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A Multi-objective Imperialist Competitive Algorithm for a Capacitated Single-allocation Hub Location Problem

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This paper presents a novel multi-objective mathematical model for a capacitated single-allocation hub location problem. There is a vehicle capacity constraint considered in this model. Additionally, our model balances the amount of the incoming flow to the hubs. Moreover, there is a set of available capacities for each potential hub, among which one can be chosen. The multiple objectives are to minimize the total cost of the networks regarding minimizing the maximum travel time between nodes. Due to the NP-hard property of this problem, the model is solved by a multi-objective imperialist competitive algorithm (MOICA). To prove its efficiency, the related results are compared with the results obtained by the well-known multi-objective evolutionary algorithm, namely NSGA-II. The results confirm the efficiency and the effectiveness of our proposed MOICA to provide good solutions, especially for medium and large-sized problems. Finally, we conclude that the proposed MOICA finds quality solutions rather than the solutions obtained by the NSGA-II algorithm.

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1. INTRODUCTION

In the network design, connecting of the terminal nods by direct links can be very costly and very busy. Therefore, it is a better traffic demand from origin nodes to destination nodes through other nodes, called hubs. For example, Figure 1 depicts the structure, in which there are direct connections between of nodes, and Figure 2 shows the network with indirect connection and three hubs. Hub facilities serve as switching and transshipment used for indirect connection between terminal nods. The aim of a hub location problem is to find location of the hubs and to assign non-hub nodes (so-called spokes) to the located hubs. In hub location problems, the cost of hub-to-hub transportation is multiplied by a reduction factor of $\alpha \in [0,1]$ due to consolidation of flows. A hub location problem is applied in airline postal [1], transportation [2], telecommunications network [3] postal network [4] and emergency services [5, 6].

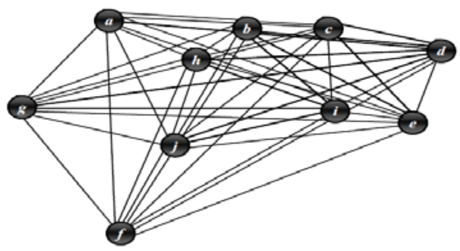

Figure 1. Fully connected network with 10 nodes and 10 origin/destination pairs

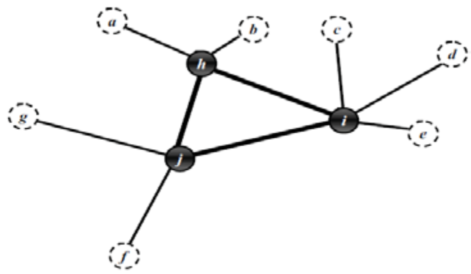

Figure 2. Network with indirect connection and three hubs

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The classical objective function in a hub location problem is divided to five major classes, namely *p*-hub median, hub location with fixed cost, *p*-hub center, hub covering and hub edge location problems. In the *p*-hub median problem, locating *p*-hubs and assigning spokes to hub nodes are the main purpose of the *p*-hub problems in such a way that the total transportation cost is minimized. The hub net work consists of two basic types, namely single and multiple allocations. In single allocation, each non-hub node is allocated to unique hub, as shown Figure 3. Many of delivery systems are in a form of single allocation. O'Kelly [7] produced the first mathematical model for the hub location problem. He proposed a quadratic integer programming for a single allocation *p*-hub median problem. Campbell [5] presented the first linear integer programming formulation for the single-allocation *p*-hub median problem. A number of researchers have discussed single allocation hub problems [4, 8-12]. In multiple allocations, each non-hub node can be allocated to more than one hub, as shown in Figure 4. In an air network, many of origins allocate to more than one hub. Campbell [13] produced the first linear integer programming formulation for multiple allocation *p*-hub median problems. In addition, other researchers studied multiple allocation hub problems [14-17].

Aim of a hub location problem with the fixed cost is similar to *p*-hub median problem, but in this problem, the fixed cost of locating hub is considered. O'kelly [18] proposed the single allocation hub location problem with fixed cost of installation hub such that the number of hub is not fixed. Since the number of hubs is not fixed, hub location problems with fixed cost can be classified according to the capacity constraint: capacitated and incapacitated hubs.

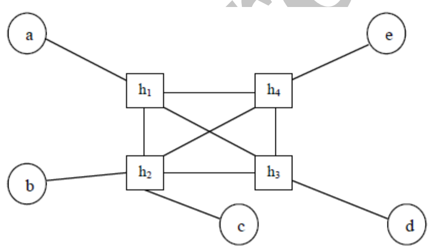

Figure 3. Single-allocation hub location problem

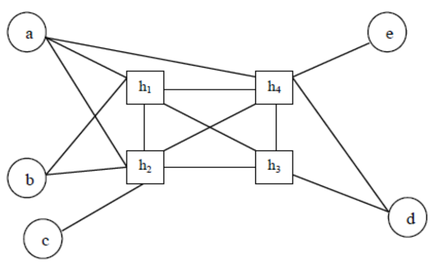

Figure 4. Multiple-allocation hub location problem

There are different kinds of capacity constraints in hub problems; a limitation on amount of incoming flow to the hub facilities. In addition of limits on amount of flow, there is another limitation on the number of spoke nodes that can be allocated to a single hub [19]. Campbel [5] and Abdinnour-Helm and Venkataramanan [20] presented incapacitated hub location models.

In this paper, we present a bi-criteria model, whose objectives conflict each other (i.e., decreasing one criterion implies an increase in other criterion. Hub location belongs to the class of NP-hard problems. Then, we propose the imperialist competitive algorithm (ICA) and NSGA-II to solve the capacitated single allocation multiple objective hub location problem**.** This paper is structured as follows. Section 2 describes the research background. A brief description of the research design and the problem formulation are provided in Section 3. Section 4 proposes the imperialist competitive algorithm. The experimental results are presented in Section 5. Finally, Section 6 presents conclusion of this paper.

2. RESEARCH BACKGROUND

Campbel [5] presented the first *p*-hub center problem and defined three types of this problem. Other studies are considered by Kara and Tansel [21]; Baumgartner [22]; Ernest et al. [23]; Sima et al. [24] and Alumur et al. [12].The aim of the hub covering problem is to design a hub-spoke network such that a distance pair of nodes do not exceed from a definite cover radius. This problem is divided two types, namely the set-covering problem and the maximal hub-covering problem. The set-covering problem is to locate hubs such that all of the nodes are covered and the cost of opening hub facilities is minimized [5, 25]. The maximal hub-covering problem is to locate a given number of hubs such that the covered demand is maximized [5, 26].

The aim of a hub edge location problem is to locate *q* hub edges such that the total cost transportation in the network is minimized. In this problem, hub networks are not necessarily complete. Nickel et al. [27] presented a hub location problem with a fixed cost for locating hub arcs. Their model is applied in urban public transportation networks. Podner et al. [28] proposed a new network design problem, in which hub arcs is located. Campbel et al. [29] presented a hub edge location problem. Yoon and Current [30] discussed multiple allocation incomplete hub location problems that minimize the total transportation costs and fixed cost of locating hubs and hub links. Calik et al. [31] presented the single allocation incomplete hub covering problem. Their model is to locate hubs and hub links to be established between the located hubs. The most of previous studies, the objective function considers only one of the cost or time criteria. Costa et al. [32]

presented a study considered a bi-criteria model with two objectives (i.e., cost and process time of hubs), simultaneously. The first and second models minimize the entire transportation cost model and the maximum processing time in the hub, respectively.

Various heuristic methods have been proposed for solving this problem. Simulated annealing (SA) for the single allocation *p*-hub median problem was presented by Ernst and Krishnamoorthy [4], Abdinnour-Helm [33]. Tabu search (TS) for the single allocation *p*-hub median problem was developed by Skorin-Kapov and Skorin-Kapov [34], Klincewicz [35]. Ernest and Krishnamoorthy [36] presented a branch-and-bound method with the initial upper bound provided by SA and random descent heuristic methods for a capacitated single allocation *p*-hub location problem. A branch-and-cut algorithm based on investigations of some polyhedral properties of the capacitated single allocation hub location problem is studied by Labbe et al. [37]. Contras et al. [38] presented the heuristic method to produce Lagrangian dual feasible solutions for a capacitated single allocation hub location problem. The first heuristic method for single allocation *p*-hub center problem is proposed by Pumak and Sepil [39]. Ernest et al. [23] presented five heuristic algorithms for a single allocation *p*-hub center problem and analyzed their worst case performances. Gavriliouk [40] studied the heuristic method for single and multiple allocation *p*-hub centers.

As mentioned above, a variety of hub location models have been studied during the last decades. However, studies on competitive hub location problems are scarce. In a real situation, several firms usually exist in a market and compete with each other in order to capture the market share. We can easily imagine that hub locations are affected by competing with rival firms. Marianov et al. [41] formulated a competitive hub location problem on a network, which seems to be the first hub location model considering competition. In their model, the sum of captured flows is maximized under some passengers' allocation rules. Sasaki et al. [16] considered a continuous hub location model, in which two firms of a similar size locate their own new hubs in an arbitrary order, and formulated a leader's problem as a bi-level programming problem. The numerical experiments reported in article [16] show that the leader firm may suffer heavy losses if it neglects to consider the competitors' strategies.

Hub location problems are NP-hardness with the exception of a few special cases. They are usually much harder to solve in comparison with other non-hub location problems. Moreover, there does not exist a general mathematical model that describes well all hub location problems. Each hub problem has its own specific structure, namely objective function, decision variables and constraints. A few additional constraints or a slight modification of the problem structure can substantially change the computational behavior of the designed solution approach. Therefore, there is no general algorithm for solving all hub problems, or at least a smaller group of them. Exact methods cannot provide solutions for large-scale hub location problems, which arise from practice in a reasonable amount of time. Therefore, heuristic methods are very promising approaches for solving hub location problems. A detailed review of hub location problems and solution methods for solving them can be found in article [41].

3. PROBLEM FORMULATION

Alumur et al. [12] presented the single allocation incomplete *p*-hub median, the incomplete hub location with fixed costs, the incomplete hub covering and the incomplete *p*-hub center problems. Our problem is an extension of the Alumur's model. Our model minimizes the cost and time between nodes simultaneously as the hubs are incomplete. This model balances the flow between hubs. In addition, the vehicle capacity constraint is considered. The problem is formulated according to the following assumptions:

- The hub network is not necessary complete.
- There are economies of scale incorporated by a discount factor $\alpha\in[0,1)$ for using inter-hub connections
- The demand for every node is fixed.
- The cost transportation of one unit of flow between a spoke and a hub based on distance is fixed and definite.
- Each spoke allocate to only one hub(i.e., single allocation)
- Different capacity levels are available for a potential hub to be located at node k, among which one can be chosen ($k\in N$).
- Fixed cost of locating a hub with capacity of level q at nod k is definite. ($k\in N$, $q\in Q_k$)
- Transportation of commodity from origin to hub is not continuous.
- The number of vehicles is limited since there is traffic.
- The capacity of vehicle is definite.
- Vehicle capacity constraint and capacity restrictions amount of incoming flow to the hub while the balancing requirements of incoming quantities of flow to the each hub are considered.

In the previously published papers it is assumed that commodities arrive to the origin node and they are sent to hubs in that moment. However, in the real world, such thing is not benefit unless hub system involvs time sensitive items. For example, if there are 50 packets in the origin node and the capacity of each vehicle is 10 packets, in this situation these packets are transported by 5 vehicles. However, if the number of packets is 55, then $[\frac{55}{10}]$ =5 vehicles is assigned and 5 commodities are remained. In this case, there are two states. In the first state, a vehicle should be waited to complete and then move that we have more time rather than the average real time between an origin and a hub. Therefore, each commodity includes the delay cost. In the second state, a vehicle should be moved with incomplete capacity moves. Since the transportation cost of a vehicle is divided by the number of commodities, so the cost of transportation of a unit flow is heavier. However, in this instance, commodities are sent to a hub from the origin with a less time rather than the average real time. Using the extended vehicle in the hub is not applicable since hubs are more congest.

In this paper, one of assumptions refers to the balance parameter. In the instance with 8 nodes, the optimal network design is given in Figure 5. In the optimal solution, nodes 3, 4 and 5 are selected to become hubs. Nodes 1 and 7 are allocated to hub 3 (incoming 262 commodities to hub 3), nodes 2 and 8 are allocated to hub 5 (incoming 159 commodities to hub 5) and node 6 is allocated to hub 4 (incoming 189 commodities to hub 4). In this case, the difference between the maximum and the minimum amount of flows assigned to some hubs (nodes 3 and 5) is equivalent to 103. Using the balance parameter,

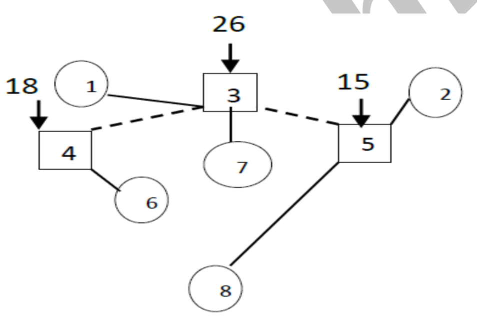

Figure 5. Optimal network design (instance)

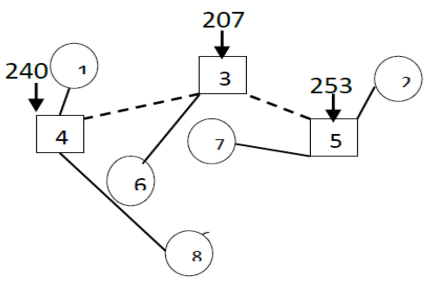

Figure 6. Optimal network design with the balance parameter

the quantity of difference can be lesser. The balancing requirement is also produced in order to reduce the traffic in some hub. If the balancing parameter constraints are used, the quantity of difference should be at most equal to 46 units. In this case, the optimal solution is changed (Figure 6). Moreover, hubs 4 and 5 now operates with their second capacity level in respect to 250 and 300, respectively.

3. 1. Notations

3. 1. 1. Subscripts

$N=\{1,\ldots,n\}$ Set of nodes.

$H=\{1,\ldots,h\}$ Set of potential hub ($H\in N$).

$Q_k=\{1,\ldots,s_k\}$ Set of the difference capacity levels available for the potential hub to be installed at node k, where $k\in$ H.

3. 1. 2. Input Parameters

w_{ij} Total flow to be sent from node i to node j ($i,j\in N$).

t_{ij} Average time of sending the flow from node i to node j ($i,j\in N$).

c_{ij} Cost for sending one unit of the flow from node i to node j ($i,j\in N$).

w_T Capacity for each vehicle

nv At most number of the extension vehicle in a path origin-hub

c_d Delay cost for one unit of the remaining flow at each hour

FH_k^q Fixed cost for installing a hub with capacity level q at node K ($k\in H$, $q\in Q_k$).

FL_{ij} Fixed cost for installing a hub link i-j ($i,j\in H$).

Γ_k^q Capacity of a hub installed at node k with a level of capacity q ($k\in H$, $q\in Q_k$).

α Constant cost discount factor for travel on the inter-hub connections.

α' Constant time discount factor for travel on the inter-hub connections.

θ Maximum value allowed for the difference between the maximum and the minimum amount of flows assigned to the some hubs.

3. 1. 3. Decision Variables

T_{ik} Time for sending the flow from node i to node k ($i\in N$, $k\in H$).

c'_{ik} Cost for sending the total flow originating at node i to hub k.

f_{ij}^k Amount of flows with origin at k that goes through hubs i and j ($k\in N$, i, $j\in H$).

a Lower limit on the minimum of the flow are allocated to some hubs.

b Upper limit on the maximum of the flow are allocated to some hubs.

d_{ij} Time of transportation between hubs i and j ($i, j \in H$).

x_{ij} 1 if node i is assigned to hub j; 0, otherwise ($i \in N, j \in H$);

y_i 1 if origin i uses of an extension vehicle; 0, otherwise ($i \in N$).

z_{ij} 1 if a link hub is established between hubs i and j; 0, otherwise ($i,j \in H$).

e_k^q 1 if node k receives a hub with capacity level q; 0, otherwise ($k \in H, q \in Q_k$).

s_{ijk} 1 if a spanning tree rooted at hub k using of links i and j; 0, otherwise ($i,j,k \in H$).

3. 2. Mathematical Model

$$\min \sum_i \sum_k (c'_{ik} + \sum_j w_{ji} c_{ik}) x_{ik} + \alpha \sum_{i\epsilon H} \sum_{j\epsilon H, i\neq j} \sum_{k\epsilon N} c_{ij} f_{ij}^k + \sum_{q\epsilon Q} \sum_{k\epsilon H} FH_k^q\, e_k^q + \sum_{i\epsilon H} \sum_{j\epsilon H; j>i} FL_{ij} z_{ij} \quad (1)$$

$$\min \max \quad r1_k + \alpha' d_{kj} + r2_j k, j\epsilon H \quad (2)$$

s.t.

$$c'_{ik} = (\left[\frac{\sum_{j\epsilon N} w_{ij}}{w_T}\right] + 1) c_{ik} W_T y_i + \ (c_{ik} \sum_{j\epsilon N} w_{ij} + (\frac{\sum_{j\epsilon N} w_{ij} - \left[\frac{\sum_{j\epsilon N} w_{ij}}{W_T}\right] W_T}{\sum_{j\epsilon N} w_{ij}})\, t_{ik} c_d (\sum_{j\epsilon N} w_{ij} \left[\frac{\sum_{j\epsilon N} w_{ij}}{W_T}\right] W_T))\, (1- y_i) if \ \sum_{j\epsilon N} w_{ij} \neq nW_T \quad (3)$$

$$c'_{ik} = \sum_j w_{ij} c_{ik} \qquad if \quad \sum_j w_{ij} = nW_T \quad (4)$$

$$T_{ik} = \frac{\sum_{j\epsilon N} w_{ij}}{(\left[\frac{\sum_{j\epsilon N} w_{ij}}{W_T}\right] + 1) W_T} t_{ik} y_i + (\frac{\sum_{j\epsilon N} w_{ij} - \left[\frac{\sum_{j\epsilon N} w_{ij}}{W_T}\right] W_T}{\sum_{j\epsilon N} w_{ij}}) \quad (5)$$

$$t_{ik}(1 - y_i) if \ \sum_{j\epsilon N} w_{ij} \neq nW_T$$

$$T_{ik} = t_{ik} \qquad if \ \sum_j w_{ij} = nW_T \quad (6)$$

$$y_i \leq 1 - x_{ii} \forall \ i\epsilon H \quad (7)$$

$$\sum_{i\epsilon N} y_i \leq nv \quad (8)$$

$$r1_k \geq T_{ik} x_{ik} \forall k\epsilon H , \ i\epsilon N \quad (9)$$

$$r2_j \geq t_{ij} x_{ij} \forall j\epsilon H \ , \ i\epsilon N \quad (10)$$

$$\sum_{k\epsilon H} x_{ik} = 1 \forall i\epsilon N \quad (11)$$

$$x_{ij} \leq x_{jj} \forall i\epsilon N \ , \ j\epsilon H \quad (12)$$

$$z_{ij} \leq x_{ii} \forall i, j\epsilon H: i < j \quad (13)$$

$$z_{ij} \leq x_{jj} \forall i, j\epsilon H: i < j \quad (14)$$

$$\sum_{j\epsilon H: j\neq i} f_{ij}^k - \sum_{j\epsilon H: j\neq i} f_{ji}^k = \sum_{l\epsilon N} w_{kl} x_{ki} - \sum_{l\epsilon N} w_{kl} x_{li} \forall i\epsilon H , k\epsilon N \quad (15)$$

$$f_{ij}^k + f_{ji}^k \leq \ \sum_{l\epsilon N} w_{kl} z_{ij} \forall i, j\epsilon H: i < j , k\epsilon N \quad (16)$$

$$\sum_{i\epsilon H: j\neq i} s_{ijk} \geq x_{kk} + x_{jj} - 1 \forall k, j\epsilon H: j \neq k \quad (17)$$

$$\sum_{i\epsilon H: j\neq i} s_{ijk} \leq x_{kk} \forall k, j\epsilon H: j \neq k \quad (18)$$

$$s_{ijk} + s_{jik} \leq z_{ij} \forall i, j, k\epsilon H: i < j \quad (19)$$

$$d_{kj} \geq (d_{ki} + t_{ij})\, s_{ijk} \forall i, j, k\epsilon H: i \neq j \ , j \neq k \quad (20)$$

$$d_{ij} = d_{ji} \forall i, j\epsilon H: i \neq j \quad (21)$$

$$d_{kk} = 0 \forall k\epsilon H \quad (22)$$

$$\sum_{i\epsilon N} \sum_{j\epsilon N} w_{ij}\, x_{ik} \leq \sum_{q\epsilon Q} e_k^q\, \Gamma_k^q \forall k\epsilon H \quad (23)$$

$$\sum_{q\epsilon Q} e_k^q \leq 1 \forall k\epsilon H \quad (24)$$

$$\sum_{i\epsilon N} \sum_{j\epsilon N} w_{ij}\, x_{ik} \ + M(1 - x_{kk}) \geq a \forall k\epsilon H \quad (25)$$

$$\sum_{i\epsilon N} \sum_{j\epsilon N} w_{ij}\, x_{ik} \ - M(1 - x_{kk}) \leq b \forall k\epsilon H \quad (26)$$

$$b - a \leq \theta \quad (27)$$

$$a, b \geq o \quad (28)$$

$$x_{ij} \epsilon \{0,1\} \forall i \ \epsilon N , j\epsilon H \quad (29)$$

$$y_i \epsilon \{0,1\} \forall i \ \epsilon N \quad (30)$$

$$d_{ij} \geq 0 \forall i, j\epsilon H \quad (31)$$

$$s_{ijk} \epsilon \{0,1\} \forall i, j, k\epsilon H: i \neq j \ , j \neq k \quad (32)$$

$$z_{ij} \epsilon \{0,1\} \forall i,j \ \epsilon H: i < j \quad (33)$$

$$e_k^q \epsilon \{0,1\} \forall \quad q \ \epsilon Q , k\epsilon H \quad (34)$$

$$f_{ij}^k \geq 0 \forall i,j \ \epsilon H: i \neq j ,k \ \epsilon N \quad (35)$$

The objective function (1) minimizes the costs for collection, transfer, distribution and the costs of establishing the hubs and the hub links. The objective (2) minimizes the maximum of travel time between pairs of origin-destination. The first term of Constraint (3) states the transportation cost from an origin to a hub where more vehicles are used. The second term is the transportation cost of the total flow originating at node i to hub k and the delay cost of the remaining flow.

Constraint (4) calculates the transportation cost of the total flow originating at origin i while the total flow originating at node i is equal to the coefficient of capacity of vehicle.

The first term of Constraint (5) is stated such that the extension vehicle is used from origin i to hub k, variable T_{ik} is equal to $\frac{\sum_{j\epsilon N} w_{ij}}{\left(\left\lfloor\frac{\sum_{j\epsilon N} w_{ij}}{W_T}\right\rfloor+1\right)W_T} t_{ik}$. In this case, the range of coefficient of variable t_{ik} is (0,1). It means that the travel time based on the amount of the flow from the origin to the hub is multiplied by a producer coefficient. Therefore, the flow is destination earlier from an average real time. The second term of Constraint (5) calculates the variable T_{ik} when we do not apply the extension vehicle. In this case, variable T_{ik} is equal to $\left(\frac{\sum_{j\epsilon N} w_{ij}-\left\lfloor\frac{\sum_{j\epsilon N} w_{ij}}{W_T}\right\rfloor W_T}{\sum_{j\epsilon N} w_{ij}}+1\right) t_{ik}$. In this paper, the range of the coefficient of variable t_{ik} is (1, 2). The term means that the major of the amount flow is transported by an integer number of vehicles. The remaining segment, which is lesser than the vehicle capacity, includes the delay time and the travel time based on amount of flow from the origin to the hub which is multiplied by a multiplier coefficient and therefore destination posterior from the average real time.

Constraint (6) computes the travel time from an origin to a hub, while the total flow originating at node i is equal to a multiple vehicle capacity. Constraint (7) states that the hub node is not applied more vehicles. Constraint (8) states that the number of the extended vehicle should be at most equal to a parameter since there is a limitation of traffic in the path. Constraints (9) and (10) states that a radius of a hub is greater than or equal to the collection and distribution time of going to any node allocated to this hub and the radius of spoke can be very small (i.e., close to zero), respectively. Constraint (11) assures that each spoke is allocated to exactly one hub. Constraint (12) states that a spoke can be only allocated to a hub node. Constraint (13) and (14) assure that a hub link i, j can be only established such that nodes i and j are hub nodes. Constraint (15) is the balance equation for the flow. The term on the left-hand side of the constraint calculates the flow within the hub network and the term on the right-hand side and calculates the flows allocated to the hub. In fact, the total entering flow originating from node k to hub i should be equal to the outgoing flow.

Constraint (16) states that variable f_{ij}^{k} to be positive if only hub link i, j is established. Constraint (17) states that the degree for each hub is at least one, and the tree rooted at hub k has an entering arc into every other hub j. Constraint (18) ensures that spanning tree rooted can be only associated with a hub and assures that each spanning tree rooted at hub k can enter at most one arc to another hub node j. The spanning tree arcs to be hub arcs stated by Constraint (19). Constraint (20) displays that the time needed to travel between hubs is calculated by the established spanning tree rooted in the hub network. Constraint (21) states that travel time inter-hub is symmetric. Constraint (22) states that travel time from a node to itself will be zero. Constraint (23) is the capacity constraints of the hub. Constraint (24) assures that for each hub one size is chosen. Constraints (25) and (26) state that the lower and upper limits on the minimum and maximum amount of the flow are allocated to some hubs, respectively. Constraint (27) states that the difference between the maximum and minimum amounts of the flow allocated to some hubs should be at most equal to parameter θ.

4. IMPERIALIST COMPETITIVE ALGORITHM

The imperialist competitive algorithm (ICA) is a new algorithm in evolutionary computation. It is inspired by a socio-political process of imperialistic competition in the real world. This algorithm starts with an initial random population, named countries. Some of the best countries (i.e., with the least cost) are chosen to be the 'imperialists' and the rest are the 'colonies' of these imperialists. All of the colonies of the initial population are divided among the imperialists based on their power. The power of an empire is equivalent to the objective function of the proposed model.

After distribution of all colonies among imperialists, these colonies belong to their relevant imperialist country. The total power of an empire is affected by both the power of the imperialist country as a central core and the power of its colonies. Therefore, we define the total power of an empire by the sum of the power of the imperialist country and a percentage of the mean power of its colonies to their imperialists. Then, imperialistic competition begins among all the empires. Any empire, which cannot succeed and increase its power in this competition, is eliminated from the competition. Therefore, in the imperialistic competition process, the powerful empires gradually increase their power and weaker empires lose their power, and ultimately they collapse. Then, all of the empires are collapsed and there is only one empire and all the other countries are colonies of the maintained empire. Following, we explain the steps of the proposed ICA.

4. 1. Solution Representation In this problem, we use the three matrices for representation of the solution. The first matrix with the 2×$n$dimension (where n is equal to the number of nodes in the network), shows the location of hub nodes and the allocation of non-hub nodes to the hubs at the first row and the chosen capacity level for each hub at the second row. For example, suppose we have 5 nodes and 2 capacity levels, as shown below.

1	2	1	2	2
2	1	2	1	1

Nodes 1 and 2 are hubs. The second and first capacity levels are chosen for hubs 1 and 2, respectively. In addition, nodes 3, 4 and 5 are allocated to hubs1, 2 and 2, respectively. The second matrix ($n \times n$) consists of binary cells. It shows that the installation hub links. In addition, the third matrix ($1 \times n$) with a binary cell shows which spoke uses more vehicles.

4. 2. Initial Population Initially, any solution is made by the designed procedure, so called construct. Then the improvement procedure, so called parallel neighborhood search method, is applied for this solution to create the initial population. If the product solution is not repeated, then it is added to the population. The construction procedure and parallel neighborhood search method are explained below.

4. 2. 1. Construction Procedure Step 0- Continue until all of the nodes is considered.

Step 1- One of the nodes, which is not considered until now, is chosen as a hub at random.

Step 1-1- One of the capacity levels is chosen for hub at random.

Step 1-2- Indices, which are not considered until now, are chosen from the remaining nodes randomly. They are allocated to the located hub based on the capacity of the mentioned hub while the zero index is not produced. In addition, the second row is quantified according to the definite capacity level for the hub.

Step 2- If all of the nodes are considered, then go to the latter step; otherwise, go to the first step.

Step 3- Randomly, select 0 or 1 for each node i and j that are hubs in the second matrix, and instead on the relevant element.

Step 4- Randomly; select 0 or 1 for the element of non-hub nodes in the third matrix. Feasibility qualification for the values for matrix (nv) should be considered.

Step 5- Finish.

4. 2. 2. Parallel Neighborhood Search Procedure This procedure consists of four neighborhood search operations, which are applied for the input solution in simultaneity or parallel. Then, one solution is selected from the input solution and solutions of these four operations on the selected solution. The first neighborhood search operation is applied for the first matrix. Randomly, two hubs is selected and one of the non-hub nodes allocated to located hubs (according to the feasibility qualification similar to the capacity limitation) is chosen and exchanged. The second neighborhood search operation is applied for the second matrix. Randomly, two indices are chosen from the hub nodes. If the value of the relevant element is 1, then it is transmuted to 0 or if it is equaled to 0, it is transmuted to 1. In addition, the third neighborhood search operation is applied for the first matrix. Randomly, the index of one of the hub nodes is selected and its capacity level is changed to one of the available levels in according to the capacity limitation. The quarter neighborhood search operation is applied for the third matrix. Randomly, one of the non-hub nodes is chosen and if the value of this element is equaled to 1, it is changed to 0 or if it is equaled to 0, it is transmuted to 1. The general structure of this procedure is as follows: the size of population=N, input solution=s, first operation=$ls1$, second operation=$ls2$, third operation=$ls3$ and quarter operation=$ls4$.

Step 0- Quantify 0 to the counter.

Step 1- Apply $ls1$ for s, and obtain $s1$.

Step 2- Apply the improvement method (parallel neighborhood search procedure) for $s1$.

Step 3- Apply $ls2$ for s, and obtain $s2$.

Step 4- Apply the improvement method for $s2$.

Step 5- Apply $ls3$ for s, and obtain $s3$.

Step 6- Apply the improvement method for $s3$.

Step 7- Apply $ls4$ for s, and obtain $s4$.

Step 8- Apply the improvement method for $s4$.

Step 9- Choose one solution from the s, $s1$, $s2$, $s3$ and instead on s.

Step 10- Add one unit to the counter. If the counter is lower than the most of the definite repetition, then go to Step 1; otherwise, go to Step7.

Step 11- Report solution s as an output and finish the procedure.

4. 3. Selection Method We select one solution from the input solution and four solutions from the produced solution according to the parallel neighborhood search procedure. In this selection method, non-dominate relation is initially applied for five solutions and we select a solution that is not dominated by other solution. When some solutions are not dominated by other, we select solutions with the maximum distance from the dynamic idealistic point.

4. 4. Dynamic Idealistic Point This point is a matrix ($1 \times n$), in which n is equal to the number of the objectives. The values of the cells are the minimum values. The dynamic idealistic point is updated by any obtained solution. The objectives of any obtained solution are compared with the relevant value in the dynamic idealistic point. If its value is less than the value in the dynamic idealistic point, then it is instead on the relevant value in the dynamic idealistic point cell.

4. 5. Improvement Method Applying the parallel neighborhood search procedure for a solution is may be the infeasible produced solution. Therefore, an improvement method is designed in this work that applies for the produced solution. It transmutes an infeasible solution to a feasible solution. The second and third structures produce feasible solution in the respective capacity; however, the constraints related to quantity of commodity may be defected. Therefore, the improvement method updates the solutions according to two constraints of the model (i.e., Constraint 15 and 16). The solutions produced by the quarter structure may be defected by Constraint (8). So improvement methods apply for the quarter matrix and produce a feasible solution.

4. 6. Evaluation Function In this paper, these solutions are classified in to the non-dominated fronts and the crowing distance is measured for each solution relevant to its rank. Following, evaluation function(c_s) is calculated for every solution [41]:

$$C_s = \frac{rank}{crowding-distance} \quad (36)$$

The evolution function C_s is computed for each solution, and then the solutions are sorted in the descending order based on C_s. $N_{imp}$of the most powerful countries (i.e., with the minimum cost) are selected to be imperialists. The remaining countries are colonies (N_{col}) that are allocated to an emperor based on their powers. To distribute the colonies among imperialists, the normalized cost for each emperor is computed by:

$$C_n = max_i c_i - c_n \quad (37)$$

where, C_n and c_n are the normalized cost for the *n*th emperor and the cost of the *n*th emperor, respectively. Then, the proportional power of each imperialist is calculated below, and based on that the colonies are allocated to the imperialist countries.

$$P_n = \frac{c_n}{\sum_{i=1}^{N_{imp}} c_i} \quad (38)$$

On one side, the normalized power of an imperialist is assessed by its colonies. Thus, the initial number of colonies of an empire is computed by:

$$N \times C_n = round\,\{P_n \times (N_{col})\} \quad (39)$$

where, $N \times C_n$ and N_{col} are the initial number of colonies of the *n*th empire and the number of all colonies, respectively. After that, to distribute the colonies among imperialist, $N \times C_n$of the colonies are chosen randomly and allocated to their imperialist.

4. 7. Assimilation Methods Imperialist countries absorb the colonies countries toward themselves based on the absorption policy. In this problem, half of the hubs of the emperor are chosen randomly and located instead of the same number of hubs with the same capacity level in each colony and the remaining nodes are allocated to the located hubs according to their capacity limitations.

4. 8. Revolution Policy One of the colonies of every emperor is chosen randomly and given to the parallel neighborhood search procedure as an input. Then, the output is instead of the colony.

4. 9. Calculation of the Cost of Imperialism The total power of an empire is affected by the power of its both parts (i.e., the emperor power and percents of its average colonies power). Therefore, the mathematical equation of the total cost is computed by:

$$T \times C_n = Cost(\text{emperor}_\text{n}) + \gamma \times mean\{Cost\,(\text{Colonies of emperor}_n)\} \quad (40)$$

where, $\mathrm{T} \times \mathrm{C_n}$ is the total cost of the *n*th imperialism? In addition, $\gamma$is a positive factor considered between 0 and 1. According to the upper relation, a little value for γcauses the cost of the *n*th imperialism closes to the cost of the *n*th emperor. In this paper, we consider γ is 0.3.

4. 10. Imperialistic Competition After computing the cost of empire, one (or some) of the weakest colonies is chosen from the weakest empires. Then, competition is made among all empires to possess this (or these) colony (or colonies). Based on the total power of empire during this competition, the most powerful empires are more likely to possess the mentioned colonies. Therefore, the weakest empire collapses in the imperialistic competition when it loses all of its colonies and their colonies are seized with a stronger imperialism. This process is continued until just one empire exists in the world.

4. 11. Comparison Metrics There are various metrics for evaluation of quality and diversity of the multi-objective evolutionary algorithm. In this paper, three metrics is considered as follows.

a. Quality Metric: This metric is used for comparison of the quality of the Pareto solutions obtained by any method. According to this metric, all of the Pareto solutions obtained by each method are ranking. The superior percent shows that the quality of this algorithm is better. Then, the percent solutions of any method in the first rank are definite.

b. Spacing Metric: This metric tests a uniform distribution of the obtained Pareto solutions in a solution front. This metric is defined by:

$$S = \frac{\sum_{i=1}^{N-1}|d_{mean} - d_i|}{(N-1) \times d_{mean}} \quad (41)$$

c. **Diversity Metrics:** This metrics specifies the measure of non-dominated solution on the optimal front. The diversity metrics is computed by:

$$D = \sqrt{\sum_{i=1}^{N} max\|x_t^i - y_t^i\|} \quad (42)$$

5. EXPERIMENTAL RESULTS

In this section, the effectiveness of the proposed ICA is compared with the non-dominated sorting genetic algorithm (NSGA-II). These algorithms are run on machine Intel Core i5-540M processor with 2.53 GHz and 4 GB of RAM.

5. 1. Test Data There are capacity levels for each potential hub in the case of the CSAMOHLP. The AP (Australian Post) data set introduced by Ernst and Krishnamoorthy [36]. Accordingly, instances with 10, 15, 20 and 25 nodes are considered. The number of capacity levels available for each hub is considered as 2, 3 and 4. The largest capacity is equal to the capacity of the AP data set instances. Then, each capacity level is equal to 70% of the capacity of the upper level ($\forall k \in H\ \Gamma_k^{s_k} = \Gamma_k$ and $\Gamma_k^q = 0.7 \times \Gamma_k^{q+1}, q = 1,..,s_k - 1$), recursively. We consider two types of the capacity of the AP data set, namely tight and loose. We took the fixed costs of opening each hub *fhk*= 100. The fixed costs of establishing hub links between the nodes of the network are calculated by:

$$L_{ij} = \frac{d_{ij}/w_{ij}}{\max_{i,j}\{d_{ij}/w_{ij}\} \times 100} \qquad \forall\, j, i \neq j \quad (43)$$

where, d_{ij} is the distance between nodes *i* and *j*, and w_{ij} is the flow between nodes *i* and *j*. For the AP data, we assumed that *H*=*N* in all at the instances.

5. 2. Parameters Tuning for ICA and NSGA-II The parameters of the ICA algorithm are the number of imperialists (i.e., Imp-Num) and the number of population (i.e., Popsize). The considered levels of the parameters are shown in Table 1. In addition, the NSGA-II algorithm has three parameters, namely mutation rate, crossover rate and maximum iteration, whose levels are shown in Table 2. The associated results are analyzed by the Taguchi design method in design of experiments (DOE) and implemented in MINITAB 16. The analysis of the ICA algorithm for the levels Imp-size and Pop-size factors are shown in Figure 7. In this figure, it can be seen that the Imp-Num=10 and Pop-size=120 results in a better output in comparison with the other values in the ICA algorithm. Furthermore, the analysis of the NSGA-II algorithm for the levels of the mutation rate, crossover rate and *Max-Iteration* factors are shown in Figure 8. The Imp-Num=10 and Pop-size=120 give a better output in comparison with the other values used in the ICA algorithm based on Figure 7. In addition, we set the parameters for the NSGA-II according to Figure 8. These parameters are the mutation rate=0.2, crossover rate=0.7 and *max_iteration*=400.

TABLE 1. Considered levels of the ICA parameters

Parameter	Level 1	Level 2	Level 3
Imp-Num	10	14	16
Pop-size	70	100	120

TABLE 2. Considered levels of the NSGA-II parameters

Parameter	Level 1	Level 2	Level 3
Mutation Rate	0.1	0.2	0.3
Crossover Rate	0.6	0.7	0.8
Max_Iteration	400	500	1000

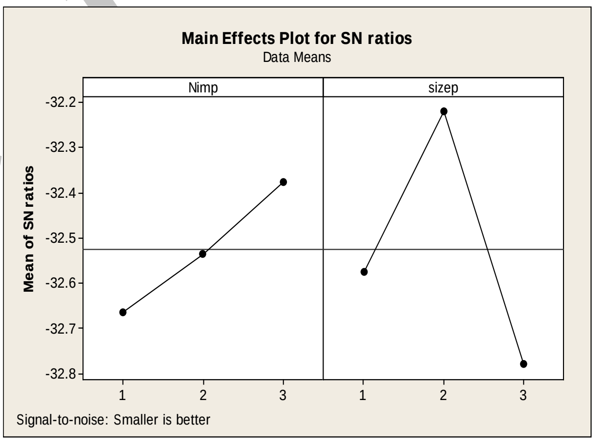

Figure 7. Analysis of the ICA algorithm for the levels of imp-size and pop-size factors

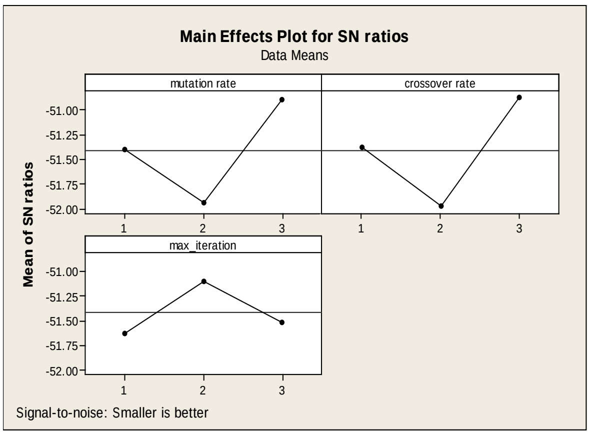

Figure 8. Analysis of the NSGA-II algorithm for the levels of mutation rate, crossover rate and max-iteration factors

5. 3. Experimental Results In this section, the effectiveness of the proposed ICA is compared with the NSGA-II algorithm for small-sized instances. There are variant metrics for comparison between the algorithms. In this paper, these algorithms are compared based on four metrics, namely quality, spacing, diversity and running time. The results of the comparison with deferent values of parameters are illustrated in Tables 3 to 6. For each instance, the first five columns are the input parameters of the problem. The sixth column is the comparison between two algorithms based on the quality metric. The seventh column is the comparison based on the spacing metric. The eighth column shows the comparison based on the diversity metrics, and the ninth column shows the comparison based on the running time analysis.

TABLE 3. Computational comparison of the proposed ICA and the NSGA-II algorithm for *n*=10

Parameters					Quality metric		Spacing metric		Diversity metric		Running time	
Costs	**α, α'**	***nv***	**s_k**	**θ**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**
Tight	0.6	1	2	$50\%\sum_{i=1}^{n} O_i$	60	40	1.44	1.76	593.13	509.13	0.46	0.25
			3		100	0	1.041	0.12	691.02	233.69	0.48	0.24
		2	2		100	0	0.61	0.82	114.26	294.94	0.45	0.25
			3		50	50	0.94	0.95	320.91	351.83	0.47	0.24
		5	2		60	40	0.97	0.89	411.09	152.19	0.49	0.23
			3		100	0	0.61	0.82	69.68	265.29	0.45	0.23
		1	2	$60\%\sum_{i=1}^{n} O_i$	100	0	1.51	0.94	414.08	446.86	0.46	0.24
			3		66.67	33.33	0.46	0.81	607.79	346.83	0.46	0.24
		2	2		100	0	0.05	1.06	211.99	482.16	0.46	0.24
			3		50	50	0.51	1.76	42712	532.94	0.46	0.23
		5	2		50	50	0.89	1.43	287.61	436.54	0.45	0.25
			3		66.67	33.33	0.6	0.96	300.82	197.85	0.44	0.24
Loose		1	2	$50\%\sum_{i=1}^{n} O_i$	80	20	0.46	1.02	224.24	88.05	0.45	0.24
			3		66.67	33.33	0.2	0.59	541.56	272.52	0.45	0.24
		2	2		50	50	0.87	0.04	182.82	171.76	0.47	0.25
			3		50	50	0.38	0.44	313.09	307.92	0.13	0.05
		5	2		75	25	1.54	1.08	265.63	257.94	0.45	0.23
			3		66.67	33.33	0.22	0.21	121.89	321.19	0.44	0.24
		1	2	$60\%\sum_{i=1}^{n} O_i$	100	0	0.85	1.17	340.69	502.33	0.47	0.23
			3		75	25	1.48	1.31	511.97	482.89	0.46	0.23
		2	2		50	50	1.32	0.14	300.5	138.59	0.47	0.24
			3		57.14	42.86	0.22	0.21	544.25	469.84	0.47	0.24
		5	2		66.67	33.33	0.45	0.14	88.03	127.56	0.46	0.24
			3		66.67	33.33	0.14	0.15	128.11	84.64	0.45	0.25

TABLE 4. Computational comparison of the proposed ICA and the NSGA-II algorithm for *n*=15

Parameters					Quality metric		Spacing metric		Diversity metric		Running time	
Costs	**α, α'**	***nv***	**s_k**	**θ**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**
Tight	0.6	1	2	$50\%\sum_{i=1}^{n} O_i$	50	50	1.29	1.56	255.71	373.29	0.87	0.43
			3		75	25	1.04	1.56	197.36	309.8	0.84	0.43
		5	2		50	50	1.30	1.61	411.67	389.75	0.82	0.44
			3		100	0	1.05	0.73	398.70	418.65	0.84	0.41
		8	2		100	0	0.46	0.37	225.75	204.05	0.86	0.43
			3		100	0	1.64	1.66	419.52	431.14	0.82	0.44
		1	2	$60\%\sum_{i=1}^{n} O_i$	100	0	1.09	1.83	210.39	202.11	0.83	0.41
			3		100	0	1.44	0.71	447.92	271.26	0.82	0.43
		5	2		100	0	094	0.95	0	0	0.83	0.42
			3		50	50	1.26	0.34	161.42	8.28	0.81	0.41
		8	2		100	0	0.11	0.79	158.52	89.73	0.83	0.44
			3		60	40	1.18	0.48	315.86	196.98	0.84	0.44
Loose		1	2	$50\%\sum_{i=1}^{n} O_i$	80	20	0.98	0.83	279.54	170.98	0.87	0.41
			3		100	0	0.29	0.64	416.62	241.96	0.87	0.43
		5	2		50	50	0.27	0.86	414.88	87.25	0.82	0.43
			3		100	0	0.86	1.03	71.69	67.45	0.77	0.41
		8	2		85.71	14.28	1.45	1.05	259.48	240.64	0.79	0.44
			3		66.67	33.33	0.32	0.63	244.25	136.21	0.78	0.43
		1	2	$60\%\sum_{i=1}^{n} O_i$	75	25	0.31	0.07	249.26	227.70	0.86	0.41
			3		100	0	0.58	0.43	77.67	75.44	0.86	0.43
		5	2		66.67	33.33	0.78	0.26	325.65	208.85	0.86	0.44
			3		75	25	0.68	0.51	158.99	112.16	0.77	0.43
		8	2		66.67	33.33	0.16	1.64	215.44	60.98	0.81	0.44
			3		100	0	0.66	0.89	125.57	70.65	0.83	0.43

TABLE 5. Computational comparison of the proposed ICA and the NSGA-II algorithm for *n*=20

Parameters					Quality metric		Spacing metric		Diversity metric		Running time	
Costs	α, α'	***nv***	s_k	θ	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**
Tight	0.6	11	2	$40\%\sum_{i=1}^{n} O_i$	50	50	0.68	0.73	1249	1643.3	0.35	0.11
				$50\%\sum_{i=1}^{n} O_i$	50	50	0.95	1.31	974.83	1951.3	0.35	0.11
				$60\%\sum_{i=1}^{n} O_i$	50	50	0.36	1.08	2560.8	2978.7	0.35	0.11
	0.7			$40\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.46	0.23	2967.4	83.64	0.36	0.11
				$50\%\sum_{i=1}^{n} O_i$	71.43	28.57	0.53	1.65	1077.5	3576.1	0.35	0.14
				$60\%\sum_{i=1}^{n} O_i$	60	40	0.42	1.31	862.42	2324.7	0.35	0.12
	0.8			$40\%\sum_{i=1}^{n} O_i$	60	40	0.39	0.34	1868.8	17729.9	0.38	0.11
				$50\%\sum_{i=1}^{n} O_i$	75	25	1.72	0.14	1273.3	1652.7	0.36	0.11
				$60\%\sum_{i=1}^{n} O_i$	66.67	33.33	1.11	0.99	2316.3	1877	0.35	0.11
	0.6		3	$40\%\sum_{i=1}^{n} O_i$	100	0	0.13	0.07	576.62	282.96	0.38	0.12
				$50\%\sum_{i=1}^{n} O_i$	100	0	0.89	0.87	723.82	2402.5	0.36	0.11
				$60\%\sum_{i=1}^{n} O_i$	60	40	1.18	0.99	3070.07	2433.5	0.36	0.11
	0.7			$40\%\sum_{i=1}^{n} O_i$	100	0	1.20	1.19	2903.6	4136.3	0.35	0.10
				$50\%\sum_{i=1}^{n} O_i$	100	0	1.84	0.66	3445.8	4561.3	0.57	0.21
				$60\%\sum_{i=1}^{n} O_i$	100	0	1.06	1.19	5327	2747.3	0.82	0.34
	0.8			$40\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.82	0.06	635.79	453.71	0.37	0.12
				$50\%\sum_{i=1}^{n} O_i$	100	0	1.29	0.26	2276.7	182.68	0.81	0.33
				$60\%\sum_{i=1}^{n} O_i$	100	0	0.53	0.26	2227.5	2116.9	0.83	0.33
	0.6		4	$40\%\sum_{i=1}^{n} O_i$	100	0	0.71	0.14	498.83	315.11	0.39	0.11
				$50\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.27	0.83	1622	2422.3	0.80	0.33
				$60\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.70	0.10	2985.6	2239.4	0.79	0.33
	0.7			$40\%\sum_{i=1}^{n} O_i$	60	40	1.06	1.16	2643.2	928.73	0.39	0.11
				$50\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.22	0.85	2404.4	1088.9	0.59	0.20
				$60\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.60	0.91	3858.9	3031.3	0.57	0.20
	0.8			$40\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.62	0.14	3204.5	1525.4	0.61	0.20
				$50\%\sum_{i=1}^{n} O_i$	66.67	33.33	1.01	0.55	3307.7	847.57	0.57	0.19
				$60\%\sum_{i=1}^{n} O_i$	100	0	1.62	1.31	1594.3	1981.6	0.57	0.19
Loose	0.6		2	$40\%\sum_{i=1}^{n} O_i$	60	40	0.27	0.99	3619.6	2499.5	0.83	0.33
				$50\%\sum_{i=1}^{n} O_i$	100	0	1.46	0.85	3190.5	1547.7	0.79	0.33
				$60\%\sum_{i=1}^{n} O_i$	60	40	0.92	0.98	1853.7	2407.8	0.82	0.33
	0.7			$40\%\sum_{i=1}^{n} O_i$	66.67	33.33	1.12	0.81	2649.9	2216.2	0.77	0.35
				$50\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.86	1.39	4046.6	2699.2	0.78	0.34
				$60\%\sum_{i=1}^{n} O_i$	60	40	0.79	1.002	1875.8	1603.5	079	0.33
	0.8			$40\%\sum_{i=1}^{n} O_i$	80	20	0.58	0.25	1456.5	109.23	0.78	0.33
				$50\%\sum_{i=1}^{n} O_i$	100	0	0.31	0.42	1358.5	2443.3	0.79	0.33
				$60\%\sum_{i=1}^{n} O_i$	100	0	0.11	0.13	2237.5	1619.6	0.82	0.33
	0.6		3	$40\%\sum_{i=1}^{n} O_i$	75	25	0.36	1.48	2268.1	1054.9	0.80	0.33
				$50\%\sum_{i=1}^{n} O_i$	50	50	0.75	0.85	3005.6	3111.6	0.77	0.34
				$60\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.70	0.74	2682	1139.6	0.78	0.33
	0.7			$40\%\sum_{i=1}^{n} O_i$	50	50	0.64	0.47	1141.3	2554.4	0.76	0.33
				$50\%\sum_{i=1}^{n} O_i$	50	50	0.26	0.19	1870.4	4270.8	0.81	0.34
				$60\%\sum_{i=1}^{n} O_i$	57.14	42.86	0.35	0.64	3680.1	3588.2	0.77	0.34
	0.8			$40\%\sum_{i=1}^{n} O_i$	50	50	0.22	1.51	2023.7	2858.2	0.36	0.11
				$50\%\sum_{i=1}^{n} O_i$	100	0	1.46	0.97	1736.4	622.08	0.35	0.11
				$60\%\sum_{i=1}^{n} O_i$	80	20	0.92	1.58	2670.4	2301.6	0.77	0.33
	0.6		4	$40\%\sum_{i=1}^{n} O_i$	66.67	33.33	0.45	0.26	1908.1	1889.9	0.81	0.34
				$50\%\sum_{i=1}^{n} O_i$	60	40	1.29	0.69	1058.2	637.94	0.76	0.33
				$60\%\sum_{i=1}^{n} O_i$	57.14	42.86	1.29	0.53	3082.9	2019.7	0.78	0.33
	0.7			$40\%\sum_{i=1}^{n} O_i$	60	40	0.86	0.63	2997.8	2189.8	0.81	0.33
				$50\%\sum_{i=1}^{n} O_i$	100	0	0.43	0.38	2000.8	1720.8	0.77	0.33
				$60\%\sum_{i=1}^{n} O_i$	!00	0	0.22	1.80	2850	1905.6	0.76	0.33
	0.8			$40\%\sum_{i=1}^{n} O_i$	66.67	33.33	1.94	0.96	2097.7	1674.8	0.76	0.34
				$50\%\sum_{i=1}^{n} O_i$	50	50	1.09	0.83	2062.8	2443.8	0.82	0.33
				$60\%\sum_{i=1}^{n} O_i$	80	20	1.27	1.52	2655.3	1810.5	0.81	0.35

TABLE 6. Computational comparison of the proposed ICA and the NSGA-II algorithm for *n*=25

Parameters					Quality metric		Spacing metric		Diversity metric		Running time	
Costs	**α, α'**	***nv***	**s_k**	**θ**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**	**ICA**	**NSGA-II**
Tight	0.6	14	2	40%$\sum_{i=1}^{n} O_i$	100	0	1.11	0.59	618.07	270.39	1.32	0.56
				50%$\sum_{i=1}^{n} O_i$	100	0	1.06	0.69	267.61	596.63	1.33	0.55
				60%$\sum_{i=1}^{n} O_i$	60	40	0.39	0.88	307	370.47	1.33	0.54
	0.7			40%$\sum_{i=1}^{n} O_i$	100	0	0.58	0.26	177.92	445.24	1.3	0.53
				50%$\sum_{i=1}^{n} O_i$	75	25	1.48	1.64	255.57	336.46	1.27	0.58
				60%$\sum_{i=1}^{n} O_i$	83.33	16.66	0.69	0.99	498.09	238.93	1.33	0.53
	0.8			40%$\sum_{i=1}^{n} O_i$	100	0	0.79	0.75	51.42	319.38	0.57	0.16
				50%$\sum_{i=1}^{n} O_i$	60	40	0.24	0.52	241.29	378.68	1.30	0.54
				60%$\sum_{i=1}^{n} O_i$	80	20	0.62	0.71	349.46	632.29	1.34	0.52
	0.6		3	40%$\sum_{i=1}^{n} O_i$	66.67	33.33	1.13	1.94	197.55	359.63	1.26	0.52
				50%$\sum_{i=1}^{n} O_i$	80	20	1.41	0.79	386.41	488.88	1.28	0.57
				60%$\sum_{i=1}^{n} O_i$	66.67	33.33	0.18	0.21	402.31	257.28	1.29	0.55
	0.7			40%$\sum_{i=1}^{n} O_i$	100	0	0.85	0.83	69.37	150.04	1.27	0.53
				50%$\sum_{i=1}^{n} O_i$	60	40	1.02	0.96	434.22	314.25	1.23	0.53
				60%$\sum_{i=1}^{n} O_i$	75	25	1.19	1.63	273.89	179.45	1.24	0.55
	0.8			40%$\sum_{i=1}^{n} O_i$	70	30	0.65	0.7	604.92	537.45	1.29	0.53
				50%$\sum_{i=1}^{n} O_i$	40	60	0.89	0.87	403.47	312.87	1.27	0.53
				60%$\sum_{i=1}^{n} O_i$	100	0	0.17	0.64	294.18	331.55	0.65	0.17
	0.6		4	40%$\sum_{i=1}^{n} O_i$	60	40	1.12	0.76	396.52	304.38	0.58	0.17
				50%$\sum_{i=1}^{n} O_i$	66.67	33.33	1.24	0.42	210.35	86.20	0.58	0.16
				60%$\sum_{i=1}^{n} O_i$	100	0	0.41	1.44	426.59	355.31	0.59	0.18
	0.7			40%$\sum_{i=1}^{n} O_i$	66.67	33.33	0.05	0.48	364.69	235.79	0.58	0.17
				50%$\sum_{i=1}^{n} O_i$	66.67	33.33	0.68	0.37	451.35	208.92	0.61	0.17
				60%$\sum_{i=1}^{n} O_i$	50	50	12	0.89	439.35	380.27	0.62	0.17
	0.8			40%$\sum_{i=1}^{n} O_i$	50	50	0.85	1.09	251.55	471.92	0.62	0.17
				50%$\sum_{i=1}^{n} O_i$	100	0	1.12	1.78	324.99	458.86	0.60	0.18
				60%$\sum_{i=1}^{n} O_i$	100	0	0.91	0.61	122.21	438.79	0.58	0.17
Loose	0.6		2	40%$\sum_{i=1}^{n} O_i$	66.67	33.33	0.59	0.17	428.54	403.94	1.24	0.57
				50%$\sum_{i=1}^{n} O_i$	60	40	1.15	0.34	341.87	260.22	1.28	0.55
				60%$\sum_{i=1}^{n} O_i$	60	40	1.31	0.96	288.17	342.16	1.29	0.55
	0.7			40%$\sum_{i=1}^{n} O_i$	62.5	37.5	0.54	1.11	400.07	253.52	1.24	0.55
				50%$\sum_{i=1}^{n} O_i$	57.14	42.86	0.63	0.14	341.46	398.37	1.27	0.55
				60%$\sum_{i=1}^{n} O_i$	75	25	127	0.81	324.74	259.02	1.28	0.54
	0.8			40%$\sum_{i=1}^{n} O_i$	71.43	28.57	0.81	0.82	422.27	623.90	1.28	0.54
				50%$\sum_{i=1}^{n} O_i$	50	50	0.23	0.51	206.62	373.25	1.30	0.53
				60%$\sum_{i=1}^{n} O_i$	66.67	33.33	0.47	0.39	193.11	254.86	1.28	0.54
	0.6		3	40%$\sum_{i=1}^{n} O_i$	83.33	16.67	0.42	0.64	427.75	373.69	1.30	0.54
				50%$\sum_{i=1}^{n} O_i$	57.14	42.86	1.19	0.71	260.54	507.56	1.29	0.52
				60%$\sum_{i=1}^{n} O_i$	100	0	0.69	0.24	110.70	65.50	1.28	0.53
	0.7			40%$\sum_{i=1}^{n} O_i$	57.14	42.86	0.76	1.09	346.24	409.98	1.27	0.53
				50%$\sum_{i=1}^{n} O_i$	57.14	42.86	0.47	0.92	337.61	547.23	1.28	0.54
				60%$\sum_{i=1}^{n} O_i$	66.67	33.33	0.07	0.24	478.79	426.07	1.24	0.56
	0.8			40%$\sum_{i=1}^{n} O_i$	62.5	37.5	0.44	0.98	604.28	406.97	1.25	0.56
				50%$\sum_{i=1}^{n} O_i$	83.33	16.67	0.65	0.85	323.22	446.46	1.28	0.54
				60%$\sum_{i=1}^{n} O_i$	60	40	0.82	1.05	413.69	601.53	1.29	0.55
	0.6		4	40%$\sum_{i=1}^{n} O_i$	55.56	44.44	0.34	0.98	492.49	287.89	1.28	0.54
				50%$\sum_{i=1}^{n} O_i$	50	50	0.68	0.37	433.79	382.23	1.28	0.54
				60%$\sum_{i=1}^{n} O_i$	100	0	0.79	0.20	297.56	185.47	1.24	0.56
	0.7			40%$\sum_{i=1}^{n} O_i$	66.67	33.33	0.16	0.4	305.24	410.63	1.28	0.54
				50%$\sum_{i=1}^{n} O_i$	60	40	0.41	0.62	357.68	227.24	1.25	0.57
				60%$\sum_{i=1}^{n} O_i$	60	40	0.96	1.22	389.84	437.07	1.25	0.54
	0.8			40%$\sum_{i=1}^{n} O_i$	50	50	0.38	1.58	349.65	367.29	1.24	0.55
				50%$\sum_{i=1}^{n} O_i$	75	25	0.61	0.18	425.88	353.13	1.28	0.54
				60%$\sum_{i=1}^{n} O_i$	57.14	42.86	0.98	0.41	479.77	346.52	1.27	0.54

The proposed ICA and the NSGA-II algorithm for n=10 A paired t-test is conducted to see whether the significant difference exists between the obtained solution of the proposed ICA and the optimal solution of the NSGA-II algorithm. The difference between the computed values of two methods for test problem i is shown by the variable Di. Therefore, the statistics are as follows:

$$t = \frac{\sqrt{n}\times\overline{D}}{S_D}; \text{ where } \overline{D} = \frac{\sum D_i}{n} \text{ and } S_D = \sqrt{\frac{\sum(D_i-\overline{D})^2}{n-1}} \qquad (44)$$

A paired t-test is conducted by 54 test problems in the SPSS software. In these Tables 8 and 9, it can be seen that, there is not statistical significant difference between solutions obtained by ICA and NSGA-II based on spacing Metrics and diversity Metrics according to the significance (2-tailed)>0.05. However, there is statistical significant difference based on quality metrics and time metrics according to the significance (2-tailed)<0.05 in Tables 7 and 10.

TABLE 7. Detailed statistics of the paired t-test for the quality metric

		Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. error mean	95% confidence interval of the difference				
					Lower	Upper			
Pair 1	ICA – NSGA-II	42.50796	34.42031	4.68401	33.11303	51.90290	9.075	53	0.000

TABLE 8. Detailed statistics of the paired t-test for the spacing metric

		Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. error mean	95% confidence interval of the difference				
					Lower	Upper			
Pair 1	ICA – NSGA-II	-1.86315	13.38053	1.82086	-5.51533	1.78903	-1.023	53	0.311

TABLE 9. Detailed statistics of the paired t-test for the diversity metric

		Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. error mean	95% confidence interval of the difference				
					Lower	Upper			
Pair 1	ICA – NSGA-II	-52.0385	297.53524	40.48942	-133.25000	29.17297	-1.285	53	0.204

TABLE 10. Detailed statistics of the paired t-test for the running time analysis

		Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. error mean	95% confidence interval of the difference				
					Lower	Upper			
Pair 1	ICA – NSGA-II	67.222	0.12869	0.01751	0.63710	0.70735	38.38	53	0.000

6. CONCLUSION

In this paper, we have studied different conflicting objectives for the capacitated single-allocation hub location problem, in which the flow has been transported non-contiguous. We have proposed two alternative bi-objectives minimizing the total transportation cost and minimizing the maximum travel time between all origin–destination pairs. In addition, the considered model has avoided from congestion in the hubs. We have proposed a new imperialist competitive algorithm (ICA) to solve the given problems. The performance of the proposed ICA has been compared with the NSGA-II algorithm based on the quality metric, spacing metric, diversity metric and running time analysis. The related results have been illustrated. According to the statistical analysis, the quality metric computed for the proposed ICA has been better than NSGA-II. However, the proposed ICA has needed more time to solve the problems. Furthermore, there has not been statistically significant difference between the solutions obtained by the proposed ICA and the NSGA-II algorithm based on the spacing and diversity metrics.

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A Multi-objective Imperialist Competitive Algorithm for a Capacitated Single-allocation Hub Location Problem

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چکیده

در این مقاله، یک مدل جدید چندهدفه با تخصیص تکی و محدودیت ظرفیت برای مکان یابی هاب ارائه می‌گردد. محدودیت وسایل نقلیه و محدودیت مقدار کالاهای وارد شده به هاب‌ها با در نظر گرفتن تعادل مورد نیاز مقدار جریان در بین هاب‌ها، در نظر گرفته می‌شود. به علاوه، برای هر هاب چندین ظرفیت موجود است که تنها یکی از سطوح موجود برای هر یک از هاب‌ها انتخاب می‌شود. این مدل چند هدفه، کل هزینه حمل در شبکه را با توجه به حداقل کردن حداکثر زمان تحویل در شبکه، حداقل می‌کند. به دلیل NP-hard بودن مسأله ارائه شده، این مدل توسط روش الگوریتم چندهدفه استعماری رقابتی حل می‌شود و برای بررسی عملکرد این الگوریتم پیشنهادی، جواب‌های به‌دست آمده با جواب‌های حاصل از الگوریتم ژنتیک با رتبه‌بندی غیرمغلوب مورد مقایسه قرار می‌گیرد که نتایج کسب شده حاکی از اثربخشی الگوریتم پیشنهادی است.

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