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Fuzzy Reliability Evaluation of a Repairable System with Imperfect Coverage, Reboot and Common-cause Shock Failure

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ABSTRACT

In the present investigation, we deal with the reliability characteristics of a repairable system consisting of two independent operating units, by incorporating the coverage factor. The probability of the successful detection, location and recovery from a failure is known as the coverage probability. The reboot delay and common cause shock failure are also considered. The times to failure of the components, time to failure due to common cause, time to repair and time to reboot are assumed to follow exponential distributions. The Markov model of the system is developed and the system state transition probabilities are determined which are further used to evaluate some reliability indices such as availability and mean time to failure. We use fuzzy logic approach for analyzing the system performance by assuming the trapezoidal membership functions of the system descriptors viz. failure rates and repair rates. The fuzzy mean time to failure and fuzzy availability have been established. A numerical experiment has been performed to validate the analytical results.

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1. INTRODUCTION

With the advancement of technology, handling diverse and critical applications with repairable systems steadily is important in many areas of day-to-day as well as industrial organizations including the manufacturing systems, computer systems, communication systems, transportation systems and power plants, etc. The reliability and availability of the systems, which are measured in terms of system state probabilities, have been identified as a major stumbling block in achieving a high or required level of system performance. In traditional reliability models a system can exist in binary states; the first one is 'up' state in which the system completely works at full capacity and another one is 'down' state in which the system does not work due to failure. The probability of system being in 'up' state is characterized by the reliability while availability is the probability that the system is operating satisfactorily at any time. The reliability refers to the system survival before the first failure while availability refers to the system survival for the repairable system. For the safe operation and assurance of the quality of the components of the system in the sense that they perform their work perfectly, a repairable system should be highly reliable.

The reliability analysis of repairable system with two independent operating units is investigated in this paper. The concepts of coverage factor, reboot and common cause shock failure are also taken into consideration. Using the trapezoidal fuzzy numbers, the failure rates and repair rates of both units have been fuzzyfied. The membership functions of availability and mean time to failure have been obtained with the help of these fuzzy fied parameters.

The remaining chapter is structured as follows. The related literature to work has been given in section 2. In section 3, we describe model and introduce the reliability and availability of the system. Section 4 presents a brief introduction of fuzzy set theory. The modeling of repairable system in extended to the fuzzy environment in section 5. In section 6, a numerical example is facilitated to explore the computational tractability. Finally, conclusions are drawn in section 7.

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2. RELATED WORK

If in a system, the failures are not successfully detected, located and recovered; then, this situation is called *imperfect coverage*. The faults, which are not covered, belong to the uncovered fault class with the probability '1-C'. A lot of work has been done in this field by many researchers. Pham [1] examined a high voltage system with imperfect coverage in which the failure rate of the fault coverage was a constant. Akhtar [2] analyzed the reliability of K-out-of-n: G system with imperfect fault coverage. Moustafa [3] studied a K-out-of-N system with imperfect coverage. Singh and Jain [4] evaluated the reliability of repairable multi-component redundant system. Trivedi [5] considered the concept of detection and imperfect coverage and their effect on the repairable systems. Myers [6] studied the reliability of a K-out-of-n: G system with imperfect fault coverage. Ke et al. [7] used Bayesian approach to predict the performance measures of a repairable system with detection, imperfect coverage and reboot. Hsu et al. [8] extended this model using asymptotic estimation. Simulation infers with an availability system with general repair distribution. Imperfect fault coverage has been studied by Ke et al. [9]. Liu [10] discussed the availability behaviour of a repairable system in which standby switched to primary is subjected to breakdowns. Wang et al. [11] compared the availability of two systems with warm standby units under the assumption that the coverage factor of the active-unit failure is different from that of the standby-unit failure.

In the real world problems, the system parameters are often imprecise due to incomplete and accurate information. In conventional reliability models, the probabilistic approach seems to be inadequate due to built-in uncertainties in data; therefore the theory of fuzzy reliability can play important role to tackle these types of difficulties. In fuzzy theory, the grade of a membership function indicates a subjective degree of preference of a decision maker with a given tolerance. The concept of fuzzy reliability has drawn the attention of many researchers. Cai et al. [12], Wu [13] and Jiang and Chen [14] studied the fuzzy system reliability. Some basic concepts of fuzzy set theory and their applications have been discussed in detail by Klir et al. [15]. Huang et al. [16] did a fuzzy analysis for steady-state availability. Ke et al. [17] analyzed a redundant repairable system with imperfect coverage and fuzzy parameter. Reliability optimization of a series-parallel system with fuzzy random lifetimes has been done by Wang and Watada [18]. Sharma and Pandey [19] considered fuzzy reliability and fuzzy availability of a three unit degraded systems. The reliability analysis of competitive failure processes under fuzzy degradation data has been done by Wang et al. [20]. Wang et al. [21] discussed an approach to predict the system reliability analysis with fuzzy random variables to represent uncertainties.

For a system working in different environments, the common cause shock failure is an important factor that should be incorporated to predict the reliability and the availability of the system. In multi-component systems, the operating units of the system may fail due to individual failure or due to common cause shock failure. Subramanian and Anantharaman [22] did the reliability analysis of a complex standby redundant system. Jain [23] analyzed the reliability of a two unit system with common cause shock failures. An availability analysis for the improvement of active/standby cluster systems using software rejuvenation has been done by Park and Kim [24]. In 2003, Vaurio [25] evaluated the common cause failure probabilities in the standby safety system using fault tree analysis with testing-scheme and timing dependencies. Vaurio [26] described the uncertainties and quantification of common cause failure rates and probabilities for the system characterization. The reliability evaluation of standby safety systems due to independent and common cause failures has been done by Lu and Lewis [27]. Xing et al. [28] did the reliability analysis of hierarchical computer based systems subject to common cause failures. The reliability of two non-identical units system with common cause shocks failure and state dependent rates has been discussed by Jain and Mishra [29]. Shen et al. [30] explored exponential asymptotic property of a parallel repairable system with common cause failure. Li et al. [31] analyzed a warm standby system with components having proportional hazard rates. Li et al. [32] did heterogeneous redundancy optimization for multi-state series–parallel systems subject to common cause failures. Ram and Singh [33] presented a mathematical model of a complex system that can fail in n-mutually exclusive ways of total failure or due to common cause failure. Distefano et al. [34] investigated dynamic reliability and availability through state-space models by considering common cause failure and load sharing.

3. MODEL DESCRIPTION

A repairable two dissimilar component system with imperfect coverage, reboot and common cause shock failure is considered.

The assumptions made for the formulation of Markov model are as follows:

Both the operating units may fail independently. The life times of the first and second units follow exponential distribution with parameters λ_1 and λ_2.

The system can also fail due to common-cause shock failure with parameter λ_C.

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- As soon as an operating unit fails, it is instantaneously detected and sent for repair.
- The operating units can be successfully recovered with probability C .
- The system needs some time for the recovery of operating units; the recovery time of operating units is exponentially distributed with parameter θ .
- In the case of unsuccessful recovery of a failed unit, the system needs to be rebooted. The reboot time is exponentially distributed with the mean $1/\beta$.
- The repair times of both the operating units are exponentially distributed with parameters μ_1 and μ_2 , respectively.
- After the failure of the second unit, the reboot or recovery cannot be performed.

The system may be in any one of the following states, at time't'.

$$\xi(t)=\begin{cases}2, & \text{Both the units works properly}\\ 1, & \text{One of the operating units is failed}\\ 0, & \text{Both the units are failed}\\ RC, & \text{Recovery takes place}\\ RB, & \text{Due to unsuccessful recovery system is rebooted}\end{cases}$$

Some more notations used in model development are as follows:

$P_n(t)$: Probability of the system being in n^{th} state (n=2, 1, 0, RB, RC) at time t

P_n: Steady state probabilities of n^{th} state (n=0, 1, 2, RC, RB)

$\tilde{P}_n(s)$: Laplace transform of $P_n(t)$ i.e.

$$\tilde{P}_n(s)=\int_0^{\infty} e^{-st}P_n(t)dt$$

3. THE RELIABILITY ANALYSIS

In this section, the reliability indices such as availability and mean time to failure are obtained for both steady and transient states of the system.

3. 1. The Reliability Function and Mean Time to Failure (MTTF) The system is initially in state '2'. When a unit fails, it is immediately detected, located and recovered with the coverage factor 'C' in state 'RB'. The recovery takes a brief time period with rate 'θ' and the system enters in state '1'. In state 1, the failed unit is repaired with repair rate 'μ_1'. If the system does not recover successfully, it goes in state RB. After the failure of second unit or due to common-cause shock failure, the system reaches in the failed state '0'. The state transition diagram, which depicts all the states, is shown in Figure 1.

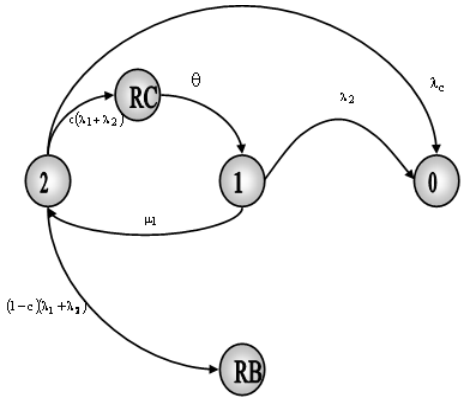

Figure 1. State transition diagram

The transient state equations governing the model are constructed as follows:

$$\frac{dP_2(t)}{dt}=-(\lambda_1+\lambda_2+\lambda_c)P_2(t)+\mu_1P_1(t) \quad (1.a)$$

$$\frac{dP_1(t)}{dt}=-(\lambda_2+\mu_1)P_1(t)+\theta P_{RC}(t) \quad (1.b)$$

$$\frac{dP_0(t)}{dt}=\lambda_2P_1(t)+\lambda_cP_2(t) \quad (1.c)$$

$$\frac{dP_{RC}(t)}{dt}=-\theta P_{RC}(t)+C(\lambda_1+\lambda_2)P_2(t) \quad (1.d)$$

$$\frac{dP_{RB}(t)}{dt}=(1-C)(\lambda_1+\lambda_2)P_2(t) \quad (1.e)$$

Let initially both units of the system be in operating state so that $P_2(0)=1$, $P_1(0)=0$, $P_0(0)=0$, $P_{RB}(0)=0$ and $P_{RC}(0)=0$.

After taking the Laplace transform of Equations 1(a-e), we obtain

$$s\tilde{P}_2(s)-1=(\lambda_1+\lambda_2+\lambda_c)\tilde{P}_2(s)+\mu_1\tilde{P}_1(s) \quad (2.a)$$

$$s\tilde{P}_1(s)=-(\lambda_2+\mu_1)\tilde{P}_1(s)+\theta\tilde{P}_{RC}(s) \quad (2.b)$$

$$s\tilde{P}_0(s)=\lambda_2\tilde{P}_1(s)+\lambda_2\tilde{P}_2(s) \quad (2.c)$$

$$s\tilde{P}_{RC}(s)=-\theta\tilde{P}_{RC}(s)+C(\lambda_1+\lambda_2)\tilde{P}_2(s) \quad (2.d)$$

$$s\tilde{P}_{RB}(s)=(1-C)(\lambda_1+\lambda_2)\tilde{P}_2(s) \quad (2.e)$$

On solving the Equations (2. a-e), we obtain Equations (3. a-e).

The Laplace transform of reliability function of the system is given by

$$\tilde{R}(s)=1-\tilde{P}_0(s)-\tilde{P}_{RB}(s)=\tilde{P}_1(s)+\tilde{P}_2(s)+\tilde{P}_{RC}(s) \quad (4)$$

The mean time to system failure (MTTF) is obtained using

$$MTTF=\lim_{s\to 0}\tilde{R}(s)=\lim_{s\to 0}\left[\tilde{P}_2(s)+\tilde{P}_1(s)+\tilde{P}_{RC}(s)\right]$$

$$=\frac{(\lambda_2+\mu_1)\left[(C(\lambda_1+\lambda_2)+\theta)+\theta C(\lambda_1+\lambda_2)\right]}{\theta\left[(\lambda_2+\mu_1)(\lambda_1+\lambda_2+\lambda_C)-\mu_1C(\lambda_1+\lambda_2)\right]} \quad (5)$$

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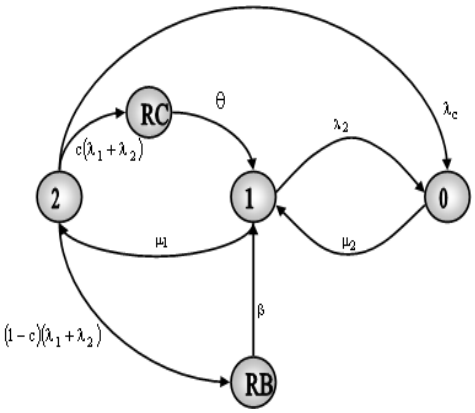

Figure 2. State transition diagram for the steady state availability

3. 2. The Steady-state Availability If recovery fails, the system goes in state 'RB'. The system needs to be rebooted and reboot of the system takes some time which is exponentially distributed with parameter β.

After rebooting, the system reaches in state '1'. Due to failure of second unit and common-cause shock failure, the system enters in state '0'. At this state, the failed unit is repaired with the repair rate 'μ_2'. In Figure 2, the state transition diagram depicting system states to evaluate steady-state availability is shown. The steady-state equations of the system are given by:

$$(\lambda_1 + \lambda_2 + \lambda_c)P_2 = \mu_1 P_1 \quad (6.a)$$

$$\theta P_{RC} = C(\lambda_1 + \lambda_2)P_2 \quad (6.b)$$

$$(\lambda_2 + \mu_1)P_1 = \theta P_{RC} + \mu_2 P_0 + \beta P_{RB} \quad (6.c)$$

$$\beta P_{RB} = (1 - C)(\lambda_1 + \lambda_2)P_2 \quad (6.d)$$

$$\mu_2 P_0 = \lambda_2 P_1 + \lambda_c P_2 \quad (6.e)$$

On solving equations (6. a-e), we find expressions for steady state probabilities as Equations (7. a-e)

The system availability is obtained using Equation (8) where, $U = \lambda_1 + \lambda_2 + \lambda_c$.

$$\tilde{P}_2(s) = \frac{(s+\theta)(s+\lambda_2+\mu_1)}{\left[(s+\theta)(s+\lambda_2+\mu_1)(s+\lambda_1+\lambda_2+\lambda_c) - \mu_1\theta C(\lambda_1+\lambda_2)\right]} \quad (3.a)$$

$$\tilde{P}_1(s) = \frac{\theta(\lambda_1+\lambda_2)}{\left[(s+\theta)(s+\lambda_2+\mu_1)(s+\lambda_1+\lambda_2+\lambda_c) - \mu_1\theta C(\lambda_1+\lambda_2)\right]} \quad (3.b)$$

$$\tilde{P}_0(s) = \frac{\lambda_2\theta C(\lambda_1+\lambda_2)}{\left[(s+\theta)(s+\lambda_2+\mu_1)(s+\lambda_1+\lambda_2+\lambda_c) - \mu_1\theta C(\lambda_1+\lambda_2)\right]} \quad (3.c)$$

$$\tilde{P}_{RC}(s) = \frac{C(\lambda_1+\lambda_2)(s+\lambda_2+\mu_1)}{\left[(s+\theta)(s+\lambda_2+\mu_1)(s+\lambda_1+\lambda_2+\lambda_c) - \mu_1\theta C(\lambda_1+\lambda_2)\right]} \quad (3.d)$$

$$\tilde{P}_{RB}(s) = \frac{(1-C)(\lambda_1+\lambda_2)(s+\theta)(s+\lambda_2+\mu_1)}{\left[(s+\theta)(s+\lambda_2+\mu_1)(s+\lambda_1+\lambda_2+\lambda_c) - \mu_1\theta C(\lambda_1+\lambda_2)\right]} \quad (3.e)$$

$$P_2 = \frac{\theta\mu_1\mu_2\beta}{\mu_2\beta\left[U\theta + \mu_1 C(\lambda_1+\lambda_2) + \mu_1\theta\right] + \theta\left[\beta\lambda_2 U + \mu_1\lambda_c + \mu_1\mu_2(1-C)(\lambda_1+\lambda_2)\right]} \quad (7.a)$$

$$P_1 = \frac{U\mu_2\theta\beta}{\mu_2\beta\left[U\theta + \mu_1 C(\lambda_1+\lambda_2) + \mu_1\theta\right] + \theta\left[\beta\lambda_2 U + \mu_1\lambda_c + \mu_1\mu_2(1-C)(\lambda_1+\lambda_2)\right]} \quad (7.b)$$

$$P_0 = \frac{(\lambda_2 U + \mu_1\lambda_c)\theta\beta}{\mu_2\beta\left[U\theta + \mu_1 C(\lambda_1+\lambda_2) + \mu_1\theta\right] + \theta\left[\beta\lambda_2 U + \mu_1\lambda_c + \mu_1\mu_2(1-C)(\lambda_1+\lambda_2)\right]} \quad (7.c)$$

$$P_{RC} = \frac{\mu_1\mu_2\beta C(\lambda_1+\lambda_2)}{\mu_2\beta\left[U\theta + \mu_1 C(\lambda_1+\lambda_2) + \mu_1\theta\right] + \theta\left[\beta\lambda_2 U + \mu_1\lambda_c + \mu_1\mu_2(1-C)(\lambda_1+\lambda_2)\right]} \quad (7.d)$$

$$P_{RC} = \frac{\mu_1\mu_2\theta(1-C)(\lambda_1+\lambda_2)}{\mu_2\beta\left[U\theta + \mu_1 C(\lambda_1+\lambda_2) + \mu_1\theta\right] + \theta\left[\beta\lambda_2 U + \mu_1\lambda_c + \mu_1\mu_2(1-C)(\lambda_1+\lambda_2)\right]} \quad (7.e)$$

$$A(\infty) = 1(P_{RB} + P_0) = \frac{\mu_2\beta\left[U\theta + \mu_1 C(\lambda_1+\lambda_2) + \mu_1\theta\right]}{\mu_2\beta\left[U\theta + \mu_1 C(\lambda_1+\lambda_2) + \mu_1\theta\right] + \theta\left[\beta\lambda_2 U + \mu_1\lambda_c + \mu_1\mu_2(1-C)(\lambda_1+\lambda_2)\right]} \quad (8)$$

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$$A=\frac{y_2\beta\left[(x_1+x_2+\lambda_c)\theta+y_1C(x_1+x_2)+y_1\theta\right]}{y_2\beta\left[(x_1+x_2+\lambda_c)\theta+y_1C(x_1+x_2)+y_1\theta\right]+\theta\left[\beta x_2(x_1+x_2+\lambda_c)+x_1\lambda_c+y_1y_2(1-C)(x_1+x_2)\right]} \quad (11.b)$$

5. FUZZY MEMBERSHIP FUNCTIONS

We have fuzzified the system parameters for extending the applicability of the system. Let us assume that the failure rates λ_1 and λ_2, repair rates μ_1 and μ_2 are fuzzified. Then, these can be represented by the fuzzy numbers denoted by $\tilde{\lambda}_1,\tilde{\lambda}_2,\tilde{\mu}_1$ and $\tilde{\mu}_2$, respectively. Let X_1,X_2,Y_1 and Y_2 are the crisp universal sets of λ_1, λ_2, μ_1 and μ_2, respectively.

Let $A_{\tilde{\lambda}_1}(x_1)$, $A_{\tilde{\lambda}_2}(x_2)$, $A_{\tilde{\mu}_1}(y_1)$ and $A_{\tilde{\mu}_2}(y_2)$ denote the membership functions of $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\mu}_1$ and $\tilde{\mu}_2$, respectively. Then, we have

$$\tilde{\lambda}_1=\left\{\left(x_1,A_{\tilde{\lambda}_1}(x_1)\right)\middle|\, x_1\in X_1\right\}$$

$$\tilde{\lambda}_2=\left\{\left(x_2,A_{\tilde{\lambda}_2}(x_2)\right)\middle|\, x_2\in X_2\right\}$$

$$\tilde{\mu}_1=\left\{\left(y_1,A_{\tilde{\mu}_1}(y_1)\right)\middle|\, y_1\in Y_1\right\}$$

$$\tilde{\mu}_2=\left\{\left(y_2,A_{\tilde{\mu}_2}(y_2)\right)\middle|\, y_2\in Y_2\right\}$$

Let $P(x_1,x_2,y_1,y_2)$ denote the system characteristics of interest (i.e. MTTF or availability). Since $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are fuzzy numbers, $\tilde{P}(\tilde{\lambda}_1,\tilde{\lambda}_2,\tilde{\mu}_1,\tilde{\mu}_2)$ is also a fuzzy number. Now, according to Zadeh's extension principle, the membership functions of these performance measures $\tilde{P}(\tilde{\lambda}_1,\tilde{\lambda}_2,\tilde{\mu}_1,\tilde{\mu}_2)$ is defined as:

$$A_{\tilde{P}(\tilde{\lambda}_1,\tilde{\lambda}_2,\tilde{\mu}_1,\tilde{\mu}_2)}(v)=\sup_{\omega}\min\left\{\begin{array}{l|l}A_{\tilde{\lambda}_1}(x_1),A_{\tilde{\lambda}_2}(x_2), & \\ A_{\tilde{\mu}_1}(y_1), & \\ A_{\tilde{\mu}_2}(y_2) & v=P(x_1,x_2,y_1,y_2)\end{array}\right\} \quad (9)$$

where, $\omega=x_1\in X_1,x_2\in X_2,y_1\in Y_1,y_2\in Y$,

On fuzzyfing the parameters in the Equation (5), we obtain the membership function of MTTF as given below:

$$A_{\tilde{P}(\tilde{\lambda}_1,\tilde{\lambda}_2,\tilde{\mu}_1,\tilde{\mu}_2)}(v)=\sup_{\omega}\min\left\{\begin{array}{l}A_{\tilde{\lambda}_1}(x_1),A_{\tilde{\lambda}_2}(x_2), \\ A_{\tilde{\mu}_1}(y_1), \\ A_{\tilde{\mu}_2}(y_2)\end{array}\middle|\, v=\text{MTTF}\right\} \quad (10.a)$$

where,

$$\text{MTTF}=\frac{(x_2+y_1)\left[(C(x_1+x_2)+\theta)+\theta C(x_1+x_2)\right]}{\theta\left[(x_2+y_1)(x_1+x_2+\lambda_C)-\tilde{\mu}_1C(x_1+x_2)\right]} \quad (10.b)$$

Similarly using the Equation (8), we obtain the membership function of availability as given below:

$$A_{\tilde{p}(\tilde{\lambda}_1,\tilde{\lambda}_2,\tilde{\mu}_1,\tilde{\mu}_2)}(v)=\sup_{\omega}\min\left\{\begin{array}{l}A_{\tilde{\lambda}_1}(x_1),A_{\tilde{\lambda}_2}(x_2), \\ A_{\tilde{\mu}_1}(y_1),A_{\tilde{\mu}_2}(y_2)\end{array}\middle|\, v=A\right\} \quad (11.a)$$

For the practical purpose, we treat the failure rate of first and second unit and the repair rates of first and second units as trapezoidal fuzzy numbers. We take $\tilde{\lambda}_1=\{a_1,a_2,a_3,a_4\}$, $\tilde{\lambda}_2=\{b_1,b_2,b_3,b_4\}$, $\tilde{\mu}_1=\{c_1,c_2,c_3,c_4\}$ and $\tilde{\mu}_2=\{c_1,c_2,c_3,c_4\}$, respectively. Their membership functions have a flat top and it is really just like a truncated triangle curve. Its membership function is given as follows:

$$A_{\tilde{\lambda}_1}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\le x<a_2\\ 1, & a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3}, & a_3<x<a_4\\ 0, & x<a_1,x>a_4\end{cases} \quad (12)$$

Similarly the membership functions $A_{\tilde{\lambda}_1}(x_1)$, $A_{\tilde{\lambda}_2}(x_2)$, $A_{\tilde{\mu}_1}(y_1)$ and $A_{\tilde{\mu}_2}(y_2)$ of $\tilde{\lambda}_2,\tilde{\mu}_1$ and $\tilde{\mu}_2$ are obtained by replacing a_i with b_i,c_i and $d_i(i=1,2,3,4)$, respectively in Equation (12).

6. NUMERICAL EXAMPLE

To demonstrate the computational tractability of the proposed model, we consider an example of an electric plant having two main coal power generators of capacities 300 MW and 150 Mw, respectively. Both of the operating units are repairable. As soon as an operating generator fails, it is immediately detected, located and recovered with a coverage probability 'C'. The intrest of the management is to know the system characteristics such as MTTF and availability for getting more profit and better performance.

The coverage probability C is fixed as 0.9. The failure rate of first and second units, repair rate of first and second units are assumed to be trapezoidal fuzzy numbers represented by $\tilde{\lambda}_1$= [0.2 0.3 0.4 0.5], $\tilde{\lambda}_2$= [0.1

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0.2 0.3 0.4], $\tilde{\mu}_1$ = [2 3 4 5] and $\tilde{\mu}_2$ =[3 4 5 6], respectively. The recovery time (θ), reboot time (β) and common cause shock failure (λ_c) are choosen as 0.5, 0.05 and 0.1, respectively. We have done all of the computational work in MATLAB 7.1 using inbuilt function of membership function in fuzzy tool. The fuzzy MTTF are presented in Table 1. The membership function graph of MTTF is shown in Figure 3. This graph shows two characteristics; the first one reveals that the approximate range of MTTF is [1.28 7.39], which indicates that the value of MTTF should not fall below 1.28 or exceed 7.39. Moreover, at level $\alpha = 1$, the most possible values of MTTF are between 2.55 and 4.50.

Table 1 summarizes the mean time to failure and availability for fuzzyfied failure rates and repair rates of both the operating units as trapezoidal fuzzy numbers. The corresponding fuzzy availability will also be a trapezoidal fuzzy number. In Figure 4, the membership function graph of availability of the system has been shown. At level $\alpha = 0$, the range of the availability is approximately [0.7501 0.9502], which indicates that the availability definitely falls in this range i.e. the availability cannot fall below 0.7501 or exceed 0.9502. At level $\alpha = 1$ as shown in Figure 4, the range of the availability is approximately [0.8532 0.9025].

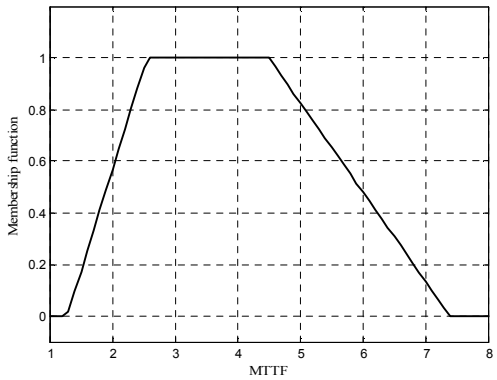

Figure 3. The membership function for the fuzzy MTTF

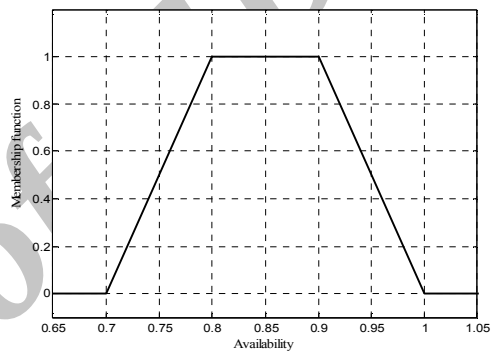

Figure 4. The membership function for the fuzzy availability

7. CONCLUSION

In this paper, a repairable system with imperfect coverage, common-cause shock failure, reboots and recovery has been considered. The fuzzified reliability, availability and mean time to failure are determined. Using the fuzzy reliability analysis a manager can decide the optimal strategy, by setting the range of MTTF and the range of availability to reflect the desired repair rates and this in turn will minimize the total cost involved. The fuzzy reliability approach provides more effective, realistic and flexible measures as compared with the traditional approach based on crisp parameter values. The proposed fuzzy reliability approach may be helpful for the prediction of precise values of the reliability indices for many systems such as power system, nuclear system, electric system, and many more.

TABLE 1. The Triangular Fuzzy Numbers

$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$	$\overline{MTTF}$	$\tilde{A}(\infty)$
0.2	0.1	2	3	1.28	0.7501
0.3	0.2	3	4	2.55	0.8532
0.4	0.3	4	5	4.50	0.9025
0.5	0.4	5	6	7.39	0.9502

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Fuzzy Reliability Evaluation of a Repairable System with Imperfect Coverage, Reboot and Common-cause Shock Failure

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چکیده

در تحقیق حاضر، میزان اعتبار خصوصیات یک سیستم قابل تعمیر که شامل دو واحد عملیاتی مستقل است را با استفاده از ترکیب فاکتور پوششی بررسی می‌کنیم. احتمال موفق بودن در تشخیص، مکان‌یابی و بازیابی از یک خطا (موجود در سیستم) به عنوان احتمال پوششی شناخته شده است. همچنین تاخیر در ریشه‌یابی و دلایل صدمه ناشی از خطا را به طور کلی مورد توجه قرار داده‌ایم. فرض می‌کنیم که تعداد خطاهای ناشی از کار اجزا، زمان خطا به علت اشتباهات رایج، زمان تعمیر و زمان ریشه‌یابی از (توابع) توزیع نمایی پیروی می نماید. از مدل مارکو برای توصیف سیستم استفاده شده است و وضعیت احتمال گذار سیستم تعیین شده است که بیشتر برای تعیین اعتبار بعضی از شاخص‌ها همانند دسترسی و متوسط زمان خطا در گذشته استفاده می‌شده است. ما از منطق فوزی استفاده کردیم تا انالیز عملکرد سیستم را به وسیله توابع اعضا ذوزنقه‌ای سیستم توصیفگر، همانند سرعت خطا و سرعت تعمیر را تعیین کنیم. زمان متوسط فوزی برای خطا و در دسترس بودن فوزی نیز مورد بررسی قرار گرفت. برای تعیین میزان اعتبار نتایج تحلیلی، یک سری ازمایشات عددی نیز انجام گردید.

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