

## “Research Note”

# LATERAL-TORSIONAL BUCKLING OF CASTELLATED BEAMS\*

H. SHOWKATI

Dept. of Civil Engineering, University of Urmia, Urmia, P.O.Box165, 57159, I. R. of Iran  
Email: h.showkati@mail.urmia.ac.ir

**Abstract**– In this paper, the bending behavior of unbraced CSB is investigated and some empirical equations are proposed to predict the bending coefficient of  $C_b$ . The acquired results are compared with some published papers in the technical literature, and by applying the proposed modification factor  $C_c$  on the relations of I-sections, a very good agreement is gained.

**Keywords**– Castellated beam, lateral-torsional buckling, bending coefficient, castellation factor

## 1. INTRODUCTION

Castellated steel beams (CSB) are lightweight sections usually made from I-sections by the castellation process. When the distortional and local buckling of section components and overall lateral buckling of such beams are prevented, it may be possible to use their full plastic bending capacities. If a local and overall lateral bracing is not provided, the bending behavior of the beam will be a function of some of the slenderness parameters [1, 3]. In this paper, a model of the eigenvalue buckling analysis is presented for unbraced CSB using the finite element method. The beam models are made of lightweight wide-flange steel sections (IPE) by the castellation process, with an original depth of  $d$  varying from 100mm to 300mm. So the ultimate section heights are between 150mm to 450mm (CPE10~CPE30). On lateral buckling of plane-webbed steel beams, several published investigations are available, like Chen and Lui [4], Hitahori and Kubo [5] and N. S. Trahair [6]. Nethercot and Kerdal [7] performed two series of tests on the lateral stability of CSB. Such a study is performed to apply the AISC-LRFD (1993) recommendations in the design of CSB [2].

## 2. LATERAL BUCKLING OF BEAMS

The governing differential equation for I-beams under the effect of uniform bending is:

$$EC_w \frac{d^4 \gamma}{dz^4} - GJ \frac{d^2 \gamma}{dz^2} - \frac{M_0^2}{EI_y} \gamma = 0 \quad (1)$$

For a nontrivial solution the smallest root is obtained as:

$$M_{0e} = \sqrt{\frac{\pi^2 EI_y}{L_b^2} \left( \frac{\pi^2}{L_b^2} EC_w + GJ \right)} \quad (2)$$

\*Received by the editors April 10, 2007; final revised form September 10, 2007.

Where  $L_b$  is the laterally unbraced span of the beam. Equation (2) could be employed to study the lateral-torsional buckling of CSB if is modified by  $C_c$  for the castellation process. It is also modified by  $C_b$  for bending gradients such as the effect of loading conditions. So we have:

$$M_e = C_b C_c \sqrt{\frac{\pi^2 EI_y}{L_b^2} \left( \frac{\pi^2}{L_b^2} EC_w + GJ \right)} \quad (3)$$

Equation (3) predicts the lateral-torsional buckling moment of CSB under the effect of various loading cases. The coefficients of  $C_b$  and  $C_c$  are calculated in this paper.

The adjacent equilibrium path in a deformed structure is used to calculate its buckling load [8]. This method of analysis, namely buckling analysis, is valid until the material remains elastic. A compact and unsupported elastic beam has a similar behavior and could be designed by the same method [9]. In this paper the bifurcation buckling method is employed to study the critical bending moment of CSB with different unbraced lengths and several loading cases. A plane four-noded finite element of SHELL181 with 6 degrees of freedom at each node has been used for the modeling of all beams. The modulus of elasticity and poisson's ratio are taken as  $E = 2.1 \times 10^5 \text{ Mpa}$  and  $\nu = 0.3$  respectively.

### 3. SIMPLY SUPPORTED CSB

In this study the results of the buckling analysis of CSB are calculated for ten groups of CPE10~CPE30 under the effect of uniform loading condition (Fig.1). It is possible to calculate the bending capacity of such beams approximately by Eq. (3). Equation (2) is derived for an elastic beam loaded by pure bending on its neutral axis. In the loading case of uniform bending the coefficient of  $C_b$  is equal to 1.0. Some of the results of this study are compared with the pure bending case of beams and shown in Fig.2. The exact values of bending coefficients are listed in Table 1 with a mean value of 0.823, from which a simple  $C_b$  equation can be obtained as:

$$C_b = 0.321 \left( \frac{L_b}{r_y} \right)^{0.174} \quad (4)$$

The  $C_b$  for plane-webbed I sections under uniform loading on the top flange is introduced as:

$$C_b = 0.373 \left( \frac{L_b}{r_y} \right)^{0.16} \quad (5)$$

Which is suitable for the range of  $150 < L_b / r_y < 400$  for usual I-beams. In Fig. 3 two equations of (4) and (5) are compared to show the effect of the castellation process. This process has been shown to reduce  $C_b$  between 6.5% and 7.7% depending on the shape properties. For the current steel material and the range of this study a simplified form is found as:

$$C_b = 0.503 \left( \frac{L_b}{h} \right)^{0.18} \quad (6)$$

This will be used to calculate the value of  $C_b$  in CSB under the effect of the uniform load on the top flange. By introducing a castellation factor  $C_c$  and a properly defined parameter of  $C_b$  for CSB, the most accurate bending capacity of beams under a uniform load will be calculated by Eq. (3). For the value of  $C_b$ , Eq. (6)

is introduced. Also, according to the acquired results in this study, it is found that  $C_e=1.056$  for CPE shapes.

Table 1.  $C_b$  Coefficients for CSB under uniform loading

CPE10	CPE14	CPE18	CPE22	CPE27
0.900	0.843	0.815	0.789	0.766

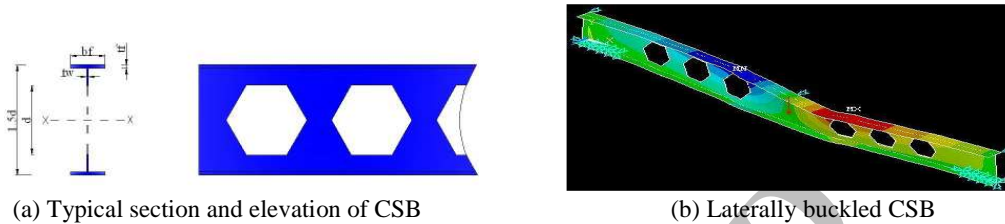


Fig. 1. Geometry and buckling form of CSB

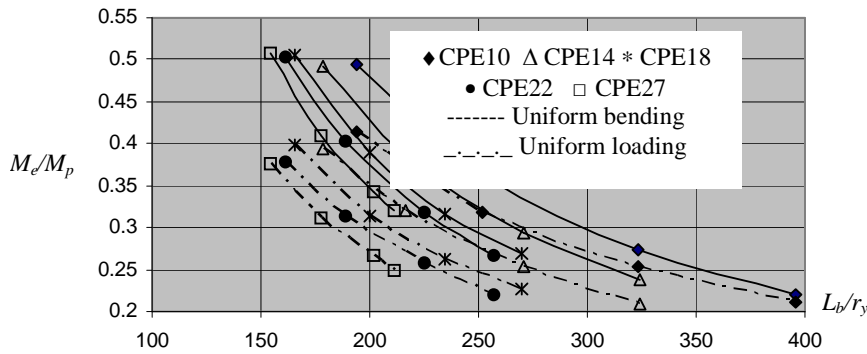


Fig. 2. Comparison of  $M_e$  in uniform bending and uniform loading

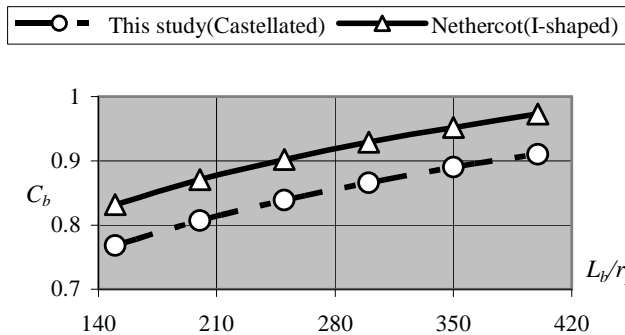


Fig. 3. Comparison of  $C_b$  between CSB and I beams

#### 4. CONCLUSION

Some of the most important remarks in this research are outlined as follows:

- Unlike the usual cases when a uniform load is applied on the top flange of a simple castellated beam, the bending moment is less than the pure bending case, with a mean bending coefficient of 0.823.
- Elastic bending capacities of I-shaped and CSB under uniform loading on the top flange have a difference of between 4.9% and 8.6%, depending on the section properties.
- Several simple expressions are derived and suggested to predict the value of the bending coefficient of  $C_b$  in different conditions.

- A new coefficient  $C_c$  is proposed for CSB to be used in the theoretical results. Its value for CPE shapes is equal to 1.056.
- The bending coefficient in a centrally-loaded simple beam is found to be 1.756, with a maximum error amount of 2%.

**Acknowledgement:** This research program has been supported by the Grant No. NRCI1437 of National Research Projects and also with the support of the National Research Council of the Islamic Republic of Iran.

#### REFERENCES

1. Showkati, H. (2002). Theoretical and numerical buckling study of CPE castellated beams. Final report of NRCI1437, Iran.
2. Showkati, H. (2002). Analysis and design of castellated beams using AISC-LRFD. Fifth International Conference on Civil Engineering, University of Ferdowsi, Iran.
3. Zirakian, T. & Showkati, H. (2006). Distortional buckling of castellated beams. *J. Construct. Steel Research*, Vol. 62, No. 9, pp. 863-871.
4. Chen, W. F. & Lui, E. M. (1987). *Structural stability*. Elsevier.
5. Hitahori & Kubo, (1998). *Thin-walled structures; research and development*. N. E. Shanmugan (ed.), Elsevier.
6. Trahair, N. S. (1993). *Flexural-torsional buckling of structures*, E & FN Spon.
7. Nethercot, D. A. & Kerdal, D. (1982). Lateral-torsional buckling of castellated beams. *The structural engineer*, Vol. 68B, No. 3.
8. Brush, D. O. & Almroth, B. O. (1975). *Buckling of bars, plates and shells*. McGraw-Hill.
9. AISC, (1993). Load and resistance factor design specification for structural steel buildings. Chicago.