PERFORMANCE MODELLING OF MACHINING SYSTEM WITH MIXED STANDBY COMPONENTS, BALKING AND RENEGING

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Abstract This paper deals with machine repair problem with balking and reneging. There is provision of mixed standby (warm and cold) components to replace the failed machines. The lifetime and repair times are assumed to have exponential distribution. Birth-death technique is suggested to obtain the queue size distribution in explicit form. A repair facility of C permanent repairmen is facilitated to repair the failed machines in FCFS basis. Some special cases are deduced which tally with earlier results. To find out the optimal number of spares and repairmen, a cost function is also mentioned.

Key Words Queue, Machine Repair, Balking, Reneging, Spares, Cost Analysis

چکیده این مقاله به مساله تعمیر ماشین گیر کننده و امتناع گر می‌پردازد. اجزای جایگزین بصورت مخلوط (گرم و سرد) تدارک می‌شوند. توزیع زمانهای کار و تعمیر، اکسپونانسیل فرض می‌شوند. برای بدست آوردن توزیع اندازه صف پیشنهاد شده است که از روش تولد / مرگ آشکار استفاده شود. از **C** تعمیر کار دائمی برای تعمیر ماشینهای از کار افتاده بر مبنای FCFS استفاده می‌شود. حالتهای خاصی نتیجه می‌شوند که با دستاوردهای قبل انطباق دارند. برای بدست آوردن تعداد بهینه قطعات یدکی و تعمیر کار، یک تابع هزینه نیز تعریف شده است.

1. INTRODUCTION

The machines are blest for solving our day-to-day problems growing in the world. Where ever people go, they have to face the rush, the machines are helpful to reduce our problems. In any manufacturing systems, if a machine fails, it causes the loss in desired demand of production. In this case standby spares/ machine/part support are used which may be of three types. If the failure rate of standby machine is zero, it is called as cold standby. In the other case when failure rate of that of functioning machine, it is referred, as warm standby while a standby machine is known as hot standby if its failure rate is same as failure rate of operating machine. The arrival of customers with machines depends upon the service/repair facility provided to renew the machines.

Machine repair problems are studied by several researchers in different frame works. Gopalan (1975) established the analysis of single server n unit system with (n-1) warm standbys. Hilliard (1976) discussed cost analysis of maintenance float system. Gross and Harris (1985) developed the M/M/C/m/m model with spares. Berg and Posner (1985) studied M/M/C repair system with spares. Daduna (1990) considered the delay times for exchangeable items in repair system. Meng (1995) compared the MTBF of four systems with standby machine is less than standby components. Gupta and Rao (1996) studied the M/G/1 machine interference model with spares. Jain (1997) analyzed (m, M) machine repair problem with spares and state dependent rates. Jain (1998) also studied machine repair problem with spares and additional repairmen. Gupta (1999) developed N – policy queuing system with finite source and warm spares. Ke and Wang (1999) considered the machine repair model with balking, reneging and server breakdowns. M/M/C/K/N machine interference model with balking, reneging, spares and additional repairman was examined by Jain et al. (2000). Jain et al. (2001a) established M/M/m/K model with additional replacement systems with minimal prediction of a flexible manufacturing system.

For smooth running of machining system, the provision of cold or warm standby components is common in real time systems, but it is recommended to facilitate mixed (cold and warm) standby components in many cases, as costs of cold and warm standbys are different and sometimes required quantity of one type of standbys is not available. For example, in power plant system, as standbys cold as well as warm spare units are used. Wang (1993) analyzed the cost of M/M/R machine repair system with mixed standbys. This paper addresses the performance issue of machining system maintained by a multi-repairmen facility by considering the provision of mixed spares along with on-line components. Considering state dependent rates also incorporates the concept of balking and reneging. In real time manufacturing and machining systems, degraded failures are frequently observed; this concept is also covered by the assumptions of state dependent rates. In section 2 we provide model description along with notations related to the model. In section 3, the governing equations and queue size distribution in product form is facilitated. Some special cases are also deduced to compare the results with earlier existing models. We obtain some performance measures in section 4. In section 5, we describe the cost function. Section 6 presents discussion and scope for future work.

2. MODEL DESCRIPTION

With a management point of view in machining systems it is not always essential to wait for a repair as it results into loss of production thus the production system needs standbys for instantaneous usage. In many situations balking and reneging of machines for repair is essential rather than creating repair facilities for a long queue of failed machines at the cost of capital investment. We consider machine repair problem with balking, reneging and mixed standby components. The failed machines arrive according to FCFS discipline and repair of failed machines and lifetime follow exponential distributions. A pool of C repairmen repairs the failed machines. The repaired machines join the standby group unless the system is short. In case of short system the repaired machines are immediately used for service. For modeling purpose, we use the following notations:

K. Capacity of the system.
N. Number of operating machines.
n. Number of failed machines in the system.
Y. Number of cold standby machines.
C. Number of permanent repairmen in the system.
λ. Failure rate of machines in the system.
μ. Service rate of permanent repairmen.
S. Number of warm standby machines.
α. Reneging rate of the machines.
β. Balking probability of the machine when all permanent.
repairmen are busy.
P_0. Steady state probability of empty state.
$P_n(t)$. Transient state probability that there are n failed machines in the system at time t (n = 0,1,2, ------ ,Y+S+K).
P_n. Steady state probability of n^{th} state.
a The parameter which denotes the degree to which repair rate is affected by the system state.

3. THE GOVERNING EQUATION AND PERFORMANCE MEASURES

We consider the two cases, which are given as follows:

Case I: When C $\leq$ Y
In this case the birth and death rates for the model

are:

$$\lambda_n=\begin{cases}N\lambda+S\alpha & 0\le n<C\\ (N\lambda+S\alpha)\beta & C\le n<Y\\ [N\lambda+(S+Y-n)\alpha]\beta & Y\le n<Y+S\\ [N+Y+S-n]\beta\lambda & Y+S\le n<Y+S+K\\ 0 & n\ge Y+S+K\end{cases}$$

and

$$\mu_n=\begin{cases}n\mu & 1\le n\le C\\ \left(\dfrac{n}{C}\right)^a C\mu & C<n\le Y+S+K\end{cases}$$

The transient state equations are as follows:

$$P_0'(t)=-[(N\lambda+S\alpha)]P_0(t)+\mu P_1(t) \tag{1}$$

$$P_n'(t)=-[N\lambda+S\alpha+n\mu]P_n(t)+ [N\lambda+S\alpha]P_{n-1}(t)+(n+1)\mu P_{n+1}(t) \quad 1\le n\le C \tag{2}$$

$$P_C'(t)=-[(N\lambda+S\alpha)\beta+C\mu]P_C(t)+ (N\lambda+S\alpha)P_{C-1}(t)+\left(\frac{C+1}{C}\right)^a C\mu P_{C+1}(t) \tag{3}$$

$$P_n'(t)=-\left[(N\lambda+S\alpha)\beta+\left(\frac{n}{C}\right)^a C\mu\right]P_n(t)+ (N\lambda+S\alpha)\beta P_{n-1}(t)+\left(\frac{n+1}{C}\right)^a C\mu P_{n+1}(t)$$

$$C+1\le n\le Y \tag{4}$$

$$P_n'(t)=-\begin{bmatrix}\{N\lambda+(S+Y-n)\alpha\}\beta\\ +\left(\dfrac{n}{C}\right)^a C\mu\end{bmatrix}P_n(t) +[N\lambda+(S+Y-n+1)\alpha]\beta P_{n-1}(t) +\left(\frac{n+1}{C}\right)^a C\mu P_{n+1}(t) \quad Y+1\le n\le Y+S \tag{5}$$

$$P_n'(t)=-\left[(N+Y+S-n)\beta\lambda+\left(\frac{n}{C}\right)^a C\mu\right]P_n(t) +(N+Y+S-n+1)\beta\lambda P_{n-1}(t) +\left(\frac{n+1}{C}\right)^a C\mu P_{n+1}(t)$$

$$Y+S+1\le n\le Y+S+K-1 \tag{6}$$

$$P_{Y+S+K}'(t)=-\left(\frac{Y+S+K}{C}\right)^a C\mu P_{Y+S+K}(t) +(N-K+1)\beta\lambda P_{Y+S+K-1}(t) \tag{7}$$

Now taking limit $t\to\infty$, the differential – difference equations (1)-(7) reduce in steady state equations as:

$$-[N\lambda+S\alpha]P_0+\mu P_1=0 \tag{8}$$

$$-\left[N\lambda + S\alpha + n\mu\right]P_n + (N\lambda + S\alpha)P_{n-1}$$

$$+ (n+1)\mu P_{n+1} = 0$$

$$1 \le n \le C - 1 \tag{9}$$

$$-\left[(N\lambda + S\alpha)\beta + C\mu\right]P_C + (N\lambda + S\alpha)P_{C-1}$$

$$+\left(\frac{C+1}{C}\right)^a C\mu P_{C+1} = 0 \tag{10}$$

$$-\left[(N\lambda + S\alpha)\beta + \left(\frac{n}{C}\right)^a C\mu\right]P_n$$

$$+ (N\lambda + S\alpha)\beta P_{n-1}$$

$$+\left(\frac{n+1}{C}\right)^a C\mu P_{n+1} = 0$$

$$C + 1 \le n \le Y \tag{11}$$

$$-\left[[N\lambda + (S + Y - n)\alpha]\beta + \left(\frac{n}{C}\right)^a C\mu\right]P_n +$$

$$[N\lambda + (S + Y - n + 1)\alpha]\beta P + \left(\frac{n+1}{C}\right)^a C\mu P_{n+1} = 0$$

$$\text{Y+1} \le n \le Y + S \tag{12}$$

$$-\left[(N + Y + S - n)\beta\lambda + \left(\frac{n}{C}\right)^a C\mu\right]P_n +$$

$$(N + Y + S - n + 1)\beta\lambda P_{n-1} + \left(\frac{n+1}{C}\right)^a C\mu P_{n+1} = 0$$

$$Y + S + 1 \le n \le Y + S + K - 1 \tag{13}$$

$$-\left(\frac{Y+S+K}{C}\right)^a C\mu P_{Y+S+K} +$$

$$(N - K + 1)\beta\lambda P_{Y+S+K-1} = 0 \tag{14}$$

Now solving equations (8) – (14), we have:

$$P_n = \begin{cases} \dfrac{(N\lambda + S\alpha)^n}{n!\,\mu^n} P_0 & 0 \le n \le C \\[2ex] \dfrac{[N\lambda + S\alpha]^n \beta^{n-c}}{C!\,\mu^n \left[\left(\dfrac{n!}{C!}\right)^a C^{(1-a)(n-C)}\right]} P_0 & C + 1 \le n \le Y \\[2ex] \dfrac{[N\lambda + S\alpha]^Y \prod\limits_{i=Y+1}^{n} [N\lambda + (S + Y - i + 1)\alpha]\beta^{n-C}}{C!\,\mu^n \left[\left(\dfrac{n!}{C!}\right)^a C^{(1-a)(n-C)}\right]} P_0 & Y + 1 \le n \le Y + S \\[2ex] [N\lambda + S\alpha]^Y \prod\limits_{i=Y+1}^{Y+S} [N\lambda + (Y + S - i + 1)\alpha]\,\lambda^{n-Y-S}\,\beta^{n-C} \\ \quad \times \dfrac{\prod\limits_{i=Y+S+1}^{n} [N + Y + S - i + 1]}{C!\,\mu^n \left[\left(\dfrac{n!}{C!}\right)^a C^{(1-a)(n-C)}\right]} P_0 & Y + S + 1 \le n \le Y + S + K \end{cases} \tag{15}$$

To find P_0, we use normalizing condition $\sum_{n=0}^{Y+S+K} P_n = 1$, which gives

$$P_0^{-1} = \left[\sum_{n=0}^{C} \frac{(N\lambda + S\alpha)^n}{n!\mu^n} + \frac{1}{C!} \right.$$

$$\sum_{n=C+1}^{Y} \frac{[(N\lambda + S\alpha)]^n \beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^a C^{(1-a)(n-C)} \mu^n\right]} +$$

$$\frac{[N\lambda + S\alpha]^Y}{C!}$$

$$+ \sum_{n=Y+1}^{Y+S} \frac{\prod_{i=Y+1}^{n} [N\lambda + (S+Y-i+1)\alpha]\, \beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^a C^{(1-a)(n-C)} \mu^n\right]} +$$

$$\frac{[N\lambda + S\alpha]^Y}{C!} \sum_{n=Y+S+1}^{Y+S+K}$$

$$\prod_{i=Y+1}^{Y+S} [N\lambda + (Y+S-i+1)\alpha]$$

$$\left. \times \frac{\prod_{i=Y+S+1}^{n} [N+Y+S-i]\, \beta^{n-C} \lambda^{n-Y-S}}{\left[\left(\frac{n!}{C!}\right)^a C^{(1-a)(n-C)} \mu^n\right]} \right] \tag{16}$$

The expected number of failed machines in the system is

$$E(N) = \sum_{n=0}^{Y+S+K} nP_n \quad = P_0 \left[\sum_{n=0}^{C} \frac{(N\lambda + S\alpha)^n}{(n-1)!\mu^n} \right.$$

$$+ \frac{1}{C!} \sum_{n=C+1}^{Y} n \frac{[(N\lambda + S\alpha)]^n \beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^a C^{(1-a)(n-C)} \mu^n\right]} +$$

$$\frac{[N\lambda + S\alpha]^Y}{C!} \sum_{n=Y+1}^{Y+S}$$

$$n \frac{\prod_{i=Y+1}^{n} [N\lambda + (S+Y-i+1)\alpha] \beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^a C^{(1-a)(n-C)} \mu^n\right]}$$

$$+ \frac{[N\lambda + S\alpha]^Y}{C!} \times$$

$$\prod_{i=Y+1}^{Y+S} [(N\lambda + (Y+S-i+1)\alpha]$$

$$\left. \sum_{n=Y+S+1}^{Y+S+K} n \frac{\prod_{i=Y+S+1}^{n} [N+Y+S-i+1]\, \beta^{n-C} \lambda^{n-Y-S}}{\left[\left(\frac{n!}{C!}\right)^a C^{(1-a)(n-C)} \mu^n\right]} \right] \tag{17}$$

Expected number of failed machines in the queue (or queue length) is

$$L_q = \sum_{n=C}^{Y+S+K} (n-C) P_n$$

$$= P_O \left[\frac{1}{C!} \sum_{n=C+1}^{Y} (n-C) \frac{[(N\lambda + S\alpha)]^n \beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^a C^{(1-a)(n-C)} \mu^n\right]} \right.$$

$$+\frac{[N\lambda+S\alpha]^{Y}}{C!}\sum_{n=Y+1}^{Y+S}$$

$$(n-C)\frac{\prod_{i=Y+1}^{n}[N\lambda+(S+Y-i+1)\alpha]\,\beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^{a}C^{(1-a)(n-C)}\mu^{n}\right]}+$$

$$\frac{[N\lambda+S\alpha]^{Y}}{C!}$$

$$\times\sum_{n=Y+S+1}^{Y+S+K}(n-C)\frac{\begin{array}{c}\prod_{i=Y+1}^{Y+S}[(N\lambda+(Y+S-i+1)\alpha]\times\\ \prod_{i=Y+S+1}^{n}[N+Y+S-i+1]\\ \beta^{n-C}\lambda^{n-Y-S}\end{array}}{\left[\left(\frac{n!}{C!}\right)^{a}C^{(1-a)(n-C)}\mu^{n}\right]}\Bigg]$$

(18)

Special Case When a = 0 and β=1 (i.e. model without balking and reneging), we get

$$P_n=\begin{cases}\frac{[N\lambda+S\alpha]^{n}}{C!\mu^{n}C^{n-C}}P_0 & C+1\le n\le Y\\ \frac{[N\lambda+S\alpha]^{Y}\prod_{i=Y+1}^{n}[N\lambda+(Y+S-i+1)\alpha]}{C!\mu^{n}C^{n-C}}P_0 & Y+1\le n\le Y+S\end{cases}$$

$$P_x=\begin{cases}\frac{[N\lambda+S\alpha]^{Y}\prod_{i=Y+1}^{Y+S}[N\lambda+(Y+S-i+1)\alpha]\prod_{i=Y+S+1}^{n}[N+Y+S-i+1]\times(\lambda^{n-Y-S})}{C!\mu^{n}C^{n-C}}P_0\\ Y+S+1\le n\le Y+S+K\end{cases}$$

(19)

Case II: When Y < C In this case, the birth and death rates are given by:

$$\lambda_n=\begin{cases}N\lambda+S\alpha & 0\le n<Y\\ N\lambda+(S+Y-n)\alpha & Y\le n<C\\ [N\lambda+(S+Y-n)\alpha]\beta & C\le n<Y+S\\ (N+Y+S-n)\beta\lambda & Y+S\le n<Y+S+K\end{cases}$$

(20)

and

$$\mu_n=\begin{cases}n\mu & 1\le n\le C\\ \left(\frac{n}{C}\right)^{a}C\mu & C<n\le Y+S+K\end{cases}$$

(21)

The transient state equation are as follows:

$$P_0'(t)=-[N\lambda+S\alpha]P_0(t)+\mu P_1(t)$$

(22)

$$P_n'(t)=-[N\lambda+S\alpha+n\mu]P_n(t)+[N\lambda+S\alpha]P_{n-1}(t)+(n+1)\mu P_{n+1}(t)\qquad 1\le n\le Y$$

(23)

$$P_n'(t) = -[N\lambda + (S+Y-n)\alpha + n\mu]P_n(t)$$

$$+ [N\lambda + (S+Y-n+1)\alpha]P_{n-1}(t)$$

$$+ [(n+1)\mu]P_{n+1}(t) \qquad Y+1 \le n \le C-1$$

(24)

$$P_C'(t) = -\begin{bmatrix}[N\lambda + (S+Y-C)\alpha]\beta \\ + C\mu\end{bmatrix}P_C(t)$$

$$+ [N\lambda + (S+Y-C+1)\alpha]P_{C-1}(t)$$

$$+ \left[\ \left(\frac{C+1}{C}\right)^a C\mu\ \right]P_{C+1}(t)$$

(25)

$$P_n'(t) = -\begin{bmatrix}\{N\lambda + (S+Y-n)\alpha\}\beta \\ + \left(\frac{n}{C}\right)^a C\mu\end{bmatrix}P_n(t)$$

$$+ [N\lambda + (S+Y-n+1)\alpha]\beta P_{n-1}(t)$$

$$+ \left[\ \left(\frac{n+1}{C}\right)^a C\mu\ \right]P_{n+1}(t)$$

$$C+1 \le n \le Y+S$$

(26)

$$P_n'(t) = -\left[(N+Y+S-n)\beta\lambda + \left(\frac{n}{C}\right)^a C\mu\right]P_n(t)$$

$$+ [(N+Y+S-n+1)\beta\lambda]P_{n-1}(t)$$

$$+ \left[\ \left(\frac{n+1}{C}\right)^a C\mu\ \right]P_{n+1}(t)$$

$$Y+S+1 \le n \le Y+S+K-1$$

(27)

$$P_{Y+S+K}'(t) = -\left(\frac{Y+S+K}{C}\right)^a C\mu P_{Y+S+K}(t)$$

$$+ (N-K+1)\beta\lambda P_{Y+S+K-1}(t)$$

(28)

The steady-state equations are obtained from equations (22)-(28) as:

$$-[N\lambda + S\alpha]P_0 + \mu P_1 = 0$$

(29)

$$-[N\lambda + S\alpha + n\mu]P_n + (N\lambda + S\alpha)P_{n-1}$$

$$+ (n+1)\mu P_{n+1} = 0$$

$$1 \le n \le Y$$

(30)

$$-[N\lambda + (S+Y-n)\alpha + n\mu]P_n +$$

$$[N\lambda + (S+Y-n+1)\alpha]P_{n-1} + (n+1)\mu P_{n+1} = 0$$

$$Y+1 \le n \le C-1$$

(31)

$$-[[N\lambda + (S+Y-C)\alpha]\beta + C\mu]P_C +$$

$$[N\lambda + (S+Y-C+1)\alpha]P_{C-1} +$$

$$\left(\frac{C+1}{C}\right)^a C\mu P_{C+1} = 0$$

(32)

$$-\left[[N\lambda + (S+Y-n)\alpha]\beta + \left(\frac{n}{C}\right)^a C\mu\right]P$$

$$n$$

$$+ [N\lambda + (S+Y-n+1)\alpha]\beta P_{n-1} +$$

$$\left(\frac{n+1}{C}\right)^a C\mu P_{n+1} = 0$$

$$C+1 \le n \le Y+S$$

(33)

$$-\left[(N+Y+S-n)\beta\lambda + \left(\frac{n}{C}\right)^a C\mu\right]P_n$$

$$+(N+Y+S-n+1)\beta\lambda P_{n-1} +$$

$$\left(\frac{n+1}{C}\right)^a C\mu P_{n+1} = 0$$

$$Y+S+1 \le n \le Y+S+K-1$$

(34)

$$-\left(\frac{Y+S+K}{C}\right)^a C\mu P_{Y+S+K} +$$

$$(N-K+1)\beta\lambda P_{Y+S+K-1} = 0$$

(35)

Now solving equations (29) – (35), we have:

$$P_n = \begin{cases} \dfrac{[N\lambda+S\alpha]^n}{n!\mu^n} P_0 & 1 \le n \le Y \\ \dfrac{[N\lambda+S\alpha]^Y}{n!\mu^n} \times \\ \prod_{i=Y+1}^{n} [N\lambda + (Y+S-i+1)\alpha] P_0 \\ Y+1 \le n \le C \end{cases}$$

$$P_n = \begin{cases} \dfrac{[N\lambda+S\alpha]^Y \times \prod_{i=Y+1}^{n} [N\lambda + (Y+S-i+1)\alpha]\, \beta^{n-C}}{C!\mu^n \left[\left(\dfrac{n!}{C!}\right) C^{(1-a)(n-C)}\right]} P_0 \\ C+1 \le n \le Y+S \\ \dfrac{[N\lambda+S\alpha]^Y \prod_{i=Y+1}^{Y+S} [N\lambda + (Y+S-i+1)\alpha] \prod_{i=Y+S+1}^{n} [N+Y+S-i+1]\lambda^{n-Y-S}\beta^{n-C}}{C!\mu^n \left[\left(\dfrac{n!}{C!}\right)^a\right] C^{(1-a)(n-C)}} P_0 \\ Y+S+1 \le n \le Y+S+K \end{cases}$$

(36)

To find P_0 using $\sum_{n=0}^{Y+S+K} P_n = 1$, we get:

$$P_0^{-1} = \left[\sum_{n=0}^{Y} \frac{(N\lambda+S\alpha)^n}{n!\mu^n} + [N\lambda+S\alpha]^Y \right.$$

$$\sum_{n=Y+1}^{C} \frac{\prod_{i=Y+1}^{n} [N\lambda + (S+Y-i+1)\alpha]}{n!\mu^n}$$

$$+\frac{[N\lambda+S\alpha]^Y}{C!} \sum_{n=C+1}^{Y+S}$$

$$\frac{\prod_{i=Y+1}^{n}\left[N\lambda+(S+Y-i+1)\alpha\right]\ \beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^{a} C^{(1-a)(n-C)}\mu^{n}\right]}$$

$$+\frac{[N\lambda+S\alpha]^{Y}\prod_{i=Y+1}^{Y+S}[N\lambda+(S+Y-i+1)\alpha]}{C!}$$

$$\times\sum_{n=Y+S+1}^{Y+S+K}$$

$$\left.\frac{\prod_{i=Y+S+1}^{n}[(N+Y+S-i+1)]\lambda^{n-Y-S}\beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^{a} C^{(1-a)(n-C)}\mu^{n}\right]}\right]$$

(37)

The expected number of machines in the system is:

$$E(N)=\sum_{n=0}^{Y+S+K} nP_n = P_0\left[\sum_{n=0}^{Y}\frac{(N\lambda+S\alpha)^{n}}{(n-1)!\mu^{n}}\right.$$

$$+[N\lambda+S\alpha]^{Y}$$

$$\sum_{n=Y+1}^{C}\frac{\prod_{i=Y+1}^{n}[N\lambda+(S+Y-i+1)\alpha]}{(n-1)!\mu^{n}}$$

$$+\frac{[N\lambda+S\alpha]^{Y}}{C!}$$

$$\sum_{n=C+1}^{Y+S} n\frac{\prod_{i=Y+1}^{n}[N\lambda+(S+Y-i+1)\alpha]\times\beta^{n-C}}{\mu^{n}\left[\left(\frac{n!}{C!}\right)^{a} C^{(1-a)(n-C)}\right]}+$$

$$\frac{[N\lambda+S\alpha]^{Y}\prod_{i=Y+1}^{Y+S}[N\lambda+(S+Y-i+1)\alpha]}{C!}\times$$

$$\left.\sum_{n=Y+S+1}^{Y+S+K} n\frac{\prod_{i=Y+S+1}^{n}[(N+Y+S-i+1)]\times(\lambda^{n-Y-S}\beta^{n-C})}{\left[\left(\frac{n!}{C!}\right)^{a} C^{(1-a)(n-C)}\mu^{n}\right]}\right]$$

(38)

Expected of machine in the queue (i.e. queue length) is:

$$L_q=\sum_{n=C}^{Y+S+K}(n-C)P_n = P_0$$

$$\left[\frac{[N\lambda+S\alpha]^{Y}}{C!}\sum_{n=C+1}^{Y+S}(n-C)\right.$$

$$\times\frac{\prod_{i=C+1}^{n}\left[N\lambda+(S+Y-i+1)\alpha\right]\beta^{n-C}}{\left[\left(\frac{n!}{C!}\right)^{a} C^{(1-a)(n-C)}\mu^{n}\right]}$$

$$+\frac{[N\lambda+S\alpha]^{Y}\prod_{i=C+1}^{Y+S}[N\lambda+(S+Y-i+1)\alpha]}{C!}$$

$$\left. \sum_{n=Y+S+1}^{Y+S+K} (n-C) \frac{\prod_{i=Y+S+1}^{n} [(N+Y+S-i+1)] \times (\lambda^{n-Y-S} \beta^{n-C})}{\left[\left(\frac{n!}{C!} \right)^{a} C^{(1-a)(n-C)} \mu^{n} \right]} \right] \tag{39}$$

Special Cases I. When $a = 0, \beta = 1$ (i.e. model with no balking and reneging)

$$P_n = \begin{cases} \dfrac{[N\lambda + S\alpha]^n}{n!\mu^n} P_0 & 1 \le n \le Y \\ \dfrac{[N\lambda + S\alpha]^Y \prod_{i=Y+1}^{n} [N\lambda + (Y+S-i+1)\alpha]}{n!\mu^n} P_0 & Y+1 \le n \le C \\ \dfrac{[N\lambda + S\alpha]^Y \prod_{i=Y+1}^{n} [N\lambda + (Y+S-i+1)\alpha]}{C!\mu^n C^{n-C}} P_0 & C+1 \le n \le Y+S \\ \dfrac{[N\lambda + S\alpha]^Y \prod_{i=Y+1}^{Y+S} [N\lambda + (Y+S-i+1)\alpha] \prod_{i=Y+S+1}^{n} [N+Y+S-i+1]\lambda^{n-Y-S}}{C!\mu^n C^{n-C}} P_0 & Y+S+1 \le n \le Y+S+K \end{cases} \tag{40}$$

II. When Y = S = 0 (i.e. model with no spares):

$$P_n = \begin{cases} \binom{N}{n} \rho^n P_0 & 0 \le n \le C \\ \binom{N}{n} \dfrac{n!}{C!C^{n-C}} \rho^n P_0 & C < n \le K \end{cases} \tag{41}$$

where $\rho = \dfrac{\lambda}{\mu}$

4. SOME MORE PERFORMANCE MEASURES

Various performance measures are obtained as given below:
Average number of operating machines is given as:

$$\mathrm{E(O)} = \mathrm{N} - \sum_{n=Y+1}^{Y+S+K} (n-Y-S)P_n \tag{42}$$

Average number of spare machines in the system is:

$$E(M) = \sum_{n=0}^{Y+S} (Y+S-n)P_n \tag{43}$$

Average number of idle repairmen is:

$$E(I) = \sum_{n=0}^{C-1} (C-n)P_n \tag{44}$$

Average number of busy repairmen is obtained as:

$$E(B) = C - E(I) \tag{45}$$

5. COST FUNCTION

In this section we provide the cost function to determine the optimal number of spares and repairmen by using various costs factors into consideration. Total average cost is given by:

$$E\,(C) = C_1 \sum_{n=Y+1}^{Y+S+K} (n - Y - S)\, P_n + C_2\, E(M) + C_3$$

$$E\,(B) + C_4\, E\,(I)$$

where:

C_1 Cost per unit time when all spares (cold and warm) are exhausted.

C_2 Cost per unit time when a spare machine is provided.

C_3 Cost per unit time per repairman when repairman is in busy state.

C_4 Cost per unit time of a repairman when he is in idle state.

6. DISCUSSION

We have incorporated mixed standby components to improve the grade of service of a machining system with balking and reneging under the care of multi-repair facility. In this paper we have provided the product type solution for queue size distribution. Some performance measures viz, expected number of operating machines/spares machines, expected number of busy and idle repairmen are suggested. The special cases deduced match with the earlier work of researchers. A cost function is also suggested which may be employed to determine the optimal number of repairmen and spares so that system engineer may grow out maximum output up to desired level. The provision of mixed standby units along with on-line units may be beneficial to achieve the pre-specified goal of production/manufacturing in machining system when on-line units are subject to failure. Such machining system wherein mixed standby components along with on-line components are employed to reduce the loss of production due to components failure can be seen in power plants, electronic/electrical component manufacturing system, computer accessories producing units, etc.

7. REFERENCES

1. Berg, M. and Posner, M. J. M., "Customer Delays in M/M/C Repair System with Spares", ***Naval Res. Logist Quart***, Vol. 32, (1985) 287-299.
2. Daduna, H., "Exchangeable Items in Repair System Delay Times", ***Oper. Res.***, Vol. 38, (1990), 349-355.
3. Gupta, S. M., "N-Policy Queuing System with Finite Source and Warm Spares", ***OPSEARCH***, Vol. 36, No. 3, (1999) 189-217.
4. Gross, D. and Harris, C. M., "Fundamentals of Queuing Theory", Second Edition, John Wiley and Sons, New York, (1985).
5. Gupta, U. C. and Srinivasa Rao, T. S. S., "M/G/1 Machine Interference Problem with Spares", ***Perf. Eval.***, Vol. 24, (1996), 265-275.
6. Gopalan, M. N., "Probability Analysis of a Single Server n Unit System with (n-1) Warm Standby", ***Oper. Res.***, Vol. 23, (1975), 591-595.
7. Hilliard, J. E., "An Approach to Cost Analysis of Maintenance Float Systems", ***AIIE Trans.***, Vol. 8, (1976), 128-133.
8. Jain, M., "(m, M) Machine Repair Problem with Spares and State Dependent Rates: A Diffusion Process Approach", ***Microelectron. Reliab.***, Vol. 37, No. 6, (1997), 929-933.
9. Jain, M., "M/M/R Machine Repair Problem with Spares and Additional Repairment", ***Ind. J. Repairmen, Ind. J. Pure Appl. Maths***., Vol. 29, No. 5, 517-524.
10. Jain, M., Singh, M. and Baghel, K. P. S., "M/M/C/K/N Machine Repair Problem with Balking, Reneging, Spares and Additional Repairmen", ***GSR***, Vol. 26-27, (2000), 49-60.
11. Jain, M., Sharma, G. C. and Singh, M., "M/M/m/K Queue with Additional Servers and Discouragements", ***International Journal of Engineering***, Vol. 15, No. 4A, (2002), 349-354.
12. Jain, M., and Baghel, K. P. S., Jadown, M. A., "Repairable System with Spares State Dependent Rates and Additional Repairmen", ***Proceedings of Thirty Fourth Annual Convention of ORSI***., (2001), 134-136.
13. Jain, M., Sharma, G. C. and Baghel, K. P. S., "Performance Prediction of a Flexible Manufacturing System", ***International Journal of Engineering***, Vol. 15, No. 4A, (2002), 355-364.
14. Jain, M., Sharma, G. C. and Kumar, A., "Maintenance Cost Analysis for Replacement Model with Perfect Minimal Repair", ***International Journal of Engineering***, Vol. 15, No. 2A, (2002), 161-168.

15. Ke, J. C. and Wang, K. H., "Cost Analysis of the M/M/R Machine Repair Problem with Balking, Reneging and Server Breakdowns", ***J. Oper. Res. Soc.***, Vol. 50, (1998), 275-282.
16. Meng, F. C,. "Comparing the MTBF of Four Systems with Standby Components", ***Microelectron. Reliab.***, Vol. 35, (1995), 1031-1035.
17. Wang, K. H., "Cost Analysis of the M/M/R Machine Repair Problem with Mixed Standby", ***Microelectron. Reliab.***, Vol. 33, (1993), 1293-1301.