TECHNICAL NOTE

RELIABILITY ANALYSIS OF K-OUT-OF N: G MACHINING SYSTEMS WITH MIXED SPARES AND MULTIPLE MODES OF FAILURE

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Abstract This paper deals with the transient analysis of K-out-of-N: G system consisting of N-operating machines. To improve system reliability, Y cold standby and S warm standbys spares are provided to replace the failed machines. The machines are assumed to fail in multiple modes. At least K-out-of-N machines for smooth functioning of the system. Reliability and mean time to failure are established in terms of transient probabilities.

Keywords K-Out-of-N: G System, Mixed Spares, Reliability, MTTF, Multiple Failure Modes

چکیده این مقاله در مورد تجزیه و تحلیل گذرای یک سیستم K از N:G شامل N ماشین عملیاتی می‌باشد. به منظور بهبود پایایی سیستم، Y ماشین رزرو فعال و S ماشین رزرو غیر فعال برای جایگزینی ماشین‌های خراب شده فراهم شده است. خرابی ماشین‌ها در چند حالت فرض شده‌است و حداقل K از N ماشین برای هموارسازی سیستم مورد نیاز می‌باشد. پایایی و متوسط زمان تا خرابی سیستم با استفاده از احتمالات گذرا تعریف شده‌اند.

1. INTRODUCTION

The performance of any machining system is highly influenced by machine failure. The machine failure may be balanced either by providing spare part support or by facilitating better repair or both so that the production may not suffer. Reliability indices of K-out of-N: G machining system with spares has been studied by many researchers. Teixeirade [1] presented multi-criteria decision models for two maintenance problems in which one is a repair contract selection and other one is a spares provisioning. Arulmozhi [2] developed a closed form solution for the system reliability of an M-out of-N warm standby system with R repairmen. Amri et al. [3] considered optimal design of k-out-of-n: G subsystems subjected to imperfect fault-coverage. Zhang et al. [4] obtained availability and reliability of k-out-of-(M+N): G warm standby systems.

In this paper, the reliability analysis of K-out-of N: G machining systems with mixed spares and multiple modes of failure is provided. A few researchers have studied various machine repair problems for multi-modes of failure; some of them have considered the two-mode failure models. Goyal and Sharma [5] gave the stochastic analysis of two unit standby systems with two failure modes. Reddy and Rao [6] obtained the optimization of parallel system subject to two modes of failure and repair provision. Sharma and

Sharma [7] considered M/M/R machine repair problem with spares and three modes of failure. Wang and Lee [8] developed the Cold-standby M/M/R machine repair problem where a group of identical and independent operating machines have $K\,(K \geq 1)$ failure modes. The cost analysis of the M/M/R machine repair problem with two modes of failure was provided by Wang and Wu [9] and Jain et al. [10]. Levitin [11] developed a model, which generalizes the linear consecutive k-out-of-r-from-n system to the case of multiple failure criteria. Assessment of reversible multi-state k-out-of-n: a G/F load-sharing system was discussed by Jenab and Dhillon [12] by using flow-graph models.

2. MODEL DESCRIPTION

A K-out-of N: G machining systems was considered with mixed spares and multiple modes of failure.

The following assumptions and notations have been used for mathematical formulation of the problem:

- The system consists of N operating machines and Y cold standbys and S warm standbys.
- The life time and repair time of the machines are exponentially distributed.
- There is a provision of cold standbys and warm standbys to replace the failed machines.
- The total number of machines in the system is given by L = N + Y + S.
- Whenever a machine is repaired, it becomes as good as a new one.
- The system works if at least K machines are working.
- The machine may fail in any one of M modes of failure. Repair times of the machine failed in m^{th} (m = 1, 2, …, M) mode are exponentially distributed with rates μ_m.
- m^{th} (m = 1, 2, …, M) failure mode of operating machines are independent Poisson processes. The state dependent rates are given by

$$\lambda(j) = \begin{cases} N\lambda_m + S\alpha_m, & 0 \leq j \leq Y \\ N\lambda_m + (Y + S - j)\,\alpha_m, & Y < j < S + Y \\ (N + S + Y - j)\,\lambda'_m, & S + Y \leq j < L - K \end{cases} \tag{1}$$

where λ_m and α_m are mean failure rates of operating and warm standby machines in m^{th} mode (m = 1, 2, …, M), respectively; λ'_m (m = 1, 2,…, M) is the degraded mean failure rate of operating machines in m^{th} mode when there are less than N operating machine in the system.

3. SYSTEM WITH REPAIR

The mathematical model for the relevant system can be formulated as a continuous time parameter. The Morkov chain with states (je_m) (j = 0, 1, …, N-K+1) representing the number of failed components due to m^{th} failure mode; here e_m is a unit row of dimension M having unity at the m^{th} position and zero elsewhere. Let $P_t(je_m)$ denote the probability of this state at time t. Also denote $\lambda = \sum_{m=1}^{M} \lambda_m$, $\alpha = \sum_{m=1}^{M} \alpha_m$.

When the system starts at time t = 0 in the state (0), the set of differential equations are as given below:

$$\frac{dP_t(0)}{dt} = -[N\lambda + S\alpha]\,P_t(0) + \sum_{m=1}^{M} \mu_m P_t(e_m) \tag{2}$$

$$\begin{aligned} \frac{dP_t(je_m)}{dt} = &-\left[N\lambda_m + S\alpha_m + j\mu_m\right] P_t(je_m) + \\ &\left[N\lambda_m + S\alpha_m\right] P_t((j-1)e_m) + \\ &(j+1)\mu_m P_t((j+1)e_m),\ (1 \leq j \leq Y) \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{dP_t(je_m)}{dt} = \\ &-\left[N\lambda_m + (Y + S - j)\,\alpha_m + j\mu_m\right] P_t(je_m) + \\ &(j+1)\mu_m P_t((j+1)e_m) + \\ &\left[N\lambda_m + (Y + S - (j-1))\,\alpha_m\right] \\ &P_t((j-1)e_m),\quad (Y < j < Y + S) \end{aligned} \tag{4}$$

$$\frac{dP_t(je_m)}{dt} = -\left[(N+S+Y-j)\lambda'_m + j\mu_m\right]P_t(je_m) + (j+1)\mu_m P_t((j+1)e_m) + \left[N\lambda_m + (Y+S-(j-1))\alpha_m\right]P_t((j-1)e_m), (j = Y+S). \tag{5}$$

$$\frac{dP_t(je_m)}{dt} = -\left[(L-j)\lambda'_m + j\mu_m\right]P_t(je_m) + \left[(L-(j-1))\lambda'_m\right]P_t((j-1)e_m) + (j+1)\mu_m P_t((j+1)e_m), \quad (Y+S<j<L-K) \tag{6}$$

$$\frac{dP_t(je_m)}{dt} = -\left[(L-j)\lambda'_m + j\mu_m\right]P_t(je_m) + \left[(L-(j-1))\lambda'_m\right]P_t((j-1)e_m), (j = L-K) \tag{7}$$

$$\frac{dP_t((N-K+1)e_m)}{dt} = K\lambda'_m P_t((N-K)e_m), \quad j = (L-K+1) \tag{8}$$

where the initial conditions are:

$$P_0(0) = 1 \text{ and } P_0(je_m) = 0 \quad \text{for } j > 0 \tag{9}$$

The reliability R(t) with repair and mean time to failure (MTTF) of the system can be calculated using

$$R_t(\text{with repair}) = P_t(0) + \sum_{j=1}^{Y}\sum_{m=1}^{M} P_t(je_m) + \sum_{j=Y+1}^{Y+S-1}\sum_{m=1}^{M} P_t(je_m) + \sum_{j=Y+S}^{L-K}\sum_{m=1}^{M} P_t(je_m) \tag{10}$$

and

$$MTTF = \int_0^\infty R_t(\text{with repair})\,dt \tag{11}$$

4. SYSTEM RELIABILITY WITHOUT REPAIR

If $\mu_m = 0$, then it is a case without repair and the following recursive formulae can be derived. It can be denoted that the Laplace transforms of P_t (je_m) by $\hat{P}_s(je_m)$; $0 \le j \le L-K+1$. Taking Laplace transform of Equations 2-8,

$$\hat{P}_s(0) = \frac{1}{[s+N\lambda+S\alpha]}, j = 0 \tag{12}$$

$$\hat{P}_s(je_m) = \frac{1}{(s+N\lambda+S\alpha)}\left\{\frac{[N\lambda_m + S\alpha_m]}{[s+N\lambda_m+S\alpha_m]}\right\}^j, \quad (1 \le j \le Y) \tag{13}$$

$$\hat{P}_s(je_m) = \frac{1}{(s+N\lambda+S\alpha)}\left\{\frac{[N\lambda_m + S\alpha_m]}{[s+N\lambda_m+S\alpha_m]}\right\}^Y \frac{\prod_{n=Y+1}^{j}\left[N\lambda_m + (Y+S-(n-1))\alpha_m\right]}{\prod_{n=Y+1}^{j}\left[s+N\lambda_m + (Y+S-n)\alpha_m\right]}, \quad (Y<j<Y+S) \tag{14}$$

$$\hat{P}_s(je_m) = \frac{1}{(s+N\lambda+S\alpha)}\left\{\frac{[N\lambda_m + S\alpha_m]}{[s+N\lambda_m+S\alpha_m]}\right\}^Y \frac{\prod_{n=Y+1}^{Y+S}\left[N\lambda_m + (Y+S-(n-1))\alpha_m\right]}{\prod_{n=Y+1}^{Y+S-1}\left[s+N\lambda_m + (Y+S-n)\alpha_m\right]} \times \frac{\prod_{n=Y+S+1}^{j}\left[\{L-(j-1)\}\lambda'_m\right]}{\prod_{n=Y+S}^{j}\left[s+(L-j)\lambda'_m\right]}, \quad (Y+S \le j < L-K) \tag{15}$$

Now inverting the Laplace transforms from Equations 9-12,

$$P_t(0) = e^{-(N\lambda + S\alpha)t}, \quad j = 0 \tag{16}$$

Using $L^{-1}\left(\frac{1}{(s+a)^n}\right) = \frac{t^{n-1}e^{-at}}{(n-1)!}$ and convolution theorem, we have

$$P_t(je_m) = \frac{\left(N\lambda_m + S\alpha_m\right)^Y e^{-(N\lambda + S\alpha)t}}{(Y-1)!} \left[\frac{t^{Y-1}e^{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]t}}{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]} - \frac{(Y-1)!\left\{e^{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]t}-1\right\}}{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]^2}\right],$$

$$(1 \le j \le Y) \tag{17}$$

$$P_t(je_m) = \frac{\left(N\lambda_m + S\alpha_m\right)^Y \prod_{n=Y+1}^{j}\left[N\lambda_m + \left(Y+S-(n-1)\right)\alpha_m\right]}{(Y-1)!}$$

$$\times\left\{\left[\frac{e^{-(N\lambda+S\alpha)t}}{\prod_{n=Y+1}^{j}\left[\left\{N\lambda_m+(Y+S-n)\alpha_m\right\}-(N\lambda+S\alpha)\right]}\times\left[\frac{t^{Y-1}e^{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]t}}{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]} - \frac{(Y-1)!\left\{e^{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]t}-1\right\}}{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]^2}\right]\right]\right\}$$

$$+\left\{\sum_{n=Y+1}^{j}\frac{e^{-\left[N\lambda_m+(N+S-n)\alpha_m\right]t}}{\left[(N\lambda+S\alpha)-\left\{N\lambda_m+(Y+S-n)\alpha_m\right\}\right]}\times\right.$$

$$\frac{1}{\prod_{\substack{n=Y+1\\ p\ne n}}^{j}\left[(Y+S-n)\alpha_m-(Y+S-p)\alpha_m\right]}\times$$

$$\left.\left[\frac{t^{Y-1}e^{\left[(N-n)\alpha_m\right]t}}{\left[(N-n)\alpha_m\right]} - \frac{(Y-1)!\left\{e^{\left[(N-n)\alpha_m\right]t}-1\right\}}{\left[(N-n)\alpha_m\right]^2}\right]\right\}\Bigg]\Bigg],$$

$$Y \le j < Y+S \tag{18}$$

$$P_t(je_m) = \left(N\lambda_m+S\alpha_m\right)^Y \prod_{n=Y+1}^{Y+S}\left[N\lambda_m+\left(Y+S-(n-1)\right)\alpha_m\right] \prod_{n=Y+S}^{j}\left[L-(n-1)\lambda'_m\right] \Big/ (Y-1)!$$

$$\times\left[\left\{e^{-(N\lambda+S\alpha)t}\Big/\right.\right.$$

$$\prod_{n=Y+1}^{Y+S-1}\left[\left\{N\lambda_m+(Y+S-n)\alpha_m\right\}-(N\lambda+S\alpha)\right] \prod_{n=Y+S}^{j}\left[(L-n)\lambda'_m\right]\times$$

$$\left[\frac{t^{Y-1}e^{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]t}}{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]} - \frac{(Y-1)!\left\{e^{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]t}-1\right\}}{\left[N(\lambda-\lambda_m)+S(\alpha-\alpha_m)\right]^2}\right]\Bigg]\Bigg]$$

$$
+\left\{\sum_{n=Y+1}^{Y+S-1} e^{-\left[N\lambda_m+(N+S-n)\alpha_m\right]t} \Big/ \left[(N\lambda+S\alpha)-\left\{N\lambda_m+(Y+S-n)\alpha_m\right\}\right]\prod_{\substack{n=Y+1\\ r\neq n}}^{Y+S-1}\left[(Y+S-n)\alpha_m-(Y+S-r)\alpha_m\right]\times\right.
$$

$$
\left[\frac{t^{Y-1}e^{\left[(N-n)\alpha_m\right]t}}{\left[(N-n)\alpha_m\right]}-\frac{(Y-1)!\left\{e^{\left[(N-n)\alpha_m\right]t}-1\right\}}{\left[(N-n)\alpha_m\right]^2}\right]
$$

$$
\left.\Big/\prod_{n=Y+S}^{j}\left[(L-n)\lambda'_m-N\lambda_m-(Y+S-n)\alpha_m\right]\right\}
$$

$$
+\left\{\sum_{n=Y+S}^{j} e^{-\left[(L-n)\lambda'_m\right]t} \Big/ \left[(N\lambda+S\alpha)-\{(L-n)\lambda'_m\}\right]\prod_{n=Y+1}^{Y+S-1}\left[\left\{N\lambda_m+(Y+S-n)\alpha\right\}_m-(L-n)\lambda'_m\right]\right.
$$

$$
\times\left[\frac{t^{Y-1}e^{\left[(L-n)\lambda'_m-\left(N\lambda_m+S\alpha_m\right)\right]t}}{\left[(L-n)\lambda'_m-\left(N\lambda_m+S\alpha_m\right)\right]}-\frac{(Y-1)!\left\{e^{\left[(L-n)\lambda'_m-\left(N\lambda_m+S\alpha_m\right)\right]t}-1\right\}}{\left[(L-n)\lambda'_m-\left(N\lambda_m+S\alpha_m\right)\right]^2}\right]
$$

$$
\left.\left.\Big/\prod_{\substack{n=Y+S\\ n\neq q}}^{j}\left[(L-n)\lambda'_m-(L-q)\lambda'_m\right]\right\}\right],
$$

$$
Y+S\leq j\leq L-K \qquad (19)
$$

The transient reliability R(t) and mean time to failure (MTTF) of the system without repair can be calculated by using the similar formulae as given in Equations 10-11.

5. SYSTEM RELIABILITY FOR MODIFIED MODEL WITH REPAIR

In this case, the reliability system is considered with repair as in Section 3 including the assumption that the relations between two failure modes are permissible. Let $\sum_{m=1}^{M} j_m e_m = J$ be the state of the system representing the number of failed components due to failure mode-m and Pt(J) be the probability of the system state at time t. For state 0, Equation 2 holds. Now other equations are constructed as follows:

$$
\frac{dP_t(J)}{dt}=-\left[N\lambda_m+S\alpha_m+\sum_{m=1}^{M} j_m\mu_m\right]P_t(J)+
\left[N\lambda_m+S\alpha_m\right]\sum_{m=1}^{M}\psi(j_m)P_t(J-e_m)+
\pi\sum_{m=1}^{M}(j_m+1)\mu_m P_t(J+e_m),\quad 1\leq j\leq Y \qquad (20)
$$

$$
\frac{dP_t(J)}{dt}=-\left[N\lambda_m+(Y+S-J)\alpha_m+\sum_{m=1}^{M} j_m\mu_m\right]P_t(J)
+\pi\sum_{m=1}^{M}(j_m+1)\mu_m P_t(J+e_m)+
\left[N\lambda_m+(Y+S-(J-1))\alpha_m\right]\sum_{m=1}^{M}\psi(j_m)P_t(J-e_m),
$$

$$
Y<j<Y+S \qquad (21)
$$

$$
\frac{dP_t(J)}{dt}=-\left[(N+S+Y-J)\lambda'_m+\sum_{m=1}^{M} j_m\mu_m\right]P_t(J)
+\pi\sum_{m=1}^{M}(j_m+1)\mu_m P_t(J+e_m)+\left[N\lambda_m+(Y+S-(J-1))\alpha_m\right]
\sum_{m=1}^{M}\psi(j_m)P_t(J-e_m),\quad j=Y+S \qquad (22)
$$

$$\frac{dP_t(J)}{dt} = -\left[(L-J)\lambda'_m + \sum_{m=1}^{M} j_m \mu_m\right] P_t(J) +$$
$$\left[(L-(J-1))\lambda'_m\right] P_t(J-e_m) +$$
$$\pi \sum_{m=1}^{M} (j_m+1)\mu_m P_t(J+e_m), \quad Y+S<j<L-K \tag{23}$$

$$\frac{dP_t(je_m)}{dt} = -\left[(L-J)\lambda'_m + \sum_{m=1}^{M} j_m \mu_m\right] P_t(J)$$
$$+\left[(L-(J-1))\lambda'_m\right] P_t(J-e_m), \quad j=L-K \tag{24}$$

$$\frac{dP_t(J)}{dt} = \left[L-(J-1)\right] \sum_{m=1}^{M} \psi(je_m)\lambda'_m \tag{25}$$
$$P_t(L-(J-e_m)), \quad j=(L-K+1)$$

Where

$$\psi(je_m) = \begin{cases} 0, & j_m = 0 \\ 1, & j_m > 0 \end{cases} \tag{26}$$

$$\pi = \begin{cases} 0, & J = N-K \\ 1, & J < N-K \end{cases} \tag{27}$$

The initial conditions are same as given by Equation 9.

The reliability R(t) and mean time to failure (MTTF) of the system can be calculated using Equation 10 and 11.

6. SPECIAL CASES

Now consider the special cases by setting appropriate parameters as follows:

Case I

Model With Two Modes Of Failure Here the machines are failed in two modes (i.e. M = 2). In this case, the formulae for reliability with and without repair, which coincide with the result obtained by Moustafa is obtained (1996).

Case II

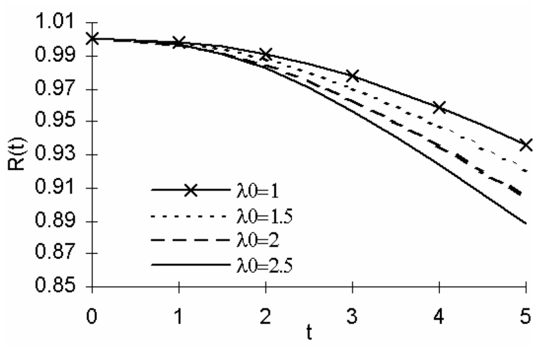
Model With Multiple Modes of Failure Without Spare When S = 0, Y = 0, in this case the system reliability without spares is found. In this case the present model reduces to the model studied by Moustafa.

7. NUMERICAL ILLUSTRION

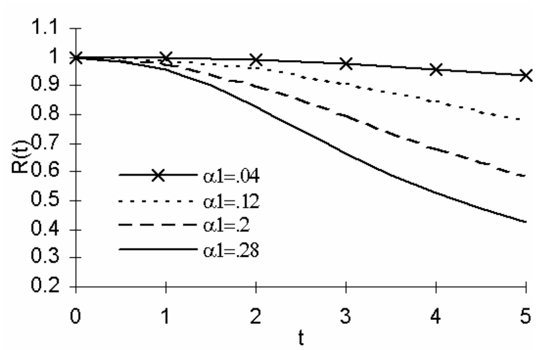
Numerical illustrations have been made to calculate system reliability. The system reliability profiles for the model with repair for different values of λ_0, α_1 and α_2 are displayed in Figures 1(a)-1(c) for heterogeneous Figures 2(a)-2(c) exhibit the system reliability for the model without repair with a heterogeneous rate. In all these figures, the default parameters are fixed as follows: From Figures 1(a)-1(c) and 2(a)-2(c) a lower value of t, R(t) is observed that decreases slowly but as t takes higher values, there is a sharp decrease in R(t). Also as λ_0, α_1 and α_2 increase, the reliability decreases, the effect is more prominent as time increases.

8. CONCLUSIONS

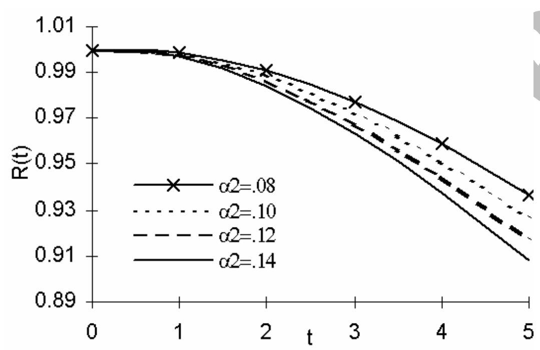
A K-out-of-N: G system has been considered as having cold as well as warm standby machines. The earlier work in the same line by Moustafa (1998) has no provision of spares whereas the present model includes cold and warm standbys. The noble feature of the present study is the sensitivity analysis via graphs to examine the effect of different parameters, while was not given by Moustafa (1998). The K-out-of-N: G system with multiple mode of failure studied seems to provide a very effective mean of improving system reliability. For example a four-engine aircraft needs only two engines to perform critical function; the operating and standby engine may fail in different modes with different rates. Other examples can be given for communication systems with three transmitters having different types of



(a)

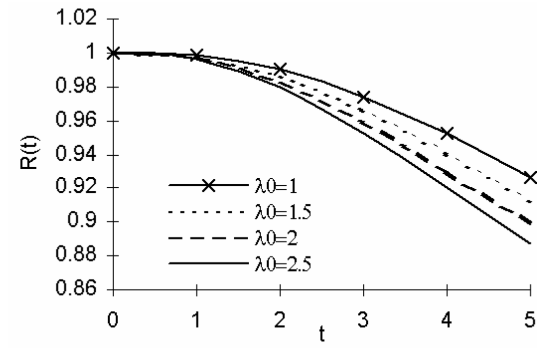


(b)

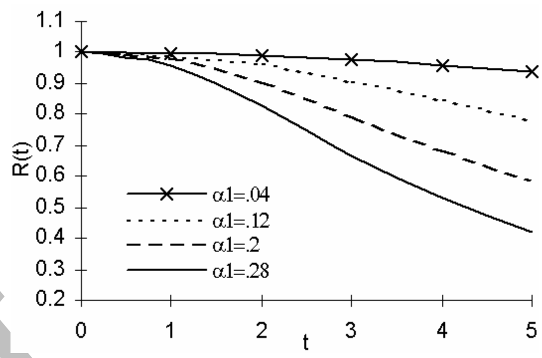


(c)

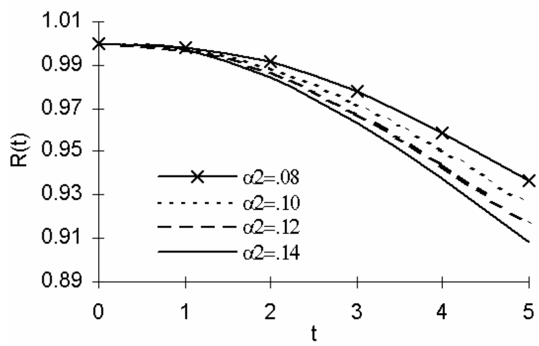
Figure 1. System reliability for model with repair and heterogeneous rate by varying (a) λ_0 (b) α_1 (c) α_2.



(a)



(b)



(c)

Figure 2. System reliability for model without repair and heterogeneous rate by varying (a) λ_0 (b) α_1 (c) α_2.

failures; the average message load may be such that at least two transmitters must be operational at all times otherwise critical messages will be lost.

The present study can be extended for linear and consecutive k-r-out-of-n: G system; the other generalization can be done by incorporating common cause of failure.

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