RESEARCH NOTE

BEHAVIOR OF PACKED BED THERMAL STORAGE

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Abstract A packed bed thermal storage has several desirable characteristics to be used for energy storage. The behavior of packed bed is predicted by set of differential equations. A numerical solution is developed for packed bed storage tank accounting to the secondary phenomena of thermal losses and conduction effect. The effect of heat loss to surrounding (k_1), conduction effect (k_2) and air capacitance (k_3) are examined in the numerical solution. It is found that the values of k_1 and k_2 are small and can practically be neglected in the solution. The solution indicates the profiles of air and rock bed temperatures with respect to time and length of the bed.

Key Words Packed Bed; Storage Tank; Numerical Solution, Heat Loss

چکیده منبع ذخیره گرما از نوع بستر ثابت دارای خواص فیزیکی و مکانیکی مناسبی برای استفاده در سیستمهای ذخیره گرمایی است. سیستمهای گرمایشی خورشیدی به دلیل محدودیت در ساعات روز و تابش نور خورشید باید دارای نوعی سیستم ذخیره گرما باشند. در این تحقیق رفتارهای مکانیکی، حرارتی و دینامیکی سیستم ذخیره گرمایی از نوع بستر ثابت را بوسیله مجموعه‌ای از معادلات دیفرانسیل انتقال جرم و حرارت مورد مطالعه قرار داده و همچنین از روش Finite Element برای حل این معادلات استفاده می‌شود. اثر ضرایب ثابت k_1 ، k_2 و k_3 در حل این معادلات بررسی شده است. نتیجه‌گیری شده است که مقدار ضرایب k_1 و k_2 اثری در حل معادلات و پیش‌گویی رفتار دینامیکی بستر ثابت ندارد.

1. INTRODUCTION

The limited amount of fossil energies has forced scientists all over the world to search for alternative renewable energy sources. The use of renewable energies has, therefore, seriously been considered in the last three decades by researchers. The sun has been the major source of renewable energy from long time ago. This energy has had a determinate contribution to the life of human being from the formation of life on the earth up to now. The transfer of energy from sun to the water took first place as the most common way of heat storage. To realize the substantial effect of the solar energy, one needs to consider its storage as electrical, chemical, mechanical or thermal energy. A few possible means of solar energy storage are mentioned here. Some of the methods need further research and development to make them technically and economically sound.

The storage of energy in solar power plant is essential for our round clock system operation. The energy storage issue is nowadays utilized to adjust the energy consumption rate or supply cut off problems. A packed bed system is considered to be convenient equipment for heat storage effort. The heat transfer to and from a flowing fluid is subjected to the theoretical investigations in order to obtain the practical information on its application to solar storage design [1-4]. The quantities of the important parameters like G (fluid flow rate per unit area), L (bed length), d (solid bed particle equivalent diameter) and A (bed surface area) are required for designing of a thermal storage bed. The assumptions made for mathematical analysis of heat transfer in a rock bed storage system are as follows: (i) negligible internal gradient within the solid particles, (ii) no internal heat generation and (iii) no mass transfer taking place in the system.

A model can be used to describe the heat transfer in a packed bed system. Various investigators based on the two sets of differential equations have proposed different models. The first is for fluid media and the second is for solid [5-7]. Schumann [1] has analytically solved the problem. Several researchers have worked on the problem with different variations and extensions; but limited information is available on its application to solar design. In this work, the system is defined by a set of partial differential equations and the backward finite difference technique is used to solve the partial differential equations. The model accounts for such phenomena as thermal loss and conduction effects. The model indicates the importance of critical parameters like airflow rate per unit surface area, rock equivalent diameter, length of bed and bed surface area for designing purpose, too.

2. MODEL

Packed bed energy storage is a particular application of a group of processes involving fluid flow through a porous media. Different industrial processes are performed in the packed columns. A number of problems are associated with the design and operation of such columns. Every process has its own specific fluid dynamic characteristics with respect to the bed physical properties that have to be considered. The design aspects include the effect of a pumping power, total energy transferred, temperature difference and noise control.

Consider a cylindrical storage rock bed with the X-direction along its axis. An elemental volume located between the abscissa x and x + dx is considered for heat transfer evaluation. The governing differential equation for the energy supplied by air to the rock bed through convection into the elemental volume during dt is:

$$h_v A (T_a - T_s) dx\, dt \qquad (1)$$

where the volumetric convective heat transfer coefficient, h_v, is the product of the heat transfer coefficient per unit area of the rock (h) and the surface area of the rock per unit volume (a). A, T_a and T_s are the bed cross sectional area, the air and the rock temperatures, respectively.

The quantity of heat carried away by the air is:

$$C_a G A (\partial T_a / \partial x) dx\, dt \qquad (2)$$

where G is the mass flow rate of air per unit cross sectional area and C_a is the heat capacity of the air.

The heat loss to the surroundings is

$$Ud\pi (T_a - T_\infty) dx\, dt \qquad (3)$$

where D, U and T_∞ are the rock bed diameter, bed heat loss coefficient and surrounding temperature, respectively.

The energy balance for the air is obtained by summing up Equations 1, 2 and 3:

$$\rho_a C_a A f (\partial T_a / \partial t) dx\, dt \qquad (4)$$

where ρ_a and f are the air density and the void fraction, respectively.

The first energy balance differential equation is derived for gaseous phase:

$$(\partial T_a / \partial t) + (G/\rho_a f)(\partial T_a / \partial x) = (-h_v / \rho_a C_a f)(T_a - T_s) - (\pi UD / \rho_a C_a f)(T_a - T_\infty) \qquad (5)$$

Heat balance for the rock bed (solid phase) is similarly obtained from:

$$(\partial T_s / \partial t) = [(h_v / \rho_s C_s (1-f)](T_a - T_s) + [k_s / \rho_s C_s (1-f)] \partial^2 T_s / \partial x^2 \qquad (6)$$

where k_s, ρ_s and C_s are thermal conductivity, density and specific heat of the rock particles, respectively. The first term in right hand side of Equation 6 is for heat transfer by convection from air, while the second describes the conduction effect within the rocks. An equivalent diameter is calculated for a spherical shape for rocks of irregular shape. In order to make Equations 5 and 6 dimensionless, the following groups of parameters are introduced:

$$y = ax = (h_v / G C_a) x \qquad (7)$$

$$z = \beta t = [h_v / \rho_s C_s (1-f)] t \qquad (8)$$

Equations 5 and 6 become:

$$\partial T_a / \partial y + k_3 (\partial T_a / \partial z) = T_s - T_a - k_1(T_a - T_\infty) \quad (9)$$

$$\partial T_s / \partial z = T_a - T_s + k_2 (\partial^2 T_s - \partial y^2) \quad (10)$$

where k_1, k_2 and k_3 are dimensionless coefficients.

$k_1 = U / h_v D$, $k_2 = h_v k_s / G^2 C^2_p$, $k_3 = \rho_a C_a f / \rho_s C_s(1\text{-}f)$

k_1, k_2 and k_3 correspond to the heat loss to the surroundings, conduction effects and air capacitance, respectively.

The initial conditions are:

$$T_a (x, t = 0) = T_a (x) \text{ or } T_a (y, z = 0) = T_a (y) \quad (11)$$

$$T_s (x, t = 0) = T_s (x) \text{ or } T_s (y, z = 0) = T_s(y) \quad (12)$$

For completely discharge rock bed, the initial condition becomes:

$$T_a (y, z = 0) = T_{a0} \text{ and } T_s (y, z = 0) = T_{s0} \quad (13)$$

The boundary conditions are:

$$T_a (y, z = 0) = T_{a0} \qquad T_s(y, z = 0) = T_{s0} \quad (14)$$

$$T_a (x = 0, t) = T_a (t) \text{ or } T_a (y = 0, z) = T_a(z) \quad (15)$$

In the solar system, $T_a(t)$ is a function of time:

$$T_a (t) = T_{a0} + (T_{amax} - T_{ao}) (\sin (\pi t / t_{coll.}))^p \quad (16)$$

$t_{coll.}$ is the time when the positive value of collector efficiency allows actual solar collection. The variation of temperature with respect to time for solar system can be expressed by Equation 16 where p can be set such that to define the temperature variation shape throughout the day since in the solar system, temperature rises from minimum to maximum and then returns to minimum value. The value of P is taken to be 1.5 in the model calculations, which conveniently defines the temperature variation:

$$\Delta T_a = T_a (\text{average}) - T_{a0}$$

If values of k_1 and k_2 are assumed to be small, Equations 9 and 10 become:

$$\partial T_a / \partial y + k_3 (\partial T_a / \partial z) = T_s - T_a \quad (17)$$

$$\partial T_s / \partial y = T_a - T_s \quad (18)$$

Considering Equations 7 and 8 and differentiating with respect to x and t, respectively:

$$\partial y/\partial x = h_v/GCa \quad (19)$$

$$\partial z/\partial t = h_v/ \rho_s C_s(1\text{-}f) \quad (20)$$

The Equations 17 and 18 can be written in the form of:

$$\partial T_a / \partial x \,.\, \partial x / \partial y + k_3 (\partial T_a / \partial t \,.\, \partial t / \partial x) = T_s - T_a \quad (21)$$

$$\partial T_s / \partial t \,.\, \partial t / \partial z = T_a - T_s \quad (22)$$

Equations 21 and 22 can finally be written as:

$$\partial T_a / \partial t + v\, \partial T_a / \partial x + k'_1 (T_a - T_s) = 0 \quad (23)$$

$$\partial T_s / \partial t - k'_2 (T_a - T_s) = 0 \quad (24)$$

where

$$V = G/(\rho_a f) \quad (25)$$

$$k'_1 = h_v / \rho_a C_a f \quad (26)$$

$$k'_2 = h_v / \rho_s C_s (1\text{-}f) \quad (27)$$

Equations 23 and 24 can be written in terms of finite difference for (n-1> x > 2) as:

$$-AT_a(x\text{-}1,t+1)+BT_a(x,t+1)+ AT_a(x+1,t+1) = CT_s(x,t) + T_a(x,t) \quad (28)$$

$$T_s(x,t+1) = (T_s(x,t)/(1+ k'_2 \Delta t)) + (k'_2 \Delta t\, T_a(x,t+1))/(1+ k'_2 \Delta t) \quad (29)$$

where

$A = v\Delta t / 2\Delta x$ $B = 1+ k'_1\Delta t - k'_1 k'_2 \Delta t^2/ (1+ k'_2\Delta t)$

$C = k'_1\Delta t/(1+ k'_2\Delta t)$ $D = v\Delta t/\Delta x = 2A$

$E = 1 + v\Delta t/\Delta x + k'_1\Delta t - k'_1 k'_2\Delta t_2/(1+ k^2_2\Delta t) = B + D = B + 2A$

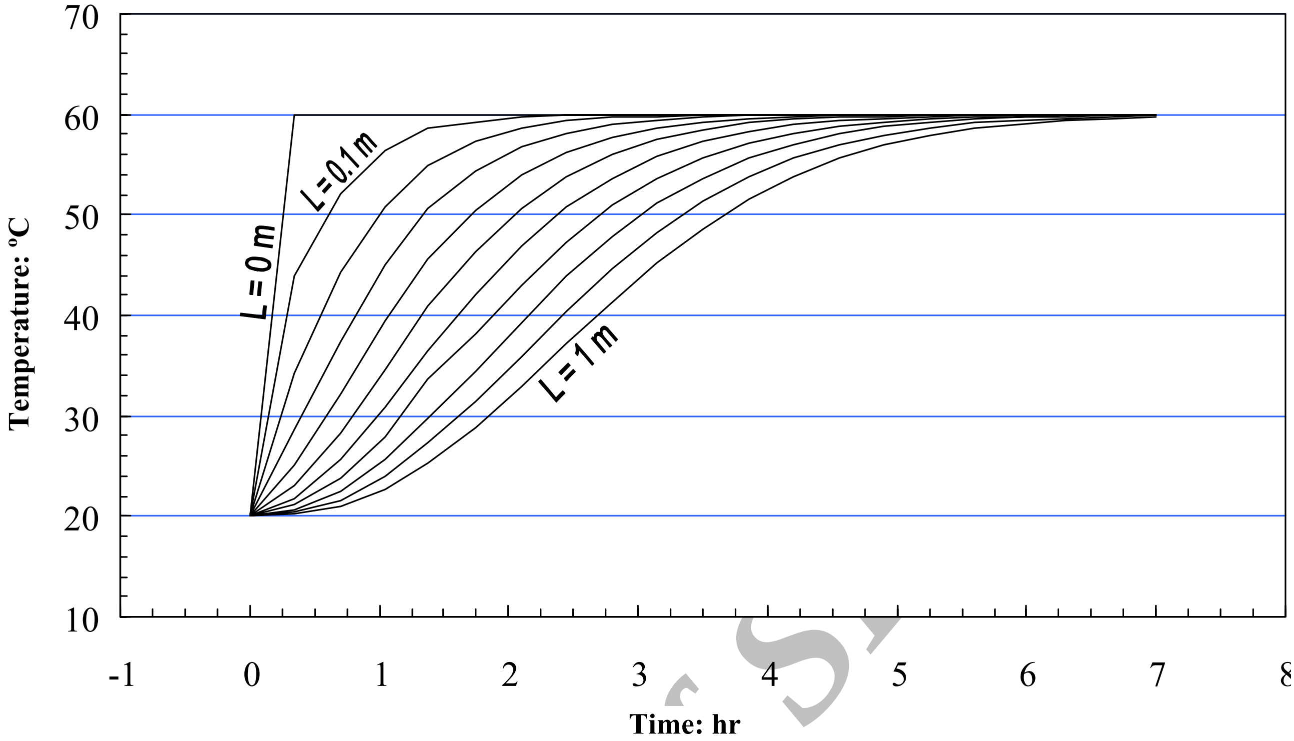

Figure 1. Variation of air temperature in different layers of rock bed.

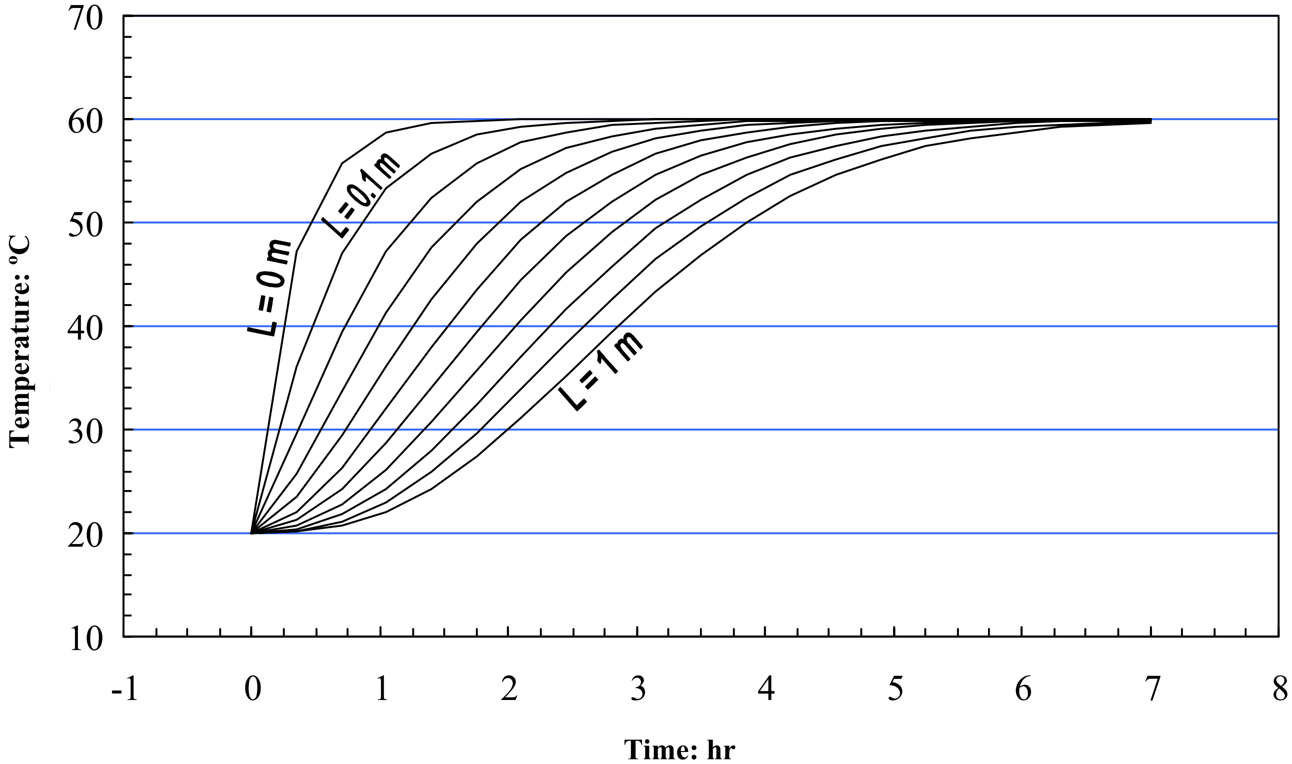

Figure 2. Variation of rock temperature in different layers of rock bed.

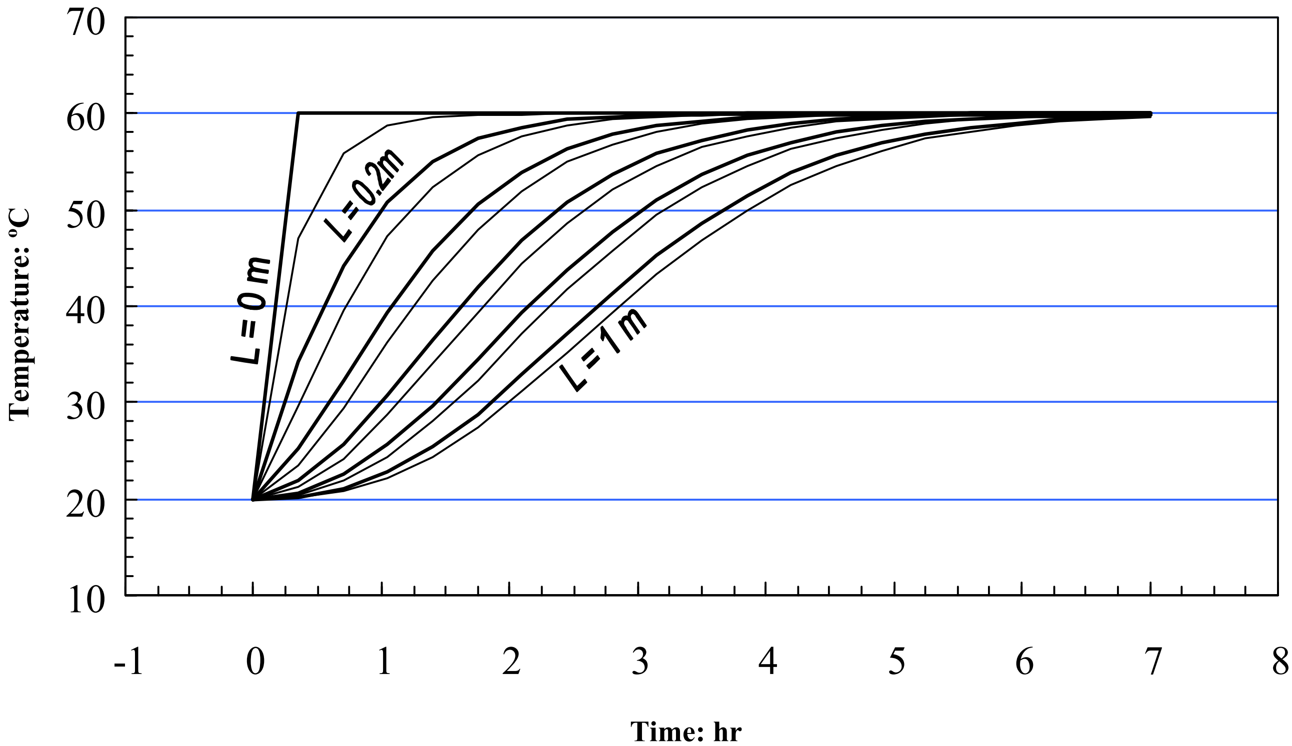

Figure 3. Comparison of rock and air temperatures variation.

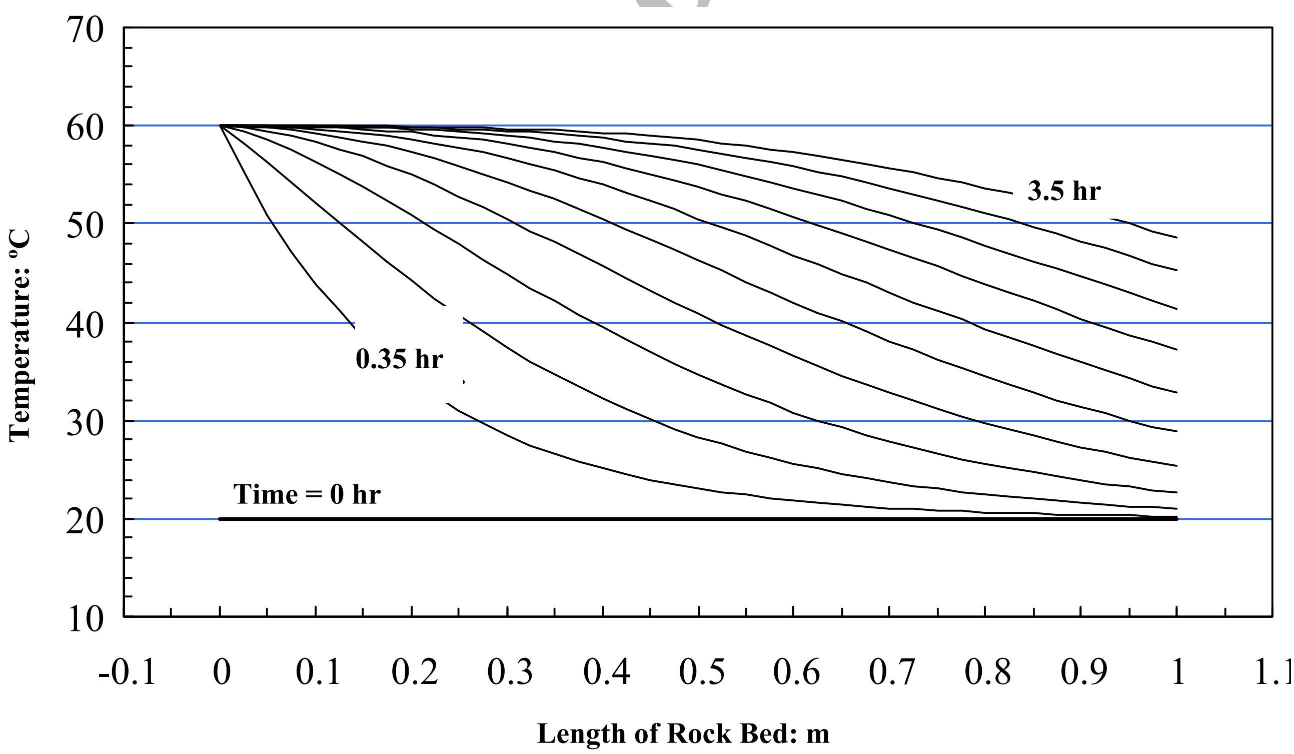

Figure 4. Variation of air temperature along bed with time.

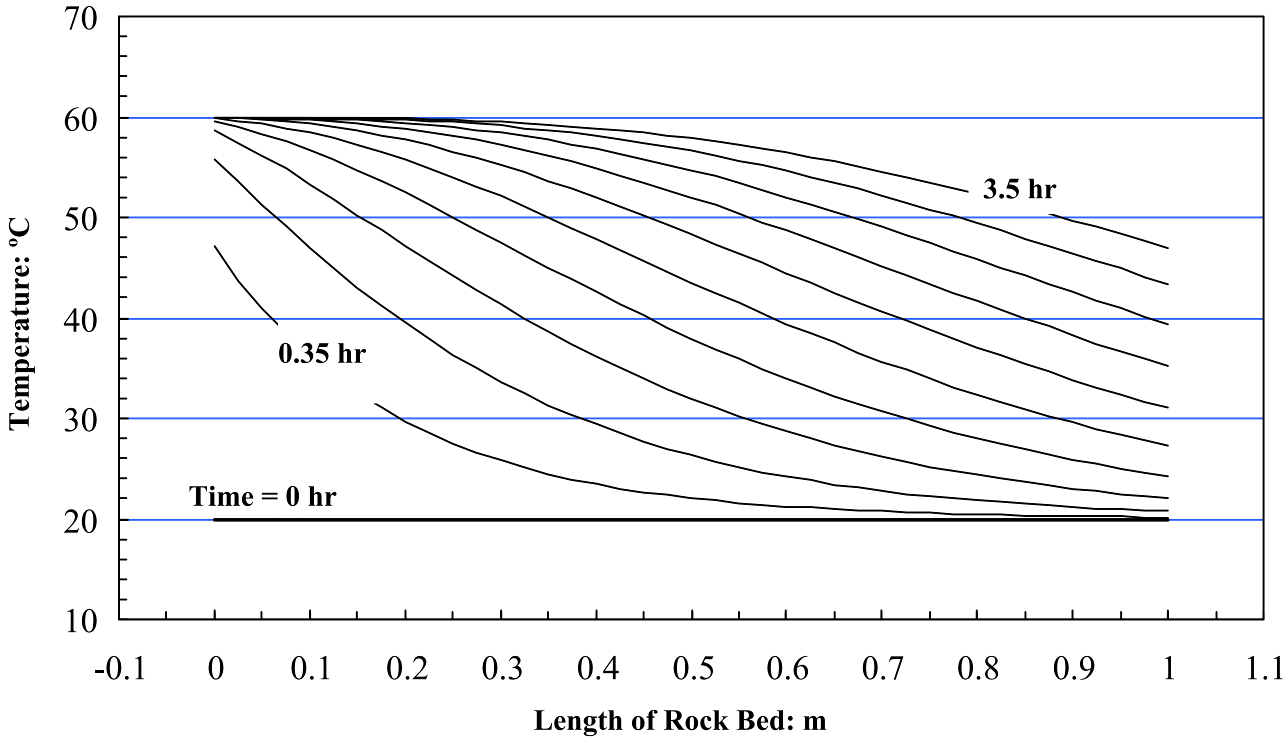

Figure 5. Variation of rock temperature along rock bed with time.

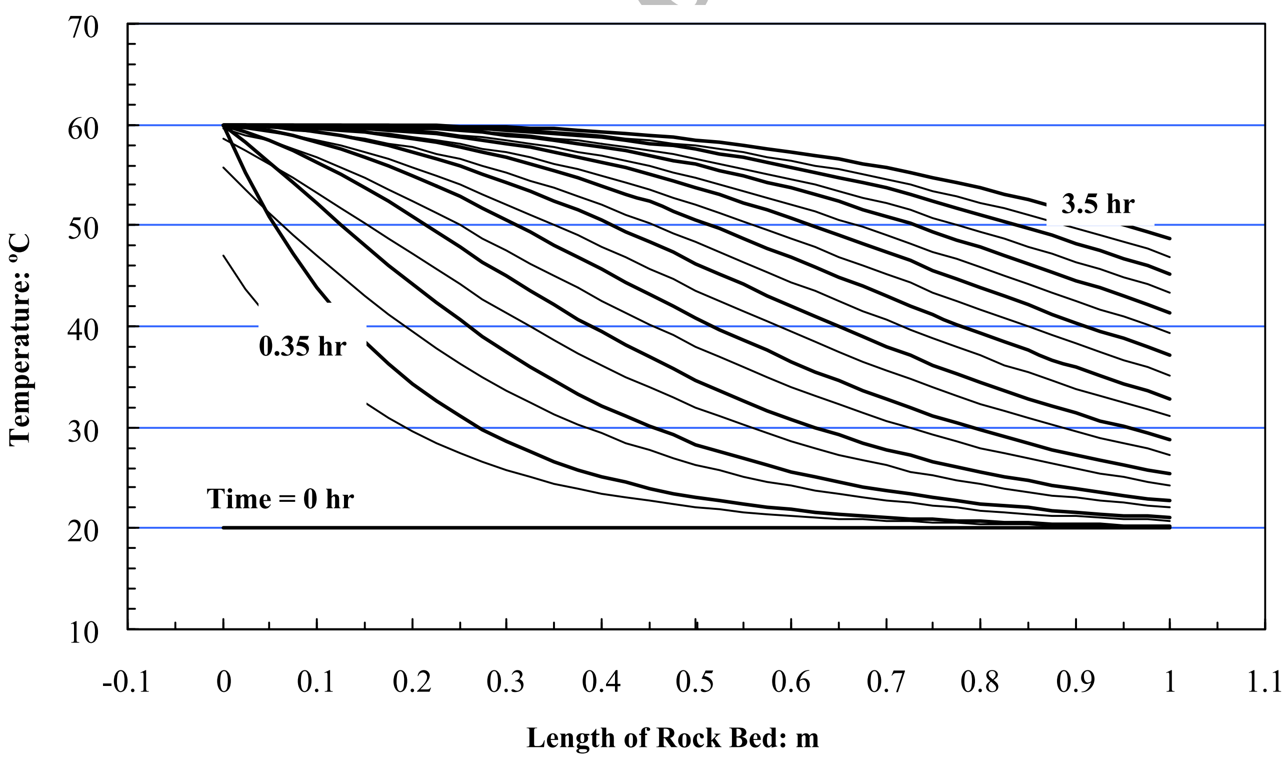

Figure 6. Comparison of rock and air temperatures variation along the rock bed length with time.

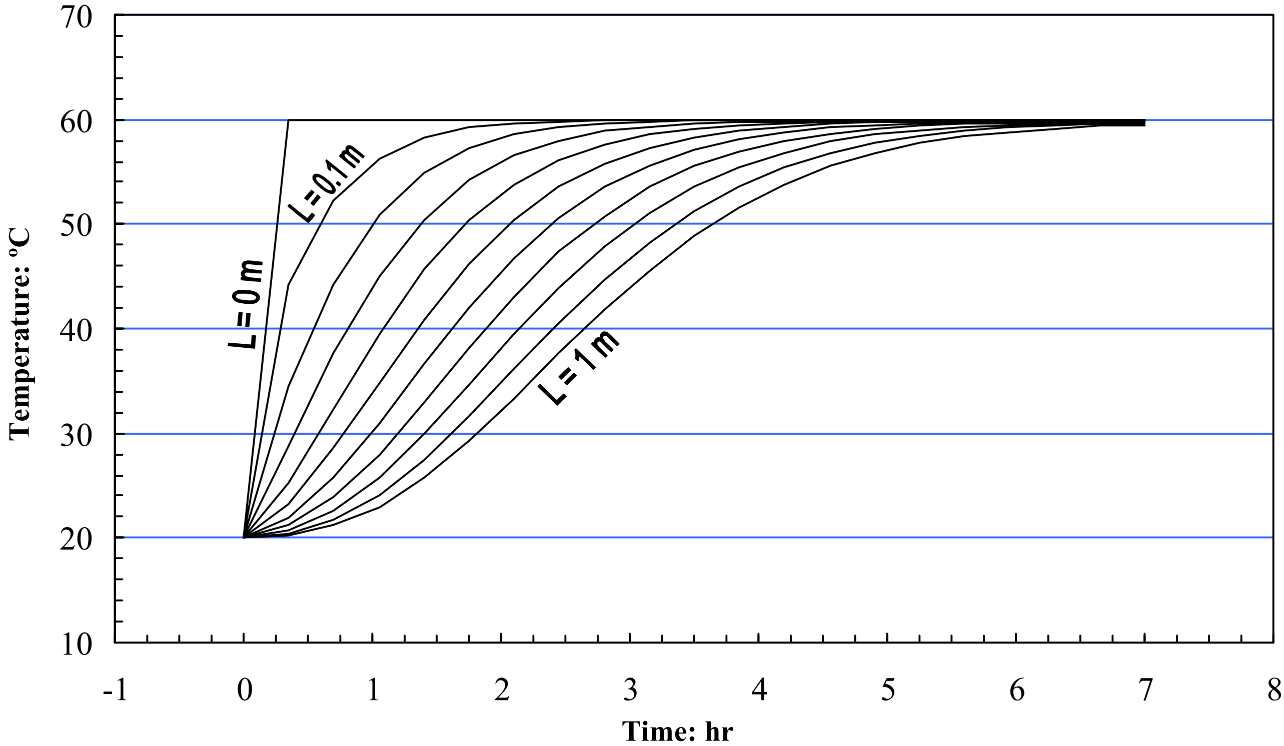

Figure 7. Variation of air temperature in different layers of rock bed.

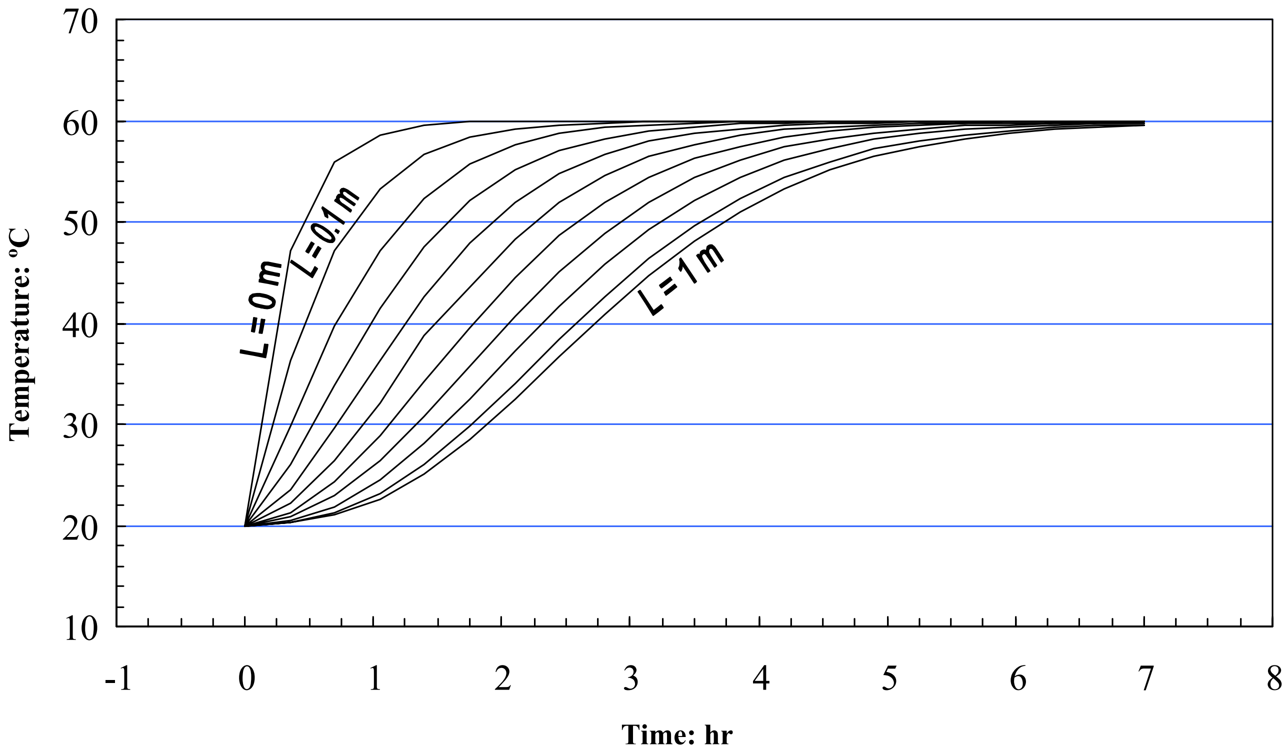

Figure 8. Variation of temperature in different layers of rock bed.

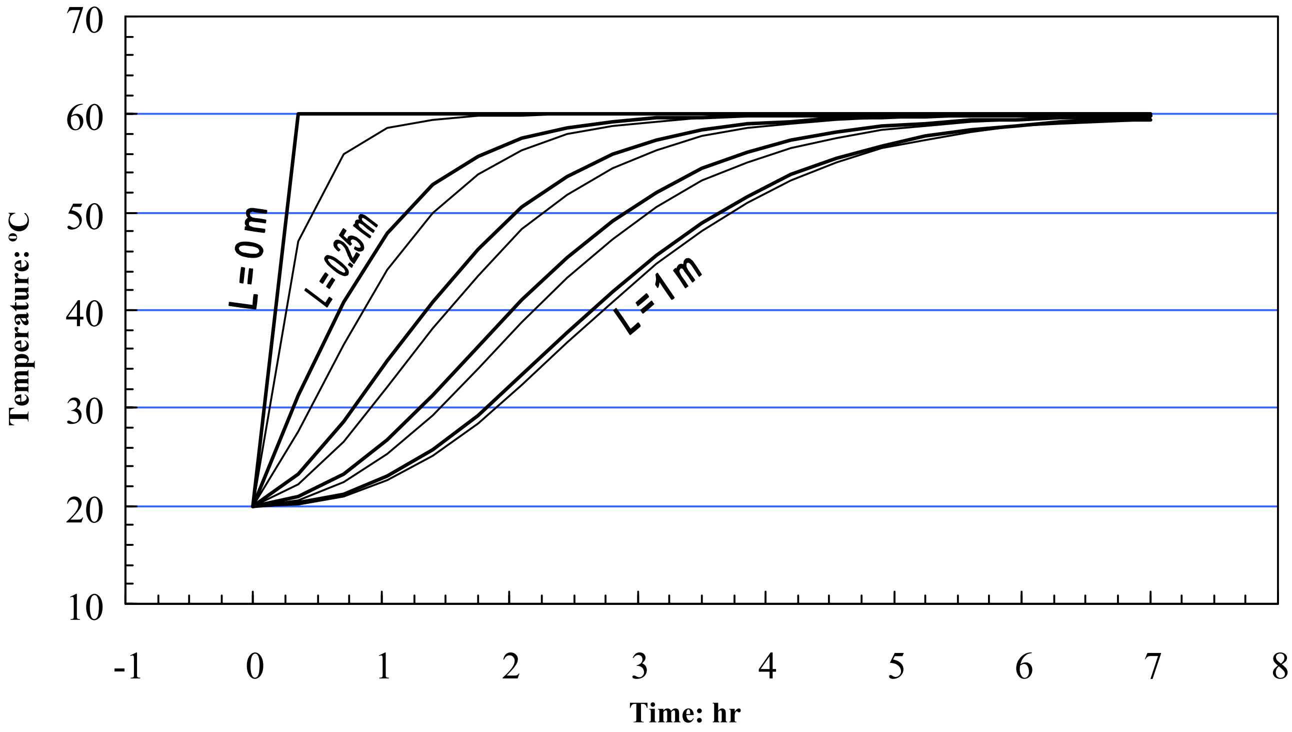

Figure 9. Comparison of air and rock temperatures variation in different Layers of rock bed.

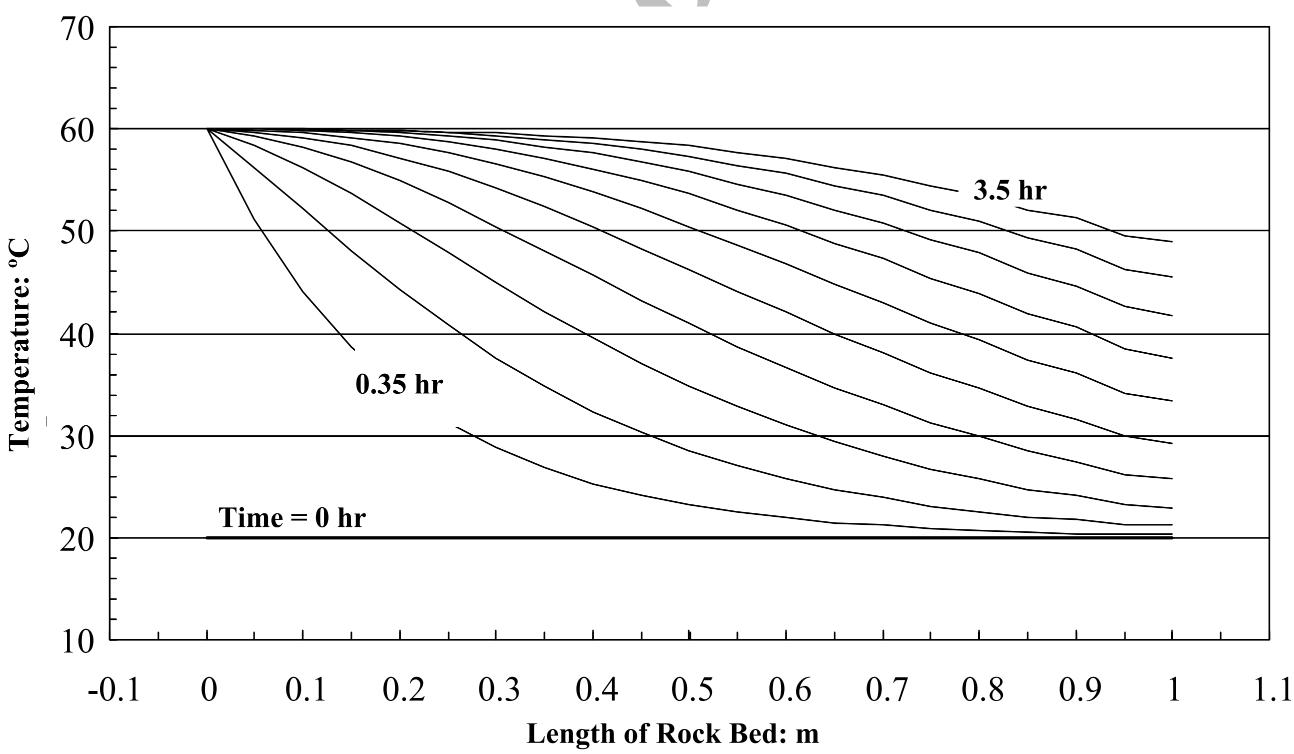

Figure 10. Variation of air temperature along the rock bed length with time.

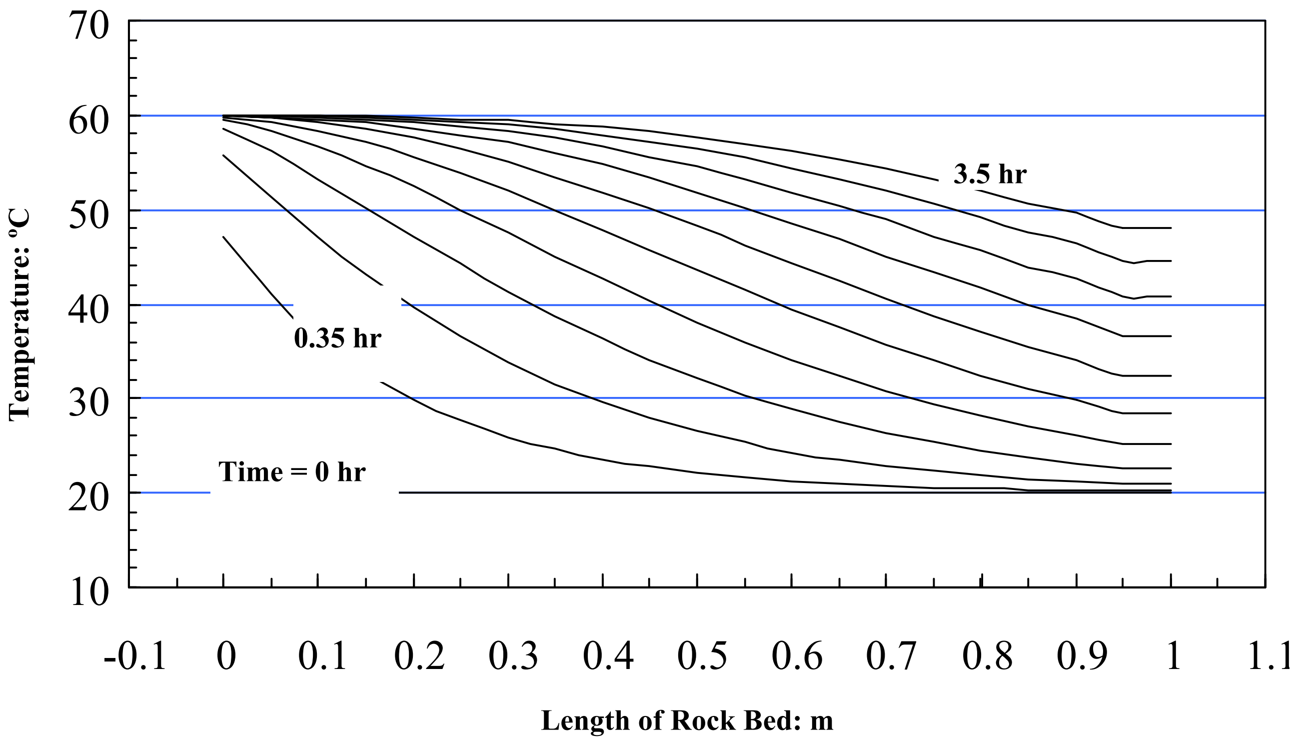

Figure 11. Variation of rock temperature along the rock length with time.

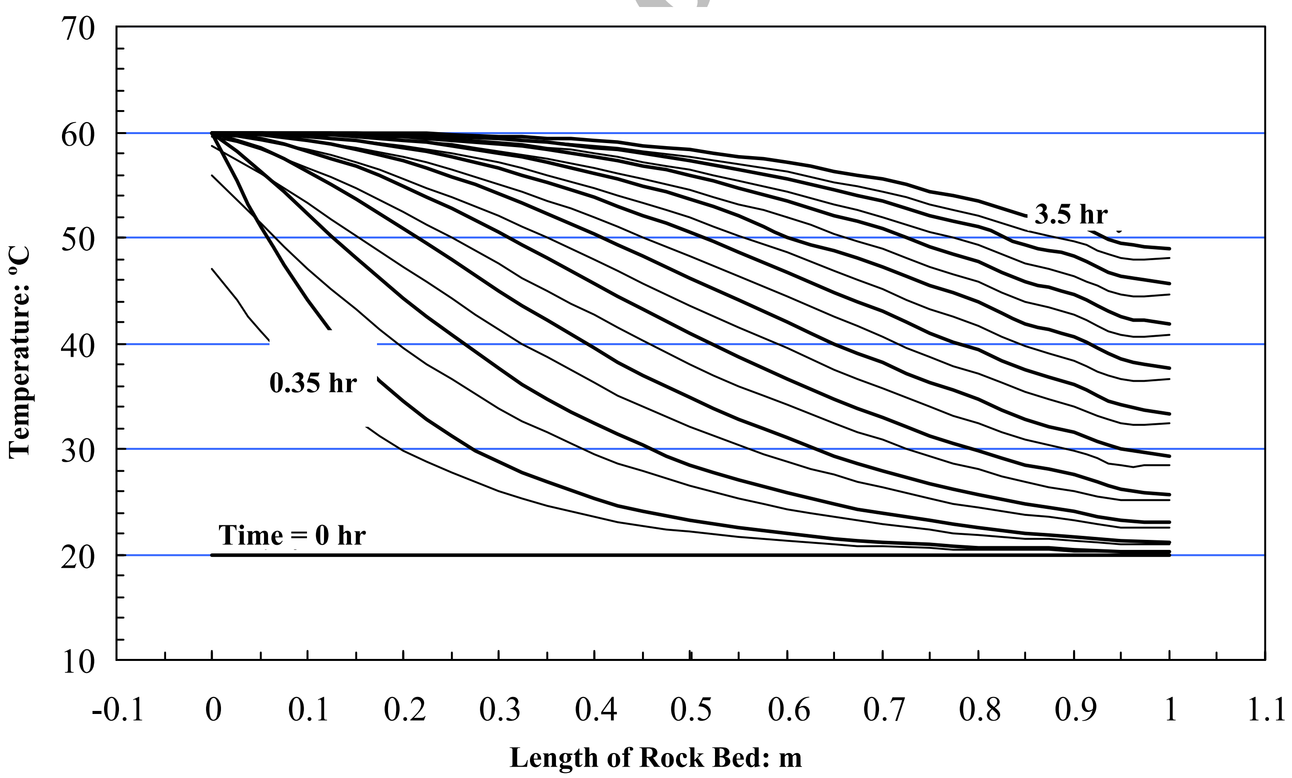

Figure 12. Comparison of air and rock temperatures along the rock bed length with time.

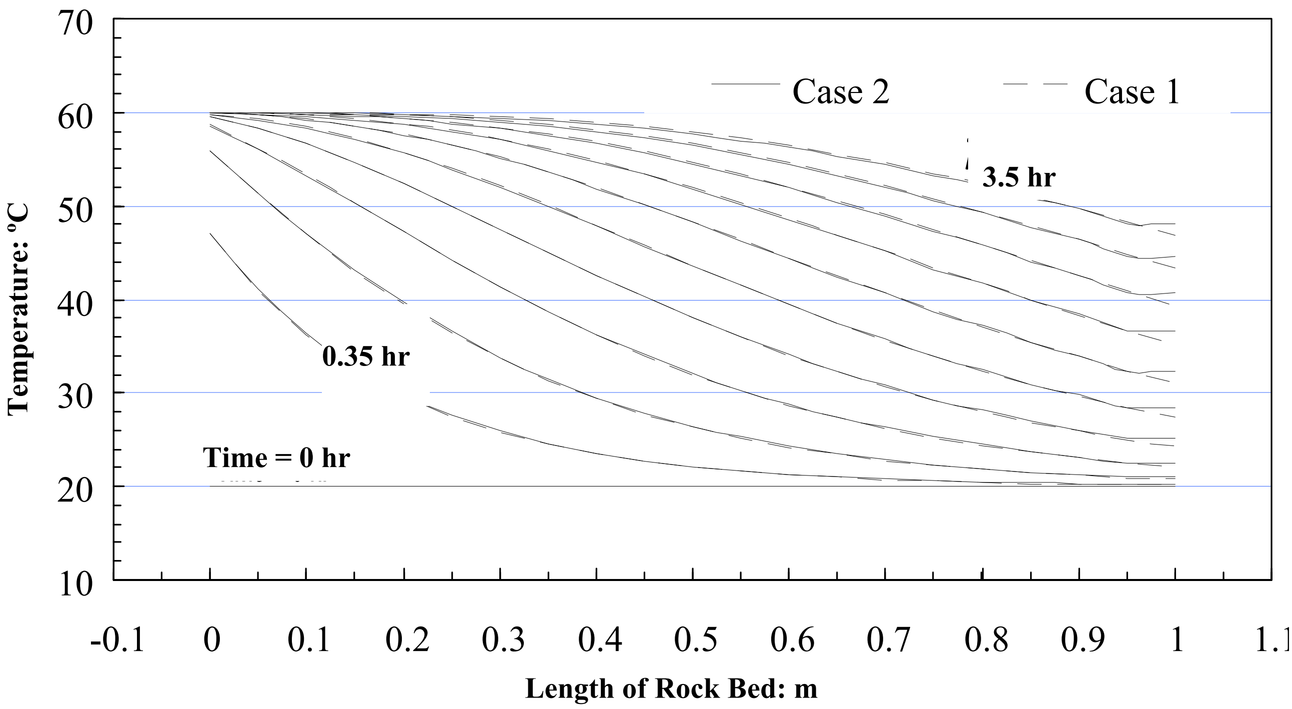

Figure 13. Comparison of rock temperature along the bed length for cases 1 and 2.

In Equation 28, considering x = 2, …, n-1, we will have a set of equations which can be written in form of a matrix. This matrix is solved for variation of air temperature at each location in the rock bed at the end of the time interval Δt, starting from initial conditions at t = 0 and using Equation 29 and Ta. The variation of rock temperature can also be calculated.

Considering the case where k_1, k_2 and $k_3 \neq 0$ in Equations 9 and 10, Equation 9 can be written in terms of finite difference as:

$$-GT_a(x-1,t+1)+HT_a(x,t+1)+GT_a(x+1,t+1)-T_s(x,t+1) = LT_a(x,t)+k_1T_\infty \tag{30}$$

for (n-1> x > 2)

where

$G= 1/2\Delta x$
$H = (1+k_1)+(k_3/\Delta t))$
$M = 1/\Delta x$
$L= k_3/\Delta t$
$N= (1/\Delta x + (1+k_1)+(k_3/\Delta t)$

Equation 10 can also be written in terms of finite difference for (n-1> x > 2):

$$-CT_s(x-1,t+1) + FT_s(x,t+1) - C\,T_s(x+1,t+1)-T_a(x,t+1) = ET_s\,(x,t) \tag{31}$$

for x = 1

$$AT_s(x,t+1) + BT_s(x+1,t+1) - C\,T_s(x+2,t+1)-T_a(x,t+1) = ET_s\,(x,t) \tag{32}$$

for x = n

$$A\,T_s(x, t+1) + BT_s(x-1,t+1) - CT_s(x-2,t+1) -T_a(x,t+1) =ET_s\,(x,t) \tag{33}$$

where

$A= ((1/ \Delta t) + 1 -(k_2/(\Delta x)^2))$

$B= 2k_2/(\Delta x)^2$

$C= k_2/(\Delta x)^2$

$E= 1/\Delta t$

$F = ((1/\Delta t) +1+ (2k_2/(\Delta t)^2)$

4. RESULTS

Two cases are considered in this work. In the first case k_1 and k_2 are considered to be negligible (i.e. k_1 and k_2 can be ignored in Equations 9 and 10). Figures 1 and 2 show the air and rock bed temperature variation with respect to the time respectively. The sets of curves in Figures 1 and 2 differ by the value of $Z = \beta t_{coll.}$. The air temperature is 60 ^{0}C at t = 0, as shown in Figure 1. This is the condition of air before introducing into the rock bed. The rock bed is divided into n layers and air temperature profile is shown in Figure 1. This figure shows the time required for complete charging of rock bed is 7 hours. The temperature profiles of rock bed layers are shown in Figure 2 at different time intervals. The hot air gives its heat quickly to the rock bed due to high heat transfer coefficient. At the beginning, the temperature of the rock bed gradually increases. Heat is transferred gradually to the other layer. After 7 hours, the temperature of the exiting air begins to rise. This is a sign of charging of the rock bed. Figure 3 compares the air and rock bed temperature profiles. As the time increased, the air and rock bed temperature profiles get closer the each other. Figure 4 shows the air temperature variation along the length of the bed. At t = 0, the air temperature is 60 ^{0}C. As is shown in Figure 4, the exit air temperature drops at very short times. As time goes on, the temperature of the exiting air is increased. After 3.5 hours, the exit temperature is about 49 ^{0}C. The rock bed temperature variation along the length of the bed is shown in Figure 5. Figure 6 compares variation of air and rock bed along the length of the bed.

In the latter, the coefficients k_1, k_2 and k_3 are not neglected in the solution. Figures 7 and 8 show the temperature variation of air and rock bed with respect to time. It shows thermal wave dispersion throughout the rock bed. The time rate of change or slope of a curve at particular time shows the impulse response in Figures 7 and 8. Figure (9) compares temperature variation of air and rock bed. As the time increased the temperature profiles of air and rock bed get closer to each other. Figures 10 and 11 show temperature variation of air and rock bed along the bed length. Figure 12 compares the temperature profiles of air and rock bed along length of the rock bed. As the time increass, the air and rock bed temperature profiles get closer to each other.

5. CONCLUSION

An analytical solution can be written for Equations 5 and 6 with mere boundary condition of T(0,t) = T(1,t) in the inlet air temperature. In this case, the solution is limited to relatively small values of time. In order to extend solution to real case where an initial non-uniform spatial temperature distribution within the bed is considered at large time, initial boundary condition 11 and 12 are to be incorporated in the solution. The solution shows the response of the rock bed during the charging period (energy recovery mode) and the profiles of air and rock bed temperatures with respect to time and length of the bed. Equations 23 and 24 must, therefore, be expressed in finite difference form and solve by numerical procedure. Since the air is used as the heat transfer medium at low temperature, the effect of k_1, k_2 and k_3 (heat loss, conduction through solid and heat capacity of fluid respectively) are found to be negligible in the solution of the case of air as a moving fluid.

6. NOMENCLATURE

A	cross section area of bed, m^2
C_a	heat capacity of the air, J/kg ^{o}C
C_s	heat capacity of the rock, J/kg ^{o}C
d	rock equivalent diameter, m
D	rock bed diameter m
f	void fraction %
G	air mass flow rate per unit cross section, kg/m^2 s
h_v	volumetric convective heat transfer coefficient, W/m^3 ^{o}C
k_s	heat conductivity of the rock W/m ^{o}C
k_1	U/h_vD
k_2	h_vk_p/G^2C_a
k_3	$\rho_aC_af//\rho_sC_s(1\text{-}f)$
T_a	air temperature, ^{o}C
T_s	rock bed temperature, ^{o}C
T_∞	surrounding temperature of rock bed, ^{o}C
t	time, s

$t_{coll.}$	solar collection time, hr
U	heat loss coefficient of the rock bed, $W/m^2\,^oC$
v_a	air velocity, m/s
x	distance along the rock bed, m
ρ_a	air density, kg/m^3
ρ_s	rock density, kg/m^3

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