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Thermo-elastic Damping in Nano-beam Resonators Based on Nonlocal Theory

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ABSTRACT

In this article, thermo-elastic damping in nano-beam resonators is investigated based on nonlocal theory of elasticity and the Euler-Bernoulli beam assumptions. To this end, governing equation of motion of the beam is obtained from stress–strain relationship of the nonlocal elasticity model and also governing equations of thermo-elastic damping are established using two dimensional non-Fourier heat conduction. Free vibration of the nano-beam resonators is analyzed using Galerkin reduced order model formulation for the first mode of vibration. In the present investigation a clamped-clamped nano-beam with isothermal boundary conditions at both ends is studied. This nonlocal model incorporates the length scale parameter (nonlocal parameter) which can capture the small scale effect. The obtained results are compared with the numerical results of the classical thermo-elastic models. Thermo-elastic damping effects on the damping ratio are studied for the various nano-beam thicknesses and ambient temperatures. In addition, the study includes computations for different values of nonlocal theory parameter. The results show that with increasing the amount of nonlocal parameter and also with decreasing the length of the nano-beam, difference between the results of classical and nonlocal theory increases.

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1. INTRODUCTION

Development of mechanical and electronic systems in a compact case such as Micro-Electro-Mechanical-System (MEMS) and Nano-Electro-Mechanical-System (NEMS) has become more important, because this essential part of technology increases the speed and compactness size of industrial equipment. In fact, MEMS/NEMS is a combination of mechanical components, electronic sensors and electronic components. The term MEMS/NEMS first started being used in the 1980s. It was used in the United States for the first time and has been applied to a broad set of technologies with the goal of miniaturizing systems through the integration of functions into small packages. This mechanical structure is very small in size, and has various applications. MEMS/NEMS has grown tremendously in the last decade and has attracted worldwide applicable attention. This is due to the wide application of MEMS/NEMS in different branches of medicine, medical engineering (analysis and synthesis of Deoxyribonucleic acid (DNA) and genetic code, drug delivery, diagnostic and imaging), transportation systems (converters, accelerometers, Gyroscopes) and production (intelligent nano-robots) and etc.

Nano-mechanical resonators are a category of NEMS devices, used widely in applications requiring high speed and accuracy [1]. Recently developing and manufacturing mobile devices with limited energy source to reduce energy consumption has been inevitable and this is available by analyzing all factors, interfering in energy consumption in these devices. To obtain high performance resonators, it is necessary to design and build a resonator that works with very low power dissipation or in other words, it works with a high quality factor. Quality factor of the resonator is a measure of the amount of energy loss. Air damping and clamping losses are the major extrinsic mechanisms of energy dissipation. Energy loss resulting from the thermo-elastic damping (TED) is the main intrinsic mechanism of energy dissipation factors in nano-beam resonators. Extrinsic losses such as air damping can be minimized by proper design and operating conditions. But intrinsic losses such as TED cannot be controlled as easily as extrinsic losses and they are almost impossible

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to eliminate [2, 3].

TED is an important loss mechanism in high quality nano-structures, especially in those using flexural vibration modes [4-7]. Therefore,it is important to find a way to reduce intrinsic losses, such as TED, as much as possible. In precise measurements, TED acts as a source of mechanical thermal noise and contributes to reduce the quality factor and this causes an increase in energy consumption.

Zener [8-10] was the first one who explained the mechanism of TED. He also derived an analytical approximation to relate the energy dissipation and the material properties of a micro-beam structure. Lifshitz and Roukes [4] studied TED of a beam with rectangular cross-section and found that after the Debye peaks, the thermo-elastic attenuation will be weakened as the size increases. The results of Lifshitz and Roukes [4] were obtained based on the classical Fourier thermal conducting equation although they did not considered the effect of boundary conditions in their research. Sun et al. [11] studied and analyzed the TED of micro-beam resonators using both the finite sine Fourier transformation method combined with Laplace transformation and the normal mode analysis. Sun et al. [12] investigated the vibration phenomenon during pulsed laser heating of a micro-beam. They also analyzed the size effect and the effect of different boundary conditions and compared the damping ratio and resonant frequency shift ratio of beams due to the air damping effect and the TED effect. Vahdat and Rezazadeh [13] studied the effects of axial and residual stresses on TED in capacitive micro-beam resonators.

Both experimental and atomistic simulation results have shown a significant size effect in mechanical properties when the dimensions of these structures become very small. For this reason, the size effect has a major role on static and dynamic behavior of micro, nanostructures and cannot be ignored. It is well known that classical continuum mechanics does not account for such size effects in micro-, nano-scale structures. In order to overcome this problem, many higher order continuum (nonlocal) theories that contain additional material constants, such as the modified couple stress theory [14], the strain gradient theory [15], the micropolar theory [16], the nonlocal elasticity theory [17], and the surface elasticity [18] have been developed to characterize size effect in micro, nano-scale structures by introducing an intrinsic length scale in the constitutive relations.

The nonlocal elasticity theory, which was introduced by Eringen [19] to account for scale effect in elasticity was used to study lattice dispersion of elastic waves. In the classical (local) elasticity theory, the stress at a point depends only on the strain at the same point whereas in the nonlocal elasticity theory, the stress at a point is a function of strains at all points in the continuum. In this way, the nonlocal continuum theory contains information about long range forces between atoms, and the internal length scale is introduced into the constitutive equations simply as material parameter to capture the small scale effect [20-23].

The aim of the present work is to investigate the thermo-elastic damping in nano-beam resonators based on nonlocal theory and Euler-Bernoulli beam assumptions. Governing equations of coupled heat conduction has been extracted from the equation of heat conduction based on non-Fourier two dimensional heat conduction based on continuum theory frame with one relaxation time in an isotropic homogeneous elastic solid by neglecting heat conduction along the width direction. The governing equations were converted to dimensionless form and the converted equations were solved by a Galerkin reduced order model.

2. PROBLEM FORMULATION

We consider small motions of a thin elastic nano-beam with dimensions of length $L(0 \le x \le L)$, width b $(-b/2 \le y \le b/2)$ and thickness h $(-h/2 \le z \le h/2)$, as is shown in Figure 1. We define the x axis along the axis of the beam and also y and z axes corresponding to the width and thickness, respectively. The usual Euler–Bernoulli assumption is made so that any plane cross-section, initially perpendicular to the axis of the beam, remains plane and perpendicular to the neutral surface during bending. Thus, the displacements can be given by:

$$u = -z\frac{\partial w}{\partial x},\ u_0 = 0;\quad v \neq 0,\ v_0 = 0;\quad w = w_0(x,t); \qquad (1)$$

$$T = T(x,z,t)$$
$$\frac{\partial T}{\partial x} \quad \frac{\partial T}{\partial z} \qquad (2)$$

where u_0, v_0 and w_0 is the displacements of the mid-plane of the nano-beam in the x, y and z direction respectively, $T = T - T_0$, T is the absolute temperature and T_0 is the temperature of the nano-beam in the natural state assumed to be equal to the ambient temperature.

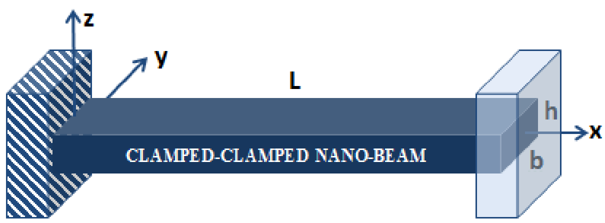


Figure 1. Schematic diagram of the physical system

The equation of constitutive relation for an isotropic homogeneous elastic solid in the absence of body forces and heat sources include [13]:

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}\left(\lambda e_{kk} - \beta_1 T\right) \tag{3}$$

where:

$$\beta_1 = \frac{E}{(1-2\nu)}\alpha_t\ ; \qquad e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right);\ i,j = x,y,z \tag{4}$$

where μ and λ are Lame's constants, α_t is the coefficient of the linear thermal expansion, σ_{ij} is the components of the stress tensor, u_i is the components of the displacement vector, e_{ij} is the components of the strain tensor, $e = e_{kk}$ is the trace of the strain tensor, ν is Poisson ratio and δ_{ij} is the Kronecker delta.

Following Sadd [24] and Vahdat and Rezazadeh [13] when the thickness (z direction) and the width (y direction) of a beam are small enough in comparison to the length (x direction) of it, based on plane stress condition it can be concluded that the stress tensor components in z and y directions are zero ($\sigma_{zz} = \sigma_{yy} = \sigma_{xz} = \sigma_{yz} = \sigma_{xy} = 0$). Therefore, the strain components can be simplified as follows:

$$\begin{aligned} e_{xx} &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} = \frac{\sigma_{xx}}{E} + \alpha_t T \\ e_{yy} &= -\nu e_{xx} + (1+\nu)\alpha_t T = \nu z\frac{\partial^2 w}{\partial x^2} + (1+\nu)\alpha_t T \\ e_{zz} &= -\nu e_{xx} + (1+\nu)\alpha_t T = \nu z\frac{\partial^2 w}{\partial x^2} + (1+\nu)\alpha_t T \\ e_{xy} &= e_{xz} = e_{yz} = 0 \end{aligned} \tag{5}$$

For a narrow beam based on Euler–Bernoulli beam assumptions, the trace of the strain tensor is as follows:

$$\begin{aligned} e_{kk} &= e_{xx} + e_{yy} + e_{zz} \\ &= -(1-2\nu)\left(z\frac{\partial^2 w}{\partial x^2}\right) + 2(1+\nu)\alpha_t T \end{aligned} \tag{6}$$

and:

$$\sigma_{xx} = -Ez\frac{\partial^2 w}{\partial x^2} - E\alpha_t T \tag{7}$$

But when the thickness of a beam is small enough in comparison to the length and width of it is considerable, based on plane strain condition it can be concluded that the stress components in z direction and strain components in y direction vanish ($\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = e_{yy} = e_{xy} = e_{xz} = e_{zy} = 0$). Therefore, the nonzero components of the strain and stress tensors in terms of the displacement field considering Euler-Bernoulli assumptions and thermal field effect can be expressed as follows [13, 24]:

$$\begin{aligned} e_{xx} &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \alpha_t T \\ e_{yy} &= \frac{\partial v}{\partial y} = 0 \\ e_{zz} &= \frac{\partial w}{\partial z} = \frac{\nu}{(1-\nu)}\left(z\frac{\partial^2 w}{\partial x^2}\right) + \frac{(1+\nu)}{(1-\nu)}\alpha_t T \end{aligned} \tag{8}$$

For a wide beam based on Euler–Bernoulli beam assumptions the trace of the strain tensor is as follows:

$$e = -\frac{(1-2\nu)}{(1-\nu)}\left(z\frac{\partial^2 w}{\partial x^2}\right) + \frac{(1+\nu)}{(1-\nu)}\alpha_t T \tag{9}$$

and:

$$\begin{aligned} \sigma_{xx} &= -\frac{E}{(1-\nu^2)}\left(z\frac{\partial^2 w}{\partial x^2}\right) - \frac{E\alpha_t}{(1-\nu)}T \\ \sigma_{yy} &= \nu\sigma_{xx} - E\alpha_t T \end{aligned} \tag{10}$$

So for both narrow and wide beam we have :

$$\sigma_{xx} = -\tilde{E}z\frac{\partial^2 w}{\partial x^2} - \beta_t T \tag{11}$$

here β_t is the thermal modulus equal to $E\alpha_t$ for the plane stress condition and equal to $E\alpha_t/(1-\nu)$ for a wide beam (the plane strain condition). Note that for a wide beam, for which $b \geq 5h$, the effective modulus $\tilde{E}$ can be approximated by the plate modulus $E/(1-\nu^2)$, otherwise $\tilde{E}$ is Young's modulus E.

3. NONLOCAL ELASTICITY THEORY OF ERINGEN

The linear theory of nonlocal elasticity leads to a set of integropartial differential equations for the displacement fields of homogeneous, isotropic bodies [25-28]. For homogenous and isotropic elastic solids, the linear theory of nonlocal elasticity is described by the following equations [19]:

$$\begin{aligned} &\sigma_{ij,i} + \rho\left(f_j - \ddot{u}_j\right) = 0 \\ &\sigma_{ij}(x) = \int_V \alpha\left(|x'-x|,\tau\right)\sigma'_{ij}(x')\,dV(x') \\ &\sigma'_{ij}(x') = \lambda e_{kk}(x')\delta_{ij} + 2\mu e_{ij}(x') \\ &e_{ij}(x') = \frac{1}{2}\left(\frac{\partial u_j(x')}{\partial x'_i} + \frac{\partial u_i(x')}{\partial x'_j}\right) \end{aligned} \tag{12}$$

where $\sigma_{ij}(x)$, ρ, f_j, and u_j are, respectively, the nonlocal stress tensor, mass density, body force density, and displacement vector at a reference point x in the body, at time t, while $\ddot{u}_j$, the second derivative of u_j with respect to time t is the acceleration vector at x. Classical or local stress tensor at x', denoted as $\sigma'_{ij}(x')$, is related to the linear strain tensor $e_{ij}(x')$ at any point x' in the body. It is clear that the classical or local constitutive relation has to be replaced by the nonlocal constitutive relative $\sigma_{ij}(x)$ at x depending not only on the classical local stress $\sigma'_{ij}(x')$ at that particular point but also on nonlocal modulus $\alpha(|x'-x|,\tau)$ where $|x'-x|$ is the Euclidean distance between. τ is a dimensionless length scale denoted by:

$$\tau = \frac{e_0 a}{L} \tag{13}$$

where a is internal characteristic length and e_0 is a material constant. Due to the difficulty in deriving an analytical solution, it is possible in an approximate sense to convert the integral-partial differential equation to a general partial differential equation [19]. The classical Hooke's law for uniaxial stress in one dimension is replaced by a nonlocal stress relation [19] as:

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = \tilde{E} e_{xx} - \beta_t T \tag{14}$$

where σ_{xx} and e_{xx} are the normal stress and strain. For limiting nanoscale $e_0 a \to 0$, the nonlocal effect can be neglected and the nonlocal stress σ_{xx} approaches that of the corresponding classical stress.

4. EQUATION OF MOTION

At this stage, multiplying both sides of Equation (14) by z and integrating over the cross section area of beam, we obtain:

$$\begin{aligned}\int_A \sigma_{xx} z dA - \int_A (e_0 a)^2 z \frac{\partial^2 \sigma_{xx}}{\partial x^2} dA \\ = -\int_A \left(\tilde{E} z^2 \frac{\partial^2 w}{\partial x^2} \right) dA - \int_A \beta_t T z dA\end{aligned} \tag{15}$$

According to the classical elastic theory for a long tube, the bending moment M_{xx} in the transverse direction and strain are denoted as:

$$M_{xx} = \int z \sigma dA \tag{16}$$

By substituting Equation (14) into Equation (15) we have:

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} + \tilde{E} I \frac{\partial^2 w}{\partial x^2} + M_T = 0 \tag{17}$$

where:

$$M_T = \int_A \beta_t T z dA \tag{18}$$

By performing the differentiation of this equation with respect to the variable x twice we obtain:

$$\frac{\partial^2 M}{\partial x^2} - (e_0 a)^2 \frac{\partial^4 M}{\partial x^4} + \tilde{E} I \frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 M_T}{\partial x^2} = 0 \tag{19}$$

According to the Euler–Bernoulli beam theory:

$$\frac{\partial^2 M}{\partial x^2} = \rho A \frac{\partial^2 w}{\partial t^2} \tag{20}$$

Substituting Equation (20) into Equation (19), we obtain the below governing nonlocal equation for vibration of nano-beams based on Euler–Bernoulli beam theory:

$$\begin{aligned}\tilde{E} I \frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 M_T}{\partial x^2} + \rho A \frac{\partial^2 w}{\partial t^2} \\ - \rho A (e_0 a)^2 \frac{\partial^4 w}{\partial x^2 \partial t^2} = 0\end{aligned} \tag{21}$$

Since in the current article free vibration of the nano-beam are investigated and referring to the Civalek and Demir [21, 22] differential equation of nano-beam motion is in order of fourth with respect to x.

5. EQUATION OF THERMO-ELASTICITY

5. 1. Classical Thermo-elasticity According to classical heat conduction theory, heat flux is directly proportional to the temperature gradient (Fourier's law) as:

$$q = -k \nabla T \tag{22}$$

k and q are the thermal conductivity and heat flux vector, respectively. Assuming small deviations of temperature from equilibrium value, the equation of coupled classical thermo-elastic (CTE) is given by [29]:

$$k T_{,ii} = \rho C_v \frac{\partial T}{\partial t} + \frac{E \alpha_t}{(1-\nu^2)} T_0 \frac{\partial e}{\partial t} \tag{23}$$

where C_v is the specific heat at constant volume.

5. 2. Generalized Thermo-elasticity Thermo-elasticity equation based on non-Fourier heat conduction equation has been proposed by Lord and

Shulman [30]. Non-Fourier or hyperbolic heat conduction equation was introduced by Maxwell [31] and Morse and Feshbach [32] to eliminate the paradox of an infinite velocity peculiar to the classical theory by extension of Fourier law of heat conduction to the most general case involving heat flux and its first time derivative [33]:

$$q_j + \tau_{0t}\dot{q}_j = -kT_{,j} \tag{24}$$

where τ_{0t} is relaxation time [33]. The constant τ_{0t} has a clear physical interpretation: it is the time required to establish the steady state of heat conduction in a volume element suddenly subjected to a temperature gradient. Chester [34] quantitatively estimated τ_{0t} in terms of measurable macroscopic parameters to be:

$$\tau_{0t} = \frac{3k}{\rho C_v \nu^2} \tag{25}$$

where ν is the phonon velocity. To the first approximation, ν can be replaced by the elastic wave velocity [35]. The coupled heat conduction equation for a thermo-elastic isotropic body with one relaxation time takes the following form [29]:

$$\rho C_v\left(\dot{T} + \tau_{0t}\ddot{T}\right) + \beta_1 T_0\left(\dot{e} + \tau_{0t}\ddot{e}\right) = KT_{,ii} \tag{26}$$

By substituting the plain stress and plain strain into strain tensor, the equation of coupled generalized thermo-elastic (GTE) is extracted:

$$\begin{aligned}&\left(\rho C_v + \gamma E\alpha_t{}^2 T_0\right)\frac{\partial T}{\partial t} + \left(\rho C_v\tau_{0t} + \gamma E\alpha_t{}^2 T_0\tau_{0t}\right)\frac{\partial^2 T}{\partial t^2}\\&-(\beta_t T_0)z\frac{\partial^3 w}{\partial x^2\partial t} - (\beta_t T_0\tau_{0t})z\frac{\partial^4 w}{\partial x^2\partial t^2}\\&-k\frac{\partial^2 T}{\partial x^2} - k\frac{\partial^2 T}{\partial z^2} = 0\end{aligned} \tag{27}$$

where γ for the plane stress condition (wide beam) is $(2(1+\nu))/(1-2\nu)$ and for the plane strain condition γ is $(1+\nu)/((1-2\nu)(1-\nu))$.

5. 3. Zener and Lifshitz Results One side of the beam is compressed and heated, while the other side is stretched and cooled. Thus, in the presence of finite thermal expansion, a transverse temperature gradient is produced. The temperature gradient generates local heat currents, which cause increase of the entropy of the beam and lead to energy dissipation. The temperature across the beam equalizes in a characteristic time τ_R, while the flexural period of the beam is ω^{-1} (ω is the vibration frequency of the beam). In the low-frequency range, $\tau_R \quad \omega^{-1}$, the vibrations are isothermal and a small amount of energy is dissipated and, for $\tau_R \quad \omega^{-1}$, adiabatic conditions prevail with low-energy dissipation similar to the low-frequency range. Whiles $\tau_R \approx \omega^{-1}$, stress and strain are out of phase and a maximum of internal friction occurs [8]. This is the so-called Debye peak. For a beam of thickness h, with a rectangular cross-section, its characteristic time is $\tau_R = (h/\pi)^2 \rho C_v / k$, the vibration frequency of a beam is [11]:

$$\omega = \frac{q^2 h}{L^2}\sqrt{\frac{E}{12\rho}} \tag{28}$$

For beams with both ends clamped $q = 4.73$. In Zener's theory, the classical Fourier thermal conduction theory is applied and there is no heat flow perpendicular to the surfaces of the beam. Thus, the internal friction, Q (Q is the quality factor defined by Zener [8]), is defined as follows [11]:

$$Q^{-1} = \frac{\alpha_t{}^2 TE}{C_P}\left(\frac{\omega\tau_R}{1+\omega^2\tau^2{}_R}\right) \tag{29}$$

C_p is the specific heat at constant pressure. Lifshitz and Roukes (LR) [4] gave another expression for the thermo-elastic damping by [11]:

$$Q^{-1} = \frac{\alpha_t{}^2 TE}{C_P}\left(\frac{6}{\eta^2} - \frac{6}{\eta^3}\frac{\sinh\eta + \sin\eta}{\cosh\eta + \cos\eta}\right) \tag{30}$$

in which $\eta = h\sqrt{\omega/2D}$.

Following dimensionless quantities are defined to transform Equations (21) and (27) into nondimensional forms:

$$\begin{aligned}&\hat{w} = \frac{w}{h},\ \hat{x} = \frac{x}{L},\ \hat{z} = \frac{z}{h},\ \hat{T} = \frac{T}{T_o},\\&\hat{t} = \frac{t}{t^*},\ t^* = L\sqrt{\frac{\rho}{\tilde{E}}},\ \hat{M}_T = \frac{M_T}{\tilde{E}bh^2},\end{aligned} \tag{31}$$

Applying these dimensionless quantities, equations will take the following forms:

$$S_1\frac{\partial^4\hat{w}}{\partial\hat{x}^4} + \frac{\partial^2\hat{M}_T}{\partial\hat{x}^2} + \frac{\partial^2\hat{w}}{\partial\hat{t}^2} - \tau^2\frac{\partial^4\hat{w}}{\partial\hat{x}^2\partial\hat{t}^2} = 0 \tag{32}$$

$$\begin{aligned}&\frac{\partial^2\hat{T}}{\partial\hat{x}^2} + S_2\frac{\partial^2\hat{T}}{\partial\hat{z}^2} - S_3\frac{\partial\hat{T}}{\partial\hat{t}} + S_4\hat{z}\frac{\partial^3\hat{w}}{\partial\hat{x}^2\partial\hat{t}} - S_5\frac{\partial^2\hat{T}}{\partial\hat{t}^2}\\&\quad + S_6\hat{z}\frac{\partial^4\hat{w}}{\partial\hat{x}^2\partial\hat{t}^2} = 0\end{aligned} \tag{33}$$

where S_i $(i = 1,\ldots,6)$ are the constants that are presented in appendix 1.

6. NUMERICAL SOLUTION

In order to analyze complex frequency of free vibration of the nano-beam coupled with thermo-elasticity equations we used a Galerkin based reduced order model [36]. Based on this model the dynamic motion and temperature of the system can be approximated in terms of linear combinations of finite number of suitable shape functions with time dependent coefficients:

$$\hat{w}\left(\hat{x},\hat{t}\right)=\sum_{k=1}^{p}\varpi_k\left(\hat{t}\right)\psi_k\left(\hat{x}\right) \tag{34}$$

$$\hat{T}\left(\hat{x},\hat{z},\hat{t}\right)=\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}\left(\hat{t}\right)\varphi_i\left(\hat{x}\right)\phi_j\left(\hat{z}\right) \tag{35}$$

Substituting Equation (35) into the M_T equation, it can be represented in non-dimensional form as follows:

$$\begin{aligned}\hat{M}_T&=\frac{M_T}{\tilde{E}bh^2}=\frac{T_0\beta_t}{\tilde{E}}\int_{-0.5}^{0.5}\hat{T}\hat{z}d\hat{z}\\&=\frac{T_0\beta_t}{\tilde{E}}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}\left(\hat{t}\right)\varphi_i\left(\hat{x}\right)\int_{-0.5}^{0.5}\hat{z}\phi_j(\hat{z})d\hat{z}\end{aligned} \tag{36}$$

Substituting Equations (34)–(36) into Equation (32) and (33) leads to following equations:

$$\begin{aligned}&S_1\sum_{k=1}^{p}\varpi_k\left(\hat{t}\right)\psi_k^{(IV)}\left(\hat{x}\right)\\&+\frac{T_0\beta_t}{\tilde{E}}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}\left(\hat{t}\right)\varphi_i^{''}\left(\hat{x}\right)\int_{-0.5}^{0.5}\hat{z}\phi_j(\hat{z})d\hat{z}\\&+\sum_{k=1}^{p}\ddot{\varpi}_k\left(\hat{t}\right)\psi_k\left(\hat{x}\right)-\tau^2\sum_{k=1}^{p}\ddot{\varpi}_k\left(\hat{t}\right)\psi^{''}_k\left(\hat{x}\right)=\epsilon_1\end{aligned} \tag{37}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}\left(\hat{t}\right)\varphi_i^{''}\left(\hat{x}\right)\phi_j\left(\hat{z}\right)\\&\quad+S_2\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}\left(\hat{t}\right)\varphi_i\left(\hat{x}\right)\phi_j^{\circ\circ}\left(\hat{z}\right)\\&\quad-S_3\sum_{i=1}^{n}\sum_{j=1}^{m}\dot{u}_{ij}\left(\hat{t}\right)\varphi_i\left(\hat{x}\right)\phi_j\left(\hat{z}\right)\\&\quad+S_4\hat{z}\sum_{k=1}^{p}\dot{\varpi}_k\left(\hat{t}\right)\psi_k^{''}\left(\hat{x}\right)\\&\quad-S_5\sum_{i=1}^{n}\sum_{j=1}^{m}\ddot{u}_{ij}\left(\hat{t}\right)\varphi_i\left(\hat{x}\right)\phi_j\left(\hat{z}\right)\\&\quad+S_6\hat{z}\sum_{k=1}^{p}\ddot{\varpi}_k\left(\hat{t}\right)\psi_k^{''}\left(\hat{x}\right)=\epsilon_2\end{aligned} \tag{38}$$

where: $\phi_j^{\circ\circ}\left(\hat{z}\right)=\partial^2\phi_j/\partial\hat{z}^2$

According to Galerkin method, following conditions should be satisfied:

$$\int_0^1\psi_f\left(\hat{x}\right)\epsilon_1 d\hat{x}=0\quad f=1,\ldots,p\ , \tag{39}$$

$$\begin{aligned}&\int_0^1\int_{-0.5}^{0.5}\varphi_q\left(\hat{x}\right)\phi_g\left(\hat{z}\right)\left(\hat{z}\right)\epsilon_2 d\hat{z}d\hat{x}=0\ ;\\&q=1,\ldots,n\ \ ,\ g=1,\ldots,m\end{aligned} \tag{40}$$

Now applying Equations (39) and (40) leads to following equations:

$$\begin{aligned}&S_1\sum_{k=1}^{p}\varpi_k K_{fk}^{(1)}+\frac{T_0\beta_t}{\tilde{E}}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}K_{fi}^{(2)}K_j^{(3)}+\sum_{k=1}^{p}\ddot{\varpi}_k K_{fk}^{(6)}\\&\quad-\tau^2\sum_{k=1}^{p}\ddot{\varpi}_k K_{fk}^{(7)}=0\end{aligned} \tag{41}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}G_{qi}^{(2)}G_{gj}^{(4)}+S_2\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}G_{qi}^{(1)}G_{gj}^{(5)}\\&-S_3\sum_{i=1}^{n}\sum_{j=1}^{m}\dot{u}_{ij}G_{qi}^{(1)}G_{gj}^{(4)}+S_4\sum_{k=1}^{p}\dot{\varpi}_k G_{qk}^{(3)}G_g^{(6)}\\&-S_5\sum_{i=1}^{n}\sum_{j=1}^{m}\ddot{u}_{ij}G_{qi}^{(1)}G_{gj}^{(4)}+S_6\sum_{k=1}^{p}\ddot{\varpi}_k G_{qk}^{(3)}G_g^{(6)}=0\end{aligned} \tag{42}$$

new parameters in Equations (41) and (42) are presented in the appendix.

Choosing suitable shape functions, which satisfy the boundary conditions, and solving Equations (41) and (42) concurrently in which:

$$\varpi_k=\bar{\varpi}_k e^{i\Omega_k\tau};\ \ u_{ij}=\bar{u}_{ij}e^{i\Omega_{ij}\tau}; \tag{43}$$

complex frequencies are obtained.

Note that $\hat{T}$ and $\hat{w}$ vibrate at the same frequency, therefore, $\Omega_k=\Omega_{ij}=\Omega$ [11]. According to complex frequency approach the TED ratio (ζ) can be calculated as [13]:

$$\zeta=\left|\frac{\Im(\Omega)}{\sqrt{\Re^2(\Omega)+\Im^2(\Omega)}}\right| \tag{44}$$

where $\Re(\Omega)$ is the real part of the complex frequency and $\Im(\Omega)$ is its imaginary part.

7. NUMERICAL RESULTS

The proposed nano-beam is a wide beam and has the following material properties as shown in Table 1 [13]. Assume coefficient of linear thermal expansion and thermal conductivity are constant.

The theoretical results obtained in the previous section of this article are employed in this part to investigate the thermo-elastic damping in the nano-beam resonators based on nonlocal elasticity. As shown in Figure 2, as the thickness increases the damping ratio first increases to attain its maximum value and the related thickness to this value can be known as the TED critical thickness and after this critical thickness damping ratio decreases.

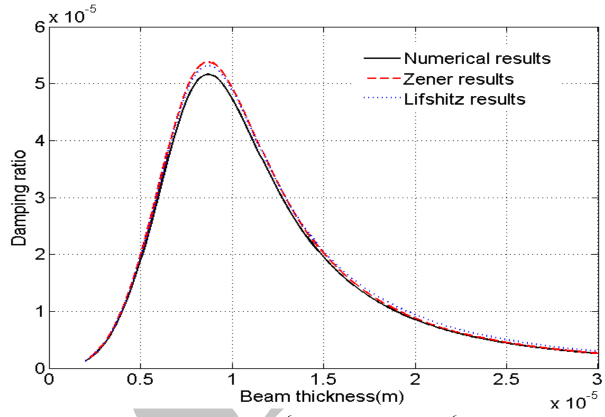
TABLE 1. Material properties of the nano-beam resonator [13]

Symbols	Values
E	169 GPa
ν	0.22
K	156 W m^{-1} K^{-1}
ρ	2330 kg m^{-3}
C_v	713 J kg^{-1} K^{-1}
α_t	2.59×10^{-6} K^{-1}
T_0	300 K

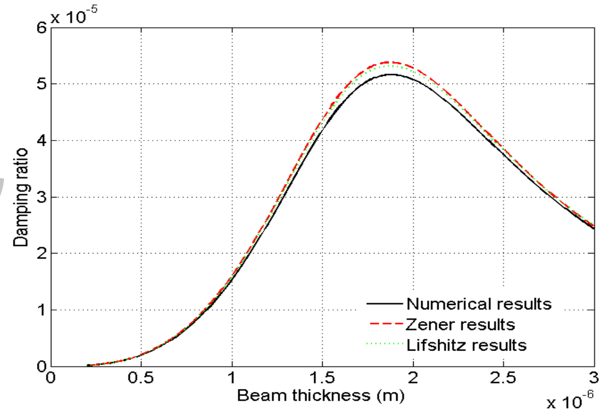
As shown in Figure 2, to provide proper verification and validation of the present numerical techniques the obtained results for TED are verified with the analytical results of Zener [8] and Lifshitz and Roukes [4]. Numerical methods have become popular with the development of the computing capabilities, and although they give approximate solutions, have sufficient accuracy for engineering purposes. Three methods were considered in Figure 2: the Zener and Lifshitz analytical methods and the Galerkin reduced order numerical method. The numerical algorithms were implemented in Matlab program. According to the obtained results, the present numerical techniques are in suitable accordance to the analytical results.

Figure 2a is the comparison of calculated results with the dimensions of $L=200\times10^{-6}\,\mathrm{m}$, $b=20\times10^{-6}\,\mathrm{m}$, Figure 2b with the dimensions of $L=200\times10^{-7}\,\mathrm{m}$, $b=20\times10^{-7}\,\mathrm{m}$ and Figure 2c with the dimensions of $L=200\times10^{-8}\,\mathrm{m}$, $b=20\times10^{-8}\,\mathrm{m}$. As shown in Figure 2 a, b and c the critical thickness changes by decreasing the length of the beam and Debye peak occur in higher amount of beam thickness and after the length of $L=200\times10^{-8}\,\mathrm{m}$ Debye peak is out of the Euler–Bernoulli assumption. Therefor, in nano scales Debye peak does not occur. It must be noted that the Debye peak takes place when the thermal characteristic time (the time necessary for temperature gradients to relax) is equal to the inverse of the beam fundamental frequency [13]. According to the relation of τ_R nano-beam vibration is in low-frequency range, $\tau_R \quad \omega^{-1}$, the vibration is isothermal and a small amount of energy is dissipated.

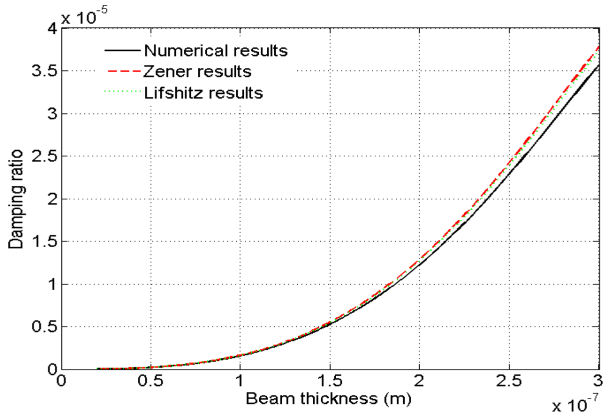
Numerical results of this article are obtained based on the Galerkin reduced order model which employs some shape functions to prepare proper solution in compare of those reported by Zener and Lifshitz but the results of Zener are obtained by a different method. Zener's exact analytical results are calculated solely from thermo-dynamical considerations. On the other hand the results of Lifshitz are obtained by neglecting some higher order terms in equations. So the differences in results of these methods are expected.



(a) $L=200\times10^{-6}\,\mathrm{m}$, $b=20\times10^{-6}\,\mathrm{m}$



(b) $L=200\times10^{-7}\,\mathrm{m}$ $b=20\times10^{-7}\,\mathrm{m}$



(c) $L=200\times10^{-8}\,\mathrm{m}$ $b=20\times10^{-8}\,\mathrm{m}$

Figure 2. Comparison of calculated results of damping ratio to Zener and LR models results for various values of beam thicknesses.

Figure 3 shows comparison of the calculated results of damping ratio to GTE and CTE models for various values of beam thicknesses with dimensions of $L = 50 \times 10^{-9}$ m , $b = 2 \times 10^{-9}$ m . Since relaxation time is very small, in micro scales the effect of relaxation time is not considerable and almost there is no difference between the results of the GTE and CTE but in nano scales it is considerable and by increasing the thickness of the beam it is more sensible in the invalid region of the Euler–Bernoulli beam assumption. So in nano scales and in the valid dimensions there is almost no difference between the GTE and CTE.

As shown in Figures 4 and 5 comparisons of calculated results of damping ratio for various values of beam thicknesses and for various values of ambient temperatures are presented. These comparisons are made for different values of the nonlocal parameter $e_0a = 0\text{nm}$, $e_0a = 2\text{nm}$, $e_0a = 4\text{nm}$ and $e_0a = 5\text{nm}$ with the dimensions of $L = 50 \times 10^{-9}$ m , $b = 2 \times 10^{-9}$ m and $h = 5 \times 10^{-9}$ m for Figure 5.

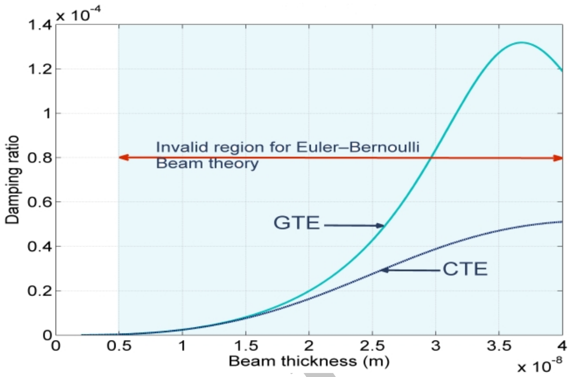


Figure 3. Damping ratio to GTE and CTE models for various values of beam thicknesses

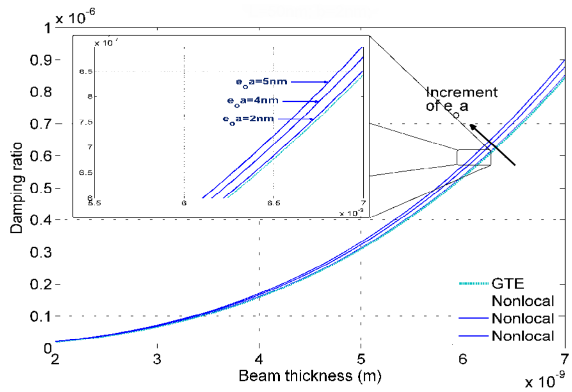


Figure 4. Damping ratio for various values of beam thicknesses and for the different values of the nonlocal parameter

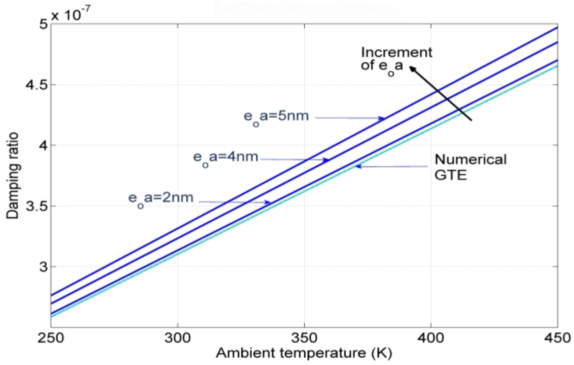


Figure 5. Comparison of calculated results of damping ratio for various values of ambient temperatures and for the different values of the nonlocal parameter

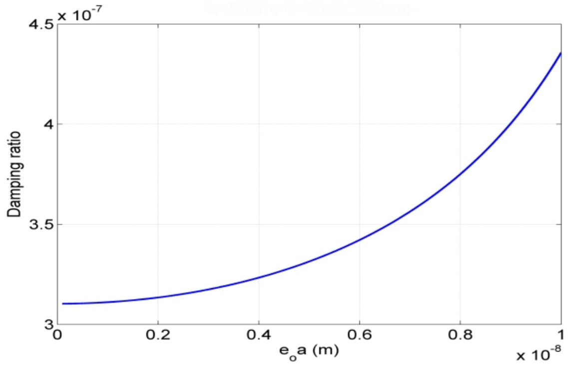


Figure 6. Damping ratio for various values of the nonlocal parameter

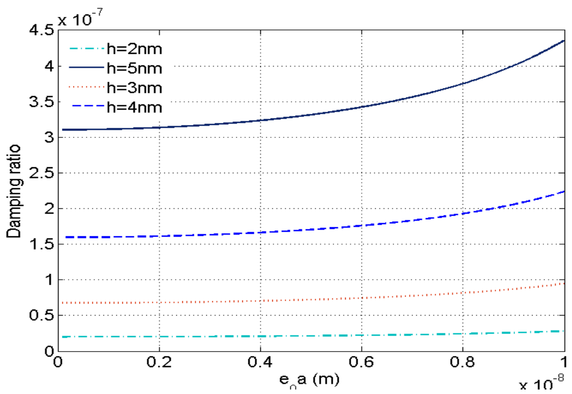


Figure 7. Damping ratio versus nonlocal parameter for different beam thicknesses

Note that when the nonlocal parameter is equal to zero, it is the same clasical model. There is considerable difference between the results of classical and nonlocal theory. By increasing the nonlocal parameter, beam thickness and ambient temperature this difference increases. As shown in the figures damping ratios predicted by the nonlocal theory are larger than those of the local (classical).

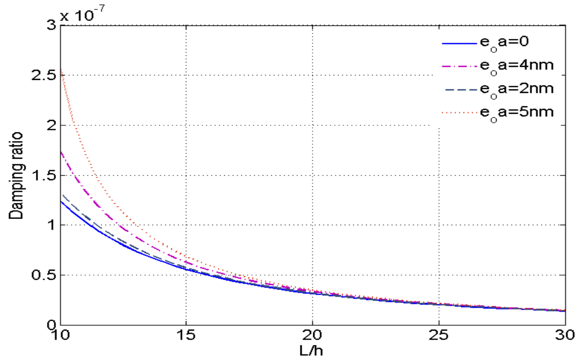


Figure 8. Damping ratio versus aspect ratio (L/h) for different ninlocal parametrs.

As shown in Figures 6 and 7 damping ratio of the beam changes considerably with changing in nonlocal parameter with the dimensions of $L = 50\times10^{-9}$ m, $b = 2\times10^{-9}$ m and $h = 5\times10^{-9}$ m for Figure 6 and various values of h such as $h = 2\times10^{-9}$ m, $h = 3\times10^{-9}$ m, $h = 4\times10^{-9}$ m and $h = 5\times10^{-9}$ m for Figure 7. Numerical results show that the nonlocal effects play an important role on the damping ratio of the nano-beam. Therefore, the small scale effects (or nonlocal effects) should be considered in the analysis of mechanical behavior of nano structures.

Figure 8 is the graphical results of the comparisons of damping ratio for various values of aspect ratio L/h and for the different values of the nonlocal parameter. In this figure we have showed the difference between the classical and nonlocal elasticity models by increasing the nonlocal parameter and also in order to magnify this difference we have plotted the figures with nonlocal parameter up to 5nm and also according to the results of Yang and Lim [37] there is no definite values for e_0a as is today and most works on nonlocal elasticity theory indicate a range of $0 \le e_0 a \le 10$. The most important observation from the figure is due to the fact that all results of nano-beam with lower aspect ratios (i.e., $L/h = 10$) are strongly affected by the nonlocal parameter. These obtained results declare that modeling based on the local (classical) beam models is not suitable, and the nonlocal beam models may offer a suitable approximation for the nano scale structures.

8. CONCLUSION

Thermo-elastic damping in a nano-beam resonator is investigated based on nonlocal elasticity and Euler-Bernoulli beam assumptions.

The effects of nonlocal parameter, aspect ratio, beam thickness and ambient temperature on the damping ratio of the nano-beam are discussed.

Numerical results showed that in nano scales and in the valid dimensions based on Euler-Bernoulli beam theory, Debye peak does not occur and also there is almost no difference between the results of GTE and CTE.

The results showed that damping ratios predicted by the nonlocal theory are larger than results of the local theory. Therefore nonlocal effects play an important role on the damping ratio of the nano-beam and results showed that with increasing the values of nonlocal parameter and also with decreasing the length of the nano-beam, difference between the results of classical and nonlocal theorise increases.

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Thermo-elastic Damping in Nano-beam Resonators Based on Nonlocal Theory

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چکیده

در این مقاله میرایی دمایی ارتجاعی در نانو تیرهای تشدید کننده بر اساس تئوری الاستیسیته غیر موضعی و فرضیات تیر اویلر–برنولی تحقیق شده است. بدین منظور، معادله حاکم بر خیز تیر از روابط تنش-کرنش مدل الاستیسیته غیر موضعی بدست آمد و نیز معادله حاکم بر میرایی دمایی ارتجاعی توسط معادله انتقال حرارت غیر فوریه دو بعدی بدست آمد. ارتعاشات آزاد نانو تیرهای تشدید کننده با استفاده از مدل کاهش مرتبه گالرکین برای مد اول ارتعاشی تحلیل شده است. در تحقیق حاضر یک نانو تیر دو سر گیردار با فرض شرایط مرزی همدمایی در هر دو انتها مطالعه شده است. این مدل غیر موضعی پارامتر طول مشخصه را در معادلات دخیل میکند که میتواند اثرات اندازه را در ریز ساختارها بررسی کند. نتایج بدست امده با نتایج عددی مدل های دمایی ارتجاعی کلاسیک مقایسه شد. اثر میرایی دمایی ارتجاعی بر روی نسبت میرایی برای ضخامت های مختلف نانو تیر و دماهای محیط مطالعه شده است. علاوه بر این، تحقیق شامل محاسبات مربوط به مقادیر متفاوت پارامتر تئوری غیر موضعی میباشد. نتایج نشان دادند که با افزایش پارامتر تئوری غیر موضعی و همچنین با کاهش طول نانو–تیر اختلاف نتایج تئوری کلاسیک و غیر موضعی بیشتر میشود.

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