A GENERALIZED LINEAR STATISTICAL MODEL APPROACH TO MONITOR PROFILES

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Abstract   Statistical process control methods for monitoring processes with univariate or multivariate measurements are used widely when the quality variables fit to known probability distributions. Some processes, however, are better characterized by a profile or a function of quality variables. For each profile, it is assumed that a collection of data on the response variable along with the values of the corresponding quality variables is measured. While the linear function is the simplest, it occurs frequently that many of the nonlinear functions may be transferred to linear functions easily. This paper proposes a control chart based on the generalized linear test (GLT) to monitor coefficients of the linear profiles and an R-chart to monitor the error variance, the combination of which is called GLT/R chart. While fixed values of the explanatory variables are cornerstones in other control charts proposed to monitor profiles, in GLT/R chart, it is not a necessary condition. In order to illustrate the robustness of the GLT/R chart a simulation study has been done in two different cases, i.e. fixed and non-fixed values of the explanatory variables. Then, the results obtained from GLT/R charts are compared to the ones from a multivariate T^2 and Exponentially Weighted Moving Average/R (EWMA/R) control charts.

Keywords   General Linear Models, Multivariate Quality Control, Profile Monitoring, Statistical Control Chart

چکیده   برای پایش فرایندهای تولید که در آنها کیفیت محصول از طریق یک یا چند مشخصهٔ کیفی با توزیعهای احتمالی معین تعریف می شود، از روش های کنترل فرایند آماری استفاده می شود. ولی بعضی از این فرایندها از طریق یک پروفایل یا تابعی از مشخصه‌های کیفی بهتر شناخته می شوند، به شرط اینکه برای این پروفایل ها داده های مناسب فراهم باشد. در بین این پروفایل‌ها، به دلیل سادگی، درک بیشتر، و اینکه بسیاری از پروفایل های غیر خطی قابل تبدیل به پروفایل‌های خطی هستند، از پروفایل‌های خطی استفاده می شود. در این تحقیق بر اساس ایدهٔ آزمون عمومی خطی یک نمودار کنترلی برای پایش ضرایب پروفایل های خطی به همراه یک نمودار کنترلی برد ارائه می کنیم تا از طریق آن قادر باشیم که میانگین و واریانس متغیر پاسخ را در فرایندهای تولیدی تحت کنترل داشته باشیم. برای ارزیابی و مقایسهٔ روش پیشنهادی با روش های مرسوم از شبیه سازی در دو حالت کلی مقدار ثابت و غیر ثابت متغیرهای مستقل پروفایل های خطی استفاده و نتایج را گزارش می کنیم.

1. INTRODUCTION

Statistical Process Control (SPC) has been successfully applied in a variety of industries since Shewhart introduced the control chart in 1924 [1]. In order to establish a control chart for a process two different approaches may be used; the first one is to determine the process state using the distribution function of a single or multiple quality characteristics, and the second one is to use a profile or a function of the quality characteristic(s). For the former case, many researches have

developed different control charts, of which some are univariate such as Shewhart, CUmulative SUM (CUSUM) and Exponentially Weighted Moving Average (EWMA), and some such as T^2, Multivariate CUSUM (MCUSUM) and Multivariate EWMA (MEWMA) are multivariate (for details see [2]). For the latter case, which has been given more attention in the recent literature, some researchers used different terminologies to express the profile. Gardner et al. [3] applied the term "signature" in their study. Jin and Shi [4] used the terms "waveform signals", the signals, such as tonnage stamping in stamping process, torque signals in tapping, and force signals in welding processes, being collected by the sensors during production processes.

In profile monitoring, for each profile it is assumed that n (n > 1) values of the response variable (Y) are measured along with the corresponding values of one or more explanatory variables (the X's), reflecting the location of the measurement on a manufactured item. As an example of a process characterized by a profile, a semiconductor manufacturing quality control problem involving calibration in which the performance of the mass flow controller was monitored by a linear function may be mentioned [5]. Also, Gardner et al. [3] studied the changes of "signature patterns" from spatial collected from a semiconductor material deposition process for detecting potential process fault. For other examples of this case, Kang and Albin [5] proposed a method to monitor aspartame (an artificial sweetener) by a profile. Moreover, Jin and Shi [6] applied linear profile monitoring to monitor stamping tonnage in one research and to monitor waveform in the other one (Jin and Shi [4]).

Literature of the profile monitoring contains different methodologies which have been applied in different situations. The simplest one is to use a simple linear regression relationship for the in-control quality of a product with known parameters. Mahmoud and Woodall [7] studied phase-one analysis of a control charting method where the quality of a process is characterized by a simple linear profile available for a fixed number of samples collected over time; a situation common in calibration applications. Kim et al. [8] proposed alternative control charts for monitoring

profile using estimated regression coefficients from each sample to construct two separate EWMA charts.

As a natural extension of the simple linear regression model used as a profile, some multiple linear regressions such as the polynomial regression model may represent the profiles. Kang and Albin [5] proposed two approaches in this category to monitor linear profiles; one was to use the multivariate T^2 chart introduced by Hotelling in 1947, and the other approach was to examine the residuals by using EWMA and R charts. For some other researches in this area, the readers are referred to the method proposed by Jensen et al. [9].

Other methods of profile monitoring have been also proposed by researchers. For example, Walker and Wright [10] used additive models to represent the curves of interest in vertical density profiles monitoring of particleboards. As another example Miller [11] and Nair et al. [12] applied linear and nonlinear types of response functions in designed experiments. As profile monitoring falls under the broad field of functional data analysis, Ramsay and Silverman [13] discussed various examples of functional data or profiles. Brill [14] applied T^2 to monitor the coefficients of a nonlinear regression function in a chemical process. His goal was to detect a change in the slope of the regression line. Kim et al. [8] proposed alternative control charts for monitoring profile using estimated regression coefficients from each sample to construct two separate EWMA charts. Also, Williams et al. [15] studied the use of the T^2 control charts to monitor the coefficients of a nonlinear regression fits to the successive sets of profile data. Lada and Wilson [16] proposed phase-two methods for a selected set of wavelet coefficients, with control limits based on a re-sampling approach under the assumption that the phase-one data were in control. William et al. [17] presented some of the general issues involved in using control charts to monitor profiles and reviewed the SPC literature on this area. Moreover, some other control charts based on the fuzzy logic approach have been proposed in the literature. For an example of these charts, which sometimes are called Possibilistic Control Chart, the reader can refer to [18].

In this paper, designing a methodology to monitor linear profiles is focused. In this regard,

the concepts of linear regression and the general linear test have been applied to provide hypothesis testing on the coefficients of linear models and simultaneously implement an R-chart to monitor the error variance. In section two a brief background on the simple linear regression models, the T^2, and the EWMA/R control charts have been provided. Then, in section three application of the general linear test and R-chart to monitor linear profiles have been discussed. Section four contains comparisons between the performances of the proposed method with the T^2 control chart through a numerical example and by a simulation study of the Average Run Length. Finally, the conclusion and recommendations for future research come in section five.

2. BACKGROUND

In Section 2.1 some brief background on the simple linear regression models, their parameter estimations, and the General Linear Test have been provided. Then in Section 2.2 and 2.3 the applications of the T^2 and EWMA/R control charts in profile monitoring have been described, respectively.

2.1. Simple Linear Regression Models As a general case assume that $X_1, X_2, ..., X_{p-1}$ and Y are the variables in a linear profile, represented by Equation 1, where Y is the quality characteristic under study, X's are the explanatory variables, the indices j and i are the sample number and the observation within the sample number, respectively, and the random variables ε_{ij}'s are independent and normally distributed with mean zero and variance σ^2.

$$Y_{ij} = \beta_0 + \beta_1 X_{1,ij} + \beta_2 X_{2,ij} + ... + \beta_{p-1} X_{p-1,ij} + \varepsilon_{ij} \quad ; \ i = 1,2,...,n \ , \ j = 1,2,...,m \quad (1)$$

For the simplest case consider a simple linear regression model in Equation 2.

$$Y_{ij} = \beta_0 + \beta_1 X_{1,ij} + \varepsilon_{ij} \quad ; \\ i = 1,2,...,n \ , \ j = 1,2,...,m \quad (2)$$

Then, in phase one of the linear profile control charting method where the process is in control, the estimated linear regression function based on all available in-control data is given in Equation 3:

$$\hat{Y} = b_0 + b_1 X_1 \quad (3)$$

Where b_0 and b_1 are the least square estimates of β_0 and β_1, respectively. When the usual assumptions hold the estimators b_0 and b_1 are normally distributed random variables with mean β_0 and β_1 respectively [19].

In phase two of the linear profile control charting method, based on information from the jth sample (different from the sample on which the parameters were originally estimated in phase one) of size n, in general linear test approach in hypothesis testing of the regression parameters the hypotheses are:

$$H_0 : \beta_0 = b_0 \ \text{ and } \ \beta_1 = b_1 \\ H_1 : \beta_0 \neq b_0 \ \text{ and } \ \beta_1 \neq b_1 \quad (4)$$

Where β_0 and β_1 are estimated by b_{0j} and b_{1j}, respectively. The test procedure involves three basic steps. In step 1, the full model (the simple linear regression model in sample j when the alternative hypothesis holds) is defined. In step two the reduced model is defined when the null hypothesis holds. Then the F test statistic is defined in step three.

2.2. The T^2 Control Chart In a quality control environment in order to monitor a process with more than one variable in which the variables are correlated, multivariate control chart such as T^2 [2] may be used. Since the least-square parameter estimates of a simple linear regression, (b_0,b_1), are correlated, then they can be monitored simultaneously through a T^2 control chart. In this chart the sample statistic is given by:

$$T^2 = (b - \beta)S^{-1}(b - \beta)^T \quad (5)$$

where b and β are the vectors of the parameter estimates and parameters respectively and S is the moment estimator of the covariance matrix of the parameter estimates vector in phase one, introduced later in Section 3.1. If the usual

assumption of a linear regression holds then the upper control limit of the T^2 control chart is $UCL = \chi^2_{\alpha, p}$, where p is the number of variables (including the y-intercept) in the regression line and $\chi^2_{\alpha, p}$ is the upper α-percentile point on a χ^2 distribution with p degree of freedom [5].

2.3. The EWMA/R Control Chart   In EWMA control chart, the jth sample statistic, (z_j), is the weighted average of the jth residual average, $(\bar{e}_j = n^{-1} \sum\limits_{i=1}^{n} e_{ij})$, and the previous residual averages; that is:

$$z_j = (1-\theta)z_{j-1} + \theta \bar{e}_j \tag{6}$$

where $0 < \theta < 1$ is the weighting constant and $z_0 = 1$. Then the control limits for the EWMA chart are:

$$LCL = -L\sigma \sqrt{\frac{\theta}{(2-\theta)n}} \quad LCL = -L\sigma \sqrt{\frac{\theta}{(2-\theta)n}} \tag{7}$$

in which σ is the error term standard deviation estimated by $\sqrt{n^{-1} \sum\limits_{j=1}^{n} MSE_j}$ where MSE_j is the mean square error of the jth fitted regression line, L is a multiple of the sample statistic standard deviation that determines the false alarm rate, and n is the sample size. Typical values are L = 3 and θ = 0.08, 0.1, 0.15 or 0.2. For detail see [5].

The Range or R chart is added to the EWMA chart to monitor the residuals and detect shifts in the process variance. For the R chart the sample statistic is $R_j = \max_i(e_{ij}) - \min_i(e_{ij})$, in which e_{ij} is the regression residual in the ith observation of the jth sample, and the control limits are:

$$LCL = \sigma(d_2 - Ld_3) \quad UCL = \sigma(d_2 + Ld_3) \tag{8}$$

where L is a multiple of the sample statistic standard deviation that determines the false alarm rate, d_2 and d_3 are constants that relate the range and standard deviation and may be found in [2] for different values of n.

3. APPLICATION OF THE GENERAL LINEAR TEST TO MONITOR PROFILES

Now the application of the general linear test to monitor the coefficients of simple linear regression model when applied to profile monitoring is explained. It is assumed that the mean of a quality characteristic (Y) of a process depends on the given value of an exploratory variable (X) through a simple linear regression function. There are two phases involved in the proposed procedure which are explained in the following sub-sections.

3.1. Phase One   In phase one of a control charting method, one analyzes a historical set of process data. The goals in this phase are to understand the variation in a process over time, to evaluate the process stability, and to model the in-control process performances. This latter step is usually accomplished by the estimation of the parameters of a parametric model. Assignable causes of variation are considered to correspond to unusual and preventable events that disrupt the process and could, for example, cause a change in the parameters of the underlying model of the profile. Samples associated with assignable causes are removed from the data if the sources of the assignable causes can be determined and they can be prevented in the future.

In the proposed methodology, a large sample of the process is collected to estimate the error variance and to validate the assumed linear profile to be used in phase two. In phase one the outliers are detected and perhaps eliminated and the usual lack of fit test is performed. Moreover a test is conducted for homogeneity of the error variance, which is a basic assumption for the general linear test applied in the next phase of the algorithm. The next phase is attempted if all of the required tests indicate that the model is valid and the undergoing assumptions hold. To do this from the sample the regression parameters (b_0 and b_1) are estimated by the usual least squares method and all of the necessary statistical tests are performed. If the conclusion is drawn that the in-control values of the regression parameter are the same as the previously known values (β_0 and β_1), then according to the general linear statistical test, in order to test whether the mean of the process is in-

control at any time, it is necessary to test the hypothesis given in (4) which is accomplished by the test statistic:

$$
F^* = \frac{\dfrac{\sum\limits_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_{1,i})^2 - \sum\limits_{i=1}^{n}(Y_i - b_0 - b_1 X_{1,i})^2}{2}}{\div \dfrac{\sum\limits_{i=1}^{n}(Y_i - b_0 - b_1 X_{1,i})^2}{n-2}}
$$

(9)

If $F^* < F_{(1-\alpha; df_R - df_f, df_f)} = F_{(1-\alpha; 2, n-2)}$, H_0 is accepted, i.e., the process mean is in-control. Otherwise, it is concluded that the process mean is out-of-control. For an in-control process, then the control limits of the F-test to detect a shift in the process mean are obtained by Equation 10.

$$
USL_F = F_{(1-\alpha; df_R - df_f, df_f)} = F_{(1-\alpha; 2, n-2)}
$$
$$
\text{and} \quad LCL_F = 0
$$

(10)

Note that by the aforementioned procedure only the mean of the process is monitored. In order to detect a shift in the error variance an R chart may be used simultaneously. The control limits of R-chart defined in phase one of the proposed method is similar to the ones defined in a usual Shewhart control chart [2]. In other words let e_{ij} be the residual in the ith observation of the jth sample. Define the jth range statistic as

$$
R_j = \max_{i}(e_{ij}) - \min_{i}(e_{ij})
$$

(11)

With the sample mean of

$$
\overline{R} = \sum_{j=1}^{m} R_j / m
$$

(12)

Then the control limits in the R-chart are:

$$
UCL_R = D_4 \overline{R} \,, \quad CL_R = \overline{R} \quad \text{and} \quad LCL_R = D_3 \overline{R}
$$

(13)

Where D_3 and D_4 are found in [2].

3.2. Phase Two   In phase two of the proposed procedure a sample of size n ($n > 3$) is collected from the process periodically at time j and the regression parameters (b_{0j} and b_{1j}) are estimated by the least square method. Suppose that the in-control values of the regression parameters estimated in phase one of the proposed procedure are b_0 and b_1. According to the background given in section two, in order to test whether the mean of the process is in-control at stage j, it is necessary to test the following:

$$
H_0 : \beta_0 = b_0 \text{ and } \beta_1 = b_1
$$
$$
H_1 : \beta_0 \neq b_0 \text{ and } \beta_1 \neq b_1
$$

In which β_0 and β_1 are estimated by b_{0j} and b_{1j}, respectively. This test is accomplished by the test statistic:

$$
F_j^* = \frac{\dfrac{\sum\limits_{i=1}^{n}(Y_{ij} - b_0 - b_1 X_{ij})^2 - \sum\limits_{i=1}^{n}(Y_{ij} - b_{0j} - b_{1j} X_{ij})^2}{2}}{\div \dfrac{\sum\limits_{i=1}^{n}(Y_{ij} - b_{0j} - b_{1j} X_{ij})^2}{n-2}}
$$

(14)

H_0 is accepted if $F_j^* < F_{(1-\alpha; df_{Rj} - df_{Fj}, df_{Fj})}$, concluding the process mean is in-control. Otherwise, it is concluded that the process mean is out-of-control. Moreover, for the sample j, the residuals are calculated and the R-chart is applied simultaneously.

In the next section the numerical example of the Kang and Albin [5] research is used and the performance of the proposed General Linear Test and R (GLT/R) control charts is compared to the ones in a T^2 and EWMA/R chart through two simulation studies of Average Run Length (ARL).

4. FIXED AND NON-FIXED VALUES OF EXPLANATORY VARIABLES

One of the assumptions in T^2 and EWMA/R charts

is that in phase one and also in each sample the values of the explanatory variables are fixed. In this case, if the range of the explanatory variable is divided to equal sub-interval, then it is possible to transform the dependent explanatory variables to the independent ones through recoding (i.e., subtracting their values from their corresponding sample mean). However, in situations when the values of the explanatory variables are not fixed, as it may occur in practice, it is not possible to transform the covariance matrix of the profile coefficients to a diagonal matrix through recoding, resulting to a reduced amount of ARL_0 and hence a restricted application of these charts. Nevertheless, this assumption is not required in the proposed GLT/R chart. Since the covariance matrix of the coefficients is not used directly, the proposed methodology is not sensitive with non-fixed value of the explanatory variables.

5. NUMERICAL EXAMPLES

For the sake of comparisons, two simulation studies are made. In the first one the explanatory variable assumes fixed values and in the second one non-fixed values of the explanatory variable are used.

5.1. Example 1. (Fixed Values of the Explanatory Variable) Consider an in-control process represented by a linear profile of $Y = 3 + 2X + \varepsilon$ for $X \in [1,10]$ with ε being a standard normal random variable. Since the parameters are known, monitoring of the process in phase two of the algorithm begins. Suppose 10 observations are collected in each sample as

$$(1, y_{1j}), (2, y_{2j}), (3, y_{3j}), (4, y_{4j}), (5, y_{5j}), (6, y_{6j}),$$
$$(7, y_{7j}), (8, y_{8j}), (9, y_{9j}), (10, y_{10j})$$

where y values are collected through a random number generator, estimate b_{0j} and b_{1j} by the least squared method, and calculate the F_j^* using Equation 5. Then, the control limits for monitoring the process mean for $\alpha = 0.005$ follows:

$$UCL_F = F_{(1 - \alpha; df_R - df_F, df_F)} =$$
$$F_{(0.995; 2, 8)} = 11.04, \quad LCL_F = 0$$

To monitor the variability of the process an R chart is applied. From the generated data the sample mean of the ranges is calculated as $\overline{R} = 2.88$. Then the control limits of this chart are

$$UCL = (2.8917)(D_4 = 1.777) = 5.138$$

$$LCL = (2.8917)(D_3 = 0.223) = 0.644.$$

In the simulation study using MATLAB 7 software, 10000 replications were generated and an in-control ARL of 201.9240 was obtained (in-control ARL of multivariate T^2 and EWMA/R charts were 206.8120 and 203.2740.). Then out-of-control ARL values were calculated by making shifts in β_0 and β_1 and the error variance using 10000 replications and were compared with the ones from the multivariate T^2 and EWMA/R charts. Also the results of the simulation study for ARL_1 are presented in Table 1 to 4.

The results of the tables show that when both intercept and slope shifts implemented simultaneously, the proposed method performs better than the EWMA/R method. In this situation the T^2 method is the best. However, for single shifts, either in the slope or in the intercept, the proposed method does not perform as good as the EWMA/R and T^2 method, especially for small shifts of intercepts and slopes.

5.2. Example 2. (Non-Fixed Values of the Explanatory Variable) Consider example one in which the values of the explanatory variable are not fixed and are allowed to be a random sample uniformly distributed on (0,1). In this case the control limits for monitoring the process mean for $\alpha = 0.005$ are as follows:

$$UCL_F = F_{(1 - \alpha; df_R - df_F, df_F)} = F_{(0.995; 2, 8)} = 11.04$$
$$, \quad LCL_F = 0$$

To monitor the variability of the process an R chart is applied. From the generated data the sample mean of the ranges as $\overline{R} = 2.8886$ are calculated. Then the control limits of the R-chart are $UCL = (2.8886)(D_4 = 1.777) = 5.1330$ and $LCL = (2.8886)(D_3 = 0.223) = 0.644$. Using 10000

TABLE 1. Comparisons of ARL Values Through Intercept Shifts (β_0 to $\beta_0 + \lambda\sigma$).

CHART	λ									
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2
GLT/R	119.129	50.782	19.598	8.642	4.642	3.045	2.021	1.543	1.247	1.124
EWMA/R	18.384	5.854	3.498	2.556	2.052	1.73	1.521	1.267	1.095	1.028
T^2	90.620	24.876	8.053	3.453	1.956	1.326	1.099	1.0340	1.006	1.001

TABLE 2. Comparisons of ARL Values Through Slope Shifts (β_1 to $\beta_1 + \eta\sigma$).

CHART	η									
	0.025	0.05	0.075	0.10	0.125	0.15	0.175	0.20	0.225	0.25
GLT/R	144.229	76.547	35.233	18.138	9.461	5.989	3.501	2.724	2.038	1.597
EWMA/R	34.343	10.966	5.786	3.899	2.985	2.475	2.115	1.905	1.702	1.538
T^2	116.764	44.308	16.881	7.444	3.833	2.222	1.601	1.248	1.100	1.038

TABLE 3. Comparisons of ARL Values Through Standard Deviation Shifts (σ to $\gamma\sigma$).

CHART	γ									
	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.0
GLT/R	21.791	5.961	3.115	2.122	1.603	1.352	1.203	1.129	1.102	1.054
EWMA/R	21.791	5.961	3.115	2.122	1.603	1.352	1.203	1.129	1.102	1.054
T^2	40.097	14.712	5.610	4.906	3.816	3.041	2.504	2.257	2.039	1.787

TABLE 4. Comparisons of ARL Values Through Both Intercept and Slope Shifts ($\lambda + \eta$ $\overline{X} = 0$; $\overline{X} = 5$).

CHART	η									
	0.025	0.05	0.075	0.10	0.125	0.15	0.175	0.20	0.225	0.25
GLT/R	181.55	149.957	110.432	83.464	59.579	39.307	27.388	3.876	20.249	14.681
EWMA/R	172.094	156.221	127.762	98.406	81.485	65.244	59.082	47.474	40.507	35.862
T^2	172.182	124.288	78.482	48.551	30.381	20.689	12.832	8.949	6.094	5.286

replications in the simulation study the ARL_0 value of the multivariate T^2 chart becomes 45.019. Therefore, this control chart can not be used in this case. Moreover, by 10000 replications the ARL_0 values for the EWMA/R and the GLT/R control charts are 204.2730 and 194.1312, respectively. The results of the comparison study based on ARL_1 values are presented in Table 5 to 8.

The results of the tables indicate that the proposed method performs not as well as the EWMA/R method for different shifts in the slopes and the intercepts. However, for those situations in which there are simultaneous shifts in the slopes and the intercepts, the proposed method performs better than the EWMA/R method.

6. CONCLUSION AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this paper the concept of the linear regression and the generalized linear test to design a control chart for profiles monitoring were employed. Moreover, in order to detect shifts in the error variance an R chart was simultaneously applied. The performance of the proposed control-charting method was compared with the ones of the T^2 and EWMA/R charts through a simulation study. The results show that both in the fixed and non-fixed values of the explanatory variable cases the proposed method performs well in situations in which there are simultaneous shifts in the slopes and the intercepts of the profile.

A sensitivity analysis on the effects of the range of the explanatory variable and also the effects of the sample size on out-of-control ARL values is an idea for future research. Moreover, developing the proposed method for nonlinear profiles will be very interesting future research in this area.

7. REFERENCES

1. Shewhart, W. A., "Economic Control of Quality of Manufactured Product", D. Van Nostrand, New York, NY., (Republished in 1980 by the American Society for Quality Control, Milwaukee, WI), (1931).

2. Montgomery, D. C.,"Introduction to Statistical Quality Control", 4th Edition, John Wiley and Sons, New York, NY, (2001).

3. Gardner, M. M., Lu, J. C., Gyurcsik, R. S., Wortman, J. J., Hornung, B. E., Heinisch, H. H., Rying, E. A., Rao, S., Davis, J. C. and Muzumder, P. K., "Equipment Fault Detection Using Spatial Signatures", *IEEE Transactions on Components, Packaging, and Manufacturing Technology-Part C*., Vol. 20, (1997), 295-304.

4. Jin, J. and Shi, J., "Automatic Feature Extraction of Waveform Signals for In-Process Diagnostic Performance Improvement", *Journal of Intelligent Manufacturing*, Vol. 12, (2001), 257-268.

5. Kang, L. and Albin, S. L., "On-Line Monitoring When the Process Yields a Linear Profile", *Journal of Quality Technology*, Vol. 32, (2000), 418-426.

6. Jin, J. and Shi, J., "Feature-Preserving Data Compression of Stamping Tonnage Information Using Wavelets", *Technometrics*, Vol. 41, (1999), 327-339.

7. Mahmoud, M. A. and Woodall, W. H., "Phase I Analysis of Linear Profiles with Calibration Applications", *Technometrics*, Vol. 46, (2004), 277-391.

8. Kim, K., Mahmoud, M. A. and Woodall, W. H., "On the Monitoring of Linear Profiles", *Journal of Quality Technology*, Vol. 35, (2003), 317-328.

9. Jensen, D. R., Hui, Y. V. and Ghare, P. M., "Monitoring an Input-Output Model for Production. I: The Control Charts", *Management Science*, Vol. 30, (1984), 1197-1206.

10. Walker, E. and Wright, S. P. "Comparing Curves Using Additive Models", *Journal of Quality Technology*, Vol. 34, (2002), 118-129.

11. Miller, A., "Analysis of Parameter Design Experiments for Signal-Response System", *Journal of Quality Technology*, Vol. 34, (2002), 139-151.

12. Nair, V. N., Taam, W. and Ye, K. Q. "Analysis of Functional Responses from Robust Design Studies", *Journal of Quality Technology*, Vol. 34, (2002), 355-370.

13. Ramsay, J. O. and Silverman, B. W., "Functional Data Analysis", Springer-Verlag, New York, NY, (1997).

14. Brill, R. V., "A Case Study for Control Charting a Product Quality Measure that is a Continuous Function over Time", *Presented at the 45th Annual Fall Technical Conference*, Toronto, Ontario, (2001).

15. Williams, J. D., Woodall, W. H. and Berch, J. B., "Phase I monitoring of nonlinear Profiles", *Presented at the 2003 Quality and Productivity Research Conference*, Yorktown Heights, New York, (2003).

16. Lada, E. K. and Wilson, J. C. L., "A Wavelet-based Procedure for Process Fault Detection", *IEEE Transactions on Semiconductor Manufacturing*, Vol. 15, (2002), 79-90.

17. Williams, J. D., Woodall, W. H., Spitzner, J., Montgomery, D. C. and Gupta, S., "Using Control Charts to Monitor Process and Product Quality Profiles", *Journal of Quality Technology*, Vol. 36, (2004), 309-320.

TABLE 5. Comparisons of ARL Values Through Intercept Shifts (β₀ to β₀ + λσ).

CHART	λ									
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2
GLT/R	117.237	48.747	19.403	8.882	4.911	2.872	2.040	1.517	1.274	1.158
EWMA/R	18.004	6.025	3.480	2.505	2.089	1.754	1.476	1.252	1.112	1.021

TABLE 6. Comparisons of ARL Values Through Slope Shifts (β₁ to β₁ + ησ).

CHART	η									
	0.025	0.05	0.075	0.10	0.125	0.15	0.175	0.20	0.225	0.25
GLT/R	150.568	87.150	44.769	24.208	13.973	8.698	5.254	3.802	2.740	2.236
EWMA/R	41.029	12.145	6.478	4.481	3.301	2.732	2.347	2.043	1.885	1.660

TABLE 7. Comparisons of ARL Values Through Standard Deviation Shifts (σ to γσ).

CHART	γ									
	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.60	2.80	3.0
GLT/R	20.248	6.084	3.057	2.028	1.574	1.334	1.188	1.122	1.094	1.063
EWMA/R	20.248	6.084	3.057	2.028	1.574	1.334	1.188	1.122	1.094	1.063

TABLE 8. Comparisons of ARL Values Through Both Intercept and Slope Shifts (λ + η $\overline{X} = 0$; $\overline{X} = 5$).

CHART	η									
	0.025	0.05	0.075	0.10	0.125	0.15	0.175	0.20	0.225	0.25
GLT/R	189.802	142.767	114.388	85.798	59.062	45.877	31.209	24.797	18.605	13.501
EWMA/R	194.366	180.881	158.829	140.990	118.772	104.788	86.094	72.979	61.682	53.202

18. Chang, S. I. and Samuel, T. R., "Intelligent CUSUM Scheme for Monitoring Process Variation Using Fuzzy Control", *International Journal of Smart Engineering System Design*, Vol. 2, (1999),

1-15.

19. Neter, M. Kutner, H., Nachtsheim, C. J. and Wasserman, W., "Applied Linear Regression Models", 3rd Edition, Irwin, Chicago, (1996).