

## A COMPLETE PROCEDURE TO DETERMINE EARTH FAULT CURRENT DISTRIBUTION AND SPLIT FACTOR FOR GROUNDING GRID DESIGN OF HV SUBSTATIONS\*

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**Abstract**– To design a safe substation grounding grid, it is necessary to compute the split factor for earth fault current and also current distribution in other possible paths. In this paper, a novel, simple and accurate method is developed for determination of this factor. In the proposed algorithm grounding grid impedance of adjacent substations, dissimilar tower footing resistances, different parallel circuits, more than one earth wires and different spans in transmission lines can be considered. In addition, the formulation for the earth faults through line towers is presented. Finally, the results are compared with the addressed cases that are comparable and show good agreement and accuracy.

**Keywords**– Split factor, earth fault current, touch and step voltages, grounding grid design, HV substations

### 1. INTRODUCTION

The earth fault currents in a power system circulate through transmission line earth wires, tower footing resistances, substation grounding grids and phase conductors connected to a faulted point as shown in Fig. 1.

To design a safe substation grounding grid and prevent any over-design, it is of prime importance to compute, correctly the split factor of an earth fault, and also current distribution in other possible paths by modeling overhead lines and adjacent substation grounding grids.

The existing grounding standard for safety in AC Substation grounding such as ANSI/ IEEE Std 80-2000[1] does not provide a direct approach for the determination of the split factor for systems with the following conditions:

- a. Different length of spans in incoming/outgoing lines,
- b. Different tower footing resistances,
- c. Existence of disjointed earth wires.

Along with these conditions, different methods have been introduced to determine this factor. Some have made different approximations and simplifications [2-6], while others have been recorded as analytical methods [7-11]. In general, the ability of methods to consider the contribution of the following parameters in a simple way may be proposed as appropriate criteria for evaluating their performances:

- Mutual inductances between phase conductors and earth wires,
- Different spans of incoming/outgoing overhead lines,
- Different tower footing resistances,
- More than one earth wire, either fully connected or disjointed,

\*Received by the editors April 10, 2007; final revised form December 30, 2007.

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- Grounding grid of adjacent substations,
- Fault location, either inside or outside the substation,
- Earth fault through short circuit to a tower of an overhead line,
- Large and complicated power system.

It can be stated that none of the methods presented can include all of the above mentioned parameters. In this paper, a novel, simple and accurate method is developed for the determination of the split factor of earth fault current, where all of the above-mentioned parameters can be considered. Accordingly, the complete formulation for general cases; first for line-to-earth short circuits inside and then outside the substation and also through any tower of outgoing overhead transmission lines are presented, and finally the closed form equations for considering special conditions (such as double-circuit lines, multi earth wires and disjointed earth wires) are given. The results for two case studies are given and compared with some previous methods and the accuracy of the proposed method is shown.

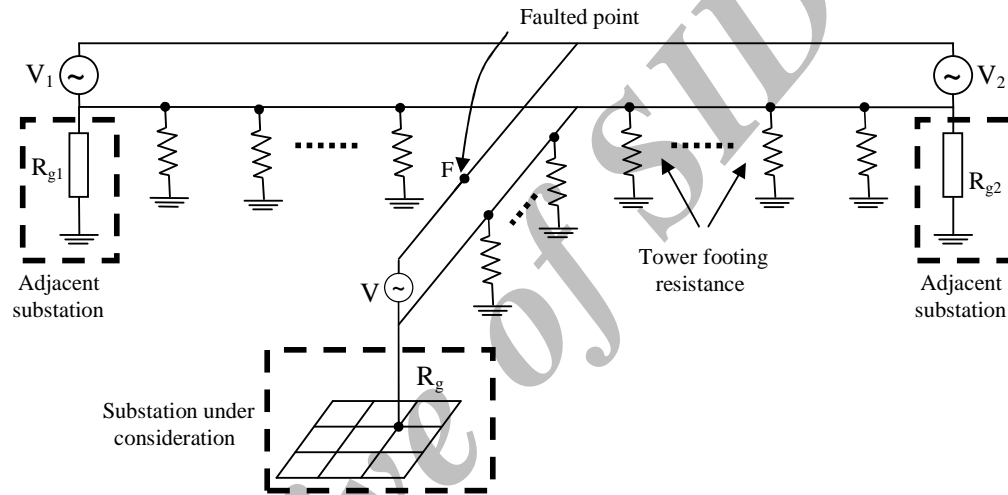


Fig. 1. The power network configuration (only the faulted phase conductor is shown)

## 2. MAIN PARAMETERS IN THE MODEL

A network containing two substations and a transmission line in-between is shown in Fig. 2, where  $I_g$  is the portion of the earth fault current passing through the substation under consideration,  $I_l$  is the section of  $I_g$  which flows through the earth wires and  $I_n$  is the outgoing current from earth wires. To determine split factor for an earth fault occurring at point F in the vicinity of the substation under consideration, different components of the mentioned network should be modeled as in the following steps. The modeling and computation for earth faults located outside the substation on an outgoing overhead line is presented in sections 3.5-b, 3.5-c.

1. Faulted phase conductor is modeled by self impedance of a phase conductor and its mutual impedances with earth wires and a series current source ( $I_r$ ) which equals to the portion of the fault current through a phase conductor ( and is calculated by any short circuit program in the power system). Modeling the case of double circuit lines is explained in 3.5.

2. The earth wire(s) and tower footing resistances are represented by a ladder network of impedances and resistances, where the impedance  $Z_{sj}$  is the self impedance of  $i$ -th span of earth wire according to its length, and  $R_{Tj}$  is the footing resistance of  $i$ -th tower. The mutual impedance between the phase conductor and  $i$ -th span of earth wire is represented by  $Z_{mj}$ .

3. Grounding grid of the substation under consideration is modeled with a resistance  $R_g$ .

4. Adjacent substations grounding grids are each modeled with a resistance, e.g.  $R_{gm}$  for  $m=1, \dots, n$ .

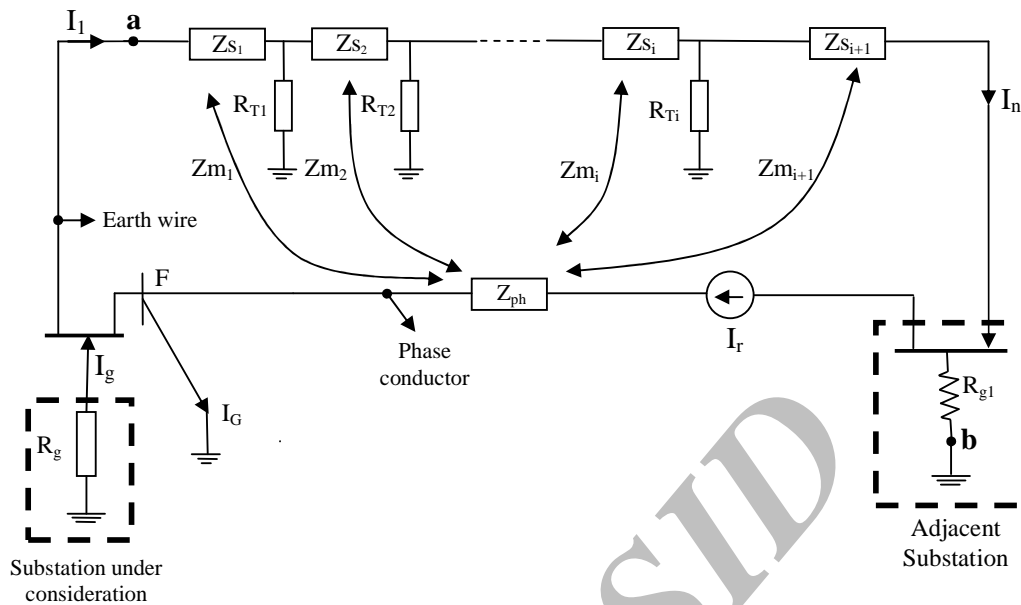


Fig. 2. Complete model for the network connected to the faulted point inside the substation (only the faulted phase conductor is shown)

It is not necessary to mention that the earth fault current, ( $I_G$ ) is also calculated through the short circuit study of the power system.

### 3. THE PROPOSED METHOD

In the proposed method, the mutual coupling between the earth wires and phase conductors are waived in the first step and the equivalent circuit of earth wires, tower footing resistances and adjacent substations grounding grid are determined. Then by compensating the effect of those mutual impedances, the fault current distribution among the grounding grid of the substation under consideration and other components of the power system is calculated.

Step by step implementation of this method is as follows:

#### a) Constructing initial equivalent circuit

Typical initial equivalent circuit of earth wire(s) and tower footing resistances is shown in Fig. 3 (without considering mutual impedances between earth wire and phase conductor). In this circuit the length of spans and, ultimately, their self impedances ( $Z_{sj}$ ) and tower footing resistances ( $R_{Tj}$ ) may be different.

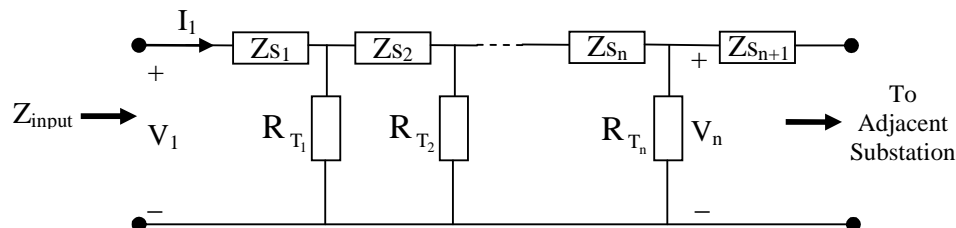


Fig. 3. Equivalent circuit of earth wire(s) and towers

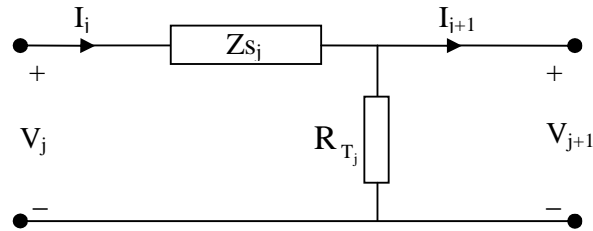


Fig. 4. Equivalent circuit of one span and tower

The overall transfer function of this ladder network can be constructed according to the transmission matrix for a two-port module, as in Fig. 4.

In Fig. 4, the transmission matrix is introduced as:

$$\begin{bmatrix} V_j \\ I_j \end{bmatrix} = \begin{bmatrix} t_{j1} & t_{j2} \\ t_{j3} & t_{j4} \end{bmatrix} \begin{bmatrix} V_{j+1} \\ I_{j+1} \end{bmatrix} = T_j \cdot \begin{bmatrix} V_{j+1} \\ I_{j+1} \end{bmatrix} \quad (1)$$

where:

$$t_{j1} = 1 + \frac{Z_{sj}}{R_{Tj}}, \quad t_{j2} = Z_{sj}, \quad t_{j3} = \frac{1}{R_{Tj}}, \quad t_{j4} = 1 \quad (2)$$

Consequently, the following relation is used to find the input impedance of the ladder network:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_n \\ I_n \end{bmatrix} = \prod_{j=1}^n T_j \cdot \begin{bmatrix} V_n \\ I_n \end{bmatrix} \quad (3)$$

and the input impedance is:

$$Z_{input} = V_1 / I_1 \quad (4)$$

#### b) Including adjacent substations grounding grid

In this stage, the contribution of an adjacent substation grounding grid in Fig. 2 is converted as in Fig. 5.

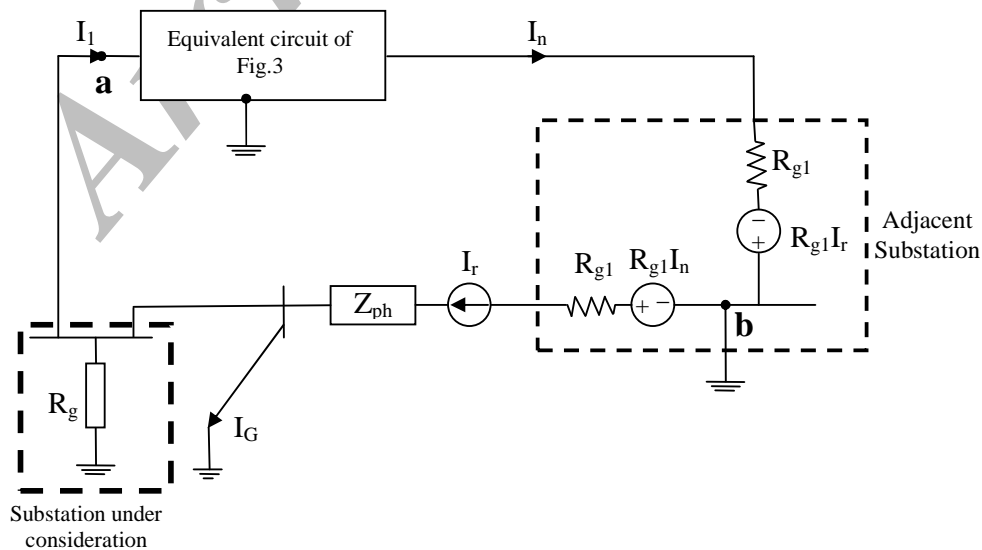


Fig. 5. Combining adjacent substation grounding grid effect with the equivalent circuit

The Thevenin's equivalent circuit, seen between points "a" and "b" in Fig. 5, is shown in Fig. 6 and its parameters are introduced by the following equations:

$$Z_e = \frac{T_{11}(Z_{s_{n+1}} + R_{gl}) + T_{12}}{T_{21}(Z_{s_{n+1}} + R_{gl}) + T_{22}} \quad (5)$$

$$k_e = \frac{T_{12}R_{gl}}{Z_{s_{n+1}} + R_{gl}} - \frac{T_{22}R_{gl}[T_{11}(Z_{s_{n+1}} + R_{gl}) + T_{12}]}{[T_{21}(Z_{s_{n+1}} + R_{gl}) + T_{22}](Z_{s_{n+1}} + R_{gl})} = \frac{-R_{gl}}{T_{21}(Z_{s_{n+1}} + R_{gl}) + T_{22}} \quad (6)$$

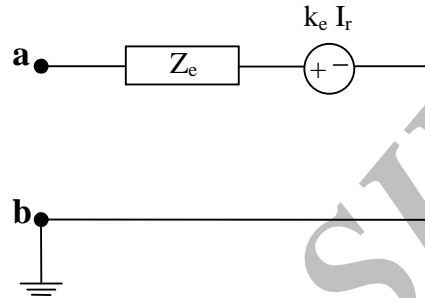


Fig. 6. Thevenin's equivalent circuit

### c) The final equivalent circuit

By substituting the equivalent circuit of Fig. 6 and the inclusion of the total mutual impedance between the earth wire(s) and the faulted phase conductor, the final equivalent circuit for the network of Fig. 2 is obtained as shown in Fig. 7.

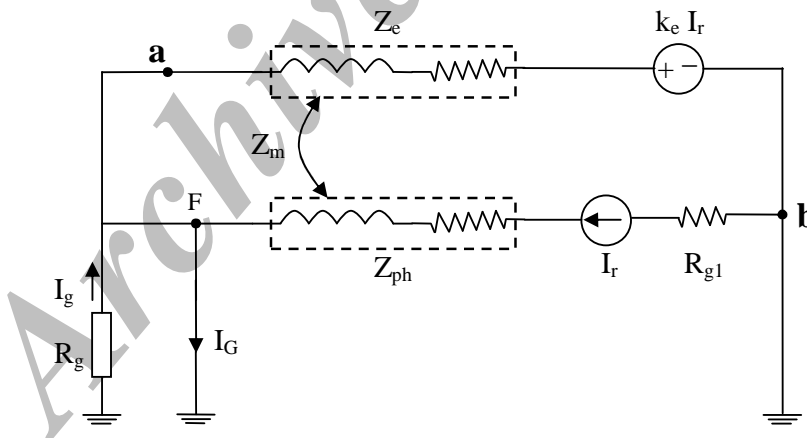


Fig. 7. The final equivalent circuit with one adjacent substation

Following the same procedure, the equivalent circuit for a substation with N outgoing transmission lines and adjacent substations can be found as shown in Fig. 8. It must be remembered that the fault currents coming through each line ( $I_{rj}$ , for the  $j$ th transmission line) are computed through short circuit study.

### d) Computing the split factor

According to Fig. 8, the mesh equations can be realized and introduced in matrix form as:

$$\begin{bmatrix} R_g + Z_{e1} & R_g & \dots & R_g \\ \vdots & \vdots & \ddots & \vdots \\ R_g & R_g + Z_{ej} & \dots & R_g \\ \vdots & \vdots & \ddots & \vdots \\ R_g & \dots & R_g & \dots & R_g + Z_{eN} \end{bmatrix} \begin{bmatrix} I_{e1} \\ \vdots \\ I_{ej} \\ \vdots \\ I_{eN} \end{bmatrix} = \begin{bmatrix} (Z_{m1} - k_{e1})I_{r1} - R_g(I_G - \sum_{i=1}^N I_{ri}) \\ \vdots \\ (Z_{mj} - k_{ej})I_{rj} - R_g(I_G - \sum_{i=1}^N I_{ri}) \\ \vdots \\ (Z_{mN} - k_{eN})I_{rN} - R_g(I_G - \sum_{i=1}^N I_{ri}) \end{bmatrix} \quad (7)$$

where  $I_{ej}$  is the part of the fault current through the earth wire of j-th incoming/outgoing transmission line. Consequently, the split factor ( $S_f$ ) in the substation under consideration is obtained by:

$$S_f = \frac{I_g}{I_{\text{fault}}} = \frac{I_G - \sum_{j=1}^n (I_{rj} - I_{ej})}{I_{\text{fault}}} \quad (8)$$

where  $I_{\text{fault}}$  in the denominator is the same as  $I_G$  whenever the split factor is computed for an earth fault outside and/or in the vicinity of the substation. In the case of an earth fault inside the substation under consideration, the same equations are used, provided that  $I_G$  and  $I_{\text{fault}}$  are set to zero and the short circuit current comes through the faulted transmission line, i.e.  $I_{rj}$ , respectively.

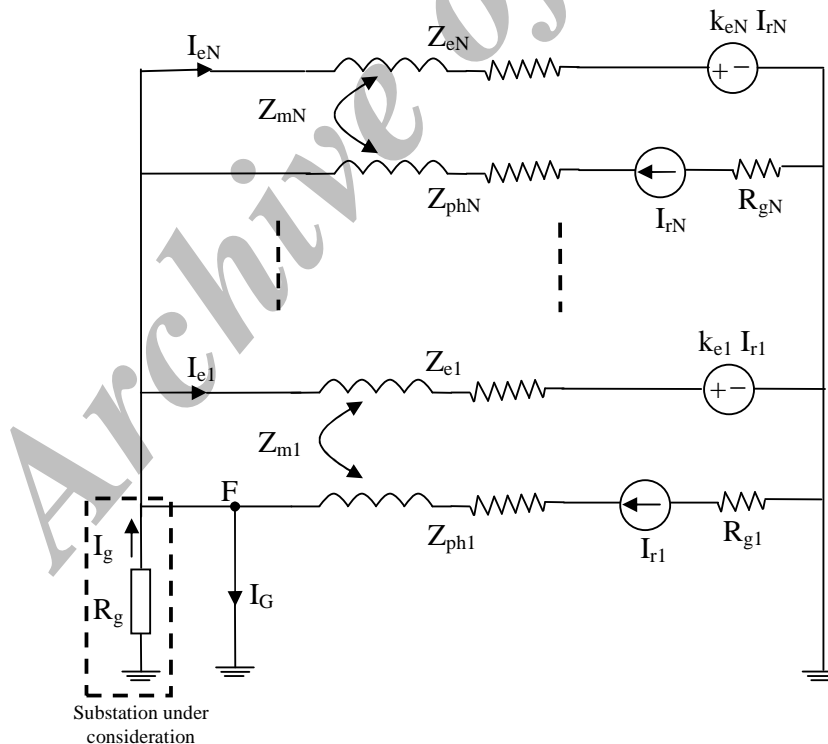


Fig. 8. General equivalent circuit

#### e) Special conditions

In addition to the introduced abilities of the proposed method, the following cases can also be analyzed by this procedure through some modifications in Eq. (7).

**1. Existence of double circuit transmission lines:** As shown in Fig. 9, Eq. (7) is valid for double circuit transmission lines provided that  $I_{rj}$  are substituted by  $I_{rj} + I'_{rj}$ , where  $I_{rj}$  is the fault current passing through the first circuit and  $I'_{rj}$  is the fault current coming through the second circuit of the  $j$ -th transmission line. This derivation assumes that the mutual impedances between each one of the circuits and earth wire are identical.

If the assumption of identical mutual impedances between similar phases of double circuit lines with earth wires are not valid or not as accurate as is desired, the  $j$ -th element of the right-hand side of Eq. (7) is replaced by:

$$\text{The } j\text{-th element of right-hand side of Eq. (7)} = (Z_{mj} I_{rj} + Z'_{mj} I'_{rj} - k_{ej} (I_{rj} + I'_{rj})) - R_g (I_G - \sum_{i=1}^N I_{ri}) \quad (9)$$

where  $Z_{mj}$ ,  $Z'_{mj}$  are the mutual impedances between the first and second circuits of the faulted line with the earth wires, respectively.

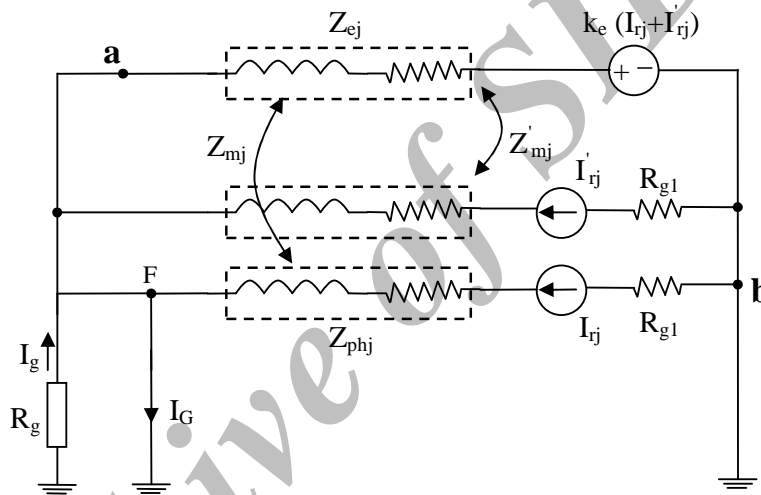


Fig. 9. The final equivalent circuit for double circuit transmission lines (with one adjacent substation)

**2. Effect of fault location on  $S_r$ :** Eq. (7) has been derived for the fault location in the vicinity of the substation under consideration. If an earth fault occurs on the  $j$ -th transmission line and in a distance  $\ell$  (as the percentage of the transmission line length), as shown in Fig. 10, the  $j$ -th element of the right-hand side vector of Eq. (7) is modified and the whole equation is substituted by:

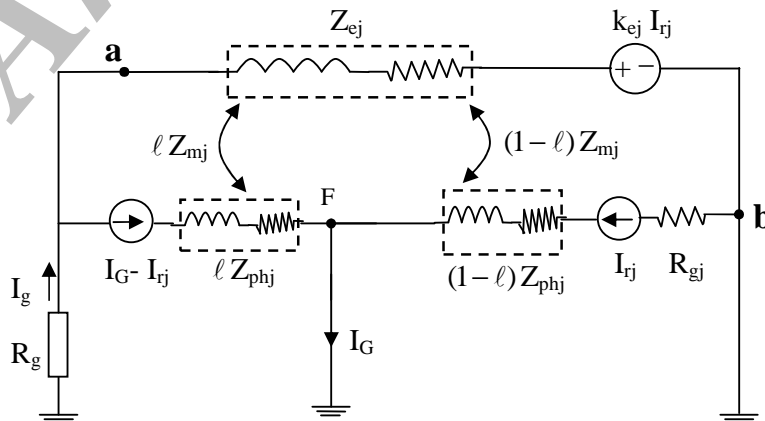


Fig. 10. The final equivalent circuit representing a fault at a distance  $\ell$

$$\begin{bmatrix} R_g + Z_{e1} & \cdots & R_g & \cdots & R_g \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_g & \cdots & R_g + Z_{ej} & \cdots & R_g \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_g & \cdots & R_g & \cdots & R_g + Z_{eN} \end{bmatrix} \begin{bmatrix} I_{e1} \\ \vdots \\ I_{ej} \\ \vdots \\ I_{eN} \end{bmatrix} = \begin{bmatrix} (Z_{m1} - k_{e1})I_{r1} - R_g(I_G - \sum_{i=1}^N I_{ri}) \\ \vdots \\ (Z_{mj} - k_{ej})I_{rj} - R_g(I_G - \sum_{i=1}^N I_{ri}) \\ \vdots \\ (Z_{mN} - k_{eN})I_{rN} - R_g(I_G - \sum_{i=1}^N I_{ri}) \end{bmatrix} - \begin{bmatrix} 0 \\ \vdots \\ \ell Z_{mj} I_G \\ \vdots \\ 0 \end{bmatrix} \quad (10)$$

**3. Earth fault through a tower of an overhead transmission line:** Usually the earth faults on transmission lines occur through a connection of the phase conductor to the metallic structure of the towers. Fig. 11 shows such a case where the faulted point is connected to the  $k$ th tower of the  $j$ -th incoming/outgoing overhead transmission line, with  $R_{Tk}$  as its footing resistance.

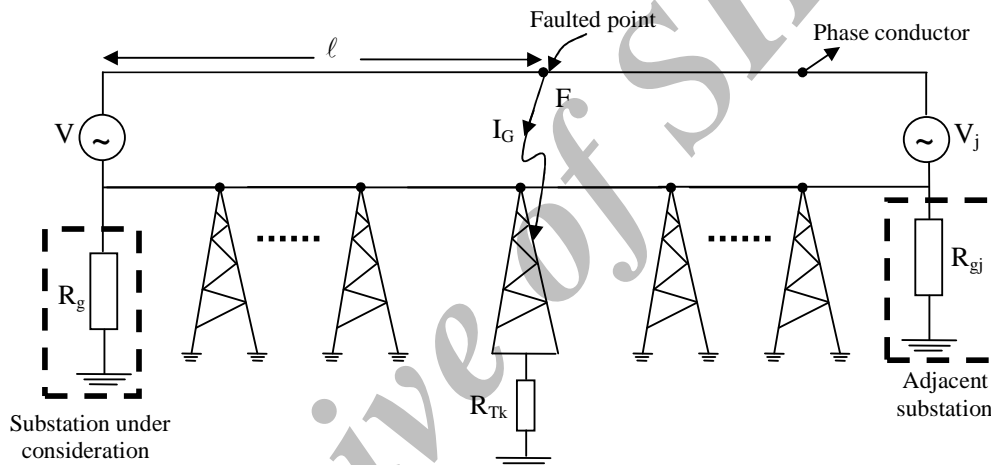


Fig. 11. The schematic diagram for an earth fault through a tower of an overhead line

Constructing the related circuit model as shown in Fig. A.2 and using the new parameters  $\psi_{ej}$ ,  $\xi_{ej}$  and  $\eta_j$  as defined in Eq. (A.1) to (A.3) in the Appendix, the  $j$ -th row in both the right hand side matrix and left hand side vector in Eq. (7) are modified and the equation is substituted by:

$$\begin{bmatrix} R_g + Z_{e1} & \cdots & R_g & \cdots & R_g \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_g & \cdots & R_g + \psi_{ej} & \cdots & R_g \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_g & \cdots & R_g & \cdots & R_g + Z_{eN} \end{bmatrix} \begin{bmatrix} I_{e1} \\ \vdots \\ I_{ej} \\ \vdots \\ I_{eN} \end{bmatrix} = \begin{bmatrix} (Z_{m1} - k_{e1})I_{r1} - R_g(I_G - \sum_{i=1}^N I_{ri}) \\ \vdots \\ (Z_{mj} - \xi_{ej})I_{rj} - R_g[(1 - \eta_j)I_G - \sum_{i=1}^N I_{ri}] \\ \vdots \\ (Z_{mN} - k_{eN})I_{rN} - R_g(I_G - \sum_{i=1}^N I_{ri}) \end{bmatrix} \quad (11)$$

**4. Disjointed earth wire:** There are cases where the earth wire of a transmission line is only installed for the spans near the substations. This condition is shown in Fig. 12.



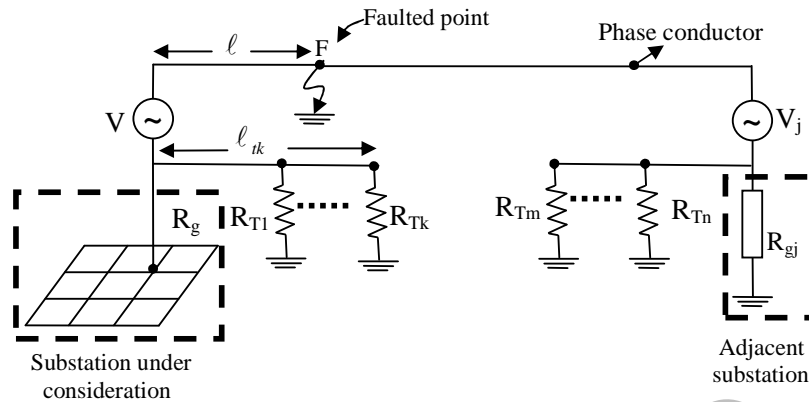


Fig. 12. The power system configuration with disjointed earth wire

As the earth wire(s) is disconnected between substations,  $I_n$  turns to zero in Eq. (3). But still the input impedance,  $Z_{ej}$  (for the earth wire and towers of  $j$ -th transmission line) in the left-hand side of Eq. (7), can be calculated by Eq. (5), provided that  $Z_{s_{n+1}}$  tends towards infinity. It must be mentioned that in Eq. (3), the ladder network is also limited to the number  $k$ . Another important point is that  $k_{ej}$  in Thevenin's equivalent circuit (Fig. 6) of a disjointed earth wire is zero, and hence, there is no current contribution from the adjacent substation.

Now, for the faulted transmission line, according to the fault location, i.e. at the  $\ell$  percentage of the length of the  $j$ -th transmission line, two cases could be recognized. The following relations show the modification in the right-hand side of Eq. (7) for each case:

1. If  $\ell \geq \ell_{tk}$  then:

$$\text{The } j\text{-th element of the right-hand side of Eq. (7)} = \ell_{tk} Z_{mj} I_{rj} - R_g (I_G - \sum_{i=1}^N I_{ri}) - \ell_{tk} Z_{mj} I_G \quad (12)$$

2. If  $\ell < \ell_{tk}$  then:

$$\text{The } j\text{-th element of the right-hand side of Eq. (7)} = \ell_{tk} Z_{mj} I_{rj} - R_g (I_G - \sum_{i=1}^N I_{ri}) - \ell Z_{mj} I_G \quad (13)$$

In the case of unfaulted incoming/outgoing transmission lines, with disjointed earth wires, the modification needed in Eq. (7) is the same as Eq. (13).

#### f) Overall procedure

Figure 13 shows the overall proposed procedure and explicitly explains its important and significant features. Finally, it is clear that this procedure can simply consider the contribution of the following conditions:

- Multi earth wires,
- Spans with different lengths,
- Different tower footing resistances,
- Double circuit transmission lines,
- Disjointed earth wires,
- Adjacent substation grounding grids,
- Faults at farther distances from the substation under consideration.

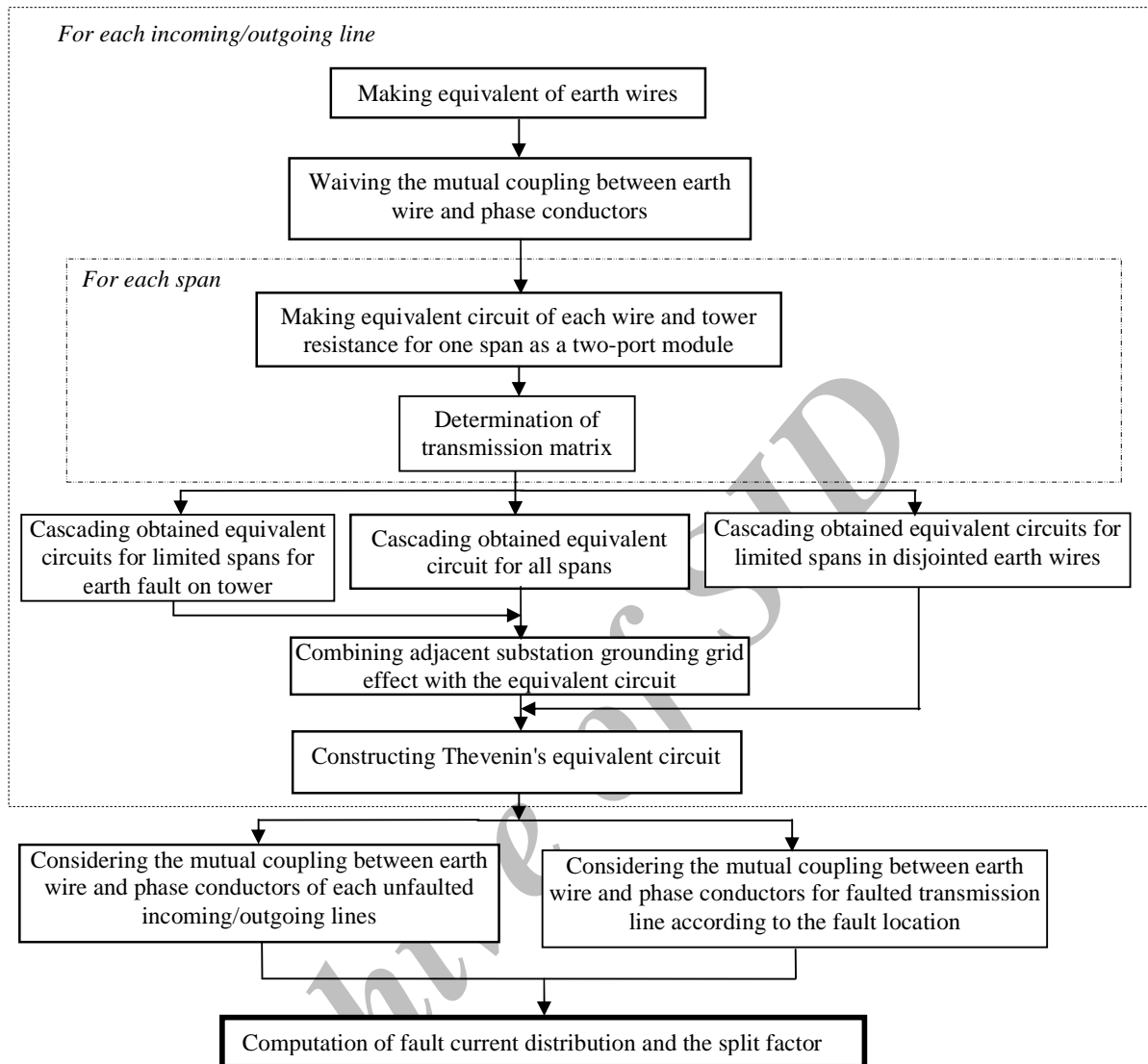


Fig. 13. Overall procedure

#### 4. RESULTS

To show the validity and accuracy of the proposed method, two problems are solved using the methods reported in [3] and [8] and the presented procedure in this paper. Also, the results of considering similar and dissimilar spans are shown by using the proposed method.

##### a) Case studies

While none of the previous reported procedures has such a capability to consider all of the parameters mentioned in section 1, in order to compare the results of the proposed method with the others, two of them are chosen, Ref. [3] because of its simple procedure for a large number of spans, and Ref. [8] to be compared as an analytical and accurate method, as mentioned in [1]. Two different case studies are performed on an example network from [12] (Appendix 7.1) and the Karkheh power plant (Appendix 7.2). The results are presented as follows:

**1. The number of spans and  $S_f$  (for earth faults inside and in the vicinity of a substation):** Figure 14 shows the effect of the number of spans on the split factor for the network in [12] by comparing the presented method with [3] and [8].

As shown in this figure, the results of the new model are similar to [8] for any number of spans, and are similar to [3] for more than 13 spans of the outgoing transmission line. Because of high computation error, the model in [3] should not be used for short transmission lines, and also due to complexity, the analytical method in [8] is impossible to be used for large numbers and complicated short transmission lines, however, the new method is able to compute the split factor for all of the cases through a simple procedure and gives accurate results.

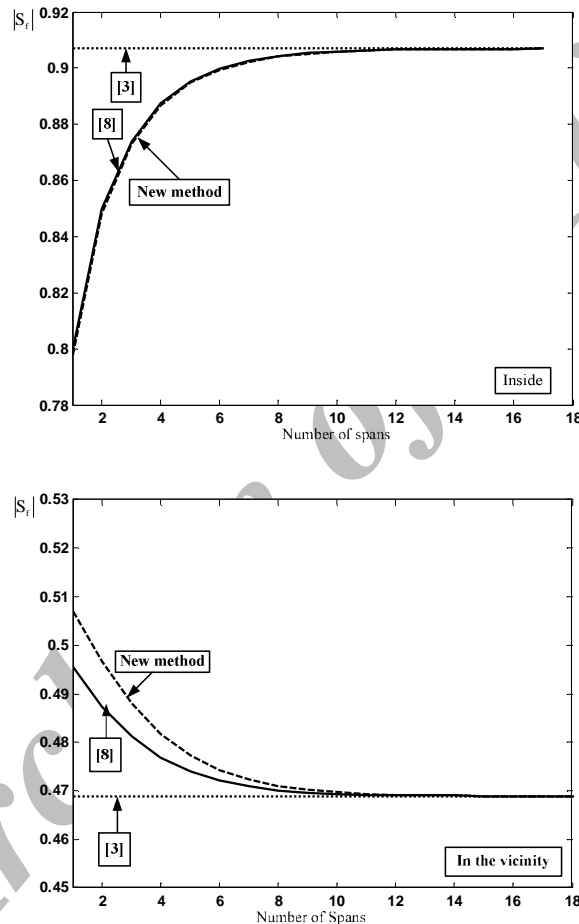


Fig. 14. Comparison between  $S_f$  from the new method and [3] and [8] for different no. of spans

**2. Adjacent grounding grids and  $S_f$  (for earth faults inside and in the vicinity of a substation):** The split factor is computed for the substation of the Karkheh power plant. In order to compare the results of the proposed method with [8], the existing double circuit transmission line in Fig. A.1 is assumed to be similar (because Ref. [8] cannot be used for dissimilar lines).

The results of the new method and [8] are compared in Fig. 15, which shows good agreement and the maximum difference between them makes the grounding grid design by the new method slightly safer (that is desired, indeed).

It is worth mentioning that [8] and other previous reported procedures cannot analyze the dissimilar double circuit outgoing lines, while the proposed approach can carry it out easily.

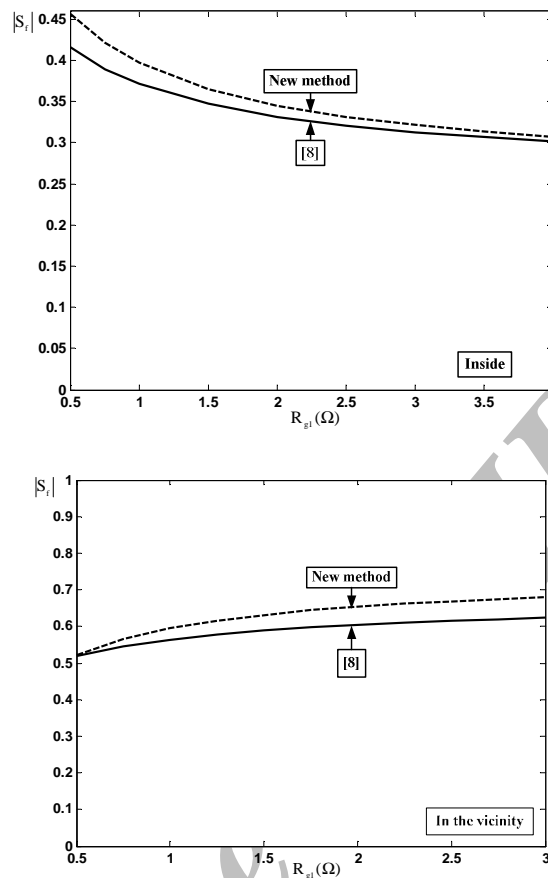


Fig.15. Comparison between  $S_f$  from the new method and [8] for Karkheh power plant

**b) Dissimilar spans and  $S_f$  (for earth faults inside and in the vicinity of a substation)**

The results of computing the split factor by considering different spans for the transmission line in Fig. A.1, as they are, and by substituting them with similar spans, are compared in Fig. 16. For the case of similar spans, the total length of the line (570 meters) is divided by the number of spans, i.e. seven. Figure 16 shows the computed split factor versus the adjacent substation grounding resistance ( $R_{g1}$ ) through two approaches. The standard deviation of the length of spans in the actual line is equal to 27.2 meters, while the mean is 81.25 meters. Further, it can be stated that the greater the difference between the spans (larger standard deviation), the higher the error in computing the split factor through assuming similar lengths for them. This error is such that the computed split factor has a lower magnitude in the case of substituting similar spans for an outgoing transmission line with dissimilar ones.

Therefore, for a safe grounding design, the dissimilarity of spans must be considered, and using any procedure that can model this condition, such as the one proposed in this paper, is obligatory.

**c) Earth faults on towers and  $S_f$  (outside a substation)**

For the network in [1] (Appendix 7.1), Fig. 17 shows the effect of different tower footing resistances on split factor with respect to different fault locations in the case of line to tower faults. It is shown that with a decrease in the tower footing resistance, the peak moves towards the towers nearer to the substation under consideration.

Also, as is shown in Fig. 18, as the contribution of the substation under consideration to the earth fault current on towers increases, the split factor becomes larger.

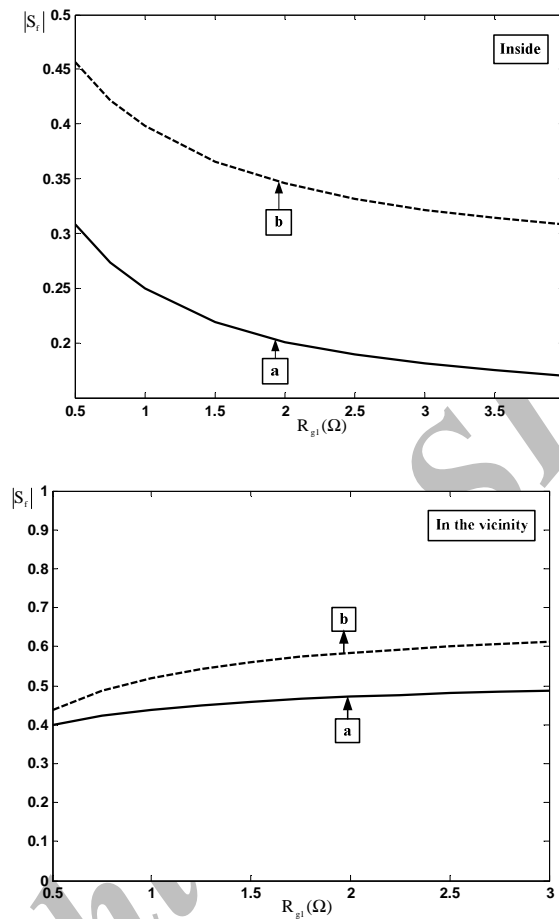


Fig. 16.  $S_f$  versus  $R_{g1}$  with (a) the assumption of similar spans and (b) dissimilar spans

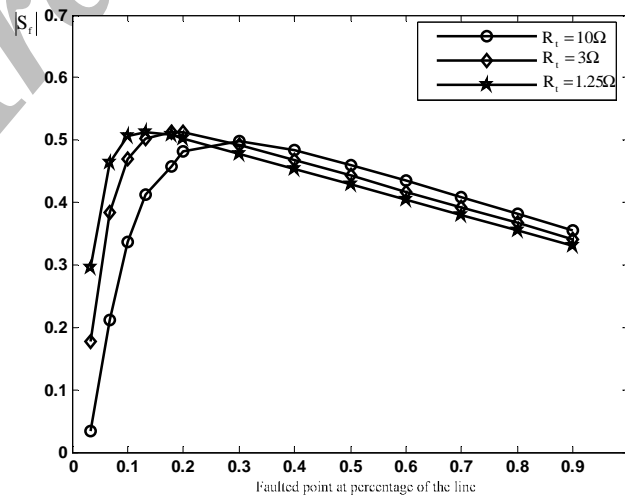


Fig. 17.  $S_f$  versus the fault location for different tower footing resistances

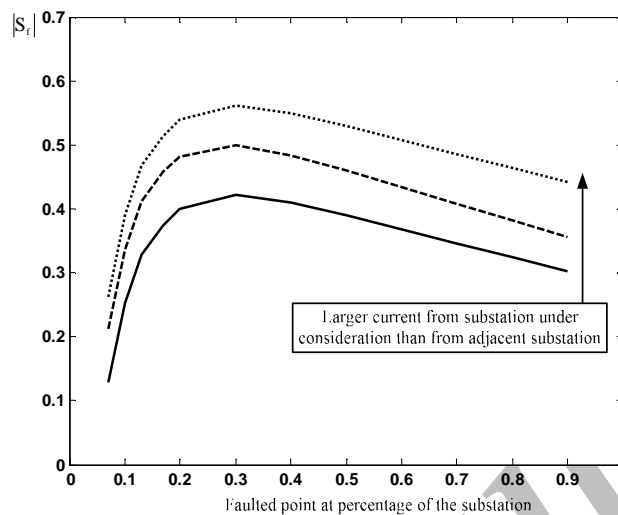


Fig. 18.  $S_f$  versus the fault location for different contribution of substation under consideration to the earth fault current

## 5. CONCLUSION

In this paper, the complete and step-by-step formulation of an accurate and simple procedure is developed for computing the split factor of the earth fault current; it is able to analyze dissimilar double circuit outgoing lines, different spans and tower footing resistances, faults on transmission lines with connected and/or disjointed earth wires and large transmission networks with multi earth wires. In addition, the proposed method is capable of simply considering the line to tower earth faults on any of the outgoing transmission lines from the substation.

The results of this method are compared with the others and show good agreement for comparable cases. Also, the results show that the outgoing lines with different spans should be modeled as they are, and may not be substituted with similar spans because of the larger split factors obtained for the real case.

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## APPENDIX 1

### a) Example network data [12]

Table A.1. The related transmission line data

$f(\text{Hz})$	50
$Z_g (\Omega/\text{km})$	$(7+j1.3)$
$Z_{gm} (\Omega/\text{km})$	$0.05+j0.38$
Span length(m)	300
$R_T (\Omega)$	10
$R_g (\Omega)$	0.5
$R_{gl} (\Omega)$	0.5

### b) The Karkheh network diagram and data

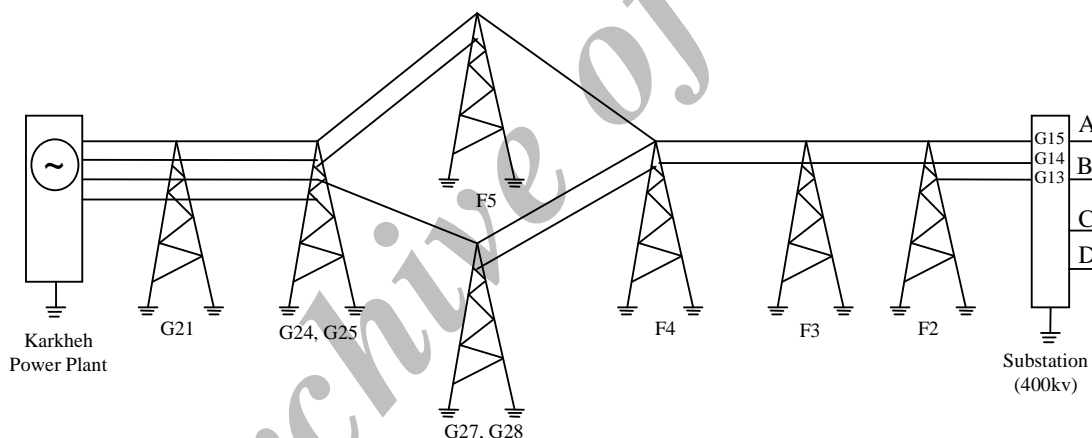


Fig. A.1. Earth wire diagram between Karkheh Power Plant and Substation

Table A.2. The transmission line data between substations

First end of span (i)	Second end of span (i+1)	Span length (m)	Number of earth wire	$Z_s(\Omega)$
P/P (Substation)	G21	55	4	$0.063+j0.049$
G21	G24 G25	61	4	$0.07+j0.054$
G24	F5	64.7	1	$0.074+j0.057$
G25	F5	54	1	$0.062+j0.048$
G25	G28	42.5	1	$0.049+j0.038$
G27	F4	44	1	$0.05+j0.039$
G28	F4	62.6	1	$0.072+j0.055$

Table A.2. Continued.

F5	F4	86	1	0.099+j0.076
F4	F3	125.9	1	0.145+j0.111
F4	F3	121.2	1	0.14+j0.107
F3	F2	111.2	1	0.13+j0.098
F3	F2	105.8	1	0.122+j0.094
F2	G13	111.2	1	0.13+j0.098
F2	G14	102.4	1	0.12+j0.091
F2	G15	97.8	1	0.112+j0.086
G13/G14	D/C	59.3	2	0.068+j0.052
G14/G15	B/A	57.3	2	0.066+j0.051

c) The equivalent circuit for computing earth fault on towers

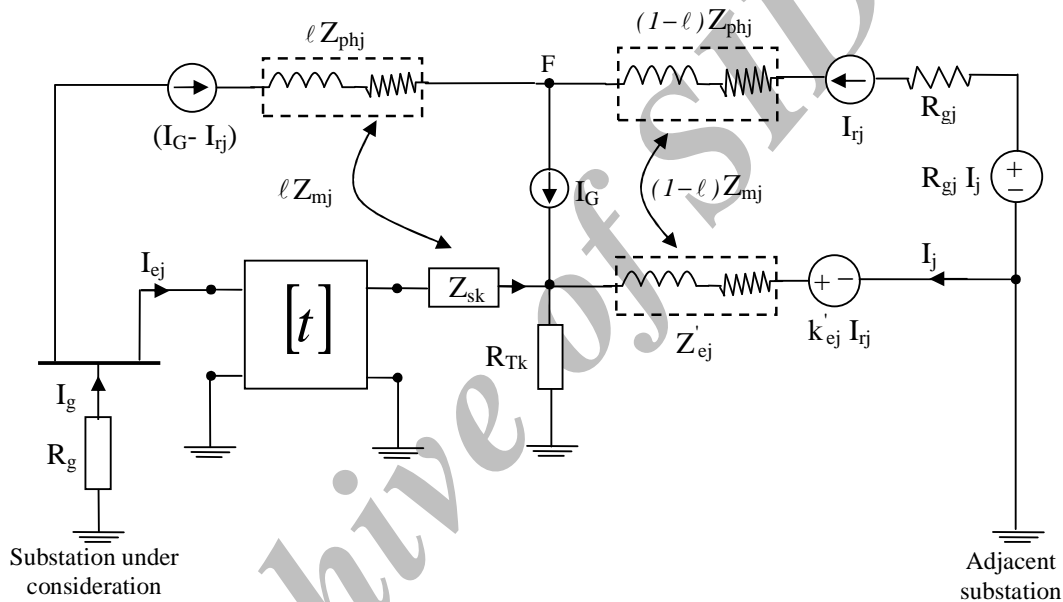


Fig. A.2. The circuit modeling of Fig. 11

The equivalent circuit of schematic diagram in Fig.11 is shown in Fig. A.2, where the Eq. (11) is derived. The introduced parameters,  $\psi_{ej}$ ,  $\xi_{ej}$  and  $\eta_j$  in Eq. (11) are defined as in the following:

$$\psi_{ej} = \frac{t_{12}}{t_{22}-1} - \frac{t_{12}^2}{\beta_1(t_{22}-1)^2} \quad (\text{A.1})$$

$$\xi_{ej} = Z_{mj} - \frac{t_{12}}{t_{22}-1} \frac{\beta_3}{\beta_1} \quad (\text{A.2})$$

$$\eta_j = \frac{1}{R_g} \frac{t_{12}}{t_{22}-1} \frac{\beta_2}{\beta_1} \quad (\text{A.3})$$

where

$$\beta_1 = \frac{(t_{12})^2}{t_{11}-1} \left[ \frac{t_{21}}{t_{22}-1} - \frac{1}{\alpha_1(t_{11}-1)} \right] \quad (\text{A.4})$$

$$\beta_2 = \frac{t_{12}}{t_{11}-1} \frac{\alpha_2}{\alpha_1} \quad (\text{A.5})$$



$$\beta_3 = \frac{t_{12}}{t_{11} - 1} \frac{\alpha_3}{\alpha_1} \quad (\text{A.6})$$

$$\alpha_1 = Z_{sk} + \frac{R_{Tk} Z'_{ej}}{R_{Tk} + Z'_{ej}} + \frac{t_{12}}{t_{11} - 1} \quad (\text{A.7})$$

$$\alpha_2 = -Z_{mk} - \frac{R_{Tk} Z'_{ej}}{R_{Tk} + Z'_{ej}} \quad (\text{A.8})$$

$$\alpha_3 = Z_{mk} - \frac{R_{Tk} (k'_{ej} - Z_{mR}) Z'_{ej}}{R_{Tk} + Z'_{ej}} \quad (\text{A.9})$$

$$Z_{mR} = (1 - \ell) Z_{mj}, \quad Z_{mk} = \ell Z_{mj} \quad (\text{A.10})$$

where  $Z_{mj}$  is the total mutual impedance between the faulted transmission line and its earth wire and  $\ell$  is the distance of the faulted point as the percentage of the transmission line length. Also, for the fault on the  $k$ -th tower,  $[t]$  is obtained from:

$$[t] = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \prod_{j=1}^{k-1} T_j \quad (\text{A.11})$$

where  $T_j$  is calculated from Eq. (1).

In the special case where the earth fault occurs on the first tower outside the substation, the introduced parameters of  $\psi_{ej}$ ,  $\xi_{ej}$  and  $\eta_j$  are defined as:

$$\psi_{ej} = Z_{sI} + \frac{R_{T1} Z'_{ej}}{R_{T1} + Z'_{ej}} \quad (\text{A.12})$$

$$\xi_{ej} = \frac{R_{T1} k'_{ej} + Z'_{ej} Z_{mR}}{R_{T1} + Z'_{ej}} \quad (\text{A.13})$$

$$\eta_j = \frac{1}{R_g} \left( -\frac{R_{T1} Z'_{ej}}{R_{T1} + Z'_{ej}} - Z_{mk} \right) \quad (\text{A.14})$$