Iranian Journal of Science & Technology, Transaction B, Engineering, Vol. 31, No. B5, pp 577-581
Printed in The Islamic Republic of Iran, 2007
© Shiraz University

"Research Note"

ELASTIC ANALYSIS OF REINFORCED SOILS USING POINT INTERPOLATION METHOD*

S. M. BINESH, N. HATAF** AND A. GHAHRAMANI

Dept. of Civil Eng., Shiraz University, Shiraz. I. R. of Iran
Email: nhataf@shirazu.ac.ir

Abstract– A recently proposed meshless method, which is called the radial basis point interpolation method (RBPIM), is used for the mechanical analysis of reinforced soils. The media of reinforced soil is divided into three parts: soil, reinforcement and interface layer. The displacement field in each part is constructed by point interpolation. A code has been developed based on RBPIM and the validity of this code has been investigated by solving some examples at the end of the paper.

Keywords– Reinforced soil, meshless method, radial basis, point interpolation

1. INTRODUCTION

The use of fabric or polymer grid reinforcing material to improve the strength and stability of geotechnical structures is increasing. Generally, the investigation of the behavior of reinforced soil structure through field observation or laboratory modeling is expensive and time consuming. So it is necessary to develop a numerical solution which may be used to predict the load-deformation characteristics of a reinforced structure. One of the most popular methods for the numerical modeling of reinforced soils is the finite element method (FEM). However this method does have some deficiencies, most of which are related to mesh definition. Therefore there is a need to develop a series of methods which are independent of mesh definition.

A new family of numerical methods globally coined as meshless or meshfree are growing rapidly. There have been various meshless methods such as smoothed particle hydrodynamics [1], finite cloud [2], diffuse element [3], element free Galerkin [4], reproducing kernel particle method [5], point interpolation method [6], etc. Their main characteristic is that there is no need for a mesh in the traditional sense.

In this paper a recently proposed radial basis point interpolation method (RBPIM) is used for the numerical analysis of reinforced soils. Firstly the basics of RBPIM are described, then the application of RBPIM to the analysis of reinforced soils is explained. Thereafter, the results of some numerical examples are presented to verify the accuracy of a developed computer code.

2. POINT INTERPOLATION METHOD

In this section, a brief introduction of the point interpolation method (PIM) is presented. PIM was first developed by Liu and Gu [6] based on the Galerkin weak form.

In this method, a scalar function $u(\mathbf{X})$ in domain Ω is approximated by a linear combination of basis functions. So

$$u(\mathbf{X}) = \sum_{i=1}^{n} p_i(\mathbf{X}) a_i = \mathbf{P}^T(\mathbf{X})\mathbf{a} \tag{1}$$

*Received by the editors May 9, 2006; final revised form April 7, 2007.
**Corresponding author

Archive of SID
www.SID.ir

where, n is the number of nodes in the support domain of point $\mathbf{X}$, $p_i(\mathbf{X})$ is a monomial in the space coordinate $\mathbf{X}^T = [x, y]$, and a_i is the coefficient for $p_i(\mathbf{X})$.

By letting the interpolation function pass through the function values at each scattered node within the defined domain of support, we have

$$\mathbf{U_s} = \mathbf{P_Q a} \tag{2}$$

where,

$$\mathbf{a} = [a_1, a_2, ..., a_n] \tag{3}$$

$$\mathbf{U_s} = [u_1, u_2, ..., u_n] \tag{4}$$

$$\mathbf{P_Q} = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 & . & . & . \\ 1 & x_2 & y_2 & x_2 y_2 & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & x_n & y_n & x_n y_n & . & . & . \end{bmatrix} \tag{5}$$

where u_i is the nodal value of u at node i. From Eq. (2)

$$\mathbf{a} = \mathbf{P_Q^{-1} U_s} \tag{6}$$

Hence, substituting Eq. (6) into Eq. (1) gives:

$$u(\mathbf{X}) = \mathbf{\Phi}(\mathbf{X}) \tag{7}$$

where the shape function $\mathbf{\Phi}(\mathbf{X})$ is defined by:

$$\mathbf{\Phi}(\mathbf{X}) = \mathbf{P^T}(\mathbf{X})\mathbf{P_Q^{-1}} = [\varphi_1(\mathbf{X}) \quad \varphi_2(\mathbf{X}) \quad . \quad . \quad . \quad \varphi_n(\mathbf{X})] \tag{8}$$

As can be seen, the shape functions constructed possess the Kronecker delta function property, which allows simple imposition of essential boundary conditions as in the conventional finite element method (FEM). However, the main challenge is the singularity of matrix $\mathbf{P_Q}$. Radial basis functions (RBF) can be used as a solution for eliminating the singularity of $\mathbf{P_Q}$ [7]. In this paper multi-quadratic RBF [7] has been used.

3. REINFORCED SOILS MODELLING

In general, for the numerical study of reinforced soils, three parts of soil, reinforcement and the interface layer are investigated separately. This research is limited to the linear elastic behavior of each part. Soil is defined as scattered nodes with two degrees of freedom. Each reinforcement is modeled by a set of nodes, arranged along two parallel lines. Interface layers are modeled by the method of Zhang et al.[8]. The main advantage of using the meshless method for the numerical analysis of reinforced soil is that there is no geometrical constrain about reinforcement modeling. It is notable that the visibility criterion [9] is used for support domain determination and supports of different parts are completely separated from each other. In other words, supports of quadrature points in soil media do not include the nodes of reinforcements and vice versa.

Having combined different stiffness matrices for different parts (i.e. soil, reinforcements, and interfaces) developed from RBPIM shape functions, the resulting system of discrete equations can be written as:

Archive of SID
www.SID.ir

$$\mathbf{KU} = \mathbf{F} \tag{9}$$

In which

$$\mathbf{K} = \mathbf{K^s} + \mathbf{K^r} + \mathbf{K^i} \tag{10}$$

where, $\mathbf{K^s}, \mathbf{K^r}, \mathbf{K^i}$ are the stiffness matrices of soil, reinforcement and interface layer respectively. $\mathbf{U}$ is the vector of the collection of degrees of freedom in the problem. $\mathbf{F}$ is the vector of external force. Solving Eq. (9) determines the unknown displacement due to external loads.

4. NUMERICAL STUDY

Based on the principles of RBPIM, a computer code is developed. In order to verify the accuracy and capability of the method, two examples are investigated in this section. The first example is related to the solution of a cantilever beam. The second one investigates a discontinuous reinforced media.

Example 1. A cantilever beam which has a 12^m height and 48^m length is subjected to a parabolic shear load of 1000^N, as shown in Fig. 1. The Young's modulus and Poisson's ratio of beam material is 3×10^7 N/m^2 and 0.3 respectively. Under plane stress conditions, the exact solution of this problem is given in [10].

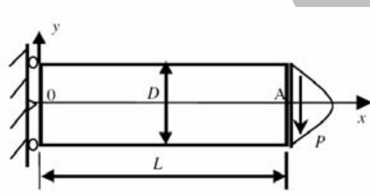


Fig. 1. Cantilever beam loaded with a parabolic shear load

The problem has been solved by the computer code developed, based on RBPIM. The results of numerical analysis for deflection and stresses are compared with the exact solution in Fig 2. The plots show excellent agreement between the exact solution and the numerical results.

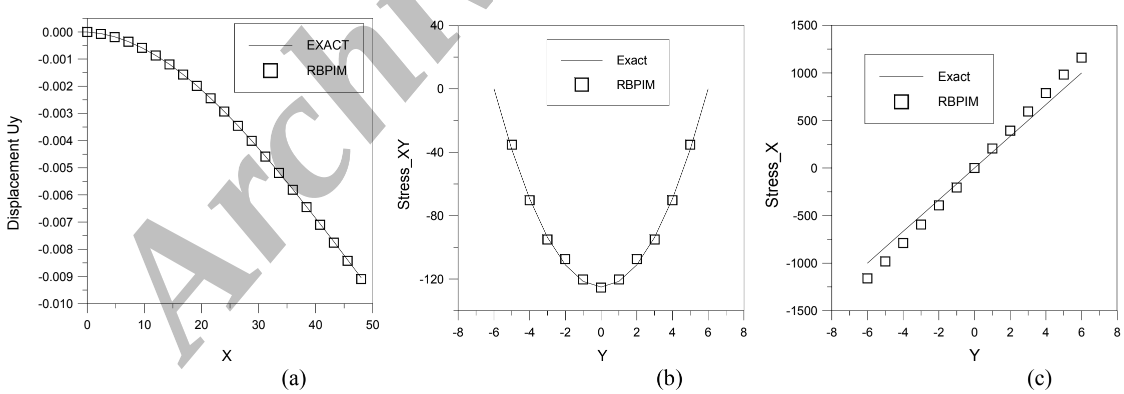
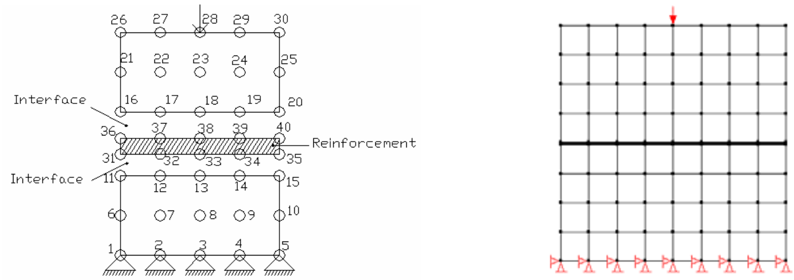


Fig. 2. Comparison between the exact solution results and RBPIM results for
a) deflection b) stress_xy c) stress_x

Example 2. Since the authors could not find any exact solution for reinforced soil problems, a hypothetical problem is introduced and the results of the code have been compared with the results of the finite element program, SIGMAW [11], for the same problem.

A 2×2 m^2 soil box, which is reinforced at mid height, is shown in Fig. 3. A concentrated 50^{KN} force is applied on top of the box. This problem has been solved by FEM and RBPIM and the results of these methods are compared in Fig. 4 for horizontal and vertical displacement at each node.

Archive of SID
www.SID.ir



(a) (b)

Fig. 3. Soil box model in a) meshless b)FEM

As can be seen from Fig. 4, there is a very good agreement between the results of the two methods and this confirms the applicability of the code for the analysis of reinforced soils in elasto-static problems.

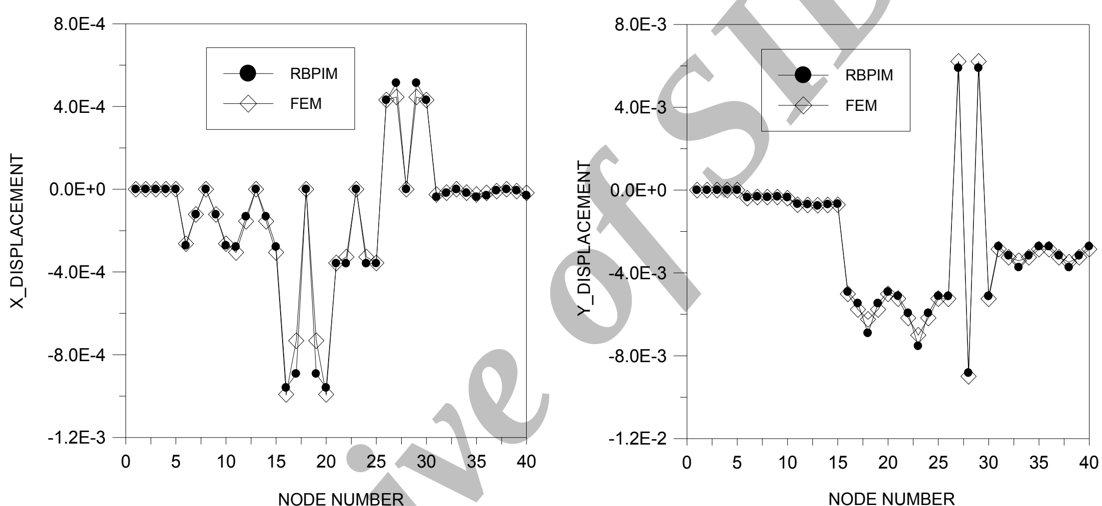


Fig. 4. Comparison between the FEM results and RBPIM results
for a) X_displacement b) Y_displacement

5. CONCLUSION

For the first time a meshless model, based on the point interpolation method, is proposed in this paper for the mechanical analysis of reinforced soils. In this model, no finite element mesh is required and only a number of points are distributed in the domain of the problem. Hence, the time-consuming mesh generation is avoided. Besides, there is no need to consider any geometrical constrains related to reinforcement modeling. The present model can be extended to three dimensional and nonlinear analyses.

NOMENCLATURE

a_i	coefficient for monomials
$\mathbf{K}$	stiffness matrix
p_i	monomial of polynomial basis function
$\mathbf{P_Q}$	polynomial basis moment matrix
u	field variable
φ	shape function

Archive of SID
www.SID.ir

REFERENCES

1. Lucy, L. B. (1997). A numerical approach to the testing of fission hypothesis. *The Astronomical Journal*, Vol. 82, No.12, pp. 1013-1024.
2. Duarte, C. & Oden, J. T. (1996). An hp adaptive method using clouds. *Computer Methods in Applied Mechanics and Engineering*, Vol. 139, pp. 237-262.
3. Nayroles, B., Touzot, G. & Villon, P. (1992). Generalizing the finite element method: diffuse approximation and diffuse elements. *Computational Mechanics*, Vol. 10, pp. 307-318.
4. Belytschko, T., Lu, Y.Y. & Gu, L. (1994). Element-free Galerkin methods. *International Journal for Numerical Methods in Engineering*, Vol. 37, pp. 229-256.
5. Liu,W. K., Jun, S. & Zhang, Y. F. (1995). Reproducing kernel particle methods. *International Journal for Numerical Methods in Engineering*, Vol. 20, pp. 1081-1106.
6. Liu, G. R. & Gu, Y. T. (2001). A point interpolation method for two-dimensional solids. *International Journal for Numerical Methods in Engineering*, Vol. 50, pp. 937-951.
7. Liu, G.R. (2002). *Mesh free methods: moving beyond the finite element method*. Florida, CRC Press, p. 171.
8. Zhang, X., Lu, M. & Wegner, J. L. (2000). A 2-D meshless model for jointed rock structures. *Int. J. Numer. Meth. Engng*. Vol. 47, pp.1649-1661.
9. Belytschko,T. & Fleming, M. (1999). Smoothing, enrichment and contact in the element-free galerkin method, *Computers and Structure*, Vol. 71, pp.173-195.
10. Timoshenko, S. P. & Goodier, J. N. (1970). *Theory of elasticity*. New York, McGraw-Hill.
11. Sigma/W Manual, (2000). *User Guide*, Geo-Slope International Ltd. Calgary, Alberta, Canada.

Archive of SID

www.SID.ir