
Cost (units/m)	2	5	8	11	16	23	32	50	60	90	130	170	300

Table 2. Nodal demand and elevation data for the two-loop network

Node	Demand (m^3/h)	Elevation (m)
1	----	210.0
2	100.0	150.0
3	100.0	160.0
4	120.0	155.0
5	270.0	150.0
6	330.0	165.0
7	200.0	160.0

Table 3. Optimal solution obtained for two-loop network

Pipe	1	2	3	4	5	6	7	8
Diameter (cm)	45.72	25.4	40.64	10.16	16	25.4	25.4	2.54

Table 4. Pipe cost data for New York network

Diameter		Pipe Cost	
(inch)	(mm)	($/ft)	($/m)
36	(910)	93.5	(306.8)
48	(1220)	134.0	(439.6)
60	(1520)	176.0	(577.4)
72	(1830)	221.0	(725.1)
84	(2130)	267.0	(876.0)
96	(2440)	316.0	(1036.8)
108	(2740)	365.0	(1197.5
120	(3050)	417.0	(1368.1
132	(3350)	469.0	(1538.7)
144	(3660)	522.0	(1712.6)
156	(3960)	577.0	(1893.0)
168	(4270)	632.0	(2073.5)
180	(4570)	689.0	(2260.5)
192	(4880)	746.0	(2447.5)
204	(5180)	804.0	(2637.8)

Table 5. Pipe and nodal data for New York tunnel network

Pipe data					Nodal data			
Pipe	Start node	End node	Length (m)	Existing diameter (mm)	Node	Demand (*l*/s)	Min. total head (ft)	(m)
1	1	2	3535.6	4570	1	reservoir	300	91.4
2	2	3	6035.0	4570	2	2616	255	77.72
3	3	4	2225.0	4570	3	2616	255	77.72
4	4	5	2529.8	4570	4	2497	255	77.72
5	5	6	2621.2	4570	5	2497	255	77.72
6	6	7	5821.6	4570	6	2497	255	77.72
7	7	8	2926.0	3350	7	2497	255	77.72
8	8	9	3810.0	3350	8	2497	255	77.72
9	9	10	2926.0	4570	9	4813	255	77.72
10	11	9	3413.7	5180	10	28	255	77.72
11	12	11	4419.6	5180	11	4813	255	77.72
12	13	12	3718.5	5180	12	3315	255	77.72
13	14	13	7345.6	5180	13	3315	255	77.72
14	15	14	6431.2	5180	14	2616	255	77.72
15	1	15	4724.4	5180	15	2616	255	77.72
16	10	17	8046.7	1830	16	4813	260	79.25
17	12	18	9509.7	1830	17	1628	272.80	83.15
18	18	19	7315.2	1520	18	3315	255	77.72
19	11	20	4389.1	1520	19	3315	255	77.72
20	20	16	11704.3	1520	20	4813	255	77.72
21	9	16	8046.7	1830				

As a final test we consider the same problem with the hydraulic constant parameters (η = 1.852, γ = -4.871, μ = 10.669) for D in meters and Q in cubic meters per second, equivalent to μ = 4.7279 for D in feet and Q in cubic feet per second, as used by Maier *et al*. [10] to solve this problem using the ACO algorithm. The best solution of 38.64M$ was obtained with p^{best}=0.15 in the 18200 evaluation which is the best ever solution reported by Maier *et al*. [10] for this problem. The convergence characteristics of the method for different values of p^{best} is shown in Fig. 8, while the optimal diameters for duplicate pipes are shown in Table 7 and compared with other solutions in the literature. Table 8 compares the hydraulic grades at critical nodes of the network with that of Maier *et al*. [10]. The method of Maier *et al*. [10], however, required six parameters α, β, ρ, R, P_{pher} and PC to be determined before the main calculations in which α, β, ρ, R are as defined in this paper and P_{pher} and PC (the same as α_p used here) are penalty parameters used for pheromone change and total cost calculation, respectively. This, of course, requires a huge amount of calculation in addition to the computational effort required to solve the problem. This should be compared with only three free parameters α_p , β and ρ used in the presented method since the value of α and R was assumed equal. The only sensitivity analysis required before the main calculation was to determine the proper value of β, since the previous experiences suggests a value of around 0.9 for ρ which was successfully used here.

Table 6. Optimal pipe diameters obtained by different methods for New York network. (Test 2)

Pipe	Present work	Savic and Walters (1997)	Boulos et al. (2000)
	(inch) (mm)	(inch) (mm)	(inch) (mm)
7	108 (2740)	108 (2740)	108 (2740)
16	96 (2440)	96 (2440)	96 (2440)
17	96 (2440)	96 (2440)	96 (2440)
18	84 (2130)	84 (2130)	84 (2130)
19	72 (1830)	72 (1830)	72 (1830)
21	72 (1830)	72 (1830)	72 (1830)
Cost (10^6\$)	37.13	37.13	37.13
Evaluations	7,000	~1,000,000	37,186

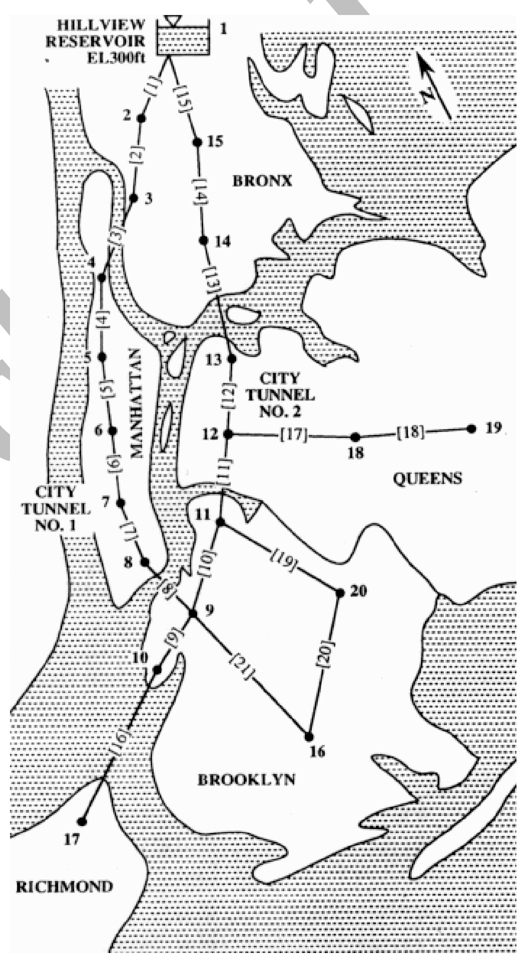

Fig. 5. New York tunnel network

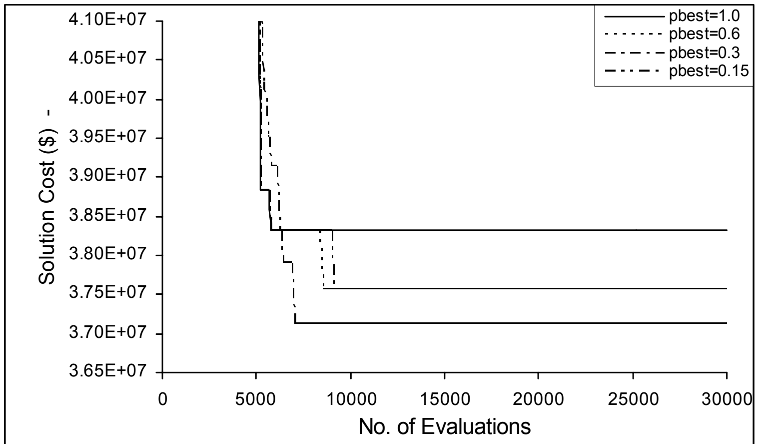

Fig. 6. Variation of the global best solution cost against the number of network analysis for different values of p^{best} (Test 2)

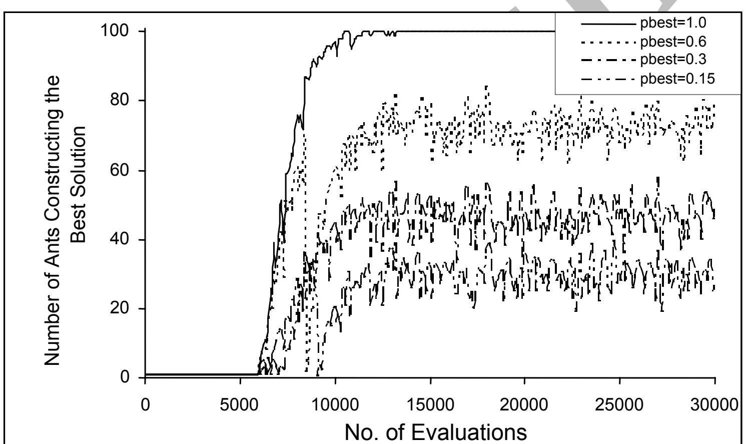

Fig. 7. The ratio of the best solution generated at each iteration for different values of p^{best} (Test 2)

Table 7. Comparison of hydraulic grades at critical nodes for New York network. (Test 2)

Node	Min. Total Head		Savic and Walters (1997)				Present Work			
			HGL		Excess		HGL		Excess	
	(ft)	(m)	(ft)	(m)	(ft)	(m)	(ft)	(m)	(ft)	(m)
16	260	79.25	260.16	(79.4)	0.16	(0.04)	260.16	(79.4)	0.16	(0.04)
17	272.80	83.15	272.86	(83.2)	0.06	(0.01)	272.86	(83.2)	0.06	(0.01)
19	255	77.72	255.21	(77.9)	0.21	(0.06)	255.21	(77.9)	0.21	(0.06)

Table 8. Optimal pipe diameters obtained by different ant algorithms for New York network. (Test 3)

Pipe	Present work		Maier et al. (2003)	
	(inch)	(mm)	(inch)	(mm)
7	144	(3650)	144	(3650)
16	96	(2440)	96	(2440)
17	96	(2440)	96	(2440)
18	84	(2130)	84	(2130)
19	72	(1830)	72	(1830)
21	72	(1830)	72	(1830)
Cost (10^6\$)	38.64		38.64	
Evaluations	18,200		13,928	

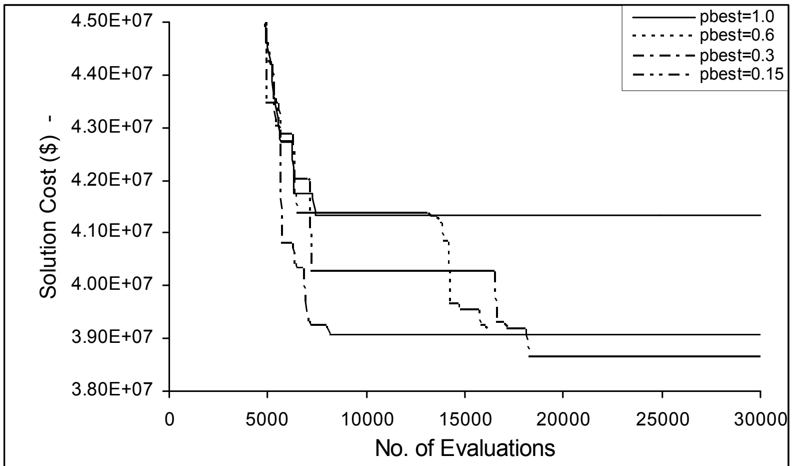

Fig. 8. Variation of the global best solution cost against the number of network analysis for different values of p^{best} (Test 3)

Table 9. Comparison of hydraulic grades at critical nodes for New York network. (Test 3)

Node	Min. total head (ft)	(m)	Maier *et al*. (2003) HGL (ft)	(m)	Excess (ft)	(m)	Present work HGL (ft)	(m)	Excess (ft)	(m)
16	260	79.25	260.08	(79.27)	0.07	(0.02)	260.07	(79.27)	0.07	(0.02)
17	272.80	83.15	272.07	(83.17)	0.06	(0.02)	272.06	(83.16)	0.06	(0.01)
19	255	77.72	255.05	(77.72)	0.04	(0.01)	255.04	(77.72)	0.04	(0.01)

4. CONCLUDING REMARKS

An ant algorithm for the optimisation of pipe networks has been presented in this paper. The relation between the pheromone change and initial pheromone strength is used for the initialisation of the pheromone trail at the start of the computation. The use of an elitist strategy is known to lead to the rapid stagnation of the ant algorithms leading to the suboptimal final solution of the method. The concept of limiting the minimum value of the pheromone trail strength is used to overcome this problem. This method, however, introduces an additional parameter, the probability of ants constructing the best found solution, to be known a priori. Numerical experiments were carried out on two benchmark examples to test the sensitivity of the method to the value of this newly introduced parameter. No conclusive conclusion regarding the proper value of the parameter could be drawn. A value in the range of 0.15-0.3, however, was found to lead to best performance of the method. The presented ant method using the proposed values of the parameter is shown capable of yielding the best ever solutions obtained for the examples considered.

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