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MODELING OF TWO-PHASE FLOW IN BOILING WATER REACTOR USING PHASE-WEIGHTED ENSEMBLE AVERAGE METHOD*

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Abstract– Investigations into boiling, the generation of vapor and the prediction of its behavior are important in the stability of boiling water reactors (BWR). The present models are limited to simplifications made to draw governing equations or lack of closure framework of the constitutive relations. The commercial codes fall into this category as well. Consequently, researchers cannot simply find the comprehensive updated relations before simplification in order to simplify them for their own works. This study offers a state of the art, phase-weighted, ensemble-averaged, two-phase flow, two-fluid model for the simulation of two-phase flow with heat and mass transfer. This approach is then used for modeling the bulk boiling (thermal-hydraulic modeling) in BWR. The resultant approach is based on using the energy balance equation to find a relation for quality of vapor at any point. The equations are solved using SIMPLE algorithm in the finite volume method and the results compared with real BWR (PB2 BWR/4 NPP) and the boiling data. Comparison shows that the present model is satisfactorily improved in accuracy.

Keywords– Two-phase flow, boiling, two-fluid model, heat and mass transfer, BWR

1. INTRODUCTION

Boiling Water Reactor (BWR) technology was established on the idea that it can become unstable under particular circumstances caused by a feedback between the thermal-hydraulics (changing in void fraction) and nuclear energy generation. The instability can result in oscillations of the power and the flow rate, which is a drawback to the smooth operation of the reactor [1]. For safety reasons, the simulation of boiling in BWR thermal-hydraulics is important in the design process. At the same time, boiling is one of the most important kinds of two-phase flow, which includes heat and mass transfer. Thus, it would be impossible to investigate the mathematical modeling of boiling without a good understanding of turbulence two-phase flow formulation.

In the past four decades, significant developments in the two-phase flow formulation have been accomplished by introducing and improving the two-fluid model. In the present state, the two-fluid model can be considered the most detailed and accurate macroscopic formulation of the thermo-fluid dynamics of two-phase systems[2-4]. In the two-fluid model, the field equations are expressed by six conservation equations, consisting of mass, momentum and energy for each phase. There are also jump conditions for all of the conserved equations at the phase interface. The field equations are obtained from an appropriate averaging of local instantaneous balance equations. The phasic interaction terms appear in each of the averaged balance equations as well.

The so-called "two-fluid model" of two-phase flow which is based on a single time-averaging has been extensively investigated by Ishii [5]. Dalhaye [6], among others, has developed both spatial and

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space/time averaging techniques. Nigmatulin [7] has derived a volumetric-averaged set of balance equations for multiphase flow. In addition, he developed a cell model as a means of deriving closure relations for his equations.

Ensemble averaging has been proposed as the fundamentally correct form of averaging by Buyevich [8] and Batchelor [9]. In addition, a number of other averaging techniques have been developed. These include a five-stage space/time averaged scheme developed by Drew[10], volume averaging using a weighting function by Iwanaga & Ishihara [11], and a volume average using a variable size averaging volume by Gray[12].

Additionally, Bataille[13] has reported on transforming the space and time coordinates to a four dimensional space. He has derived a technique whereby flow parameters are averaged over the four dimensions, while being weighted by a smoothing function, which enhances the smoothness of derivatives of the averaged parameters.

The ensemble averaged two-phase flow equations using the interfacial forces between two phases were developed by many authors such as Arnold [14], Park [15], Arnold et al. [16], Antal [17], Drew & Lahey[18, 19]. Drew & Passman [20] give a very broad and detailed overview of averaging methods for multi-component flows. They discuss the ensemble-averaging model in detail as well. Drew and Lahey et al. have continued their research in the ensemble averaging model [21-25].

Work has been done to the present mathematical models for BWR thermal-hydraulic as well. The analyses of the important improvements are provided in section 2.f.

In the thermal-hydraulics section of BWRs, the subcooled water enters the reactor core [26] and flows upward in the channels between the fuel elements. The heat generation of the nuclear fuel elements is approximately cosine shape in the axial direction. Due to the convective heat transfer, the subcooled water, after passing a short zone of subcooled boiling, reaches the saturation temperature and bulk boiling occurs along the channel. As a consequence, more vapor volume is generated in the flow direction. Because of the physical domain and boiling situation, the pressure difference between the upper plenum (outlet of the channel) and the lower plenum (inlet of the channel) in a BWR is small [27, 28]. The $\triangle P/P$ is so small (about 0.02) that even linear assumption offers an acceptable result [29]. In the present study, in order to investigate the bulk boiling in BWR, a vertical duct with the same operating condition is considered. For validity of the model, the simulation is compared with real BWR (PB2 BWR/4 NPP) [27, 28] at operating conditions and also with boiling data [30, 31]. The following physical and operating conditions have been used for comparison with the boiling data; channel cross section flow area 0.0098 m^2, the channel length 3.81m, the water mass flow rate 17.3797 kg/s, inlet subcooling 10 K, system pressure 72 bar, and the channel heat generation 4.6791 MW (cosine shape) in steady state condition.

2. MATHEMATICAL MODEL

The two-fluid models of two-phase flow are formulated based on space, time or ensemble averaging of the local instantaneous phasic balance equations. As a consequence, the two-fluid model can only simulate the average flow behavior. It provides sufficiently accurate empirical correlations for mass, momentum and heat transfer processes at the phasic interface and at the boundary walls. The correct formulation of the basic two-fluid conservation equations, and the most appropriate closure laws, are still subject to debate and depend strongly on the particular problem as to which terms should or should not be included [32].

a) Ensemble average

An ensemble is the set of all experiments with the same initial and boundary conditions and some (undefined) properties, which are associated with the mean and distribution of the particles and their

velocities. These sets are valuable for performing averages because variations in the details of the flows are assured in all situations, while at the same time variations in the gross flows cannot occur. The ensemble average allows the interpretation of phenomena in terms of the "repeatability" of multicomponent flows. Any particular exact experiment or realization cannot be repeated; however, any repetition of the experiment will lead to another member of the ensemble.

The ensemble average for a field, f(x, t) over a particular realization μ of the process is given as [20]

$$\langle f(x,t)\rangle = \int_{\varepsilon} f(x,t;\mu)P(\mu)d\mu, \tag{1}$$

where *P(μ)* is a probability density function of observing realization μ, and ε is the set of all realizations of the process of interest. It is worth noting that the probability density function satisfies:

$$\int_{\varepsilon} P(\mu)d\mu = 1 \tag{2}$$

In the two-fluid model, both of the phases must be continuous, but it is obvious that in the phase change process or in a dispersed phase existing in a continuous fluid, the discontinuity appears. Thus, for solving this difficulty, and also to determine which phase is present at a particular point (function of space and time), the phase indicator function χ(x, t) is defined by [5]

$$\chi_k(x,t) = \begin{cases} 1 & if\ phase\ "k"\ is\ found\ at\ (x,t) \\ 0 & otherwise \end{cases} \tag{3}$$

It follows that the ensemble averaged phase indication function, summed over all phases, must equal one. Therefore, the volume fraction of each phase can be defined by

$$\begin{aligned} \alpha_k &= \langle \chi_k \rangle \\ \sum_{k=1}^{n} \alpha_k &= 1 \end{aligned} \tag{4}$$

where n is the number of phases. For taking the average process in the conserved equations, the "χ - Phase weighted ensemble average ($\langle f_k \rangle^{\chi\rho}$)" needs to be defined:

$$\langle f_k \rangle^{\chi} = \frac{\langle \chi_k f_k \rangle}{\alpha_k} \tag{5}$$

and "χρ - Phase weighted ensemble average ($\langle f_k \rangle^{\chi\rho}$)" as:

$$\langle f_k \rangle^{\chi\rho} = \frac{\langle \chi_k \rho_k f_k \rangle}{\alpha_k \langle \rho_k \rangle^{\chi}} \tag{6}$$

b) Governing equations

The approach followed is based on the phase-weighted ensemble average two-fluid model described by Drew & Passman [20] which has been adopted and improved by many researchers [21-25] for simulation in multiphase flow systems. The following conservation equations are phasic equations, namely for the liquid or vapor phase. Here, to obtain equations for each phase of the boiling case, one needs to change the subscript "k" to "l" or "v" for liquid phase and vapor phase respectively.

The phasic mass equation,

$$\frac{\partial}{\partial t}\left(\alpha_k \langle \rho_k \rangle^{\chi}\right)+\nabla\cdot\left(\alpha_k \langle \rho_k \rangle^{\chi}\langle v_k \rangle^{\chi\rho}\right)=\dot{m}_k'''-\dot{\not{m}}_k''' \,. \tag{7}$$

The phasic momentum equation,

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\alpha_k \langle \rho_k \rangle^{\chi}\langle v_k \rangle^{\chi\rho}\right)+\nabla\cdot\left(\alpha_k \langle \rho_k \rangle^{\chi}\langle v_k \rangle^{\chi\rho}\langle v_k \rangle^{\chi\rho}\right)=\\&-\nabla\left(\alpha_k \langle p_k \rangle^{\chi}\right)+\nabla\cdot\left[\alpha_k\left(\langle \tau_k \rangle^{\chi}+\tau_k^{t}\right)\right]+\sum F_k+\alpha_k \langle \rho_k \rangle^{\chi}\langle b_k \rangle^{\chi\rho}+v_{ki}\dot{m}_k'''-v_k\dot{\not{m}}_k'''\end{aligned} \,. \tag{8}$$

The phasic energy equation,

$$\begin{aligned}&\frac{\partial}{\partial t}\left[\alpha_k \langle \rho_k \rangle^{\chi}\left(\langle u_k \rangle^{\chi\rho}+\frac{1}{2}\langle v_k \rangle^{\chi\rho}.\langle v_k \rangle^{\chi\rho}+u_k^{t}\right)\right]+\\&\nabla\cdot\left[\alpha_k \langle \rho_k \rangle^{\chi}\langle v_k \rangle^{\chi\rho}\left(\langle u_k \rangle^{\chi\rho}+\frac{1}{2}\langle v_k \rangle^{\chi\rho}.\langle v_k \rangle^{\chi\rho}+u_k^{t}\right)\right]=\\&\nabla\cdot\alpha_k\left[\left(\langle \tau_k \rangle^{\chi}+\tau_k^{t}\right).\langle v_k \rangle^{\chi\rho}-\langle q_k'' \rangle^{\chi}-q_k^{t}\right]+\alpha_k \langle q_k''' \rangle^{\chi}+\alpha_k \langle \rho_k \rangle^{\chi}\langle b_k \rangle^{\chi\rho}.\langle v_k \rangle^{\chi\rho}+\\&E_k+W_k+\left[u_{ki}+\frac{1}{2}\left(v_{ki}.v_{ki}\right)\right]\dot{m}_k'''-\left[u_k+\frac{1}{2}\left(v_k.v_k\right)\right]\dot{\not{m}}_k'''\end{aligned} \tag{9}$$

where $\dot{m}_k'''$ is phasic interfacial mass generation rate (such as phase change and/or mass source and/or flashing), $\dot{\not{m}}_k'''$ is phasic interfacial mass reduction rate during the process, and $\sum F_k$ is the sum of interfacial momentum transfer. In the above equations the components are assumed to be chemically inert, nonpolar, and not under the influence of electromagnetic fields.

c) Jump conditions

The Jump conditions of mass, momentum and energy are

$$\sum\ (\dot{m}_k'''-\dot{\not{m}}_k''')=0 \tag{10}$$

$$\sum\left(\sum F_k+v_{ki}\dot{m}_k'''-v_k\dot{\not{m}}_k'''\right)=M \tag{11}$$

$$\sum\left(E_k+W_k+\left[u_{ki}+\frac{1}{2}\left(v_{ki}.v_{ki}\right)\right]\dot{m}_k'''-\left[u_k+\frac{1}{2}\left(v_k.v_k\right)\right]\dot{\not{m}}_k'''\right)=\in \tag{12}$$

where M and $\in$ are surface tension source and interfacial energy source respectively. They are defined fundamentally as [20]

$$M=\left(H\sigma n+\nabla_i\,\sigma\right) \tag{13}$$

$$\in=H\sigma\, n.v_i+\nabla.(\sigma v_i)-\frac{du_i}{dt}-u_i\,\nabla_{i.}.v_i \tag{14}$$

where H is the mean curvature of the interface, σ is the surface tension, n is the unit normal to the interface, ∇_i denotes the gradient in the surface coordinates, and u_i the surface internal energy source.

In order to get closure, it is necessary to be able to express all the parameters in the two-fluid model and the associated jump conditions in terms of the dependent variables.

d) Interfacial momentum transfer

The interfacial momentum transfer is modeled with the interfacial forces. For boiling flow in vertical channels, the following forces must be taken into account: the drag force F^{drag}, the lift force F^{lift}, the turbulent dispersion force $F^{turbulent}$, the virtual mass force $F^{virtual}$ and the wall lubrication force F^{wall}. The total interfacial force per unit volume is the sum of the forces,

$$\sum F_k = F_k^{drag} + F_k^{lift} + F_k^{turbulent} + F_k^{\text{virtual}} + F_k^{wall} \tag{15}$$

The drag force is a vector directed along the relative velocity of the vapor phase that is exerted by the vapor phase on the liquid phase. Thus, it depends strongly on the relative velocity of the phases and the interfacial area. Some studies offer an interfacial area only for spherical bubbles [25, 33-35] (then it couldn't be perfect for void fraction more than 0.25 [36]). In this study, it is used as an expression which includes the interfacial area term in common form as [20-25, 37-39].

$$F_v^{drag} = -F_l^{drag} = -\frac{1}{8}\rho_l A''' C_D (v_v - v_l)\,|\,v_v - v_l\,| \tag{16}$$

where C_D and A''' are the drag force coefficient and the interfacial area density, respectively. The C_D is flow-regime dependent, and is usually calculated by using a correlation, but for boiling in the general case, it needs a C_D for such a wide range of void fraction as [40].

$$C_D = \begin{cases} 24(1+0.1\,\mathrm{Re}_b^{0.75})\,\mathrm{Re}_b^{-1} & when \quad 0 < \alpha_v \le 0.1 \\ \dfrac{2D_b}{3}\sqrt{\dfrac{g(\rho_l-\rho_v)}{\sigma}}\left[\dfrac{1+17.67(1-\alpha_v)^{1.285}}{18.67(1-\alpha_v)^{1.5}}\right]^2 & when \;\; 0.1 < \alpha_v \le 0.25 \\ 9.8(1-\alpha_v) & when \;\; 0.25 < \alpha_v \end{cases} \tag{17}$$

However, some of the valuable works did not mention any relation for C_D [19, 21, 22 & 24]. For boiling flow, the interfacial area density should also be determined for a large range of vapor volume fraction by using [41]

$$A''' = \frac{4.5(\alpha_v - \alpha_{sb})}{D(1-\alpha_{sb})} + \frac{6\alpha_{sb}(1-\alpha_v)}{D_b(1-\alpha_{sb})} \tag{18}$$

where α_{sb} is the small bubble void fraction (thus, it is equal to void fraction "α_v"within bubbly flow region) and it can be calculated by [42]

$$\alpha_{sb} = \begin{cases} \alpha_v & when \quad \alpha_v < 0.25 \\ 0.3929 - 0.5714\alpha_v & when \quad 0.25 \le \alpha_v < 0.6 \\ 0.05 & when \quad 0.6 \le \alpha_v \end{cases} \tag{19}$$

The lift force arises from a velocity gradient of the continuous phase in the lateral direction. It acts perpendicular to the main flow direction and is proportional to the gradient of the liquid velocity field. It is important to obtain correct radial distributions of the two phases in 2D and 3D analyses. The following expression is offered by Drew & Lahey [43], and has been addressed by recent acceptable works [24, 33, 34, 39, 44].

$$F_v^{lift} = -F_l^{lift} = C_L \alpha_v \rho_l (v_v - v_l) \times (\nabla \times v_l) \tag{20}$$

where C_L is the lift force coefficient and depends on Eötvos number $E\ddot{o}$ as [45]

$$C_L = 0.288\tanh(0.121\,\mathrm{Re_b}) + \begin{cases} 0, & when \quad \mathrm{E\ddot{o}} < 4 \\ -0.096\mathrm{E\ddot{o}} + 0.384, & when \quad 4 < \mathrm{E\ddot{o}} < 10 \\ -0.576 & when \quad 10 < \mathrm{E\ddot{o}} \end{cases} \tag{21}$$

where bubble Reynolds number $\mathrm{Re_b}$ and Eötvos number $\mathrm{E\ddot{o}}$ are:

$$\mathrm{Re_b} = \frac{|v_v - v_l|\,\mathrm{D_b}\rho_l}{\mu_l} \tag{22}$$

and

$$\mathrm{E\ddot{o}} = \frac{\mathrm{g}\mathrm{D_b^2}(\rho_l - \rho_v)}{\sigma} \tag{23}$$

where σ is surface tension, and a good way to predict it is offered by Tahery & Modarress [46] for different substances.

The turbulent dispersion force accounts for the effect of the dispersion of vapor bubbles caused by liquid turbulence and then smoothes the void fraction distribution. The following turbulent dispersion force was derived by Lopez de Bertodano [47] and has been widely used [19, 24, 32, 37, 48, 49].

$$F_v^{turbulent} = -F_l^{turbulent} = -C_{TD}\rho_l k_l \nabla\alpha_v \tag{24}$$

In the present model, C_{TD} and k_l are the turbulent dispersion force coefficient and the kinetic energy of the turbulence of the liquid phase, respectively. Where, the turbulent dispersion force coefficient C_{TD} is 0.1 for bubbles.

The virtual mass force comes into calculation whenever one phase is accelerating relative to the other one. In the case of a bubble accelerating in a continuous liquid phase, this force can be described by the following expression [24, 25 & 37].

$$F_v^{\mathrm{virtual}} = -F_l^{\mathrm{virtual}} = \alpha_v\rho_l C_{VM}\left[\left(\frac{\partial v_l}{\partial t} + v_l.\nabla v_l\right) - \left(\frac{\partial v_v}{\partial t} + v_v.\nabla v_v\right)\right] \tag{25}$$

where the virtual mass coefficient C_{VM} is equal to 0.5 for spherical bubbles.

The wall lubrication force is introduced to eliminate the effect of vapor sticking to the wall, thus it is also another parameter which is influential in obtaining the correct radial distribution of the two phases. The most detailed model is [24]

$$F_v^{wall} = -F_l^{wall} = \left\{\max\left[\frac{2\alpha_v\rho_l v_{axial}^2}{\mathrm{D_b}}\left(\mathrm{C_{W1}} + \mathrm{C_{W2}}\frac{\mathrm{D_b}}{2y}\right), 0\right]\right\}\delta_w n_{\mathrm{w}} - \frac{0.3\rho_l\alpha_v}{D_b}\sqrt{\frac{\tau_w}{\rho_l}}(v_v - v_l)\delta_w \tag{26}$$

where n_{w} is the unit vector normal to the wall and the other parameters are defined as

$$v_{axial} = v_r - (n_{\mathrm{w}}.v_r)n_{\mathrm{w}}, \quad \mathrm{C_{W1}} = -0.104 - 0.06|v_r|, \quad \mathrm{C_{W2}} = 0.147$$
$$y = (x - x_w).n_w \text{ and } \delta_w = \begin{cases} 1.0 & y \le D_b \\ 0.0 & otherwise \end{cases} \tag{27}$$

The first term on the right hand side of Eq. (26) is a force normal to the wall, and the second term is a force parallel to the wall. They are important for the nodes, which are adjacent to the wall.

The momentum jump condition needs to get closure as well. Thus, *M*, which is the surface tension source, can be written for the bubbles as [21, 22]

$$M = \nabla(\alpha_v \tau_v) \tag{28}$$

where τ_v

$$\tau_v = \rho_l \left[-\frac{9}{20}(v_r v_r) + \frac{3}{20}|v_r|^2 I \right] + (p_v - p_l)_i I \tag{29}$$

e) Turbulent model

In the present study, an elaborate extended k–ε turbulent model for the two-phase flow which includes extra source terms is used [24]. The extra source terms represent the increased generation of turbulence by the presence of the bubbles. Zboray & Cachard [32] also utilized almost the same model in their work. However, no standard model such as the k–ε for single-phase flow exists for two-phase flow, and many authors argue that if the dispersed phase elements are small and/or the void fraction is low, the standard k–ε formulation can be used.

The turbulent kinetic energy for liquid phase

$$\frac{D(\alpha_l k_l)}{Dt} - \nabla.(\alpha_l \frac{\upsilon_l^T}{\sigma_k} \nabla k_l) = \alpha_l (P_l - \varepsilon_l) + \alpha_l S_k \tag{30}$$

Turbulent dissipation for liquid phase

$$\frac{D(\alpha_l \varepsilon_l)}{Dt} - \nabla.(\alpha_l \frac{\upsilon_l^T}{\sigma_\varepsilon} \nabla \varepsilon_l) = \frac{\alpha_l}{k_l}\left(C_{1\varepsilon} \varepsilon_l P_l - C_{2\varepsilon} \varepsilon_l^2\right) + \alpha_l S_\varepsilon \tag{31}$$

The turbulence viscosity induced by the bubbles in the liquid phase is modeled as [50]

$$\upsilon_l^T = C_\mu \frac{k_l^2}{\varepsilon_l} + C_{\mu b} R_b \alpha_v |v_v - v_l| \tag{32}$$

The interaction terms are defined as [24]

$$S_k = \frac{k_l}{C_{\varepsilon 2} \varepsilon_l} S_\varepsilon = 0.25 \alpha_v (1 + C_D^{4/3}) \frac{|v_v - v_l|^3}{D_b} \tag{33}$$

and other parameters [32]

$$k_l = \frac{1}{2} v_l' . v_l', \quad P_l = \mu_{eff} \nabla v_l . (\nabla v_l + (\nabla v_l)^T) - \frac{2}{3} \nabla . v_l (\mu_{eff} \nabla . v_l + \rho_l k_l)$$
$$C_{1\varepsilon} = 1.44,\ C_{2\varepsilon} = 1.92,\ \sigma_\varepsilon = 1.3,\ \sigma_k = 1.0\ C_\mu = 0.09,\ C_{\mu b} = 1.2 \tag{34}$$

Similar turbulent transport equations could be utilized for the vapor phase.

f) Previous mathematical models for boiling in BWR

Significant works have been done for modeling of boiling with different constitutive arrangements in general cases or specific purposes; such as the works of Li et al. [33, 34], Kurul & Podowski [42]

Esmaeilzadeh & Abbasi [51]. Here, the analysis of some of the works which have modelled boiling in the BWR domain in particular are presented.

Anglart et al. [37] worked on CFD prediction of flow and phase distribution in fuel assemblies of BWR. They offered the two-fluid model governing equations, but the turbulence model was the classical k–ε without extra source terms, and also the turbulent viscosity term that they addressed from Sato's work has a missed " ρ_l " in the second term. They offered a constant lift force coefficient (C_L) too.

Aktas [52] intended to offer a model for simulating two-phase flows in the thermal-hydraulics of light water reactors. In his work, only the drag force is mentioned instead of interfacial forces. Moreover, the author offered the interfacial mass exchange term in the phasic mass equations, but this term is not affected in momentum equations. Furthermore, the jump conditions for momentum and energy equations are not defined clearly.

Manera et al. [53] worked on modeling and simulation of flashing-induced instabilities in natural-circulation systems of BWRs. The authors offered a 4-equation two-phase model by using FLOCAL software in their work.

Ferng et al. [54] offered a methodology that includes two-phase hydrodynamic CFD models and Flow Accelerated Corrosion (FAC) models in order to predict severe FAC wear sites for BWR. The two-phase flow model includes only lift and virtual mass forces. Further, the standard k–ε turbulent model is used.

Ustinenko et al. [44] offered a new two-phase computational fluid dynamics model for boiling water reactor analysis. A set of governing equations is proposed but details of the kind of averaging used are not provided. Moreover, the reader could not find any jump condition. In addition, it seems that the governing equation could predict only the equilibrium cases. In addition, the authors represented simple convective equation between liquid and vapor for the boiling model, but did not mention how the heat transfer coefficient for the phases are obtained.

Consequently, the researchers could not simply find the comprehensive updated relations before simplification to simplify them for their own works.

g) Simplification and procedure

If subcooled water enters into a vertical heated channel, the water is heated up to the saturated water, and then saturated bulk boiling starts. It is obvious that in the case of saturation boiling, the pressure or temperature is unknown. Thus, for a steady state case, the energy balance equation between the inlet and a saturated boiling section could be

$$\dot{Q}(z) + \dot{m}_{in}\left[h + V^2/2 + gz\right]_{in} = \dot{m}_l[h + V^2/2 + gz]_l + \dot{m}_v[h + V^2/2 + gz]_v \tag{35}$$

Hence, the quality at any point of the saturated boiling section could be derived by solving for $\dot{m}_v / \dot{m}_{in}$ as

$$x = \frac{\dot{Q}(z)/\dot{m}_{in} + h_{in} - h_l - gz + 0.5V_{in}^2 - 0.5V_l^2}{(h_{fg} - 0.5V_l^2 + 0.5V_v^2)} \tag{36}$$

where the subscript "*in*" indicate the inlet condition, and the other parameters are related to the cross section. In the Eq. (36), the quality is a function of z, the average phasic velocities (i.e. V_l, V_v), and the enthalpies (enthalpies are function of pressure or temperature in saturated boiling). The average phasic velocities in three dimensions could be obtained as

$$V_k^2 = \left\langle u_k^2 \right\rangle + \left\langle v_k^2 \right\rangle + \left\langle w_k^2 \right\rangle \tag{37}$$

On the other hand, the quality could be found from [36]

$$x = \frac{1}{1+\left(\frac{\rho_l}{\rho_v}\right)\left(\frac{V_l}{V_v}\right)\left(\frac{\alpha_l}{\alpha_v}\right)} \tag{38}$$

and the equations (36) and (38) should be equal in iterative sequence.

3. RESULTS AND DISCUSSION

a) Computation details

In this paper, a SIMPLE algorithm of the finite volume method is used for the differencing of the various conservation one-dimensional equations, and the associated interfacial jump conditions. The equations are discretised on a collocated mesh using standard differencing techniques. The sets of discretised equations were solved iteratively in a sequential manner. The solution algorithm utilizes a procedure similar to the SIMPLE method in which the pressure–velocity correction technique is extended to two-phase flows [55]. In the present approach, energy balance equation (i.e. Eq. 36) is used instead of usual energy Eq. (9) for bulk boiling, iteratively. This saves significant computational time. An overall computational flow chart is shown in Fig.1.

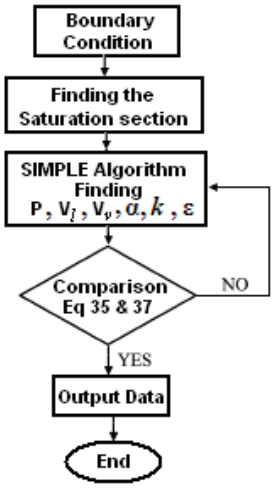

Fig. 1. Overall computational flow chart

b) Sensitivity analysis

1. Influence of grid arrangement: Prior to evaluating the model, the influence of grid density on the precision of numerical results was analyzed. Due to simplification of the flow field, a one-dimensional computational domain was built and four different grid arrangements (5, 10, 20 and 40 uniform length cells per meter) were tested. No significant difference between the predicted results of the 20 and the 40 grid arrangements (per meter length) was found. Therefore, it is confirmed that the 20 grid arrangement per meter is adequate to the issue of the present study.

2. Influence of Interfacial momentum transfer: It is important to note that the only non-zero terms of the interfacial momentum in the axial direction are the drag force and virtual mass force; while the lift, wall force and turbulent dispersion terms are important to distribute the bubbles in the lateral direction. The most important force acting on the solution is drag force. Because of this, its proportion to $\rho_l v_r^2$ and also C_D and A''' vary strongly by void fraction. In this study, the influence of forces is investigated by

eliminating each one, while other models and parameters were kept unchanged. The elimination of the drag force and virtual mass force exert an 11.5 % and 2.3 % average error (respectively) for the void fraction along the path.

c) Modeling of experimental data

The most challenging part of the BWR steady-state analysis is the prediction of the void fraction distribution [28]. To validate the proposed model, comparisons have been made with available real BWR (PB2 BWR/4 NPP) data [27, 28] and also boiling data [30, 31]. Fig. 2 shows that the proposed theoretical axial void fraction distribution in the saturated boiling region agrees well with the real BWR. Fig.3 indicates the axial power distribution of the BWR which is used for simulation (it is approximately cosine shape) [28].

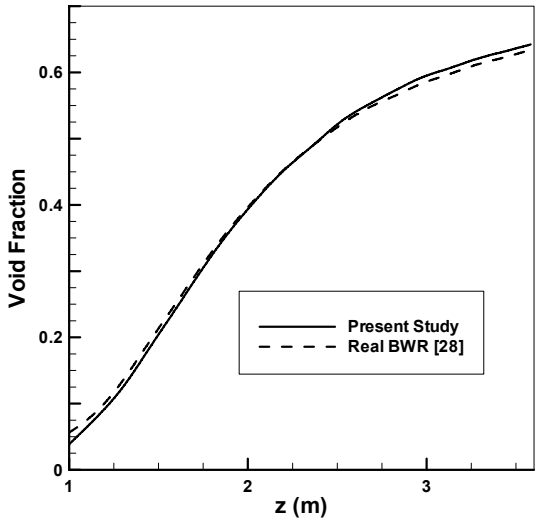

Fig. 2. The void fraction comparison with real BWR

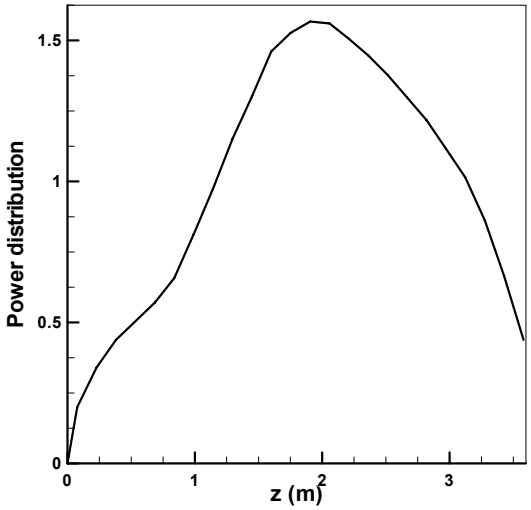

Fig. 3. Power distribution of the BWR [28]

Figures 4 and 5 show the results of the current study which are compared with boiling data for void fraction and quality, respectively. The power distribution for generating the data is cosine shape.

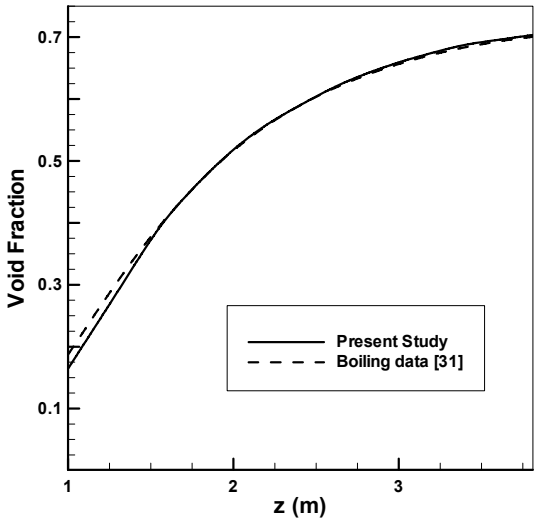

Fig. 4. The void fraction comparison with boiling data

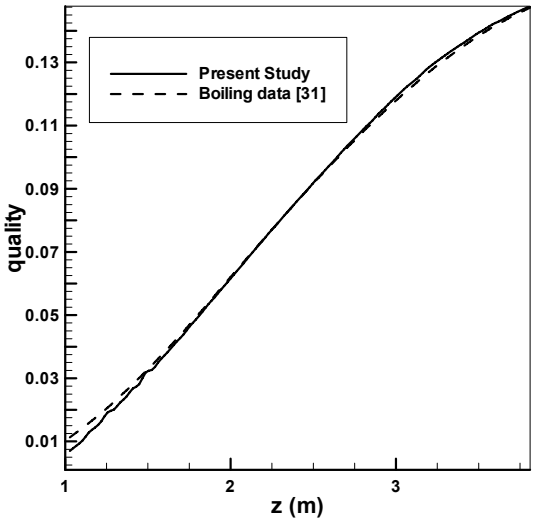

Fig. 5. The comparison of quality with boiling data

In the boiling systems, one of the most important parameters is the accuracy of the pressure drop. Figure 6 shows the comparisons of the pressure drop along the channel, where the relative error is less than 1%. As it is shown, the result of the present study is close to the boiling data. In Fig. 7, the liquid phase superficial velocity (J_l) and vapor phase superficial velocity (J_v) are shown within a two-phase region (saturated boiling zone) of the channel. The comparison shows that the results are in good agreement with the boiling data. Since the temperature and density of the phases in the saturation zone depend on pressure; and from Fig. 6 it is clear that the pressure along the channel is available and accurate, so the densities and temperature for any location can be predicted.

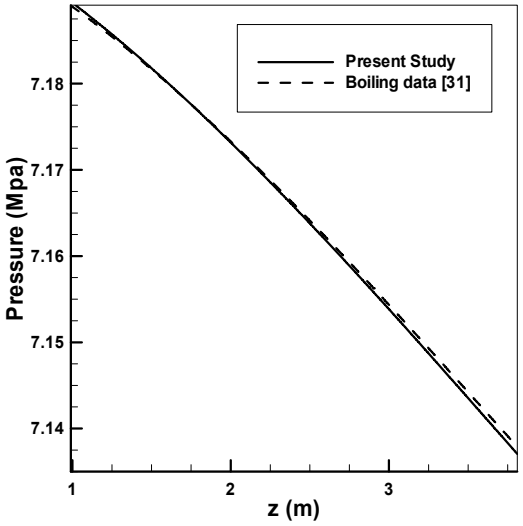

Fig. 6. The pressure changes along the boiling part of the channel

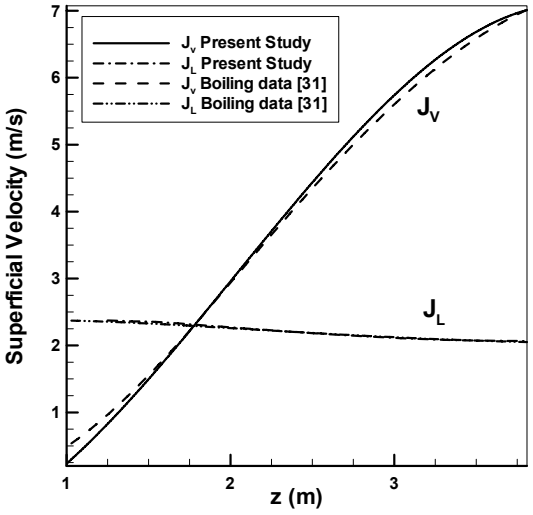

Fig. 7. The variation of the superficial velocities along the channel

As a consequence of the comparison of the present study shown in Figs. 2-7, it is clear that in the case of saturated boiling, using Eq.36 instead of general energy Eq.9 is reasonable.

The results of the parametric study on how the outlet superficial velocities depend on various input heat flux and mass flow rates are shown in Figs.8 and 9, respectively. In this study, their influence is investigated by changing one while keeping the other parameters unchanged.

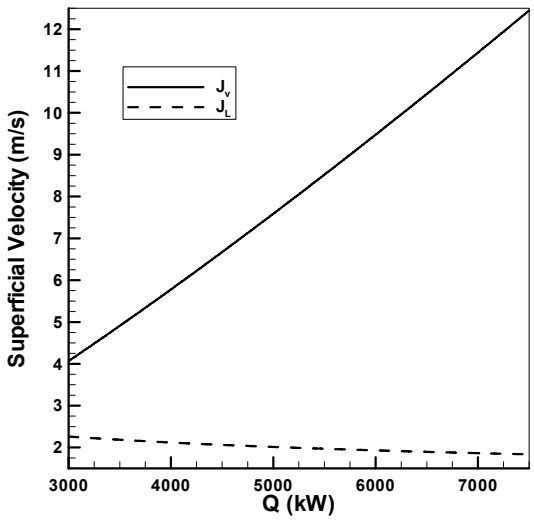

Fig. 8. The variation of the superficial velocities versus channel heat flux

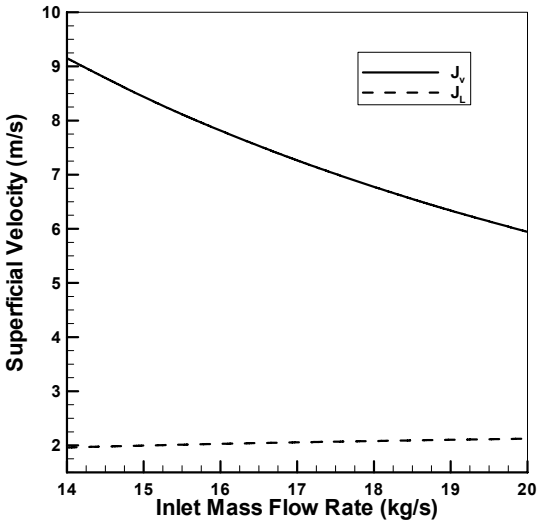

Fig. 9. The superficial velocities versus different inlet mass flow rate

Figure 8 shows that by increasing the heat flux of the channel, the vapor superficial velocity increases sharply, but the liquid superficial velocity decreases a little. This is due to the increase of α_v, so that α_l decreases. Moreover, increasing the heat flux makes the distance between the inlet and saturation section shorter, as a consequence, a greater amount of vapor has a longer path to accelerate. On the other hand, the inlet velocity of the subcooled water is 2.35 m/s and the exit liquid (water) superficial velocity for all different heat flux cases is about 2 m/s. Therefore, one phase inlet velocity can be used as a first guess for the liquid superficial velocity while performing iterative studies. Figure 9 denotes that whenever the inlet mass flow rate of the channel increases, the vapor superficial velocity decreases, but the liquid superficial velocity increases a little. This is due to less time to transfer the heat to the water, therefore α_v decrease.

4. CONCLUSION

In the present study, a three-dimensional, phase-weighted ensemble averaged two-phase flow (turbulence), two-fluid model for simulating a wide range of void fractions with heat and mass transfer has been developed. The model is simplified for saturated boiling in a heated channel with BWR conditions. The simplification is based on using an overall energy balance analysis instead of usual two-phase flow energy equation. The results have been validated by available real BWR (PB2 BWR/4 NPP) data and also boiling data. The computational results highlight that in the case of saturated boiling it is possible to use an overall energy balance analysis instead of the usual two-phase flow energy equation. This saves significant computation time and simplifies turbulence bulk boiling in BWR. As future work, the authors will work to extend this simplification method for subcooled boiling as well.

NOMENCLATURE

A'''	interfacial area density
b_k	phasic body force, $k = l, v$
BWR	boiling Water Reactor
D	channel equivalent diameter
2D	two dimensions
3D	three dimensions
E_k	phasic interfacial heat source, $k = l, v$
FCA	flow Accelerated Corrosion
f	any variable for taking averaging
F_k	phasic interfacial force density, $k = l, v$
g	gravitational acceleration
h	enthalpy
h_{fg}	latent heat
I	identity matrix
J	superficial velocity
$\dot{m}_k'''$	phasic interfacial mass generation, $k = l, v$
$\dot{m}_k'''$	phasic interfacial mass reduction, $k = l, v$
$\dot{m}$	mass flow rate
M	surface tension source
NPP	nuclear Power Plant
PB	peach Bottom
p_k	phasic pressure, $k = l, v$
$P(\mu)$	probability density function of observing realization μ
q''	heat flux

q'''	volumetric heat source
$\dot{Q}(z)$	``Total heat transferred to channel from inlet to z
Re_b	bubble Reynolds number
u	velocity component in x axis direction
u_k	phasic internal energy, $k = l, v$
v	velocity component in y axis direction
V	total phasic average velocity of the channel cross section at z
v_k	phasic velocity, $k= l, v$
v_r	relative velocity ($v_v - v_l$)
w	velocity component in z axis direction
W_k	phasic interfacial work, $k = l, v$
x	quality
z	upward axial location in the channel

Greek

α_k	phasic volume fraction, $k = l, v$
υ^T	the turbulence viscosity induced by the bubbles
χ	phase indicator function
ρ_k	phasic density, $k= l, v$
σ_s	surface tension
τ_k	phasic stress tensor, $k= l, v$
τ_k^t	phasic ensemble averaged Reynolds stress tensor, $k = l, v$
$\in$	interfacial energy source
∇	del operator

Subscripts

b	bubble
in	at the inlet of the channel
ki	phasic Interface, $k = l, v$
k	each phase, $k = l, v$
l	liquid phase
r	relative
v	vapor phase

Superscripts

t	related to Reynolds stress
χ	weighted average with phase indicator function
χρ	weighted average with phase indicator function and density

Notation Convention

$\langle\ \rangle$	ensemble average
$\langle\ \rangle^{\chi}$	χ - Phase weighted ensemble average
$\langle\ \rangle^{\chi\rho}$	χρ - Phase weighted ensemble average

REFERENCES

1. Aguirre, C., Caruge, D., Castrillo, F., Dominicus, G., Geutjes, A. J., Saldo, V., van der Hagen, T. H. J. J., Hennig, D., Huggenberger, M., Ketelaar, K. C. J., Manera, A., Munoz-Cobo, J. L., Prasser, H. M., Rohde, U., Royer, E. & Yadigaroglu, G. (2005). Natural circulation and stability performance of BWRs (NACUSP). *J. Nuclear Eng. and Design,* Vol. 235, pp. 401–409.
2. Ishii, M., Hiblki, T., Revankar, S. T., Kim, S. & Le Corre, J. M. (2002) Interfacial area and interfacial transfer in two-phase flow systems. *DOE Final Report* - PU/NE-02-06- U.S. Department of Energy, Office of Basic Energy Science.
3. Lahey Jr., R. T. (2005). The simulation of multidimensional multiphase flows. *J. Nuclear Eng. and Design,* Vol. 235, pp. 1043–1060.
4. Zboray, R. & Cachard, F. de (2005). Simulating large-scale bubble plumes using various closure and two-phase turbulence models. *J. Nuclear Eng. and Design,* Vol. 235, pp. 867–884.
5. Ishii, M. (1975). *Thermo-fluid dynamic theory of two-phase flow*. Eyrolles Publications, France.
6. Delhaye, J.M. (1976). *Local time averaged equations*, *Two- phase flows and heat transfer*. Vol. 1, pp. 91-100.
7. Nigmatulin, R. I. (1979). Spatial averaging in the mechanics of heterogeneous and dispersed systems. *Int. J. Multiphase Flow*, Vol. 5, pp.353 - 385.
8. Buyevich, Y. A. (1966). Kinematics of mass transfer between a polydispersed particle system and surroundings media. *J. Appl. Mech. And Tech. Phys.*, Vol. 7, pp. 32-36.
9. Bachelor, G. K. (1970). The stress system in a suspension of force-free particles. *J. Fluid Mech.*, Vol. 42, pp.545 - 570.
10. Drew, D.A. (1971). *Averaged field equations for two- phase media. Studies in applied math,* Vol. 1, pp.133-166.
11. Iwanaga, M. & Ishihara, T. (1980). Volume averaged expression of two – phase flow. *Bull. JEME,* Vol. 23, pp. 1124-1131.
12. Gray, W. C. (1983). Local volume averaging of multiphase systems using a non–constant averaging volume. *Int. J. Multiphase flow*. Vol. 9, pp. 755-761.
13. Bataille, J. (1981). Averaged field equations for multiphase flows (Report No. GEOFLO/9, USA.
14. Arnold, G. S. (1988). Entropy and objectivity as constraints upon constitutive equations for two-fluid modeling of multiphase flow. PhD Dissertation, Rensselaer Polytechnic Institute, Troy, NY 12180-3590, USA.
15. Park J. W. (1992). Void wave propagation in two- phase Flow. PhD Dissertation, Rensselaer Polytechnic Institute, Troy, NY 12180-3590, USA.
16. Arnold, G. S., Drew, D. A. & Lahey, R. T. (1989). Derivation of constitutive equations for interfacial force and Reynolds stress for a suspension of sphere using Ensemble cell averaging. *Chemical Eng. Communication*, Vol. 86, pp. 376-389.
17. Antal, S. P. (1994). Phase distribution in bubbly two-phase flows. PhD Dissertation, Rensselaer Polytechnic Institute, Troy, NY 12180-3590, USA.
18. Drew, D.A. & Lahey, R.T. (1999). An ensemble Averaging Model for dilute Invisid Two-Phase flow. *Center for Multiphase Research* RPI, Troy, NY, USA.
19. Lahey, Jr., R. T. & Drew, D. A. (1999). An analysis of two-phase flow and heat transfer using a multi-dimensional, multi-field, two-fluid computational fluid dynamics (CFD) model. *Proc. of Japan/US Seminar on Two-Phase Flow Dynamics, UC-SB.*
20. Drew, D. A. & Passman, S. L. (1999). *Theory of multicomponent fluid*. Springer Publications.
21. Drew, D. A. & Lahey, R. T. (2000). An analysis of two-phase flow and heat transfer using a multidimensional multi-field, two-fluid computational fluid dynamics (CFD) model. *Japan/US Seminar on Two-Phase Flow Dynamic, Santa Barbara, CA (June 5-8), USA.*

22. Lahey, R. T. & Drew, D. A. (2001). An analysis of two-phase flow and heat transfer using a multidimensional four-field, two-fluid model. *J. Nuclear Eng. and Design,* Vol. 204, pp. 29–44.
23. Galimov, A. Y., Drew, D. A., Lahey, R. T. & Moraga, F. J. (2005). The analysis of interfacial waves. *J. Nuclear Eng. and Design,* Vol. 235 pp. 1283-1292.
24. Lahey Jr., R. T. (2005). The simulation of multidimensional multiphase flows. *J. Nuclear Eng. and Design*, Vol. 235, pp. 1043–1060.
25. Moraga, F. J., Larreteguy, A. E., Drew, D. A. & Lahey Jr., R. T. (2006). A center-averaged two-fluid model for wall-bounded bubbly flows. *Computers & Fluids,* Vol. 35, pp. 429–461.
26. EL-Wakil, M. M. (1982). Nuclear Energy Conversion. ANS Publication, USA.
27. Solis, Jorge, Kostadin N., Ivanov, Baris, Sarikaya, Olson, Andy M. & Hunt, Kenneth W. (2001). *Boiling water reactor turbine trip (TT) benchmark. Volume I: Final specifications*. Nuclear Energy Agency, US Nuclear Regulatory Commission.
28. Bedirhan, Akdeniz, Kostadin, N. Ivanov & Olson Andy M. (2006). *Boiling water reactor turbine trip (TT) benchmark*. Volume III: Summary Results of Exercise 2. Nuclear Energy Agency, NEA No. 5437, US Nuclear Regulatory Commission.
29. Sharifpur, M., Salehi, M., Nouri Brojerdi, A. & Arefmanesh, A. (2003). Ensemble averaged two- phase flow numerical simulation in vertical ducts for the void-studying behavior in BWRs. *Proceeding of 11th International Conference on Nuclear Engineering ICONE 11 (ASME), Japan.*
30. Yadigaroglu, G. & Askari, B. (2003). Linear stability analysis of BWRs (NACUSP Project-unpublished report). Nuclear Engineering Laboratory, Swiss Federal Institute of Technology, Institute of Energy Technology, Zurich, Switzerland.
31. Yadigaroglu, G. & Askari, B. (2004). *Personal communication*. Nuclear Engineering Laboratory, Swiss Federal Institute of Technology, Institute of Energy Technology, Zurich, Switzerland.
32. Zboray, R. & Cachard, F. (2005). Simulating large-scale bubble plumes using various closure and two-phase turbulence models. *J. Nuclear Eng. and Design,* Vol. 235, pp. 867–884.
33. Li, X., Wang, R., Huang, R. & Shi, Y. (2006). Numerical investigation of boiling flow of nitrogen in a vertical tube using the two-fluid model. *Applied Thermal Engineering*, Vol. 26, pp. 2425–2432.
34. Li, X., Wang, R., Huang, R. & Shi, Y. (2007). Numerical investigation of boiling flow of nitrogen in a vertical tube using the two-fluid model. *International Journal of Heat and Mass Transfer,* Vol. 50, pp. 1971–1981.
35. Politano, M. S., Carrica, P. M. & Converti, J. (2003). A model for turbulent polydisperse two-phase flow in vertical channels. *International Journal of Multiphase Flow*, Vol. 29, pp. 1153–1182.
36. Collier, J. G. & Thome, J. R. (1994). Convective boiling and condensation. Oxford Presses Publication.
37. Anglart, H., Nylund, O., Kurul, N. & Podowski, M. Z. (1997). CFD prediction of flow and phase distribution in fuel assemblies with spacers. *J. Nuclear Eng. and Design,* Vol. 177, pp. 215-228.
38. Yeoh, G. H. & Tub, J.Y. (2004). Population balance modelling for bubbly flows with heat and mass transfer. *Chemical Engineering Science,* Vol. 59, pp. 3125 – 3139.
39. Yeoh, G. H. & Tub, J. Y. (2006). Numerical modelling of bubbly flows with and without heat and mass transfer. *Applied Mathematical Modelling,* Vol. 30, pp. 1067–1095.
40. Ishii, M. & Zuber, N. (1979). Drag coefficient and relative velocity in bubbly, droplet or particulate flows. *AIChE J.* Vol. 25, pp. 843–855.
41. Ishii, M. & Mishima, K. (1984). Two-Fluid Model and hydrodynamic constitutive relations. *J. Nuclear Eng. and Design,* Vol. 82, pp. 107-126.
42. Kurul, N. & Podowski, M.Z. (1991). On the modeling of multidimensional effects in boiling channels. *ANS Proc., 1991 National Heat Transfer Conference, Minneapolis, MN.*
43. Drew, D. A. & Lahey Jr., R. T. (1979). Application of general constitutive principles to the derivation of multidimensional two-phase flow equation. *Int. J. Multiphase Flow* 5, 243–264.

44. Ustinenko, V., Samigulin, M., Ioilev, A., Lob, S., Tentner, A., Lychagin, A., Razin, A., Girin, V. & Vanyukov, Ye. (2008). Validation of CFD-BWR, a new two-phase computational fluid dynamics model for boiling water reactor analysis. *J. Nuclear Eng. and Design*, Vol. 238, pp. 660-670.
45. Tomiyama, A., Tarnai, H., Zun, I. & Hosokama, S., (2002). Transverse migration of single bubbles in simple shear flow. *Chem. Eng. Sci.,* Vol. 57, pp. 1849–1858.
46. Tahery, R. & Modarress, H., (2005). A new and a simple model for surface tension prediction. *Iranian Journal of Science & Technology, Transaction B, Engineering*, Vol. 29, No. B5.
47. Lopez de Bertodano, M., Lahey Jr., R. T. & Jones, O. C. (1994). Phase distribution in bubbly two-phase flow in vertical ducts. *Int. J. Multiphase Flow,* Vol. 20, No. 5), pp. 805–818.
48. Lahey, Jr., R. T., Bertodano, M. & Jones, O. C. (1993). Phase distribution in complex geometry conduits. *J. Nuclear Eng. and Design*, Vol. 141, pp. 177–201.
49. Alajbegovic, A., Drew, D. A. & Lahey Jr., R. T., (1999). An analysis of phase distribution and turbulence structure in disperse particle/liquid flow. *Chem. Eng. Comm.* Vol. 174, pp. 85–133.
50. Sato, Y., Sadatomi, M., Sekoguchi, K., (1981). Momentum and heat transfer in two-phase bubble flow-I. Theory. *Int. J. Multiphase Flow* 7, 167–177.
51. Esmaeilzadeh, E. & Abbasi, M., (2005). A numerical investigation on convective film boiling of the single component refrigerant as alterative of CFCs in a vertical pipe. *Iranian Journal of Science & Technology, Transaction B, Engineering*, Vol. 29, No. B2.
52. Aktas, Birol (2003). Level tracking in thermal-hydraulic simulations of nuclear reactors. PhD Dissertation, The Pennsylvania State University, USA.
53. Manera, A., Rohdea, U., Prasser H. M. A. & Hagenb, T. H. J. J. van der, (2005), Modeling of flashing-induced instabilities in the start-up phase of natural-circulation BWRs using the two-phase flow code FLOCAL, *J. Nuclear Eng. and Design,* Vol. 232, pp. 1517-1535.
54. Fernga, Yuh Ming, Tsenga, Yung Shin, Pei A, & Bau Shei, LongWang, S. (2008), A two-phase methodology to predict FAC wear sites in the piping system of a BWR. *J. Nuclear Eng. and Design,* Vol. 238, pp. 2189–2196.
55. Oliveira, P.J., Issa, R.I., (2001). Numerical aspects of an algorithm for the eulerian simulation of two-phase flows. *Proceedings of ECCOMAS 2001*, Swansea, UK.