



## Availability of k-out-of-n: F Secondary Subsystem with General Repair Time Distribution

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### ABSTRACT

In this paper we study the steady state availability of main k-out-of-n: F and secondary subsystems. When more than k units of main subsystem fail, then the main subsystem shuts off the secondary subsystem. The life time distributions of the main units and that of secondary subsystem are exponentially distributed. A repair facility having single repairman is facilitated to restore the failed unit. The repair time is assumed to be i.i.d. general distributed. The failure of the main unit shuts off the operation of secondary subsystem, but not the other way around. The steady state availability is obtained by solving the linear ordinary differential equations governing the model by using supplementary variable technique. Four special cases of the repair distribution namely exponential, Gamma, Weibull and Pareto have been examined numerically for the illustration purposes.

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## 1. INTRODUCTION

In real time multi-component systems, the k-out-of-n structure is a very popular type of redundancy. Such type of systems is widely applicable in many industrial and military systems. The term k-out-of-n system is often used to indicate either a good (G) system or a failed (F) system or both. Since the value of n is usually larger than the value of k, the redundancy is generally built into a k-out-of-n system. An n-component system that works (or is good) if and only if at least k out of the n components work is called a k-out-of-n: G system. The parallel and series systems can be considered as particular cases of the k-out-of-n: G system. A system having n-component is called a k-out-of-n: F if and only if at least k out of the n components fail. Based on these two definitions, a k-out-of-n: G system is equivalent to an (n-k+1)-out-of-n: F system. A series system is equivalent to a 1-out-of-n: F system and also has the structure of n-out-of-n: G system while a parallel system is equivalent to an n-out-of-n: F system and is also the same as a 1-out-of-n: G system. The examples of k-out-

of-n systems include the multi pump system in a hydraulic control system, multi display system in a cockpit, the multi-engine system in an airplane, and many more. To cite a common application, we consider the example of driving an automobile having a V8 engine which works only if at least four cylinders are firing; if less than four cylinders fire, it is not possible to drive the automobile. Since for minimal functioning of the engine, the system tolerates failures up to four cylinders, it can be treated as a 4-out-of-8: G system. Another example is related to a data processing system for which out of five video displays at least three operable displays are required for full data display. Thus the display sub-system behaves as a 3-out-of-5: G system. In a 2-out of 3: G communication system at least two transmitters out of total three transmitters are required to be operational at all the times, otherwise correct message will not reach the destination.

The reliability analysis of a series-parallel system under different types of repair was considered by the Nikolov [1]. Arulmozhi [2] studied the reliability of an M-out-of-N warm standby system with R repair facilities. A repairable circular consecutive-k-out-of-n: F system with one repairman was studied by Richard et

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al. [3]. Malwane and Ananda [4] considered the confidence intervals for steady state availability of a system with exponential operating time and lognormal repair time. Malwane and Gamage [5] studied the steady state availability and statistical inference for two-parameter failure time distributions namely Weibull, Gamma and Lognormal. The system level strength interference based reliability modeling of k-out-of-n system was done by Xie et al. [6]. Sharma et al. [7] studied the reliability analysis of k-out-of-n: G machining system with spares and imperfect coverage. For R-out-of-n system with several repairmen, the exponential life time and phase type repair times, an algorithmic approach was presented by Barron et al. [8]. The limiting availability or steady state availability of a repairable system was investigated by Mi [9]. They have considered the life times and the repair times for the failed system as independent but not necessarily identically distributed. Redundancy or standby support is a technique that has been widely applied to improve the system reliability and availability for the system design of k-out-of-(M+N): G warm standby system. A detailed account on the topic was given by Zhang et al. [10]. Xia and Zhang [11] explored the reliability equivalence factors in Gamma distribution. Xing et al. [12] studied the reliability of k-out-of-n system with phased mission requirements and imperfect fault coverage. The reliability analysis for a k-out-of-n: G system with redundant dependency and repairmen having multiple vacations was suggested by Li [13]. Amari [14] presented the reliability of k-out-of-n cold standby systems with Erlang distributions. Krishnan and Somasundaram [15] provided the reliability and profit analysis of repairable k-out-of-n system with sensor. Peng et al. [16] studied the reliability of 1-out-of-(n+1) warm standby system subject to fault level coverage. The reliability analysis of k-out-of-n system with components having random weights was given by Eryilmaz. Wang et al. [17] using the multi-living agent concept investigated complex information systems.

In many situations, the system consists of main as well as secondary subsystems operating under certain shut-off rules. Two shutoff rules are commonly used (i) to eliminate the possibility of additional failures of operating units when the system is broken down, and (ii) all operating units are subject to failure due to stress during the broken down state of the system. The availability for two unit non independent series systems subject to shut off rules, where the non independence can be described by a bivariate exponential life time distribution was provided by Osaki et al. [18]. Zhang et al. [19] studied the consecutive-k-out-of-n: G system and secondary subsystem subject to shut-off rules. Chen et al. [20-23] considered a model with main standby redundancy and secondary subsystems having preemptive repeat priorities. If the secondary subsystem fails when the main subsystem is operating with one

functioning unit and one failed unit, then the repair of the secondary subsystem is started according to preemptive priority rule. The availability analysis of non-independent main unit and n-unit series subsystem subject to shut-off rules was treated by Chen [24]. The reliability analysis of k-out-of-n: G redundant system in the presence of common cause shock failures with imperfect fault coverage was suggested by Mallikarjunudu et al. [25]. Moghaddass [26] considered the reliability and availability of a repairable k-out-of-n: G system with repairmen subject to shut off rules.

In order to study the system having general repair time distribution, the non-Markovian model can be transformed to the Markovian model by introducing the supplementary variables corresponding to elapsed repair time. The supplementary variable techniques was introduced by Cox and the same has been employed by a few researchers to study the availability of the system whose components are subject to governed by shut off rules (cf. Hudes, [27]; Khalil, [28]. Chen et al. [29] studied the model with general repair times which has main and a secondary subsystem subject to shut-off rules. Chaudhury [30] investigated Poisson queue with general setup time and vacation period using the supplementary variable technique. Jain and Mishra [31] suggested the reliability analysis of unreliable server retrieval queue with bulk arrivals via supplementary variables technique. Ozaki and Kara [32] studied the reliability measures of 1-for-2 shared protection system with general repair time distributions.

In this paper, we analyze a system consisting of a main unit and n-unit series subsystem with imperfect repair, shut-off rule, and arbitrary distributions of times to failure and repair. The rest of the work is organized as follows. Section 2 provides the assumptions and notations to formulate the mathematical model. Section 3 contains the governing equations and analysis. In Section 4, the steady state availability and special cases of distributions are taken into consideration. Sections 5 and 6 contain the numerical results and conclusions, respectively.

## 2. MODEL DESCRIPTIONS

The system is composed of main k-out-of-n: F subsystem and a secondary subsystem. We develop the reliability model for analyzing the main and secondary subsystems subject to shut off rules as follows. The system can be considered as a main unit and an n identical components series-parallel subsystem. When more than k units of the subsystem fail, then the main subsystem shuts off the secondary subsystem. The life time of the main subsystem and secondary subsystem are independent and exponentially distributed. The repair time distributions of the main unit and that of any one of the n identical units of series-parallel subsystem

are assumed to be general independent and identically distributed. The failure of the main unit shuts off the operation of secondary subsystem, but not vice versa. The switchover of standby unit to operational unit in the main subsystem is perfect. For modeling purpose, the following notations are used:

$N(t)$	number of failed units in the main subsystem at time $t$ where $N(t) = 0, 1, 2, \dots, k$ .
$M(t)$	the number of failed units in the secondary subsystem at time $t$ where $M(t) = 0, 1$ .
$X(t)$	elapsed repair time for a failed unit of main subsystem at time $t$ .
$Y(t)$	elapsed repair time for the failed unit of secondary subsystem at time $t$ .
$P_{ij}(x, t) dt$	$\Pr\{N(t) = i, M(t) = j, x \leq X(t) \leq x + dt, i = 0, 1, 2, \dots, k; j = 0, 1\}$
$P_{00}$	$\lim_{t \rightarrow \infty} P_{00}(t)$
$P_{ij}(x)$	$\lim_{t \rightarrow \infty} P_{ij}(x, t), (i = 0, 1, 2, \dots, k; j = 0, 1)$
$P_{ij}$	$\int_0^{\infty} P_{ij}(x) dx, (i = 0, 1, 2, \dots, k; j = 0, 1)$
$k$	minimum number of components required for a successful operation of the main subsystem.
$n$	total number of components in the main subsystem
$\lambda_1 (\lambda_2)$	constant failure rate of the units of main subsystem (secondary subsystem).
$\mu_1(x), \mu_2(y)$	the repair rate of the unit of main subsystem and repair rate of secondary subsystem.
$f_1(x), f_2(y)$	pdf of repair time distribution of any unit of main and secondary subsystems, respectively.
$f_i^*(s)$	Laplace transform of $f_i(x), f_2(y)$ , $i = 1, 2$
$A_v$	steady state system availability

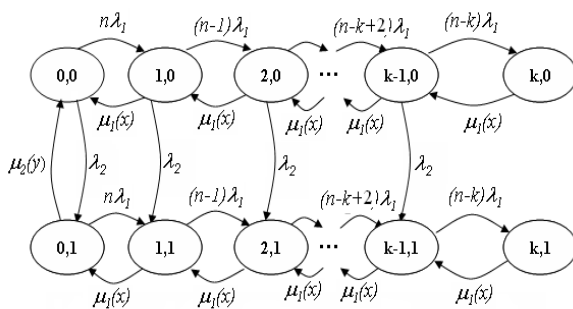


Figure 1. Transition rate diagram of k-out-of-n: F system

### 3. GOVERNING EQUATIONS AND ANALYSIS

The transition flow of k-out-of-n: F system is shown in Figure 1. Using the appropriate transition rates, the differential equations for the probabilities of system states are constructed as follows:

$$\frac{d}{dt} P_{00}(t) = -(n\lambda_1 + \lambda_2) P_{00}(t) + \int_0^{\infty} P_{10}(x, t) \mu_1(x) dx + \int_0^{\infty} P_{01}(y, t) \mu_2(y) dy \quad (1)$$

$$\frac{\partial}{\partial t} P_{r0}(x, t) + \frac{\partial}{\partial x} P_{r0}(x, t) = -[(n-r)\lambda_1 + \lambda_2 + \mu_1(x)] P_{r0}(x, t), \quad 1 \leq r \leq k-1 \quad (2)$$

$$\frac{\partial}{\partial t} P_{k0}(x, t) + \frac{\partial}{\partial x} P_{k0}(x, t) = -\mu_1(x) P_{k0}(x, t) + (n-k+1)\lambda_1 P_{k-1,0}(x, t) \quad (3)$$

$$\frac{\partial}{\partial t} P_{01}(y, t) + \frac{\partial}{\partial y} P_{01}(y, t) = -[n\lambda_1 + \mu_2(y)] P_{01}(y, t) \quad (4)$$

$$\frac{\partial}{\partial t} P_{r1}(x, t) + \frac{\partial}{\partial x} P_{r1}(x, t) = -[(n-r)\lambda_1 + \mu_1(x)] P_{r1}(x, t) + \lambda_2 P_{r0}(x, t), \quad 1 \leq r \leq k-1 \quad (5)$$

$$\frac{\partial}{\partial t} P_{k1}(x, t) + \frac{\partial}{\partial x} P_{k1}(x, t) = -\mu_1(x) P_{k1}(x, t) + (n-k+1)\lambda_1 P_{k-1,1}(x, t) \quad (6)$$

As  $t$  approaches infinity, from Equations (1)-(6) the following equations are obtained for the steady state probabilities as:

$$(n\lambda_1 + \lambda_2) P_{00} = \int_0^{\infty} P_{10}(x) \mu_1(x) dx + \int_0^{\infty} P_{01}(y) \mu_2(y) dy \quad (7)$$

$$\frac{d}{dx} P_{r,0}(x) = -[(n-r)\lambda_1 + \lambda_2 + \mu_1(x)] P_{r,0}(x), \quad 1 \leq r \leq k-1 \quad (8)$$

$$\frac{d}{dx} P_{k,0}(x) = -\mu_1(x) P_{k,0}(x) + (n-k+1)\lambda_1 P_{k-1,0}(x) \quad (9)$$

$$\frac{d}{dy} P_{01}(y) = -[n\lambda_1 + \mu_2(y)] P_{01}(y) \quad (10)$$

$$\frac{d}{dx} P_{r,1}(x) = -[(n-r)\lambda_1 + \mu_1(x)] P_{r,1}(x) + \lambda_2 P_{r,0}(x), \quad 1 \leq r \leq k-1 \quad (11)$$

$$\frac{d}{dx} P_{k,1}(x) = -\mu_1(x) P_{k,1}(x) + (n-k+1)\lambda_1 P_{k-1,1}(x) \quad (12)$$

The normalization conditions is given by:

$$P_{00}(t) + \int_0^{\infty} P_{01}(y, t) dy + \sum_{r=1}^k \int_0^{\infty} P_{r0}(x, t) dx + \sum_{r=1}^k \int_0^{\infty} P_{r1}(x, t) dx = 1 \quad (13)$$

The boundary conditions are as follows:

$$P_{r0}(0) = (n-r+1)\lambda_1 P_{r-1,0} + \int_0^{\infty} P_{r+1,0}(x) \mu_1(x) dx, \quad 1 \leq r \leq k-1 \quad (14)$$

$$P_{k,0}(0) = 0 \quad (15)$$

$$P_{01}(0) = \lambda_2 P_{00} + \int_0^{\infty} P_{11}(x) \mu_1(x) dx \quad (16)$$

$$P_{r1}(0) = (n-r+1)\lambda_1 P_{r-1,1} + \int_0^{\infty} P_{r+1,1}(x) \mu_1(x) dx, \quad 1 \leq r \leq k-1 \quad (17)$$

$$P_{k,1}(0) = 0 \quad (18)$$

For brevity of notations, denote:

$$\Lambda_r = (n-r)\lambda_1 + \lambda_2,$$

$$\theta_r = (n-r)\lambda_1; 1 \leq r \leq k-1$$

$$\phi = n\lambda_1$$

$$H_1(x) = \int_0^{\infty} \mu_1(t) dt \quad \text{and} \quad H_2(y) = \int_0^{\infty} \mu_2(t) dt$$

Solving Equations (8) and (10), we have:

$$P_{r0}(x) = P_{r0}(0) e^{-\Lambda_r x - H_1(x)} \quad (19)$$

$$P_{01}(y) = P_{01}(0) e^{-\phi y - H_2(y)} \quad (20)$$

Substituting values from Equations (19) and (20) into Equation (7), we get:

$$(n\lambda_1 + \lambda_2) P_{00} = P_{10}(0) f_1^*(\Lambda_1) + P_{01}(0) f_2^*(\phi) \quad (21)$$

Using Equations (11) & (19), and (12) and (22), we get:

$$P_{r,1}(x) = [P_{r,0}(0) \{1 - e^{-\lambda_2 x}\} + P_{r,1}(0)] e^{-\theta_r x - H_1(x)}, \quad 1 \leq r \leq k-1 \quad (22)$$

$$P_{k,1}(x) = \frac{1}{\Lambda_{k-1}} \left[ \{P_{k-1,0}(0) (\lambda_2 - \Lambda_{k-1} e^{-\theta_{k-1} x} + \theta_{k-1} e^{-\Lambda_{k-1} x}) + \Lambda_{k-1} P_{k-1,1}(0) (1 - e^{-\theta_{k-1} x})\} e^{-H_1(x)} \right] \quad (23)$$

Using Equations (19) and (14), we obtain:

$$P_{r,0}(0) = \frac{\phi \prod_{r=1}^{k-1} \Lambda_r \theta_{r-1}}{\prod_{r=1}^{k-1} [\lambda_2 + \theta_r f_1^*(\Lambda_r)]} P_{00}, 1 \leq r \leq k-1 \quad (24)$$

Equations (9) and (19) yield:

$$P_{k,0}(x) = \frac{\theta_{k-1} P_{k-1,0}(x)}{\Lambda_{k-1}} \{1 - e^{-\Lambda_{k-1} x}\} e^{-H_1(x)} \quad (25)$$

From Equations (21) and (25), we obtain  $P_{01}(0)$  as:

$$P_{01}(0) = \frac{[(n\lambda_1 + \lambda_2)(\lambda_2 + \theta_1 f_1^*(\Lambda_1)) + \phi \Lambda_1 f_1^*(\Lambda_1)] P_{00}}{f_2^*(\phi)(\lambda_2 + \theta_1 f_1^*(\Lambda_1))} \quad (26)$$

Now Equations (17) and (26) provide:

$$P_{r,1}(0) = \frac{[\Lambda_r (n-r+1)(n\lambda_1 + \lambda_2)(\lambda_2 + \theta_1 f_1^*(\Lambda_1)) \phi \Lambda_1 f_1^*(\Lambda_1) + (\lambda_2 + \theta_r f_1^*(\Lambda_r) - \Lambda_r f_1^*(\theta_r)) (\phi \prod_{r=1}^{k-1} \Lambda_r \theta_{r-1})]}{f_1^*(\phi) [1 - \Lambda_r + \Lambda_r f_1^*(\theta_r)] [\lambda_2 + \theta_r f_1^*(\Lambda_r)]} P_{00}, \quad 1 \leq r \leq k-1 \quad (27)$$

Putting values from Equations (24), (26) and (27) in normalizing condition, we get:

$$P_{00} = \frac{a}{b} \quad (28)$$

where

$$a = \sum_{r=1}^{k-1} \left[ (f_2^*(\phi) \{1 - \Lambda_r + \Lambda_r f_1^*(\theta_r)\}) \prod_{r=1}^{k-1} [\lambda_2 + \theta_r f_1^*(\Lambda_r)] \right]$$

$$b = [(n\lambda_1 + \lambda_2)(\lambda_2 + \theta_1 f_1^*(\Lambda_1)) + \phi \Lambda_1 f_1^*(\Lambda_1)] + \sum_{r=1}^{k-1} \left[ \phi \prod_{r=1}^{k-1} \Lambda_r \theta_{r-1} \right] + \sum_{r=1}^{k-1} \left[ f_2^*(\phi) (1 - \Lambda_r + \Lambda_r f_1^*(\theta_r)) \prod_{r=1}^{k-1} [\lambda_2 + \theta_r f_1^*(\Lambda_r)] \right] + \sum_{r=1}^{k-1} \left[ \Lambda_r (n-r+1)(n\lambda_1 + \lambda_2) \{ \lambda_2 + \theta_1 f_1^*(\Lambda_1) \phi \Lambda_1 f_1^*(\Lambda_1) + \{ \lambda_2 + \theta_r f_1^*(\Lambda_r) - \Lambda_r f_1^*(\theta_r) \} \left\{ \phi \prod_{r=1}^{k-1} \Lambda_r \theta_{r-1} \right\} \} \right]$$

#### 4. SYSTEM AVAILABILITY ( $A_v$ )

The system availability is obtained as:

$$A_v = P_{00} + \sum_{r=1}^{k-1} P_{r0} = \frac{c}{b} \quad (29)$$

where

$$c = \sum_{r=1}^{k-1} \left[ \phi \{ f_2^*(\phi) (1 - \Lambda_r + \Lambda_r f_1^*(\theta_r)) \} \times \prod_{r=1}^{k-1} \{ (\Lambda_r \theta_{r-1}) (\lambda_2 + \theta_r f_1^*(\Lambda_r)) \} \right]$$

#### Special Cases:

In this section we discuss the system availability for special repair time distributions. The general repair time distribution  $f_i(x)$  of the main and secondary subsystems for special cases is given as follows:

$$f_i(x) = \begin{cases} \mu_i e^{-\mu_i x}, & \text{Exponential distribution} \\ \frac{\mu_i^m}{(m-1)!} x^{m-1} e^{-\mu_i x}, & \text{Gamma distribution} \\ \mu_i m x^{m-1} e^{-\mu_i x^m}, & \text{Weibull distribution} \\ \frac{\mu_i}{(1+x)^{\mu_i+1}}, & \text{Pareto distribution} \end{cases} \quad (30)$$

The Laplace transform of these distributions are determined as:

$$f_i^*(s) = \begin{cases} \frac{\mu_i}{s + \mu_i}, & \text{Exponential distribution} \\ \left( \frac{\mu_i}{s + \mu_i} \right)^m, & \text{Gamma distribution} \\ 1 - \frac{s}{2} \sqrt{\frac{\pi}{\mu_i}} e^{\frac{s^2}{4\mu_i}}, & \text{Weibull distribution} \\ \mu_i s^{\mu_i} e^{s^{\mu_i}} \Gamma(-\mu_i, s), & \text{Pareto distribution} \end{cases} \quad \dots(31)$$

Now we give the values  $f_1^*(\Lambda_r)$ ,  $f_1^*(\theta_r)$  and  $f_2^*(\phi)$  for different distributions used in the formula for availability given in Equation (29) as follows:

#### Case 1: Exponential distribution

$$\begin{aligned} f_1^*(\Lambda_r) &= \left( \frac{\mu_1}{\Lambda_r + \mu_1} \right); \\ f_1^*(\theta_r) &= \left( \frac{\mu_1}{\theta_r + \mu_1} \right) \\ \text{and } f_2^*(\phi) &= \left( \frac{\mu_2}{\phi + \mu_2} \right) \end{aligned} \quad (32)$$

#### Case 2: Gamma distribution

$$\begin{aligned} f_1^*(\Lambda_r) &= \left( \frac{\mu_1}{\Lambda_r + \mu_1} \right)^m; f_1^*(\theta_r) = \left( \frac{\mu_1}{\theta_r + \mu_1} \right)^m \\ \text{and } f_2^*(\phi) &= \left( \frac{\mu_2}{\phi + \mu_2} \right)^m \end{aligned} \quad (33)$$

#### Case 3: Weibull distribution

$$\begin{aligned} f_1^*(\Lambda_r) &= 1 - \frac{\Lambda_r}{2} \sqrt{\frac{\pi}{\mu_1}} e^{\frac{\Lambda_r^2}{4\mu_1}}; f_1^*(\theta_r) = 1 - \frac{\theta_r}{2} \sqrt{\frac{\pi}{\mu_1}} e^{\frac{\theta_r^2}{4\mu_1}} \\ \text{and } f_2^*(\phi) &= 1 - \frac{\phi}{2} \sqrt{\frac{\pi}{\mu_2}} e^{\frac{\phi^2}{4\mu_2}} \end{aligned} \quad (34)$$

#### Case 4: Pareto distribution

$$\begin{aligned} f_1^*(\Lambda_r) &= \mu_1 \Lambda_r^{\mu_1} e^{\Lambda_r^{\mu_1}} \Gamma(-\mu_1, \Lambda_r) \\ f_1^*(\theta_r) &= \mu_1 \theta_r^{\mu_1} e^{\theta_r^{\mu_1}} \Gamma(-\mu_1, \theta_r) \quad 1 \leq r \leq k-1 \\ f_2^*(\phi) &= \mu_2 \phi^{\mu_2} e^{\phi^{\mu_2}} \Gamma(-\mu_2, \phi) \end{aligned} \quad (35)$$

To perform numerical experiments, using results of Equations (32)-(35) in Equation (29), we get system availability in particular cases when  $n = 5$ ,  $k = 1, 2, 3$ .

## 5. NUMERICAL RESULTS

In this section, we illustrate how system parameters such as failure rates  $\lambda_1$  and  $\lambda_2$  and repair rates  $\mu_1$  and  $\mu_2$  will affect the system availability for k-out-of-n: F

system. For computational purpose, we consider four types of repair time distributions of main and secondary subsystems, which are (a) Exponential, (b) Gamma, (c) Weibull, and (d) Pareto. We examine the effect of failure rates on the system availability in Figures 2 and 3 and that of repair rates on the system availability in Figures 4 and 5, respectively for  $k = 1, 2, 3$ .

For the sake of convenience, in Figures 2 and 3 we set default parameters for the computation purpose as,  $n = 5$ ,  $\lambda_1 = 0.03$ ,  $\lambda_2 = 0.09$ ,  $\mu_1 = 2$ ,  $\mu_2 = 3$  and  $m = 3$  for Gamma distribution. In these figures, we observe that the system availability shows nearly same trend for exponential and gamma repair time distributions when plotted against either failure rates  $\lambda_1, \lambda_2$  or repair rates  $\mu_1, \mu_2$  respectively; whereas it shows different trends for Weibull and Pareto repair time distributions when plotted against the same.

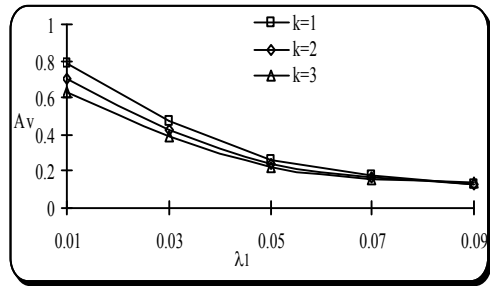
In Figures 2(a-b), we notice that the system availability decreases sharply when plotted against  $\lambda_1$  for exponential and gamma distributions respectively, and for different values of  $k$ . In Figures 2(c) and 3(a-c), the system availability decreases almost linearly with the increase in failure rates  $\lambda_1$  and  $\lambda_2$  respectively, for different repair time distributions and for different values of  $k$ . Figure 2(d) shows that the system availability first decreases slowly and then sharply afterwards with the increase in the failure rate  $\lambda_1$  in case of Pareto repair time distribution. Figure 3(d) shows that the system availability decreases sharply with the increase in failure rates  $\lambda_2$  in case of Pareto repair time distribution. As expected, the system availability decreases with the increase in failure rate for different repair time distributions. Figures 2(a-d)-3(a-d) depict that for a k-out-of-n: F system, the system availability is higher for lower values of  $k$  and vice versa on increasing either failure rates or repair rates of the subsystem; this trend is what we expect in real time system.

For Figures 4 and 5, we set default parameters  $n = 5$ ,  $\lambda_1 = 0.003$ ,  $\lambda_2 = 0.09$ ,  $\mu_1 = 2$ ,  $\mu_2 = 1$  and  $m = 3$  in case of gamma distribution to illustrate the effect of repair rates  $\mu_1$  and  $\mu_2$  respectively, over system availability. In Figures 4(a-c) and 5(a-c) for different repair time distributions, we observe that the system availability first increases linearly then becomes almost constant afterwards with the increase in the repair rates  $\mu_1$  and  $\mu_2$ , respectively. It is clear from Figures 4(d) and 5(d) that in case of Pareto repair time distribution, the system availability increases sharply with the increase in repair rates  $\mu_1$  and  $\mu_2$  respectively. As expected, the system availability increases with the increase in repair rate for different repair time distributions. Finally, we conclude that :

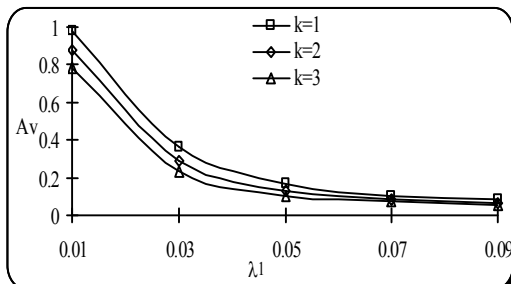
- The system availability shows decreasing (increasing) trends with the increase in failure (repair) rate. The effects of failure rates as well as

repair rates are more prominent in case of Weibull repair time distributions as compared with exponential and gamma repair time distributions.

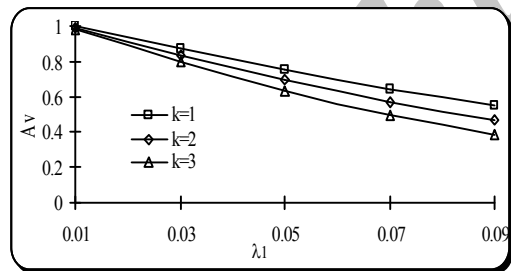
- The system availability is higher for lower values of  $k$ , but lesser for higher values of  $k$ . Moreover, the effect of  $k$  over system availability is more significant for high failure rates or repair rates in comparison to low failure rates or repair rates.



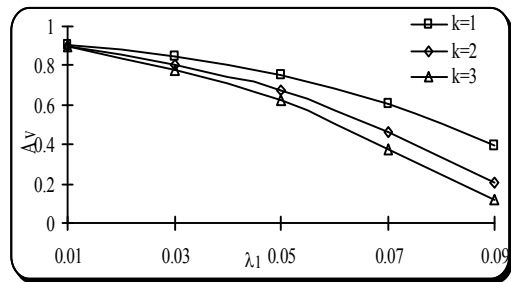
(a)



(b)

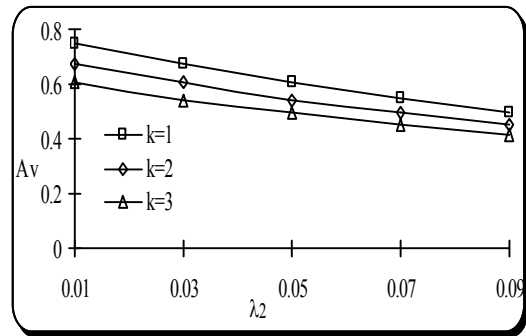


(c)

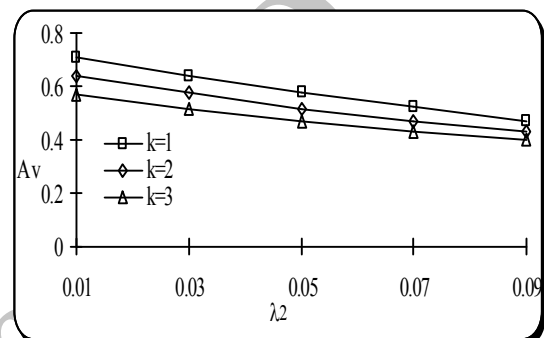


(d)

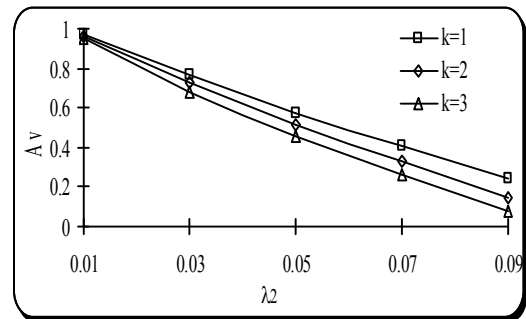
**Figure 2.** System availability vs  $\lambda_1$  for: (a). Exponential (b). Gamma (c). Weibull (d). Pareto, repair time distributions of main subsystem



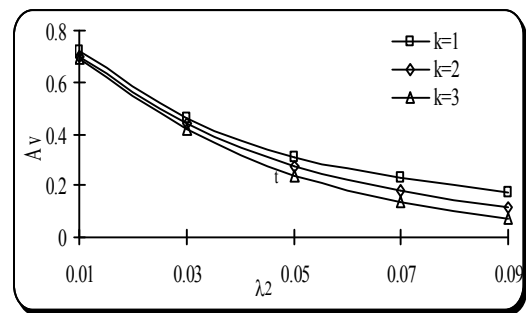
(a)



(b)

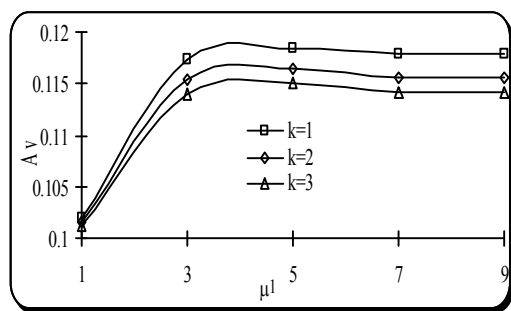


(c)

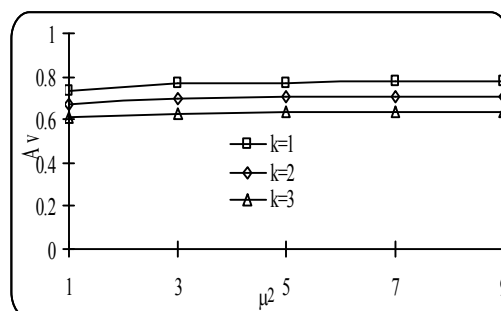


(d)

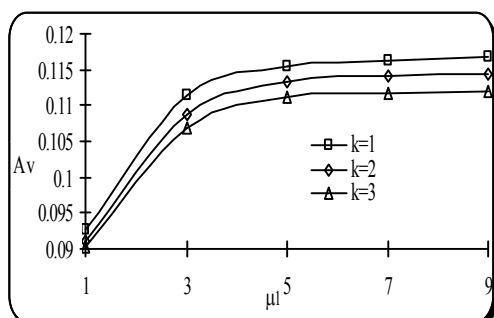
**Figure 3.** System availability vs  $\lambda_2$  for: (a). Exponential (b). Gamma (c). Weibull (d). Pareto, repair time distributions of secondary subsystem



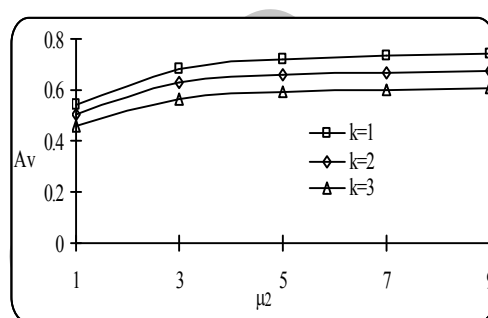
(a)



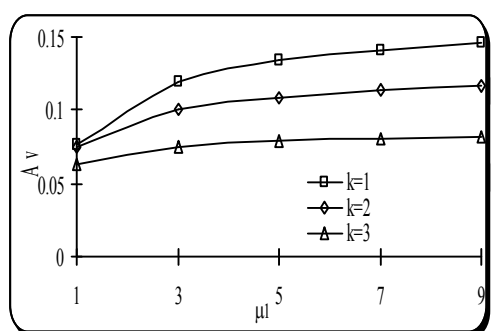
(a)



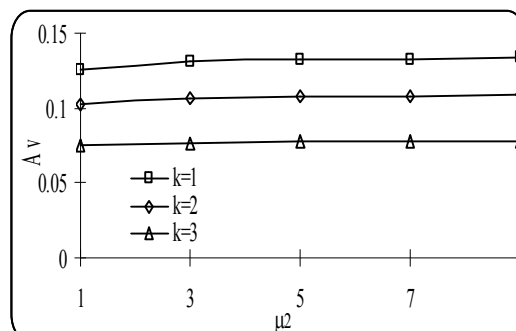
(b)



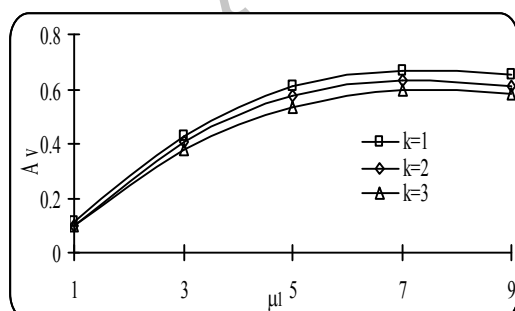
(b)



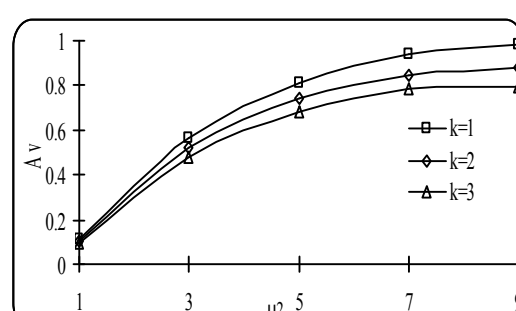
(c)



(c)



(d)



(d)

**Figure 4.** System availability vs  $\mu_1$  for: (a). Exponential (b). Gamma (c). Weibull (d). Pareto, repair time distributions of main subsystem

**Figure 5.** System availability vs  $\mu_2$  for: (a). Exponential (b). Gamma (c). Weibull (d). Pareto, repair time distributions of secondary subsystem

## 6. CONCLUSIONS

The system composed of two subsystems connected with a single repair facility is common in many machining systems including computer system, manufacturing system, transmission lines, etc. In this investigation, we have studied a repairable multi-component system having a main  $k$ -out-of- $n$ :  $F$  and secondary subsystem under the assumption of exponential life time distribution and general distributed repair time of components and main unit. The steady state availability established can be easily computed as shown by taking numerical illustration. The knowledge of availability measures may be helpful to the system designers and reliability engineers to operate the concerned system to achieve desired efficiency in terms of availability.

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## Availability of k-out-of-n: F Secondary Subsystem with General Repair Time Distribution

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در این مقاله ما به مطالعه در دسترس بودن حالت یکنواخت اصلی  $K: F$  از  $N$  و زیرسیستم‌های ثانویه می‌پردازیم. هنگامی که بیش از  $K$  واحد زیرسیستم اصلی از کار می‌افتد، زیرسیستم اصلی زیرسیستم ثانویه را خاموش می‌کند. توزیع عمر مفید واحدهای اصلی و زیرسیستم‌های ثانویه به صورت نمایی است. مرکز تعمیر با داشتن یک تعمیرکار مسوول بازگرداندن واحد از کار افتاده است. توزیع زمان تعمیر به صورت عمومی  $i.i.d$  فرض می‌شود از کار افتاده‌گی واحد اصلی عملکرد زیرسیستم ثانویه را قطع می‌کند، اما بر عکس آن اتفاق نمی‌افتد. در دسترس بودن حالت یکنواخت با حل معادلات خطی دیفرانسیل معمولی حاکم بر مدل با استفاده از روش متغیر مکمل به دست آمده است. به منظور ارایه‌ی مثال‌های عملی، چهار حالت‌خاص توزیع یعنی تعمیر نمایی، گاما، وایبول و پارتو به صورت عددی مورد بررسی قرار گرفته است.

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