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The Effect of Anisotropy on Free Vibration of Rectangular Composite Plates with Patch Mass

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ABSTRACT

In this paper, the effect of anisotropy on the free vibration of laminated rectangular plate supporting a localized patch mass is investigated. The two variable refined plate theory is applied to define the third order displacement field of a composite rectangular plate. The plate is considered to have simply supported boundaries. The equations of motion for rectangular plate are obtained by calculus of variation. Parametric study of non-dimensional natural frequencies is carried out and the influences of geometrical parameters such as aspect ratio of the plate, size and location of the patch mass on these frequencies are also studied. First, the results obtained are compared with those reported using several plate theories. In the next step, the effect of anisotropy on free vibration of plates for different types of lamination are studied. The numerical results are found to be in a very good agreement with well known published papers for the case of vibration analysis of loaded and unloaded plates.

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1. INTRODUCTION

Composite materials are simply a combination of two or more different materials that may provide superior and unique mechanical and physical properties. These materials have high strength to weight ratio compared with other materials. Practically, these materials are exposed to various loading conditions such as distributed patch mass, transverse and in-plane loadings and etc. hence it is necessary to investigate their response to these loading conditions. Srinivas and Rao [1] studied the bending, vibration and buckling behavior of simply supported thick orthotropic rectangular laminated plates and obtained normal and shear stresses under the effect of uniformly distributed transverse load. Withney and Pagano [2] investigated free vibration response of a composite plate using first order shear deformation theory (FSDT) and employed the Yang-Norris-Stavski (YNS) theory to study the cylinrical bending of anti-symmetric cross-ply and angle-ply plate strips with sinusoidal loading. Bert and Chen [3] presented a closed form solution for the free vibration of simply supported anti-symmetric rectangular plates based on the YNS theory. Shankara and Iyengar [4] obtained finite element solutions for free vibration of laminated composite plates by higher-order shear deformation theory. Reddy [5] carried out free vibration analysis of anti-symmetric angle-ply laminated plates considering the effect of transverse shear deformation using finite element method (FEM). In other work, Reddy [6] classified a set of equilibrium equations for the kinematic models proposed by Levinson and Murthy. Khdeir and Reddy [7] obtained the free vibration response of angle-ply and cross-ply laminated composite plate using second order shear deformation theory. Wong [8] studied the effect of distributed patch mass on the plate vibration response. In his work, the effects of shear deformation and rotary inertia were not considered and Rayleigh-Ritz method was used to find the response of a rectangular plate. Shimpi and Patel [9-10] used the two variable refined plate theory for simply supported orthotropic plates and the results obtained were compared with non-dimensional central displacement in the through thickness direction. Alibeigloo et al. [11] studied the vibration response of anti-symmetric rectangular plates with distributed patch mass using third order shear deformation theory (TSDT) and obtained the first natural frequency of the plate considering the size and location of the distributed mass on the top surface of the plate. Alibeigloo and Kari [12] also studied the forced

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vibration response of anti-symmetric laminated rectangular plates with distributed patch mass. Seung-Eock et al. [13] employed the two variable refined plate theory (RPT2) for plates which are under the action of the transverse and in-plane forces and obtained the stiffness and mass matrices using Hamilton principle. They compared the non-dimensional deflection obtained by various theories namely the classical laminate plate theory, the first order shear deformation theory, the higher order shear deformation theory and the refined plate theory. They showed that the RPT2 gives more accurate results of deflection and buckling load than the HSDT in comparison with the three-dimensional elasticity solution. Seung-Eock et al. [14] also carried out buckling analysis of isotropic and orthotropic plates using the two variable refined plate theory. A closed form solution of a simply supported rectangular plate subjected to in-plane loading was obtained using Navier's method. In this paper, the non-dimensional first natural frequency of vibration with simply supported boundary conditions and under the effect of a patch mass in arbitrary dimensions and positions is obtained. The effect of various parameters such as position of the patch mass and the aspect ratio of the plate on free vibration are also studied.

2. BASIC FORMULATION

A rectangular plate with length, width and thickness equal to a, b and h respectively is considered. The plate supports a distributed patch mass, M_{mass}, with dimensions equal to c and d in the x and y-direction, respectively which is located in arbitrary position (x', y') in Figure 1. The mass is considered to be placed on the upper surface of the plate. The global Cartesian coordinate system is chosen with the origin at the corner and on the middle plane of the plate, z=0. Therefore, the domain of plate is defined as $0 \le x \le a$, $0 \le y \le b$ and $-h/2 \le z \le h/2$.

In order to proceed with the formulation of the problem using the two variable refined plate theory (RPT2), it is assumed that the displacements (u, v, w) of the plate are small in comparison with the thickness of the plate, hence the strains involved are considered to be infinitesimal. On the other hand, the transverse normal stress in the z-direction, σ_z, is very small in comparison with the in-plane stresses, σ_x and σ_y. As a consequence of the above definition, the stress–strain relations can be reduced from a 6×6 matrix to a 5×5 matrix which can reduce the complexity of the problem. The total displacement of the plate in the z-direction (W) is assumed to be consisting of three components, w_a (extension), w_b (bending) and w_s (shear) which are functions of x, y and the time [13].

$$W(x,y,z,t) = w_a(x,y,t) + w_b(x,y,t) + w_s(x,y,t) \tag{1}$$

The displacements in the x and y-directions are also defined as [13]:

$$\begin{aligned} U(x,y,z,t) &= u(x,y,t) + u_b(x,y,t) + u_s(x,y,t) \\ V(x,y,z,t) &= v(x,y,t) + v_b(x,y,t) + v_s(x,y,t) \end{aligned} \tag{2}$$

The bending components of displacements i.e. u_b and v_b are considered to be similar to their corresponding components of displacements, u and v (component of extension) as defined in the classical plate theory (CPT). It means that:

$$u_b = -z(\partial w_b/\partial x), \qquad v_b = -z(\partial w_b/\partial y) \tag{3}$$

Considering the fact that the shear stresses τ_{zx} and τ_{zy} are zero at upper and lower faces of the plate, $z=+h/2$ and $z=-h/2$, respectively, the shear displacement u_s and v_s can be written as [13]:

$$u_s = z\left(\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right)\frac{\partial w_s}{\partial x}, \qquad v_s = z\left(\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right)\frac{\partial w_s}{\partial y} \tag{4}$$

Each layer is assumed to have orthotropic material property, hence the stress-strain relations in the direction of the principle axes of orthotropy are found to be as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{5}$$

where Q_{ij} are the components of the reduced stiffness matrix and are expressed in terms of material properties of each layer.

$$\begin{aligned} &Q_{11} = \frac{E_1}{1-\upsilon_{12}\upsilon_{21}},\ Q_{22} = \frac{E_2}{1-\upsilon_{12}\upsilon_{21}},\ Q_{12} = \upsilon_{12}Q_{22}, \\ &Q_{13} = Q_{23} = Q_{33} = 0, Q_{44} = G_{23},\ Q_{55} = G_{13}\ \ and\ \ Q_{66} = G_{12}. \end{aligned} \tag{6}$$

Equation (5) represents the stress-strain relations in an especially orthotropic material, where the principle axes of orthotropy are parallel to the geometric axes of the plate (x, y), i.e. the direction of application of the load.

In order to define the stress-strain relations in the geometrical coordinate system of the plate, that is the global Cartesian coordinate system, the components of the reduced stiffness tensor should be transformed according to the transformation law of fourth order tensors. Hence, the stress-strain relations in the global coordinate system are:

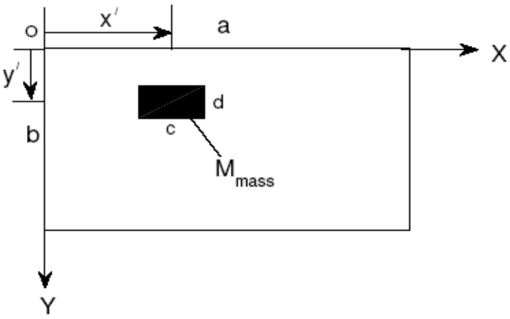

Figure 1. A rectangular plate with a localized patch mass

$$\begin{Bmatrix} \sigma^{(k)}_x \\ \sigma^{(k)}_y \\ \tau^{(k)}_{xy} \\ \tau^{(k)}_{yz} \\ \tau^{(k)}_{xz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon^{(k)}_x \\ \varepsilon^{(k)}_y \\ \gamma^{(k)}_{xy} \\ \gamma^{(k)}_{yz} \\ \gamma^{(k)}_{xz} \end{Bmatrix} \tag{7}$$

where k indicates the layer number and $\overline{Q}_{ij}$ are the material constants of the *k*th lamina in the laminate coordinate system.

In order to obtain the equations of motion by the Hamilton principle, the strain energy and the kinetic energy of the plate are first defined. The definition of the strain energy is as follows:

$$U_{plate} = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}\,dV \tag{8}$$

The strain energy of the plate can be written as [13]:

$$\begin{aligned} U_{plate} = \frac{1}{2}\int_{A_{plate}} & N_x\varepsilon_x^0 + N_{xy}\gamma_{xy}^0 + N_y\varepsilon_y^0 + M_x^b\kappa_x^b + M_{xy}^b\kappa_{xy}^b \\ & + M_y^b\kappa_y^b + M_x^s\kappa_x^s + M_{xy}^s\kappa_{xy}^s + M_y^s\kappa_y^s + Q_{yz}^s\gamma_{yz}^s + Q_{xz}^s\gamma_{xz}^s \\ & + Q_{yz}^a\gamma_{yz}^a + Q_{xz}^a\gamma_{xz}^a \end{aligned} \tag{9}$$

where $\{N\}$, $\{M\}$ and $\{Q\}$ are the stress resultants of the total N layers of the plate and defined in Appendix.

The total kinetic energy is the summation of the kinetic energy of the plate and the kinetic energy of the uniformly distributed patch mass with dimensions *c* and *d* acting on the top surface of the plate.

$$T = T_{mass} + T_{plate} \tag{10}$$

The kinetic energy of plate is defined as:

$$T_{plate} = \frac{1}{2}\int_V \rho\left(\left(\frac{\partial U}{\partial t}\right)^2 + \left(\frac{\partial V}{\partial t}\right)^2 + \left(\frac{\partial W}{\partial t}\right)^2\right)dxdydz \tag{11}$$

Substituting Equations (1) to (4) into Equation (11), and considering the limits of integration in the plate, the kinetic energy of plate can be written as:

$$\begin{aligned} T_{plate} &= \frac{1}{2}\int_0^b\int_0^a I_0\left[\dot{u}^2 + \dot{v}^2 + \left(\dot{w}_a + \dot{w}_b + \dot{w}_s\right)^2\right]dxdy \\ &+ \frac{1}{2}\int_0^b\int_0^a I_2\left[\left(\frac{\partial \dot{w}_b}{\partial x}\right)^2 + \left(\frac{\partial \dot{w}_b}{\partial y}\right)^2\right]dxdy \\ &+ \frac{1}{2}\int_0^b\int_0^a \frac{I_2}{84}\left[\left(\frac{\partial \dot{w}_s}{\partial x}\right)^2 + \left(\frac{\partial \dot{w}_s}{\partial y}\right)^2\right]dxdy. \end{aligned} \tag{12}$$

where I_0, I_2 are the inertia terms as below:

$$\left(I_0, I_2\right) = \int_{-h/2}^{h/2} \rho\left(1, z^2\right)dz \tag{13}$$

The kinetic energy of the distributed patch mass (M_{mass}) which is located on the top surface of the plate $(z = h/2)$ can be written as:

$$T_{mass} = \frac{1}{2}\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2} \rho_m h_m \left(\begin{aligned} &\left[\dot{u} - \frac{h}{2}\left(\frac{\partial \dot{w}_b}{\partial x}\right) - \frac{h}{12}\left(\frac{\partial \dot{w}_s}{\partial x}\right)\right]^2 + \\ &\left[\dot{v} - \frac{h}{2}\left(\frac{\partial \dot{w}_b}{\partial y}\right) - \frac{h}{12}\left(\frac{\partial \dot{w}_s}{\partial y}\right)\right]^2 + \\ &\left[\dot{w}_a + \dot{w}_b + \dot{w}_s\right]^2 \end{aligned} \right) dydx \tag{14}$$

where ρ_m and h_m are the density of the patch mass and its thickness in the *z*-direction, respectively. It is observed that the ranges of integration for Equations (12) and (14) are different. Hamilton principle is employed to obtain the coefficients of mass and stiffness matrices.

Substituting the displacements field in the relevant strain energy and kinetic energy terms, integrating the results and obtaining their first variation, the equations of motion are found. Substituting Equations (9), (12) and (14) into Equation (15) that called Hamilton principle and carrying out integration by parts, the equations of motion are derived as:

$$\int_{t_1}^{t_2} \delta\left(V + U - T\right)dt = 0 \tag{15}$$

where δ presents a variation with respect to x and y. Here V denotes the work done due to applied loads. Since primary aim is in the free vibration analysis, the energy due to applied forces is zero. Substitution of displacements into Equation (15) and integrating the equation by parts, the equations of motion are obtained as:

$$\begin{aligned} \delta u &\rightarrow \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u} \\ \delta v &\rightarrow \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_0\ddot{v} \end{aligned} \tag{16}$$

$$\delta w^b \rightarrow \frac{\partial^2 M_x^b}{\partial x^2} + \frac{\partial^2 M_y^b}{\partial y^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} = I_0 \ddot{W} - I_2 \nabla^2 \ddot{w}_b$$

$$\delta w^s \rightarrow \frac{\partial^2 M_x^s}{\partial x^2} + \frac{\partial^2 M_y^s}{\partial y^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y}$$

$$= I_0 \ddot{W} - \frac{I_2}{84}\nabla^2 \ddot{w}^s$$

$$\delta w^a \rightarrow \frac{\partial Q_{xz}^a}{\partial x} + \frac{\partial Q_{yz}^a}{\partial y} = I_0 \ddot{W}$$

Finally, by collecting the coefficients of parts, the governing equation of plate vibration is obtained as below:

$$([S] - [M]\omega^2)\{\lambda\} = \{0\} \tag{17}$$

where [S], [M], ω and λ are the stiffness [13], mass matrices, natural frequency and the vector of unknown coefficients, respectively.

For convenience, the non-dimensional natural frequency of plate is defined as [11]:

$$\varpi = \omega \frac{a^2}{h}\sqrt{\frac{\rho}{E_2}} \tag{18}$$

3. PROBLEM DEFINITION

Now, a set of boundary conditions are considered and it is called the SS-2 boundary condition that is applied for an anti-symmetric angle-ply laminate and is defined by . K. Seung-Eock et al. [13].

In order to satisfy the boundary conditions, the following displacement fields are assumed:

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \\ w_a \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos\alpha_m x \sin\beta_n y \\ V_{mn} \sin\alpha_m x \cos\beta_n y \\ W_{bmn} \sin\alpha_m x \sin\beta_n y \\ W_{smn} \sin\alpha_m x \sin\beta_n y \\ W_{amn} \sin\alpha_m x \sin\beta_n y \end{Bmatrix} \tag{20}$$

where $\alpha = m\pi/a$, $\beta = n\pi/b$ and U_{mn}, V_{mn}, W_{bmn}, W_{smn}, W_{amn} are coefficients.

4. RESULTS AND DISCUSSION

Three sets of dimensionless material properties are considered:

MAT1:

$$E_1/E_2 = 40, G_{12}/E_2 = 0.6, G_{23}/E_2 = G_{13}/E_2 = 0.5, \nu_{12} = 0.25$$

MAT2:

$$E_1/E_2 = open, G_{23}/E_2 = 0.4, G_{12}/E_2 = G_{13}/E_2 = 0.6, \nu_{12} = 0.3$$

MAT3:

$$E_1/E_2 = open, G_{23}/E_2 = 0.2, G_{12}/E_2 = G_{13}/E_2 = 0.5, \nu_{12} = 0.25$$

At first, the effect of the distributed patch mass is not considered and vibration response of a plate without the patch mass is studied. The results are then compared with the results obtained using the second order shear deformation theory for angle-ply and cross-ply square laminated plates as reported by Khdeir and Reddy [7]. The results obtained with a/h as a parameter for four layers anti-symmetric angle-ply, MAT1 with $G_{13} = 0.6E_2$, $[45/-45]_2$ and two layer cross-ply $(0/90)$, MAT3 are presented in Table 1. Comparison of the results indicates that the results obtained using the present method are lower than those of the second order shear deformation theory. This difference reduces as the ratio a/h increases. Table 2 shows the effect of aspect ratios on the natural frequencies of a four-layer composite plate. The accuracy of the RPT2 is presented with comparing of well known published results. In order to evaluate the accuracy of the RPT2, the % error is calculated as:

$$\%\ error = 100 \times \left(\frac{value\ \ obtained\ \ by\ the\ RPT\ 2}{Correspond\ \ ing\ value\ by\ other\ theory} - 1 \right) \tag{21}$$

The % error in values of the non-dimensional first natural frequencies for a/h=10 and a/h=50 is shown in Figure 2. It is observed from Figure 2 that the % error of the RPT2 with the YNS and HSDT has the same results (when $a \approx 2b$ and a/h=10). The ranges of the % error with corresponding values of the FEM, YNS, HSDT and TSDT are inside the intervals [0.1664 3.4017], [0.6591 2.7373], [0.0268 1.8973] and [0.0002 1.6531], respectively. It is interesting to know that the % error increases with increasing the a/h ratio for the FEM and also decreases for YNS, HSDT and TSDT.

Now, a distributed patch mass, M_{mass}, at the centre of the plate $(x' = a/2, y' = b/2)$ with the following properties is considered:

$M_{mass}/M_{plate} = 0.5,\ c/a = d/b = 0.4 \rightarrow$

$\rho_m h_m cd/\rho hab = 0.5 \rightarrow\ \rho_m h_m = 3.125\rho h = 3.125 I_0$.

With above definition of patch mass properties, the kinetic energy of mass is obtained as below:

$$\begin{aligned} T_{mass} &= \frac{1}{2}\int_{0.3a}^{0.7a}\int_{0.3b}^{0.7b} 3.1250 I_0 \left(\dot{u}^2 + \dot{v}^2 + \left(\dot{w}_a + \dot{w}_b + \dot{w}_s \right)^2 \right) dxdy \\ &+ \frac{1}{2}\int_{0.3a}^{0.7a}\int_{0.3b}^{0.7b} 9.3750 I_2 \left[\left(\frac{\partial \dot{w}_b}{\partial x} \right)^2 + \left(\frac{\partial \dot{w}_b}{\partial y} \right)^2 \right] dxdy \\ &+ \frac{1}{2}\int_{0.3a}^{0.7a}\int_{0.3b}^{0.7b} 0.2604 I_2 \left[\left(\frac{\partial \dot{w}_s}{\partial x} \right)^2 + \left(\frac{\partial \dot{w}_s}{\partial y} \right)^2 \right] dxdy \end{aligned} \tag{22}$$

Using Equation (14), the arrays of mass matrix are:

$$M_{11} = 1.2136 I_0,\quad M_{22} = 1.2136 I_0,\quad M_{34} = 2.5436 I_0 \tag{23}$$

$M_{35} = 2.5436 I_0$, $M_{45} = 2.5436 I_0$,
$M_{55} = 2.5436 I_0$

$$M_{33} = 2.5436 I_0 + 1.65 I_2 ((\pi/a)^2 + (\pi/b)^2)$$

$$M_{44} = 2.5436 I_0 + I_2 (\frac{1}{84} + 0.167)((\pi/a)^2 + (\pi/b)^2)$$

The effect of the distributed patch mass on vibration response of isotropic plate is studied.

Then, the results are compared with the results obtained using the three order shear deformation theory for angle-ply laminates as reported by Alibegloo [11]. The results obtained with (a/h) as a parameter are presented in Table 3. In Figure 3, the results obtained by the present study (RPT2) are compared with the results obtained by (TSDT) method [11]. It is considered the same position of patch mass for anisotropic plate. In Table 4, the effect of anisotropy on free vibration of rectangular different laminated plates with patch mass at the centre of the plate for MAT2 is presented (E_1/E_2 aspect ratio is also variable). As shown by Table 4, the results of FNF for anisotropic plate $[\alpha/\alpha/-\alpha/\alpha]$ are greater than the results for anisotropic plate $[\alpha/\alpha/\alpha/-\alpha]$ and also isotropic plate $[\alpha/-\alpha/\alpha/-\alpha]$ with patch mass. In the next step, the distributed patch mass in three different positions of the plate is considered and the results of FNF for anisotropic different laminated plates are compared together in Table 5. These three positions of the distributed patch mass are assumed as:

$x'_1 = a/4, y'_1 = b/2$, 2) $x'_2 = a/2, y'_2 = b/4$, 3) $x'_3 = a/6, y'_3 = b/5$

and properties of the distributed patch mass are considered as $M_{mass}/M_{plate} = 0.3, c/a = d/b = 0.2$.

The arrays of mass matrix for these three positions are defined in Appendix. Table 5 shows the effect of a/h and a/b ratios on free vibration of four-layer anisotropic plates with patch mass. Increasing of the foregoing ratios leads to increase of the non-dimensional first natural frequencies. On the other hand, the effects of these ratios on anisotropic laminated plate $[\alpha/\alpha/-\alpha/\alpha]$ are greater than anisotropic laminated plate $[\alpha/\alpha/\alpha/-\alpha]$. Due to symmetry imposed by the boundary conditions of the plate, it is observed that patch mass in position 1 and position 2 would have similar natural frequencies of vibration.

5. CONCLUSION

In this study, the two variable refined plate theory for vibration study of laminated composite isotropic and anisotropic plates with patch mass was developed. Firstly, the governing equation of rectangular plate vibration with a patch mass was obtained by this theory. On the other hand, the effect of various parameters such as size and location of distributed patch mass, different aspect ratios and different types of anisotropy on the natural frequency of plate vibration was studied. To illustrate the accuracy of RPT2, the results obtained by this theory were compared with the results obtained by the well known theories. The main conclusions are as follows:

1) The RPT2 is variationally consistent and number of unknown functions involved in this theory is only two.
2) As seen in section 4, the RPT2 has more consistency with TSDT compared to FEM, YNS and HSDT.
3) The % error of the RPT2 in frequency value approaches to zero in respect of FEM from $a/b \succ 1$ to $a \approx 2b$ at $a/h = 10$.
4) The % error of the RPT2 in frequency value approaches to zero in respect of HSDT and TSDT at $a/b = 1$ and $a/h = 10$.
5) Increasing the ratio of *a/h* can cause to increase the % error of RPT2 with FEM and decrease the % error for YNS, HSDT and TSDT.
6) Considering the large values of *a/h* ratio, the RPT2 has more consistency with YNS, HSDT and TSDT than FEM for rectangular composite plates.
7) Amount of patch mass and its location leads to change fundamental frequencies.
8) The lowest natural frequency of plate with symmetric boundary conditions occurs with the patch mass at the centre of the plate. The natural frequency increases with the patch mass moving towards the corner of the plate.
9) The influences of aspect ratios on free vibration of anisotropic plates $[\alpha/\alpha/-\alpha/\alpha]$ are greater than anisotropic plates $[\alpha/\alpha/\alpha/-\alpha]$ and also isotropic plates $[\alpha/-\alpha/\alpha/-\alpha]$

TABLE 1. The non-dimensional first natural frequency

	a/h				
Theory/Laminate	**5**	**10**	**20**	**50**	**100**
SSDT / Angle ply[7]	12.928	18.665	21.954	23.252	23.458
RPT2 / Angle ply	12.534	18.320	21.803	23.223	23.451
SSDT / Cross ply[7]	7.609	8.997	9.504	9.665	-
RPT2 / Cross ply	7.545	8.963	9.494	9.663	9.688

TABLE 2. The non-dimensional first natural frequency, MAT1, $[45/-45]_2$

		a/b						
a/h	**Sources**	**0.2**	**0.6**	**0.8**	**1**	**1.2**	**1.6**	**2**
10	Reddy [5]	8.7240	12.9650	15.7120	18.6090	21.5670	27.7360	34.2470
	Bert [3]	8.6640	12.8200	15.5400	18.4600	21.5100	27.9500	34.8700
	Shankara [4]	8.5557	12.5588	15.1802	17.9735	20.8797	26.9916	33.5534
	Alibeigloo [11]	8.5587	12.5646	15.1873	17.9829	20.8947	27.0306	33.6340
	Present	8.6060	12.6424	15.2101	17.9784	20.9214	27.2818	34.1900
20	Reddy [5]	9.4750	14.8960	18.5570	22.5840	26.8570	36.2490	46.7890
	Bert [3]	9.3000	14.4500	17.9700	21.8700	26.1200	35.5600	46.2600
	Shankara [4]	9.3011	14.3856	17.8458	21.6808	25.8363	35.0421	45.4096
	Alibeigloo [11]	9.2661	14.3594	17.8217	21.6588	25.8174	35.0365	45.4305
	Present	9.2825	14.3887	17.8302	21.6552	25.8278	35.1772	45.8175
30	Reddy [5]	9.6670	15.3850	19.3040	23.6760	28.3810	38.9400	51.1320
	Bert [3]	9.4360	14.8400	18.5600	22.7400	27.3500	37.8200	49.9800
	Shankara [4]	9.4880	14.8427	18.5390	22.6911	27.2555	37.5907	49.5474
	Alibeigloo [11]	9.4196	14.7896	18.4866	22.6371	27.1995	37.5341	49.4992
	Present	9.4274	14.8038	18.4908	22.6352	27.2047	37.6112	49.7263
40	Reddy [5]	9.7590	15.8530	19.6040	24.1180	29.0030	40.0710	53.0120
	Bert [3]	9.4850	14.9800	18.7800	23.0800	27.8300	38.7200	51.5200
	Shankara [4]	9.5724	15.0248	18.8134	23.0940	27.8286	38.6523	51.3324
	Alibeigloo [11]	9.4754	14.9500	18.7384	23.0137	27.7409	38.5499	51.2217
	Present	9.4799	14.9583	18.7408	23.0125	27.7439	38.5969	51.3642
50	Reddy [5]	9.8160	15.6890	19.7590	24.3430	29.3210	40.6530	53.9890
	Bert [3]	9.5070	15.0400	18.8900	23.2400	28.0600	39.1700	52.2900
	Shankara [4]	9.6216	15.1177	18.9510	23.2956	28.1168	39.1932	52.2539
	Alibeigloo [11]	9.5016	15.0261	18.8586	23.1948	28.0031	39.0503	52.0860
	Present	9.5045	15.0315	18.8602	23.1940	28.0051	39.0815	52.1822

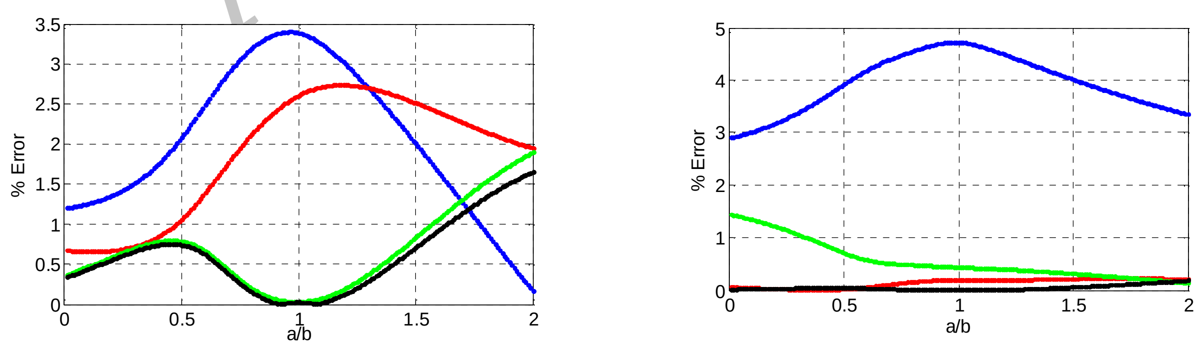

Figure 2. The % error of the RPT2 in non-dimensional first natural frequencies with corresponding value by FEM [5] (*blue*), corresponding value by YNS [3] (*red*), corresponding value by HSDT [4] (*green*) and corresponding value by TSDT [11] (*black*) (Left, a/h=10 and Right, a/h=50)

TABLE 3. The non-dimensional first natural frequency, MAT1, $[45/-45]_2$

		a/h				
a/b	**Theory**	**10**	**20**	**30**	**40**	**50**
0.2	TSDT [11]	5.3728	5.8127	5.9078	5.9423	5.9585
0.2	RPT2	5.4030	5.8227	5.8488	5.9122	5.9446
0.4	TSDT [11]	6.4359	7.1545	7.3177	7.3777	7.4060
0.4	RPT2	6.5018	7.1774	7.3283	7.3835	7.4095
0.6	TSDT [11]	7.8768	9.0055	9.2748	9.3751	9.4227
0.6	RPT2	7.9366	9.0264	9.2845	9.3803	9.4258
0.8	TSDT [11]	9.5102	11.1744	11.5923	11.7503	11.8256
0.8	RPT2	9.5487	11.1861	11.5973	11.7527	11.8269
1	TSDT [11]	11.2448	13.5765	14.1934	14.4304	14.5442
1	RPT2	11.2868	13.5870	14.1974	14.4321	14.5448
1.2	TSDT [11]	13.0434	16.1777	17.0518	17.3932	17.5585
1.2	RPT2	13.1349	16.2067	17.0646	17.4000	17.5624
1.4	TSDT [11]	14.8956	18.9654	20.1612	20.6369	20.8689
1.4	RPT2	15.0849	19.0383	20.1967	20.6571	20.8816
1.6	TSDT [11]	16.8026	21.9349	23.5225	24.1660	24.4826
1.6	RPT2	17.1292	22.0784	23.5961	24.2092	24.5103
1.8	TSDT [11]	18.7684	25.0836	27.1374	27.9863	28.4075
1.8	RPT2	19.2592	25.3224	27.2646	28.0625	28.4573
2	TSDT [11]	20.7951	28.4084	31.0067	32.1021	32.6506
2	RPT2	21.4663	28.7637	31.2024	32.2214	32.7294

TABLE 4. The non-dimensional first natural frequency, MAT2, $\alpha = 45$, $a/h = 10$, $b/a = 5$

		E_1/E_2					
Lamination		**3**	**10**	**20**	**30**	**40**	**50**
$[\alpha/\alpha/\alpha/-\alpha]$	Anisotropic	2.3777	3.1280	3.8942	4.4884	4.9798	5.4003
$[\alpha/\alpha/-\alpha/\alpha]$	Anisotropic	2.4509	3.4579	4.4488	5.1890	5.7836	6.2802
$[\alpha/-\alpha/\alpha/-\alpha]$	Isotropic	2.4170	3.3041	4.1924	4.8643	5.4082	5.8650

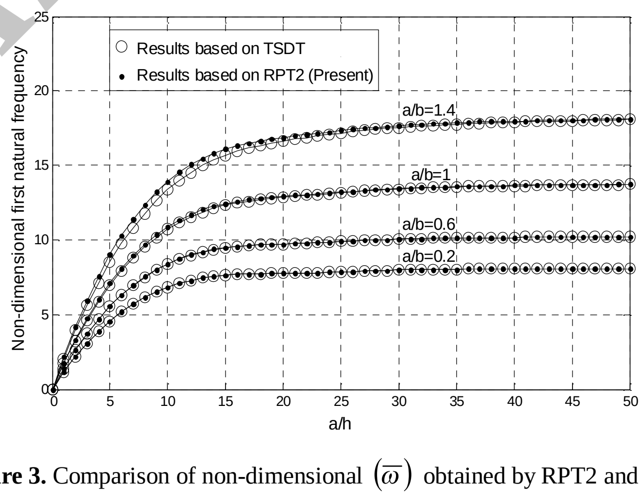

Figure 3. Comparison of non-dimensional $(\overline{\omega})$ obtained by RPT2 and TSDT

TABLE 5. The non-dimensional first natural frequency, MAT1, $[45/-45]_2$

a/b	Lamination	a/h 10	20	30	40	50
Position 1						
0.2	$[\alpha/\alpha/\alpha/-\alpha]$	5.1686	5.4812	5.5464	5.5699	5.5808
	$[\alpha/\alpha/-\alpha/\alpha]$	5.9248	6.3796	6.4770	6.5123	6.5288
0.8	$[\alpha/\alpha/\alpha/-\alpha]$	9.6697	10.9556	11.2588	11.3713	11.4247
	$[\alpha/\alpha/-\alpha/\alpha]$	10.5704	12.2834	12.7075	12.8672	12.9432
1.4	$[\alpha/\alpha/\alpha/-\alpha]$	15.6051	18.8574	19.7285	20.0647	20.2264
	$[\alpha/\alpha/-\alpha/\alpha]$	16.7904	20.9722	22.1700	22.6425	22.8722
2	$[\alpha/\alpha/\alpha/-\alpha]$	22.4899	28.7611	30.6523	31.4126	31.7851
	$[\alpha/\alpha/-\alpha/\alpha]$	23.9509	31.7737	34.3236	35.3791	35.9031
Position 2						
0.2	$[\alpha/\alpha/\alpha/-\alpha]$	5.1873	5.4874	5.5493	5.5715	5.5819
	$[\alpha/\alpha/-\alpha/\alpha]$	5.9419	6.3857	6.4799	6.5139	6.5299
0.8	$[\alpha/\alpha/\alpha/-\alpha]$	9.6783	10.9595	11.2608	11.3725	11.4254
	$[\alpha/\alpha/-\alpha/\alpha]$	10.5776	12.2872	12.7095	12.8684	12.9440
1.4	$[\alpha/\alpha/\alpha/-\alpha]$	15.5793	18.8416	19.7198	20.0593	20.2229
	$[\alpha/\alpha/-\alpha/\alpha]$	16.7713	20.9576	22.1614	22.6372	22.8686
2	$[\alpha/\alpha/\alpha/-\alpha]$	22.4069	28.6941	30.6119	31.3870	31.7678
	$[\alpha/\alpha/-\alpha/\alpha]$	23.8950	31.7157	34.2856	35.3542	35.8860
Position 3						
0.2	$[\alpha/\alpha/\alpha/-\alpha]$	6.1358	6.5294	6.6089	6.6372	6.6504
	$[\alpha/\alpha/-\alpha/\alpha]$	7.0136	7.5981	7.7175	7.7602	7.7801
0.8	$[\alpha/\alpha/\alpha/-\alpha]$	11.3638	13.0331	13.4101	13.5479	13.6128
	$[\alpha/\alpha/-\alpha/\alpha]$	12.2959	14.5948	15.1317	15.3289	15.4218
1.4	$[\alpha/\alpha/\alpha/-\alpha]$	18.0563	22.3655	23.4754	23.8945	24.0943
	$[\alpha/\alpha/-\alpha/\alpha]$	19.0972	24.7965	26.3612	26.9582	27.2435
2	$[\alpha/\alpha/\alpha/-\alpha]$	25.5836	33.9495	36.4142	37.3800	37.8470
	$[\alpha/\alpha/-\alpha/\alpha]$	26.6909	37.3022	40.7176	42.0805	42.7428

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APPENDIX

The arrays of Mass matrix

Position 1 $\rightarrow M_{11} = 1.0194 I_0$,

$M_{22} = M_{34} = M_{35} = M_{45} = M_{55} = 1.5820 I_0$

$$M_{33} = 1.5820 I_0 + I_2\left(\left(\pi/a\right)^2 + \left(\pi/b\right)^2\right) + 90 I_2 \begin{pmatrix} 0.0194 \times \left(\pi/a\right)^2 + \\ 6.4511e-4 \times \left(\pi/b\right)^2 \end{pmatrix}$$

$$M_{44} = 1.5820 I_0 + I_2/84\left(\left(\pi/a\right)^2 + \left(\pi/b\right)^2\right) + 6 I_2 \begin{pmatrix} 0.0194 \times \left(\pi/a\right)^2 + \\ 6.4511e-4 \times \left(\pi/b\right)^2 \end{pmatrix}.$$

Position 2 $\rightarrow M_{11} = 1.5820 I_0, M_{22} = 1.0194 I_0$,

$M_{34} = M_{35} = M_{45} = M_{55} = 1.5820 I_0$

$$M_{33} = 1.5820 I_0 + I_2\left(\left(\pi/a\right)^2 + \left(\pi/b\right)^2\right) + 90 I_2 \begin{pmatrix} 0.0194 \times \left(\pi/a\right)^2 + \\ 6.4511e-4 \times \left(\pi/b\right)^2 \end{pmatrix}$$

$$M_{44} = 1.5820 I_0 + I_2/84\left(\left(\pi/a\right)^2 + \left(\pi/b\right)^2\right) + 6 I_2 \begin{pmatrix} 0.0194 \times \left(\pi/a\right)^2 + \\ 6.4511e-4 \times \left(\pi/b\right)^2 \end{pmatrix}.$$

Position 3 $\rightarrow M_{11} = 1.2070 I_0, M_{22} = 1.3120 I_0$,

$M_{34} = M_{35} = M_{55} = 1.1140 I_0, M_{45} = 1.5820 I_0$,

$$M_{33} = 1.1140 I_0 + I_2\left(\left(\pi/a\right)^2 + \left(\pi/b\right)^2\right) + 90 I_2 \begin{pmatrix} 0.0104 \times \left(\pi/a\right)^2 + \\ 0.0069 \times \left(\pi/b\right)^2 \end{pmatrix}$$

$$M_{44} = 1.5820 I_0 + I_2/84\left(\left(\pi/a\right)^2 + \left(\pi/b\right)^2\right) + 2.5 I_2 \begin{pmatrix} 0.0104 \times \left(\pi/a\right)^2 + \\ 0.0069 \times \left(\pi/b\right)^2 \end{pmatrix}.$$

$$\begin{Bmatrix} \left(N_x, N_y, N_{xy}\right) \\ \left(M_x^b, M_y^b, M_{xy}^b\right) \\ \left(M_x^s, M_y^s, M_{xy}^s\right) \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left(\sigma_x, \sigma_y, \sigma_{xy}\right) \begin{Bmatrix} 1 \\ z \\ J \end{Bmatrix} dz, \; J = -z\left(\frac{1}{4} - \frac{5}{3}(z/h)^2\right)$$

$$\begin{Bmatrix} \left(Q_{xz}^a, Q_{yz}^a\right) \\ \left(Q_{xz}^s, Q_{yz}^s\right) \end{Bmatrix} = \sum_{k=1}^{N} \int_{Z_k}^{Z_{k+1}} \left(\sigma_{xz}, \sigma_{yz}\right) \begin{Bmatrix} 1 \\ I \end{Bmatrix} dz, \; I = \frac{5}{4} - 5(z/h)^2$$

$$\left(\gamma_{xz}^a, \gamma_{yz}^a, \gamma_{xz}^s, \gamma_{yz}^s\right) = \left(\partial w_a/\partial x, \partial w_a/\partial y, \partial w_s/\partial x, \partial w_s/\partial y\right)$$

$$\left(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right) = \left(\partial u/\partial x, \partial v/\partial y, \partial u/\partial y + \partial v/\partial x\right)$$

$$\kappa_x^b = -\left(\partial^2 w_b/\partial x^2\right), \kappa_y^b = -\left(\partial^2 w_b/\partial y^2\right), \kappa_{xy}^b = -2\left(\partial^2 w_b/\partial x \partial y\right)$$

$$\kappa_x^s = -\left(\partial^2 w_s/\partial x^2\right), \kappa_y^s = -\left(\partial^2 w_s/\partial y^2\right), \kappa_{xy}^s = -2\left(\partial^2 w_s/\partial x \partial y\right)$$

The Effect of Anisotropy on Free Vibration of Rectangular Composite Plates with Patch Mass

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چکیده

در این مقاله، تأثیر ناهمسانگردی برروی ارتعاشات آزاد صفحات مستطیلی حامل توزیع جرم پیوسته بررسی می‌شود. از تئوری دو متغیر اصلاح شده صفحه برای تعریف میدان جابجایی مرتبه سه صفحات کامپوزیتی مستطیلی استفاده می‌گردد. معادلات حرکت صفحه توسط محاسبه واریاسیون بدست می‌آید. مطالعه پارامتریک فرکانس‌های طبیعی بدون بعد و نیز تأثیرات سایر پارامترهای هندسی از قبیل نسبت ظاهر صفحه، اندازه و مکان جرم پیوسته برروی این فرکانس‌ها بررسی می‌شود. در ابتدا نتایج بدست آمده با نتایج گزارش شده توسط چندین تئوری صفحات مقایسه شده و در مرحله بعد اثر ناهمسانگردی روی ارتعاشات آزاد صفحه برای چندین نوع لایه‌چینی متفاوت مورد تحلیل قرار می‌گیرد. نتایج حاصله نشان دهنده دقت بالای تئوری بکار رفته با نتایج متشر شده در زمینه تحلیل ارتعاشات صفحات بارگذاری شده و بارگذاری نشده می‌باشد.

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