Iranian Journal of Science & Technology, Transaction B, Engineering, Vol. 31, No. B5, pp 567-571
Printed in The Islamic Republic of Iran, 2007

"Research Note"

NONLINEAR ANALYSIS OF SEMI-RIGID FRAMES WITH RIGID END SECTIONS*

S. SEREN AKAVCI
Cukurova University, Department of Architecture, 01330 Balcali, Adana, Turkey
Email: akavci@mail.cu.edu.tr

Abstract– This work presents a computer-based analysis of semi-rigid steel frames. The geometric nonlinearity of the structure and the material nonlinearity of the connections are considered in the analysis. The critical load has been searched as a suitable load parameter for the loss of stability of the system. Several examples are presented to demonstrate the validity of the analysis procedure.

Keywords– Semi-rigid, nonlinear, plane frame, rigid-end section, critical load

1. INTRODUCTION

Connection flexibility affects both the force distribution and the deformation in the beams and columns of a frame and must be included in stuctural analysis. The behavior of flexible connections is usually described by their moment-rotation curves, in which the slope of the curve corresponds to the rotational rigidity of the connections. Several analytical and experimental studies have been done on this subject in recent years [1, 7].

In this study, the nonlinear analysis of frames with rigid end sections and nonlinear semi-rigid connections has been carried out taking into consideration the effect of geometric nonlinearity. The element matrices of members are formulated by using the second order theory. The analysis is then carried out by an incremental procedure. In the analysis, the critical load of the system has been obtained by using a double iteration process. The calculation of the critical load is purely a search for the value of a suitable load parameter λ for the loss of stability of the system.

2. ELEMENT STIFFNESS MATRIX AND STABILITY ANALYSIS

In this study, to describe the connection behavior, different mathematical models have been proposed. The chosen connection model to describe the nonlinear M-θ_r curve of semi-rigid connections is given by King and Chen [6] as follows:

$$K_t = \frac{dM}{d\theta_r} = K_i \left[1 - \left(\frac{M}{M_u} \right)^c \right] \tag{1}$$

where K_t is the tangent stiffness of the connection, K_i is the initial connection stiffness, M_u is the ultimate bending moment capacity of the connection, M is connection moment, and c is the shape factor.

*Received by the editors May 10, 2004; final revised form January 23, 2007.

The nonlinear stiffness matrix of a homogenous, flexibly connected member having rigid end sections has been obtained from the solution of the second order analysis, where axial force is taken into account as

$$k=\begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix} \quad \text{where;} \quad k_{ii}=\begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{EIu^3H_1}{L^3H} & \frac{EIu^2H_2}{L^2H} \\ 0 & \frac{EIu^2H_2}{L^2H} & \frac{EIuH_3}{LH} \end{bmatrix} \quad k_{ij}=\begin{bmatrix} -\frac{EA}{L} & 0 & 0 \\ 0 & -\frac{EIu^3H_1}{L^3H} & \frac{EIu^2H_4}{L^2H} \\ 0 & -\frac{EIu^2H_2}{L^2H} & \frac{EIuH_5}{LH} \end{bmatrix} \quad k_{jj}=\begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{EIu^3H_1}{L^3H} & -\frac{EIu^2H_4}{L^2H} \\ 0 & -\frac{EIu^2H_4}{L^2H} & \frac{EIuH_6}{LH} \end{bmatrix} \tag{2}$$

In expression (2)

$$\begin{aligned}
H_1 &= (k_Ak_B + {*}N\,EI)S + \alpha\, EI(k_A + k_B)C \\
H_2 &= \alpha\, EI\, k_A\; S - {*}k_Ak_B\,(1-C) \\
H_3 &= (N\,L\,k_A - {*}k_Ak_B)S + {*}(\alpha L)k_Ak_B\;C \\
H_4 &= \alpha\, EI\, k_B\;\; S - {*}k_Ak_B\;(1-C) \\
H_5 &= -{*}(\alpha L)\; k_A\; k_B + {*}k_Ak_B\; S \\
H_6 &= (N\,L\,k_B - {*}k_Ak_B)S + {*}(\alpha L)k_Ak_B\; C \\
H &= [\alpha\, EI(NL - {*}(k_A + k_B)) + {*}(\alpha L)k_Ak_B]S + {*}[NL\;(k_A + k_B) - 2{*}k_Ak_B]C + 2\,k_Ak_B
\end{aligned} \tag{3}$$

where E is Young's modulus, A is the cross-sectional area of the member, I is the moment of inertia of the member, L is the length of the member, k_A is the moment value necessary for the spring at the left support to rotate one radian, k_B is the moment value necessary for the spring at the right support to rotate one radian, N is the axial force in the member,C is [cos (αL)] for compressive and [cosh (αL)] for tensile axial force, S is [sin (αL)] for compressive and [sinh (αL)] for tensile axial force, * is -1 for compressive and +1 for tensile axial force, and u = α L.

The point of frame instability occurs when the determinant of the overall stiffnes matrix becomes equal to zero.

$$Det\, K(N) = 0 \tag{4}$$

3. INCREMENTAL ITERATIVE SOLUTION METHOD

The computational technique, namely The Newton-Raphson method, used to obtain the nonlinear response of a frame is shown graphically in Fig. 1. The loading is applied incrementally to the frame and within each load step an iterative solution is performed until a required accuracy is attained.

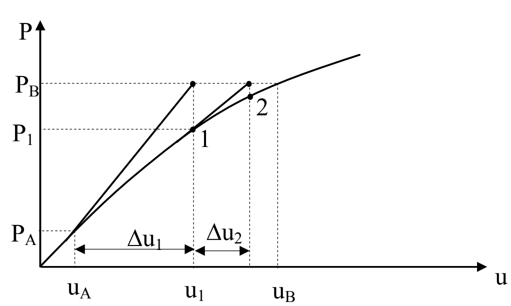

Fig. 1. The Newton-Raphson iteration procedure

The Newton-Raphson iteration procedure for each increment:

$$K_i^{j-1}\,\Delta u_i^j = P_i - F_i^{j-1} = R_i \tag{5}$$

$$\Delta u_i = u_{i+1} - u_i \tag{6}$$

where K_i^{j-1} is the nonsingular tangent stiffness matrix, Δu_i^j is the incremental structure nodal displacement vector, P_i is the incremental load vector, F_i^{j-1} is the structure internal force vector, and i, j are the numbers of the load step and iteration.

The right side of Eq. (5) is known as a vector of out-of-balance (residue) loads due to nonlinearity. Convergence is assumed when

$$R_i < \varepsilon_R \ , \qquad \Delta u_i < \varepsilon_u \tag{7}$$

where R_i is the out-of-balance load, and ε_R and ε_u are tolerances.

4. NUMERICAL RESULTS

In the first example, the two-story one-bay frame as shown in Fig. 2 is analyzed with rigid and semi-rigid connections, with and without geometric nonlinearity. The connections have an initial stiffness of 206667 in-kip/rad, and an ultimate moment of 761 in-kip with a shape factor of 0.525 . The results of linear and nonlinear analysis are presented in Figs. 3-4.

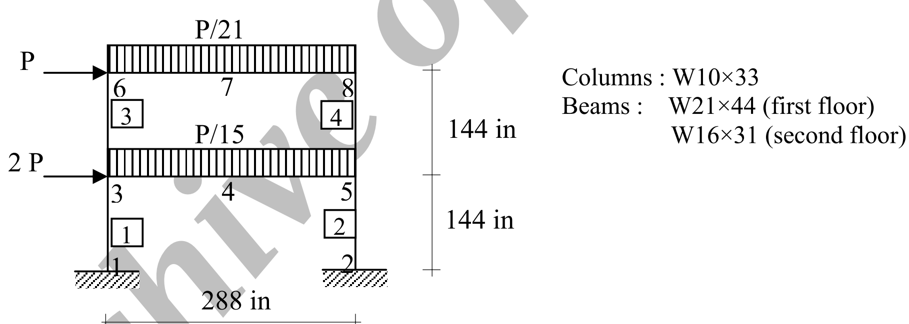

Fig. 2. Two-story one-bay frame with uniform and concentrated loads

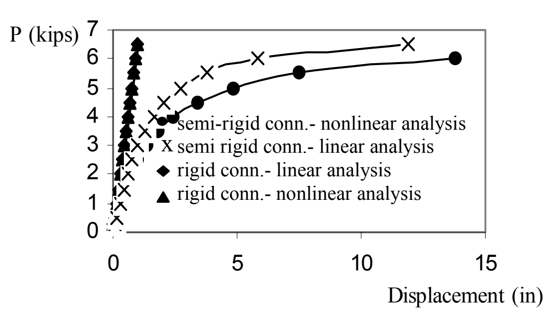

Fig. 3. The load-lateral displacement curves of node 8

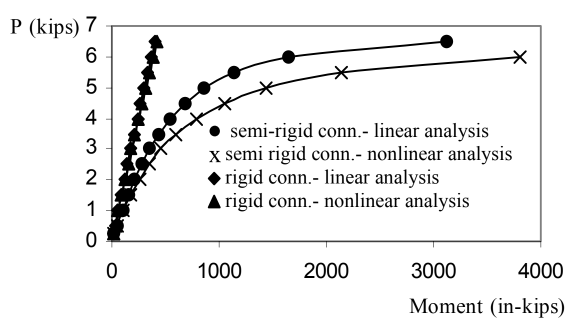

Fig. 4. The load-moment curves of node 1

As the second example, the frame with concentrated loads in Fig. 5 is analyzed for different connection stiffnesses and rigid end lengths. The definition of the fixity factor is given as $\gamma_i = \dfrac{1}{1+3EI/k_i L}$, where k_i is the rotational stiffness of the connection.

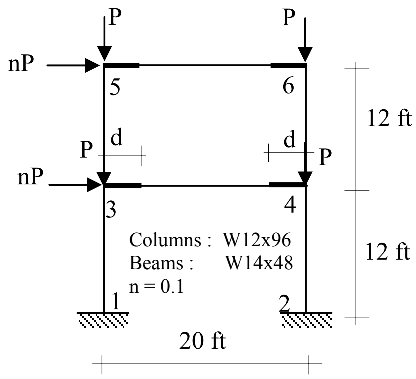

Fig. 5. Two-story one-bay frame with rigid end sections

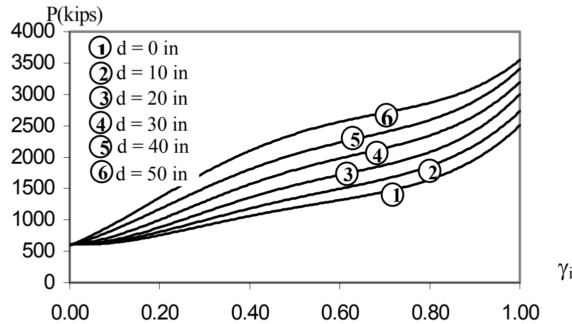

Fig. 6. The critical loads for the frame as a function of the fixity factor and the length of the rigid end sections

The variation of the critical load of the frame with the connection sitffness and the rigid end length is given in Fig. 6. The variations of the displacement and moment with the connection stiffness and rigid end length are given in Figs. 7-8 for P = 100 kips. The displacements and moments are normalized by dividing them by the corresponding results for the case of fully pinned joints.

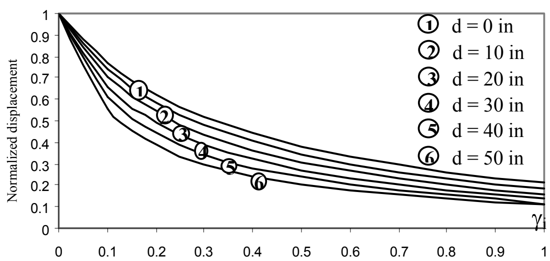

Fig. 7. The variation of displacement with connection stiffness and rigid end length at node 6

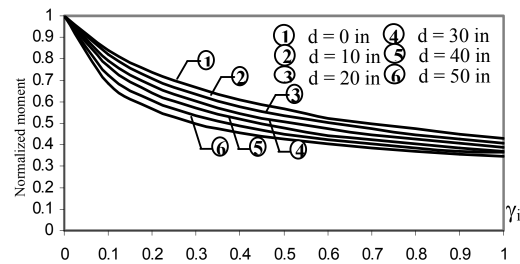

Fig. 8. The variation of moment with connection stiffness and rigid end length at node 1

The last example (Fig. 9), which is selected for comparison, is taken from Sekulovic and Salatic[7]. The critical loads of bifurcation type stability loss of a single bay frame with a different number of storeys found by the present method has been compared with those of [7].

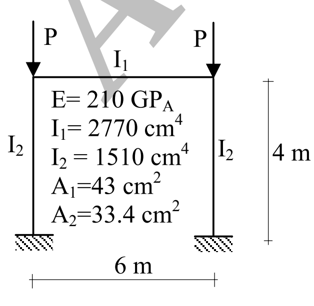

Fig. 9. The single bay frame with one

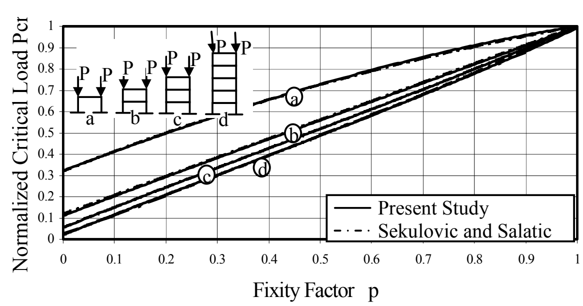

Fig. 10. Influence of connection flexibility on the critical load of the frame

5. CONCLUSIONS

In the numerical part of this study, some examples have been solved for observing the influence of the semi rigid connections on the bending response of frames. For frames with rigid connections and low levels of loading, the difference between the displacements and moments of linear and nonlinear analyses is very slight. However, for frames with semi-rigid connections, the displacements and moments take significantly fewer values in the linear case than in the nonlinear one for the same load, even when it is at a low level. Furthermore, as the load level increases, it is observed that, the linear analysis yields much lower critical loads compared to the nonlinear one.

The results of examples show that when the connections are pinned joints, the effect of the lengths of the rigid end sections on the critical load of the system is very slight, almost none. However, when the connections are semi-rigid, the critical load of the system increases with the increase in the lengths of the rigid end sections and the stiffness of the springs.

REFERENCES

1. Baniotopoulos, C. C. & Ivanyi, M. (2000). Semi-rigid joints in structural steelwork. CISM Lecture Notes 419, Springer-Verlag Vienna, ISBN 3-211-83331-5.
2. Ivanyi, M. & Skaloud, M. (1993). Stability problems of steel structures. CISM Lecture Notes, 323, Springer-Verlag, ISBN 0-387-82398-0.
3. Lightfoot, E. and Messurier, A. P. (1979). Elastic analysis of frame-works with elastic connections. *J. of Struct. Div. ASCE,* Vol. 100, pp. 1297-1309.
4. Ang, K. M. & Morris, G. A. (1984). Analysis of three-dimensional frames with flexible beam-column connections. *Can. J. Civil Eng.,* Vol. 11, pp. 241-254.
5. Barakat, M. & Chen, W. F.(1990). Practical analysis of semi-rigid frames. *Eng. J. AISC,* Vol. 27, No. 2, pp. 54-68.
6. King, W. S. & Chen, W. F. (1993). LRFD analysis for semi-rigid frame design. *Eng. J. AISC*, Vol. 30, No. 4, pp. 130-140.
7. Sekulovic, M. & Salatic, R. (2001). Nonlinear analysis of frames with flexible connections. *Computers and Structures*, Vol. 79, pp. 1097-1107.