THE EFFECT OF ADDED MASS FLUCTUATION ON HEAVE VIBRATION OF TLP

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Abstract The resulting motion in waves can be considered as a superposition of the motion of the body in still water and the forces on the restrained body. In this study the effect of added mass fluctuation on vertical vibration of TLP in the case of vibration in still water for both free and forced vibration subjected to axial load at the top of the leg is presented. This effect is more important when the amplitude of vibration is large. Also this is important in fatigue life study of tethers. The structural model used here is very simple. Perturbation method is used to formulate and solve the problem. First and second order perturbations are used to solve the free and forced vibration respectively.

Key Words Added Mass Fluctuation, Wave, TLP, Heave Vibration, Perturbation Method

چکیده حرکت ایجاد شده بر اثر موج را می توان به صورت ترکیب خطی حرکت جسم در آب ساکن و حرکت ناشی از نیروهای وارده به جسم ثابت در نظر گرفت. در این مطالعه اثر جرم افزوده متغیر در حالت آب ساکن بر پاسخ دینامیکی قائم سکوی دارای پایه کششی، برای دو حالت ارتعاش آزاد و ارتعاش اجباری تحت بار محوری در بالای پایه ارائه شده است. این امر بویژه در شرایطی که دامنه ارتعاش قائم سازه، دارای مقدار قابل ملاحظه ای باشد، از اهمیت بیشتری برخوردار است. همچنین این اثر در مطالعه عمر خستگی تاندونها دارای اهمیت می باشد. مدل سازه ای مورد استفاده بسیار ساده می باشد. به منظور فرمول بندی و حل مساله از روش اغتشاش استفاده شده است. در حالت ارتعاش آزاد، از اختلال مرتبه دوم و در مورد ارتعاش اجباری از اختلال مرتبه اول استفاده شده است.

1. INTRODUCTION

Tension leg platforms (TLPs) are well-known structures for oil exploitation in deep water and are becoming increasingly popular for oil drilling at very deep water sites. Figure. (1) shows different components of the TLP made up of vertical and horizontal elements on the upper structure and vertical tendons connecting the structure to a foundation on the seabed. These structures consist of semi-submersible platforms with sufficient buoyancy to develop the required tension in the tethers. The tension leg platform (TLP) is a moored floating structure whose buoyancy is more than its weight. The mooring system of TLP consists of number of tensioned tethers connected to the columns at the top and anchored to the seabed at the bottom. These tethers are vulnerable to failure due to fatigue produced by fluctuation of tension. Many studies have been carried out to understand the structural behavior of TLP and determine the effect of several parameters on dynamic response and average life time of the structure [1-6]. The tether system is a critical and basic component of the TLP. The most important point in the design of TLP is the pretension of the legs. The pretension causes that the platform behaves like a stiff structure with respect to the vertical degrees of freedom (heave, pitch and roll), whereas with respect to the horizontal degrees of freedom (surge, sway and yaw) it behaves as a floating structure. Therefore the periods of the vertical degrees of freedom are lower than the others. Among the various degrees of freedom, vertical motion (heave) is very important because of the direct effect on the stress fluctuation that leads to fatigue and fracture. Therefore the conceptual studies to understand the dynamic vertical response of TLP, can be useful for designers.

Simple models for heave response of tension leg platform under harmonic vertical load has been proposed [7]. The effect of added mass fluctuation on the heave response of tension leg platform has been investigated by using perturbation method for discrete [8] and continues model [9]. Added mass fluctuation has important effect on fatigue life of tethers [8].

In this study the effect of added mass fluctuation in the case of vibration in still water for both free and forced vibration is discussed. A similar formulation can be developed for motion analysis of the restrained body in waves. The problem is solved by means of perturbation method [10-11].

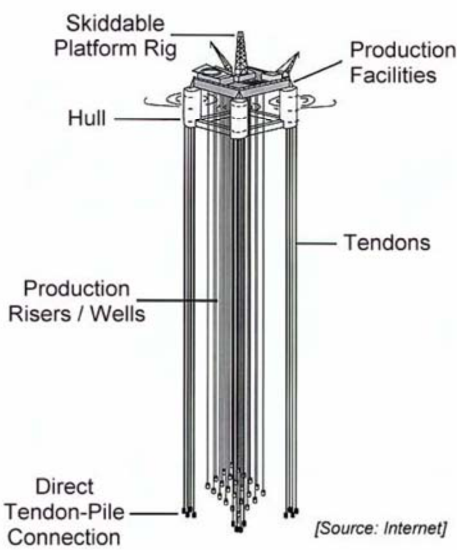

Figure 1. TLP configuration and components [12]

2. FREE VIBRATION ANALYSIS

The resulting motion in waves can be seen as a superposition of the motion of the body in still water and the forces on the restrained body in waves (Figure. 2).

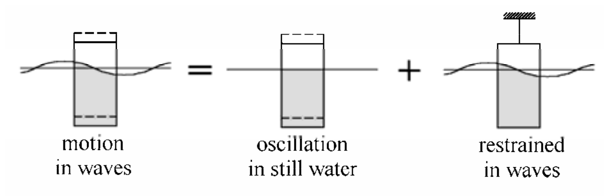

Figure 2. Superposition of hydromechanical and wave loads [12]

In this paper the oscillation in still water is considered. Also in the case of small diameter, the forces of restrained body in waves are similar to the oscillation in still water. Structural modeling of a TLP as a moored structure is shown in Figure. (3). Free vibration equation of motion is as follows

$$m\ddot{y} + ky = 0 \tag{1}$$

where

$$m = m_s + m_a(t) \tag{2}$$

m_s is the structural mass and $m_a(t)$ is the time varying added mass, and

$$k = k_t + k_b \tag{3}$$

where k_t and k_b are mooring and buoyancy stiffness respectively. If the structure is not being vertically moored, k_t is equal to zero.

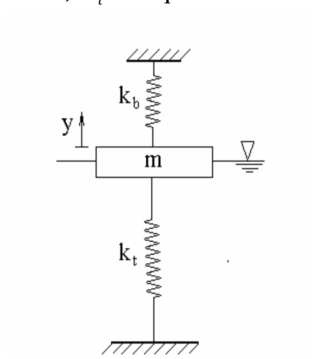

Figure 3. Dynamic structural model

Total added mass can be considered as summation of two parts

$$m_a(t) = m_{a0} + \varepsilon m_{a0} y(t) \tag{4}$$

where m_{a0} is constant and $\varepsilon m_{a0} y(t)$ is the time varying part. ε is the perturbation parameter and $y(t)$ is the vertical motion. Defining $m_{a0}/(m_s + m_{a0}) = a$ and $\omega_n^2 = \sqrt{k/m}$, the equation of motion can be written as

$$(1 + \varepsilon a y)\ddot{y} + \omega_n^2 y = 0 \tag{5}$$

Structural damping is assumed to be equal to zero. Perturbation method is used to solve the Eq. (5). A solution in the form of an infinite series of the perturbation parameter ε is assumed as follows

$$y(t) = y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) + \varepsilon^3 y_3(t) + \cdots \tag{6}$$

The frequency of nonlinear vibration depends on the amplitude of vibration and perturbation parameter

$$\omega^2 = \omega_n^2 + \varepsilon\alpha_1 + \varepsilon^2\alpha_2 + \varepsilon^3\alpha_3 + \cdots \tag{7}$$

where α_i are as yet undefined functions of amplitude. In this study, the second order perturbation method is used to solve the differential equation, therefore the response and frequency of vibration are considered as follows

$$y(t) = y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) \tag{8}$$

$$\omega^2 = \omega_n^2 + \varepsilon\alpha_1 + \varepsilon^2\alpha_2 \tag{9}$$

Substituting Eqs. (8) and (9) into Eq. (5), one obtains

$$\begin{aligned} &\left\{1 + \varepsilon a\left[y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t)\right]\right\} \times \\ &\left[\ddot{y}_0(t) + \varepsilon\ddot{y}_1(t) + \varepsilon^2\ddot{y}_2(t)\right] + \\ &(\omega^2 - \varepsilon\alpha_1 - \varepsilon^2\alpha_2) \times \\ &\left[y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t)\right] = 0 \end{aligned} \tag{10}$$

Since the perturbation parameter ε could have been chosen arbitrarily, the coefficients of the various powers of ε must be equated to zero. This leads to a system of equations which can be solved successively

$$\ddot{y}_0 + \omega^2 y_0 = 0 \tag{11}$$

$$\ddot{y}_1 + \omega^2 y_1 = -a y_0 \ddot{y}_0 + \alpha_1 y_0 \tag{12}$$

$$\ddot{y}_2 + \omega^2 y_2 = -a(y_1\ddot{y}_0 + y_0\ddot{y}_1) + \alpha_1 y_1 + \alpha_2 y_0 \tag{13}$$

The solution of the Eq. (11), subjected to the initial conditions $y_0(0) = A$ and $\dot{y}_0(0) = 0$, is

$$y_0 = A\cos\omega t \tag{14}$$

Substituting Eq. (14) into the right hand side of the Eq. (12), one obtains

$$\ddot{y}_1 + \omega^2 y_1 = \alpha_1 A\cos\omega t + \frac{aA^2\omega^2}{2}(\cos 2\omega t + 1) \tag{15}$$

The forcing term $\cos\omega t$ leads to a secular term $t\cos\omega t$ in the solution of y_1. Such terms violate the initial stipulation that the motion is to be periodic. Therefore one must impose the following condition

$$\alpha_1 = 0 \tag{16}$$

Now imposing the initial conditions $y_1(0) = \dot{y}_1(0) = 0$, the solution of the Eq. (15) is as follows

$$y_1 = \frac{aA^2}{2}\left(1 - \frac{2}{3}\cos\omega t - \frac{1}{3}\cos 2\omega t\right) \tag{17}$$

Substituting Eqs. (14) and (17) into Eq. (13), one obtains

$$\begin{aligned} &\ddot{y}_2 + \omega^2 y_2 = \\ &\quad \frac{a^2A^3\omega^2}{2}\left(\cos\omega t - \frac{4}{3}\cos^2\omega t - \right. \\ &\quad\quad \left. \frac{5}{3}\cos\omega t\cos 2\omega t\right) + \alpha_2 A\cos\omega t \end{aligned} \tag{18}$$

Equation (18) can be rewritten as

$$\begin{aligned} \ddot{y}_2 + \omega^2 y_2 = &-\frac{a^2A^3\omega^2}{12} \times \\ &\left(4 + 4\cos 2\omega t + 5\cos 3\omega t\right) + \\ &\left(\frac{a^2A^2\omega^2}{12} + \alpha_2\right)A\cos\omega t \end{aligned} \tag{19}$$

where

$\cos^2\omega t = \dfrac{\cos 2\omega t + 1}{2}$ and

$\cos\omega t\cos 2\omega t = \dfrac{\cos\omega t + \cos 3\omega t}{2}$ have been used. Similarly the forcing term $\cos\omega t$ would lead to a secular term in the solution; therefore one must impose the following condition

$$\alpha_2 = -\frac{a^2 A^2 \omega^2}{12} \tag{20}$$

Now Eq. (19) becomes

$$\ddot{y}_2 + \omega^2 y_2 = -\frac{a^2 A^3 \omega^2}{12} \times \left(4 + 4\cos 2\omega t + 5\cos 3\omega t\right) \tag{21}$$

Imposing the initial conditions $y_2(0) = \dot{y}_2(0) = 0$, the solution of the Eq. (21) is as follows

$$y_2 = \frac{a^2 A^3}{288}\left(-96 + 49\cos\omega t + 32\cos 2\omega t + 15\cos 3\omega t\right) \tag{22}$$

Substituting Eqs. (14), (17) and (22) into Eq. (8), the response of the system is obtained as follows

$$y(t) = A\cos\omega t + \varepsilon\frac{aA^2}{2}\left(1 - \frac{2}{3}\cos\omega t - \frac{1}{3}\cos 2\omega t\right) + \varepsilon^2\frac{a^2 A^3}{288} \times \left(-96 + 49\cos\omega t + 32\cos 2\omega t + 15\cos 3\omega t\right) \tag{23}$$

Substituting Eq. (20) into Eq. (7), the vibration frequency becomes

$$\omega^2 = \frac{\omega_n^2}{1 + \varepsilon^2\dfrac{a^2 A^2}{12}} \tag{24}$$

The frequency is found to decrease with the amplitude, as expected because of increasing in mass. Moreover considering the second order perturbation, the frequency ω decreases with the perturbation parameter. Therefore Eq. (23) is used to determine the response of the system with second order perturbation in which frequency ω is calculated from Eq. (24). First order perturbation response is determined from Eq. (23) in which $\omega = \omega_n$, and the terms having ε^2 are vanished.

3. FORCED VIBRATION ANALYSIS

Forced vibration equation of motion is as follows

$$\left\{1 + \varepsilon a\left[y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t)\right]\right\} \times \left[\ddot{y}_0(t) + \varepsilon\ddot{y}_1(t) + \varepsilon^2\ddot{y}_2(t)\right] + (\omega^2 - \varepsilon\alpha_1 - \varepsilon^2\alpha_2) \times \left[y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t)\right] = \frac{F_0}{m}\cos(\Omega t) \tag{25}$$

This leads to a system of equations which can be solved successively

$$\ddot{y}_0 + \omega^2 y_0 = \frac{F_0}{m}\cos(\Omega t) \tag{26}$$

$$\ddot{y}_1 + \omega^2 y_1 = -a y_0 \ddot{y}_0 + \alpha_1 y_0 \tag{27}$$

$$\ddot{y}_2 + \omega^2 y_2 = -a(y_1\ddot{y}_0 + y_0\ddot{y}_1) + \alpha_1 y_1 + \alpha_2 y_0 \tag{28}$$

The solution to the Eq. (26), subjected to the initial conditions $y_0(0) = 0$ and $\dot{y}_0(0) = 0$, is

$$y_0 = -\frac{F_0}{m}\frac{\cos(\omega t) - \cos(\Omega t)}{\omega^2 - \Omega^2} \tag{29}$$

Substituting Eq. (29) into the right side of the Eq. (27), one obtains

$$\ddot{y}_1 + \omega^2 y_1 = -\alpha_1\frac{F_0}{m}\frac{\cos(\omega t) - \cos(\Omega t)}{\omega^2 - \Omega^2} - \frac{a}{2}\left(\frac{F_0/m}{\omega^2 - \Omega^2}\right)^2 \times \left[(\omega^2 + \Omega^2)\left(\cos(\omega - \Omega)t + \cos(\omega + \Omega)t - 1\right) - \omega^2\cos 2\omega t - \Omega^2\cos 2\Omega t\right] \tag{30}$$

Like previous one has

$$\alpha_1 = 0 \tag{16}$$

The homogenous solution of Eq. (30) is

$$y_1^{(h)} = C_1\sin\omega t + C_2\cos\omega t \tag{31}$$

and the particular solution is

$$y_1^{(p)} = D_1\cos(\omega - \Omega)t + D_2\cos(\omega + \Omega)t + D_3\cos 2\omega t + D_4\cos 2\Omega t + D_5 \tag{32}$$

Imposing the initial conditions $y_1(0) = \dot{y}_1(0) = 0$, results in

$$D_1 = -\frac{a}{2}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2 \frac{\omega^2+\Omega^2}{\Omega(2\omega-\Omega)},$$

$$D_2 = \frac{a}{2}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2 \frac{\omega^2+\Omega^2}{\Omega(2\omega+\Omega)},$$

$$D_3 = -\frac{a}{6}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2,$$

$$D_4 = \frac{a}{2}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2 \frac{\Omega^2}{\omega^2-4\Omega^2},$$

$$D_5 = \frac{a}{2}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2 \frac{\omega^2+\Omega^2}{\omega^2}, \qquad C_1 = 0,$$

$$C_2 = -\frac{a}{3}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2 \frac{\omega^6+4\omega^4\Omega^2-11\omega^2\Omega^4+6\Omega^6}{\omega^2(\omega^2-4\Omega^2)(4\omega^2-\Omega^2)}$$

Now the solution of the Eq. (30) is as follows

$$\begin{aligned} y_1 = -\frac{a}{6}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2 & \Bigg\{2\frac{\omega^6+4\omega^4\Omega^2-11\omega^2\Omega^4+6\Omega^6}{\omega^2(\omega^2-4\Omega^2)(4\omega^2-\Omega^2)}\cos\omega t + \\ & 3\frac{\omega^2+\Omega^2}{\Omega(2\omega-\Omega)}\cos(\omega-\Omega)t - \\ & 3\frac{\omega^2+\Omega^2}{\Omega(2\omega+\Omega)}\cos(\omega+\Omega)t + \\ & \cos 2\omega t - 3\frac{\Omega^2}{\omega^2-4\Omega^2}\cos 2\Omega t - 3\frac{\omega^2+\Omega^2}{\omega^2}\Bigg\} \end{aligned} \tag{33}$$

Substituting Eqs. (29) and (33) into Eq. (28), one obtains

$$\ddot{y}_2 + \omega^2 y_2 = -a(y_1\ddot{y}_0 + y_0\ddot{y}_1) - \alpha_2 \frac{F_0}{m}\frac{\cos(\omega t)-\cos(\Omega t)}{\omega^2-\Omega^2} \tag{34}$$

where

$$\begin{aligned} y_1\ddot{y}_0 + y_0\ddot{y}_1 = & -\frac{5}{6}\frac{aF_0^3}{m^3\omega^2\Omega\left(-4\omega^{10}+29\omega^8\Omega^2-67\omega^6\Omega^4+67\omega^4\Omega^6-29\omega^2\Omega^8+4\Omega^{10}\right)}\times \\ & \Big\{\omega^9\big(12\cos(2\omega+\Omega)t-12\cos(2\omega-\Omega)t-6\cos(\omega+2\Omega)t+6\cos(\omega-2\Omega)t\big)+ \\ & \omega^8\Omega\begin{pmatrix}-4\cos 2\omega t+2\cos(\omega+\Omega)t+2\cos(\omega-\Omega)t+22\cos(2\omega+\Omega)t+12\cos\Omega t- \\ 9\cos(\omega-2\Omega)t-4-20\cos 3\omega t+22\cos(2\omega-\Omega)t-14\cos\omega t-9\cos(\omega+2\Omega)t\end{pmatrix}+ \\ & \omega^7\Omega^2\big(-12\cos(\omega-2\Omega)t+36\cos(2\omega-\Omega)t+12\cos(\omega+2\Omega)t-36\cos(2\omega+\Omega)t\big)+ \\ & \omega^6\Omega^3\begin{pmatrix}-16\cos 2\omega t+10\cos(\omega+\Omega)t+10\cos(\omega-\Omega)t-85\cos(2\omega+\Omega)t-66\cos\Omega t+ \\ 45\cos(\omega-2\Omega)t-16+85\cos 3\omega t-85\cos(2\omega-\Omega)t+73\cos\omega t+45\cos(\omega+2\Omega)t\end{pmatrix}+ \\ & \omega^5\Omega^4\big(-42\cos(\omega-2\Omega)t+48\cos(2\omega-\Omega)t+42\cos(\omega+2\Omega)t-48\cos(2\omega+\Omega)t\big)+ \\ & \omega^4\Omega^5\begin{pmatrix}44\cos 2\omega t-14\cos(\omega+\Omega)t-14\cos(\omega-\Omega)t-16\cos(2\omega+\Omega)t-12\cos\Omega t-60\cos 3\Omega t \\ +63\cos(\omega-2\Omega)t-16-20\cos 3\omega t-16\cos(2\omega-\Omega)t-62\cos\omega t+63\cos(\omega+2\Omega)t\end{pmatrix}+ \\ & \omega^3\Omega^6\big(-24\cos(\omega-2\Omega)t+24\cos(\omega+2\Omega)t\big)+ \\ & \omega^2\Omega^7\begin{pmatrix}-24-24\cos 2\omega t-10\cos(\omega+\Omega)t-10\cos(\omega-\Omega)t+16\cos(2\omega+\Omega)t+117\cos\Omega t \\ +15\cos 3\Omega t-36\cos(\omega-2\Omega)t+16\cos(2\omega-\Omega)t-24\cos\omega t-36\cos(\omega+2\Omega)t\end{pmatrix}+ \\ & \Omega^9\big(12\cos(\omega-\Omega)t-24\cos\Omega t+12\cos(\omega+\Omega)t\big)\Big\} \end{aligned} \tag{35}$$

Similarly one has

$$\alpha_2 = \frac{5}{6}\left(\frac{aF_0}{m}\right)^2 \frac{(\omega^2-\Omega^2)\left(-14\omega^6+73\omega^4\Omega^2-62\omega^2\Omega^4-24\Omega^6\right)}{\left(-4\omega^{10}+29\omega^8\Omega^2-67\omega^6\Omega^4+67\omega^4\Omega^6-29\omega^2\Omega^8+4\Omega^{10}\right)} \tag{36}$$

and

$$\omega^2 = \omega_n^2 - \frac{5}{6}\varepsilon^2\left(\frac{aF_0}{m}\right)^2 \frac{(\omega^2-\Omega^2)\left(-14\omega^6+73\omega^4\Omega^2-62\omega^2\Omega^4-24\Omega^6\right)}{\left(-4\omega^{10}+29\omega^8\Omega^2-67\omega^6\Omega^4+67\omega^4\Omega^6-29\omega^2\Omega^8+4\Omega^{10}\right)} \tag{37}$$

Now the first order perturbation solution becomes

$$\begin{aligned} y(t) = & -\frac{F_0}{m}\frac{\cos(\omega t)-\cos(\Omega t)}{\omega^2-\Omega^2} - \varepsilon\frac{a}{6}\left(\frac{F_0/m}{\omega^2-\Omega^2}\right)^2 \times \\ & \left\{2\frac{\omega^6+4\omega^4\Omega^2-11\omega^2\Omega^4+6\Omega^6}{\omega^2(\omega^2-4\Omega^2)(4\omega^2-\Omega^2)}\cos\omega t + 3\frac{\omega^2+\Omega^2}{\Omega(2\omega-\Omega)}\cos(\omega-\Omega)t - 3\frac{\omega^2+\Omega^2}{\Omega(2\omega+\Omega)}\cos(\omega+\Omega)t + \right. \\ & \left. \cos 2\omega t - 3\frac{\Omega^2}{\omega^2-4\Omega^2}\cos 2\Omega t - 3\frac{\omega^2+\Omega^2}{\omega^2}\right\} \end{aligned} \tag{38}$$

in which $\omega = \omega_n$.

4. CASE STUDY

A numerical study has been carried out to understand the effect of parameters ε and a on the amplitude and frequency of vibration. It is supposed that the period of the structure is equal to 1, therefore the frequency of system is $\omega = 2\pi$, and the initial condition $A = 1$ is imposed. In Figures (4-6) the time histories and phase planes of system are shown for various values of ε and *a*. Figure. (4) shows the response time history phase plane for $\varepsilon = 0.25$ and *a*= 0.25, 0.5 and 0.75. It is observed that the solution of first and second order perturbations are close together and for *a*= 0.25, 0.5 and 0.75, the differences between the amplitudes of the first and second order perturbations are 5%, 9% and 14% respectively.

Similar results for $\varepsilon = 0.5$ and $\varepsilon = 0.75$ are obtained from Figures. (5) and (6). In the case of $\varepsilon = 0.5$ the differences between the amplitudes related to various values of *a* are increased clearly. Also in the case of $\varepsilon = 0.75$ the differences between the amplitudes related to various values of *a* is more clear rather than previous values.

Phase planes illustrated in Figures (4-6) show the stability and periodicity of the solutions. The differences between the amplitudes related to various values of *a* are shown in other way.

It is seen that for small values of *a*, the difference between the first and second order perturbations is small and it increases with increasing *a*. Also increase in perturbation parameter ε, leads to increase the difference between linear response and both the first and second order perturbation solutions.

The analytical solution shows that considering second order perturbation leads to change in the frequency of vibration. Time history of response for $\varepsilon = a = 0.75$, is shown in Figure. (7). It is seen that there is not shift in period of vibration in the case of first order perturbation in spite of a slowly varying shift in the period of vibration in the case of second order perturbation as mentioned in the text because of being function of perturbation parameter.

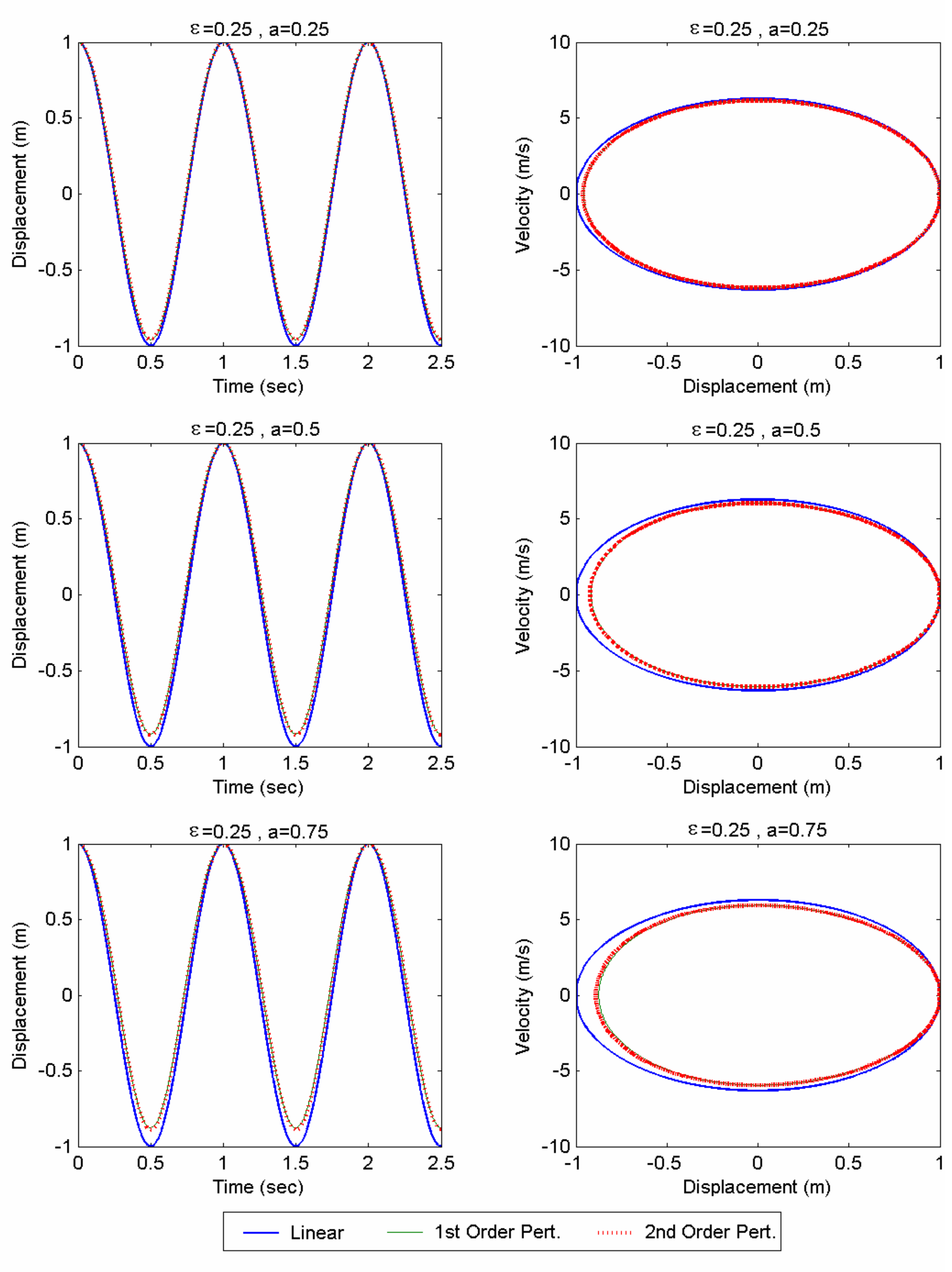

Figure 4. Response and phase plane of the system ($\varepsilon = 0.25$ and $a = 0.25, 0.5, 0.75$)

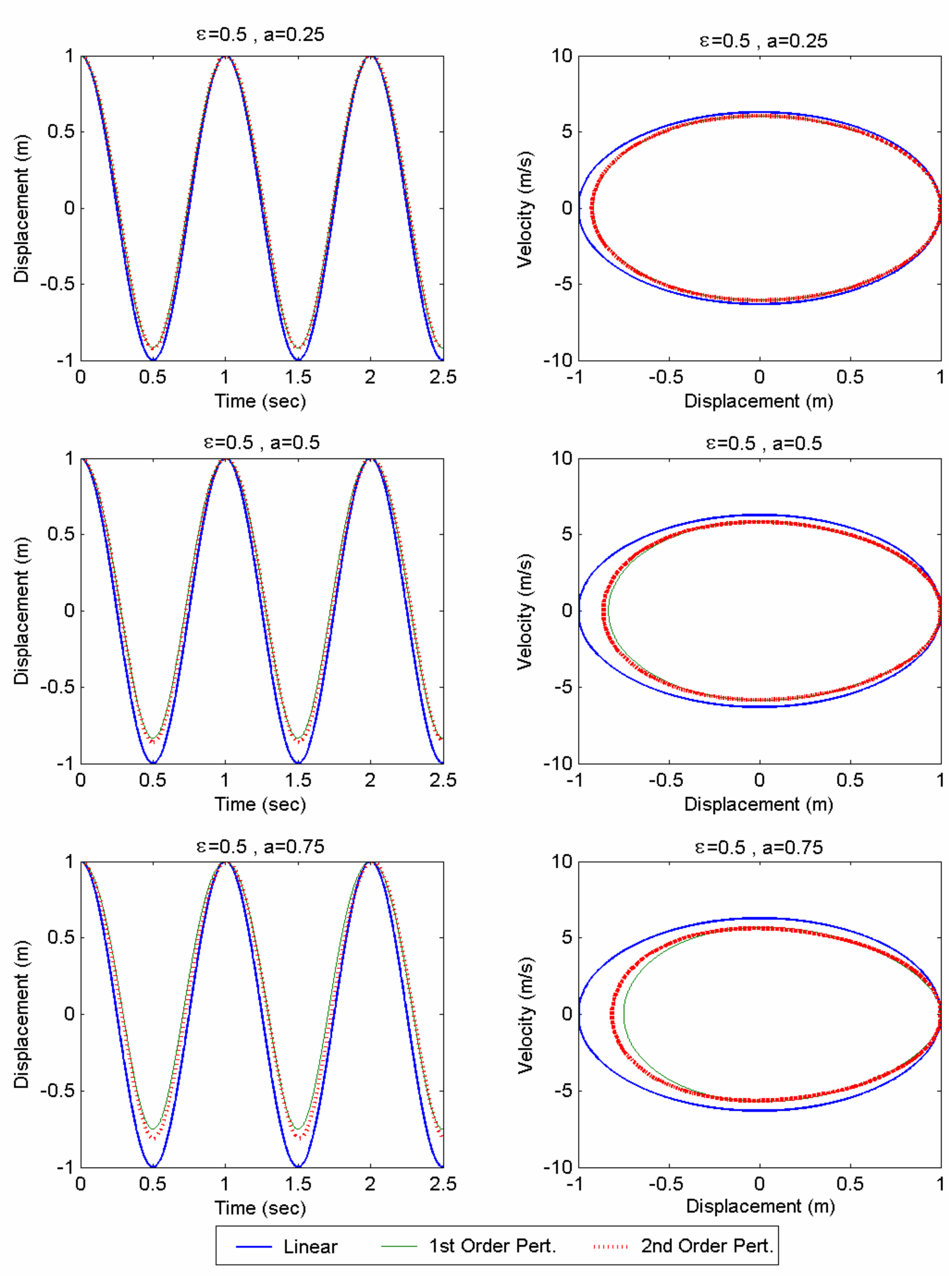

Figure 5. Response and phase plane of the system (ε = 0.75 and *a*= 0.25, 0.5, 0.75)

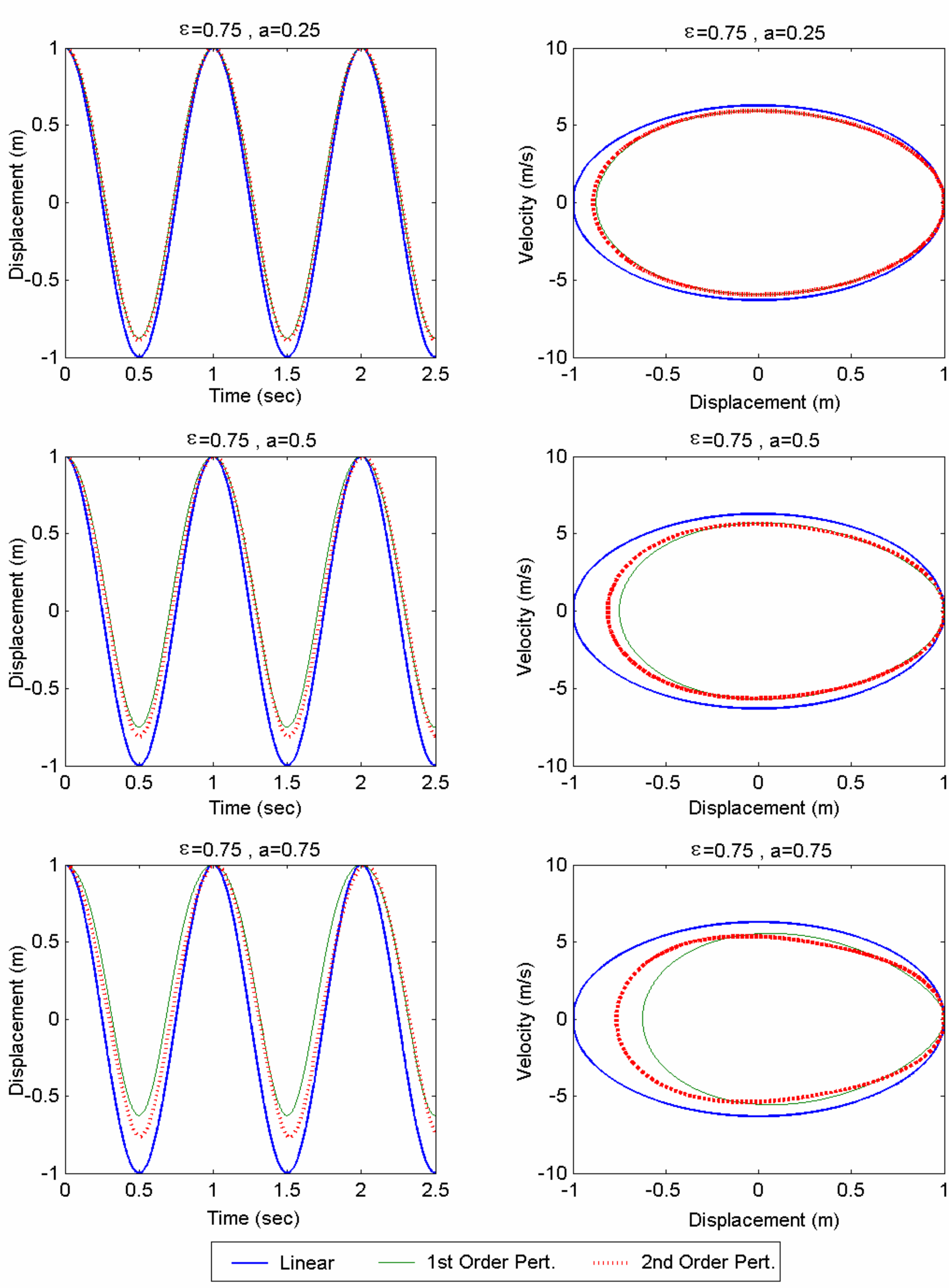

Figure 6. Response and phase plane of the system ($\varepsilon = 0.75$ and $a = 0.25, 0.5, 0.75$)

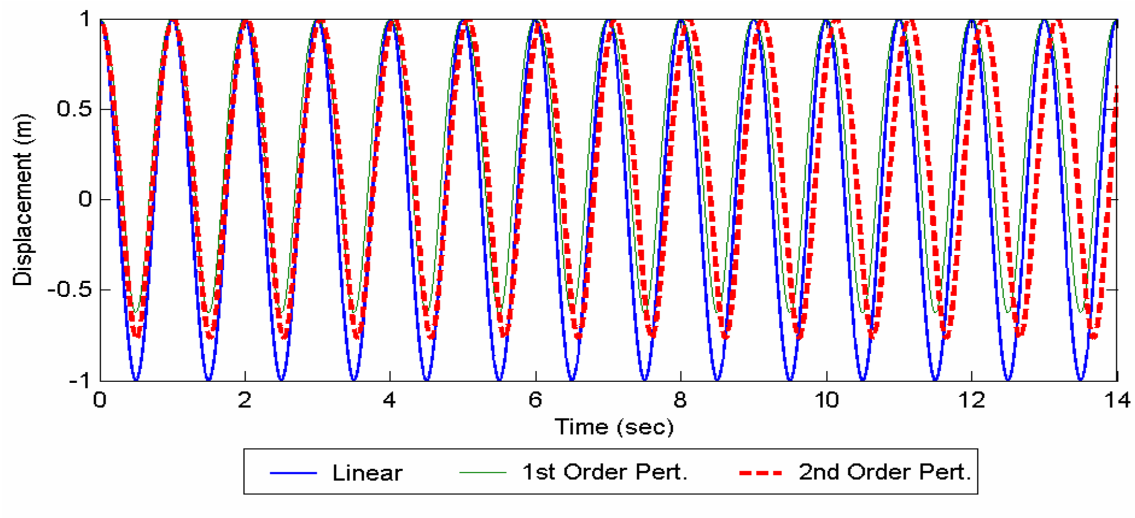

Figure 7. Time history response of the system ($\varepsilon = a = 0.75$)

5. CONCLUSION

The effect of added mass fluctuation on the heave motion of a TLP subjected to axial load (or initial conditions) at the top of the leg has been investigated. Perturbation method has been used to formulate and solve the problem. The solution gives a conceptual view of the heave motion of a TLP, also it is important in fatigue life study of mooring lines. The parametric study shows the effect of some parameters on the response in the case of first and second order perturbation.

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