International Journal of Engineering

Journal Homepage: www.ije.ir

Multi-objective Efficient Design of *np* Control Chart Using Data Envelopment Analysis

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PAPER INFO

Paper history:
Received 20 September 2012
Received in revised form 18 November 2012
Accepted 24 January 2013

Keywords:
Economic-statistical Design
Data Envelopment Analysis (*DEA*)
MOESD
np Control Chart
Maxmin Weight Model

ABSTRACT

Control charts are the most important tools of statistical process control used to discriminate between assignable and common causes of variation and to improve the quality of a process. To design a control chart, three parameters including sample size, sampling interval, and control limits should be determined. The objectives are hourly expected cost, in-control average run length, power of the control chart, and average time to signal. Different approaches such as statistical design, economic design, and economic-statistical design of control charts have been considered by many researchers. Recently, multi-objective design of control chart has been investigated in the literature. In this paper we propose a multi-objective economic-statistical design of *np* control chart (*np-MOESD*). To solve the multi-objective model, a method is used to find the Pareto optimal solution and then a combined method based on Data Envelopment Analysis (*DEA*) is proposed to determine the most efficient design parameters. A numerical example of Duncan [1] illustrates the proposed approach. Sensitivity analysis is performed to evaluate the proposed model. In addition, the proposed model is compared with pure economic design (Duncan's model) as well as another model in the literature. Results show that the proposed *np-MOESD* model improves statistical properties of *np* control charts.

doi: *10.5829/idosi.ije.2013.26.06c.07*

1. INTRODUCTION

Control charts are powerful tools of statistical process control for monitoring and improvement of processes. To design control charts properly, we need to find optimal value of parameters, sample size (n), sampling interval (h), and the control limit (d). Selecting a combination of these parameters is called the design of control charts. A control chart which is used to monitor nonconforming fraction is p control chart. np-chart is the best alternative for the p-chart. This chart is used to monitor the number of nonconforming items. Different approaches such as statistical design, economic design, and economic-statistical design of control charts have been considered by many researchers to determine design parameters. The first economic model to determine design parameters of $\bar{X}$ control charts was presented by Duncan [2]. After that economic design of different control charts have been investigated by many researchers. Ladany [3] presented a model for the economic design of the p control chart. Chiu [4] proposed the economic model of the np control chart. In the proposed model, he considered the cost of process adjustment. In addition, he assumed that process is stopped when the signal is taken by the control chart. Chiu [5] developed the economic model in Chiu [4] under multiple assignable causes. Montgomery et al. [6] proposed the economic model for the fraction nonconforming control chart under multiple assignable causes. In addition, they performed sensitivity analysis on the economic model. Duncan [1] proposed the economic model for p (np) control charts under one assignable cause. Chung [7] improved Chiu's study by presenting an algorithm in the procedure. Lo [8] assumed that the input parameters of the economic design are not always known. He used two methods including maximum likelihood estimation and a naïve method in his paper. Wang and Chen [9] presented an economic-statistical model to design np control chart under the fuzzy situation. They used a fuzzy mathematical programming model and a heuristic method to determine the parameters.

Woodall [10] stated that the economic design of control chart without considering statistical constraints

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leads to poor results in statistical properties. To overcome this problem, researchers proposed economic-statistical design of control charts, for example see review paper by Celano [11]. Another approach to account for this problem is using multi-objective economic-statistical design of control charts. Evans and Emberton [12] suggested the multi-objective model for designing the joint $\bar{X}$ and *R* control charts. Then, Del Castillo et al. [13] presented a multi-objective model for design of $\bar{X}$ control chart. This model is including three objective functions, two statistical objectives and an economic cost function. They used nonlinear optimization algorithm to solve the multi-objective model. Chen and Liao [14] proposed optimal design of $\bar{X}$ control chart as a multiple criteria decision-making and solved this model using data envelopment analysis (*DEA*). Celano and Fichera [15] presented a model similar to Del Castillo et al. [13] and solved this model by genetic algorithm. Zarandi et al. [16] suggested a fuzzy multi-objective model for economic-statistical design of $\bar{X}$ control chart when the input parameters are fuzzy. They used a genetic algorithm (GA) to find the optimal design parameters. Asadzadeh and Khoshalhan [17] presented a multi-objective model for $\bar{X}$ control charts. This model is similar to that used in Chen and Liao [14]. However, they added an average time to signal (ATS) constraint to the model and generalized the cost function from one assignable cause to multiple assignable causes. Recently, Safaei et al. [18] used a multi-objective model for economic-statistical design of $\bar{X}$ control chart considering Taguchi loss function. Furthermore, they used non-dominated sorting genetic algorithm II (NSGA-II) to solve the applied model.

Data envelopment analysis (*DEA*) is an optimization approach for measuring the relative efficiency of a batch of competing decision-making units (*DMU*s), whenever there are multiple inputs and outputs for each *DMU*s (Thanassoulis, [19]). The application of the *DEA* is in different areas including designing control charts. Chen and Liao [14] used *DEA* for determining the parameters of $\bar{X}$ control charts when there is only one assignable cause in the model. Asadzadeh and Khoshalhan [17] used *DEA* approach to solve a multi-objective model presented for designing $\bar{X}$ control charts.

In Table 1, comparison between different researches on multi-objective design of control charts is done. Furthermore, the scope of our research among the other multi-objective researches is illustrated. To the best of our knowledge, there is no research on multi-objective design of *np* (*p*) control charts. In this paper, we design the *np* control chart with multiple objectives and determine design parameters, namely sample size (*n*), sampling interval (*h*), and sample number of nonconforming items (*d*). In the proposed multiple-objective model, there are three statistical objectives and one economic objective. To solve the multi-objective model, a method is used to find the Pareto optimal solution and then a combined method based on *DEA* is proposed to determine efficient design parameters. Finally, the most efficient combination of design parameters among *DEA* efficient units is selected using the maximin weight model (See Wang et al., [20]).

TABLE 1. Comparison of studies on multi-objective (criteria) design of control charts.

Authors (year)	Assumptions	Objectives	Constraint	Control Chart Type	Solution Approach	Remarks
Del Castilo et al. [13]	Single assignable cause	Cost, Type *I* error, *ATS*	Type *I* error, Power	$\bar{X}$control chart	Markin [21]	Weak optimization tool
Celano and Fishera [15]	Single assignable cause	Cost, Type *I* error, Power	Without constraint	$\bar{X}$control chart	*GA*	Aggregative multi-objective function approach
Chen and Liao [14]	Single assignable cause	Cost, ARL_0, Power	Type *I* error, Power	$\bar{X}$control chart	*DEA*	Inability in determining the most efficient unit
Zarandi et al. [16]	Single assignable cause, Fuzzy parameters	Cost , ARL_0, Power	Type *I* error, Power	$\bar{X}$control chart	*GA*	Aggregative multi objective function approach
Asadzadeh and Khoshalhan [17]	Multiple assignable causes	Cost, ARL_0, Power	Type *I* error, Power, *ATS*	$\bar{X}$control chart	*DEA*	Inability in determining the most efficient unit
Safaei et al. [18]	Single assignable cause	Cost, ARL_1	ARL_0	$\bar{X}$control chart	NSGA-II	Non-aggregative multi-objective function approach
Our research	Single assignable cause	Cost, ARL_0, Power, *ATS*	Cost, Type *I* error, Power, *ATS*	*np*(*p*) control chart	*DEA* and maximin weight model (Wang et al., [20])	Ability in determining the most efficient unit

There are different methods for ranking *DEA* efficient units. For example, Andersen and Petersen [22] proposed the super-efficiency method for ranking the *DEA* efficient units. Sinuany-Stern et al. [23] proposed an *AHP/DEA* method for ranking decision making units (DMUs). Wang et al. [20] used a minimum weight restriction in *DEA* for determining the most efficient units and ranking *DEA* efficient units.

2. THE *np-MOESD* MODEL

In this section, the Duncan's economic model is introduced. Then, the proposed *np-MOESD* model is presented.

2. 1. Economic Cost Function for *np* Control Charts

Duncan [1] presented economic model with one assignable cause for *np (p)* control charts. This model is similar to Duncan model [2] presented for $\bar{X}$ control charts. In Duncan model [1], there are in-control and one out-control states. The time between occurring two assignable causes is randomly exponentially distributed with mean time $1/\lambda$. Moreover, the distribution of the number of nonconforming items in a sample is binomial with mean of np_0. When a shift (δ) occurs in the process, the process mean changes from p_0 to p_1 as follows:

$$p_1 = p_0 + \delta\sqrt{p_0(1-p_0)} \tag{1}$$

If the average time in hours to looking for the assignable cause is *D*, and the time to take and testing the sample is *gn*, hence, the average time of the process which is in out-of-control state (*B*) will be

$$B = h/P - \tau + gn + D, \tag{2}$$

where, *P* is the probability of a signal (power) given by

$$P = 1 - \sum_{x=0}^{d} \frac{n!}{x!(n-x)!}(p_1)^x (1-p_1)^{n-x}. \tag{3}$$

In Equation (3), *d* (the number of nonconforming items) is upper control limit (*UCL*) of the *np* control chart. When the number of nonconforming items exceeds the *d*, the process is in the state of out-of control. τ is the average time of occurrence of an assignable cause between samples and given by

$$\tau = \frac{\left[1-(1+\lambda h)e^{-\lambda h}\right]}{\lambda\left(1-e^{-\lambda h}\right)}. \tag{4}$$

Since the average time a process is in-control state equals to $1/\lambda$, hence, the expected length of a cycle is

$$\frac{1}{\lambda} + B. \tag{5}$$

Moreover, probability of Type I error (α) is given by

$$\alpha = 1 - \sum_{x=1}^{d} \frac{n!}{x!(n-x)!}(p_0)^x (1-p_0)^{n-x}. \tag{6}$$

The average number of false alarms occurs per cycle is as follows:

$$A = \alpha e^{-\lambda h}/(1-e^{-\lambda h}). \tag{7}$$

Thus, the expected cost per hour is

$$L = \frac{\lambda MB + \lambda AT + \lambda W}{1+\lambda b} + \frac{b}{h} + \frac{cn}{h}, \tag{8}$$

where, *M* is the loss per hour due to producing nonconforming items in the out-of-control state, *T* is the average cost of verifying a false alarm, *W* is the average cost of detecting an assignable cause, *b* is the fixed cost and *c* is the variable cost of sampling.

2. 2. Multi-objective Design of *np* Control Chart

The main aim of multi objective decision making (*np-MOESD*) is to find a solution which can provide a harmony between all objectives (Hwang and Masud, [24]). In this paper, we propose an *np-MOESD* model with four objectives. One of them is the economic cost function which presented in the previous section, and the other objectives are statistical functions. The multi-objective model is given as follows:

$$\begin{aligned} &\text{Max } ARL_0(U) \\ &\text{Max } P(U) \\ &\text{Min } ATS(U) \\ &\text{Min } L(U) \\ &\text{s.t.} \\ &P \ge P_s \\ &\alpha \le \alpha_s \\ &ATS \le ATS_s \\ &L \le L_s, \end{aligned} \tag{9}$$

where, ARL_0 is the in-control average run length, inverse of false alarm rate (α), *P* is the probability of detecting a shift in a process (power of control chart), *ATS* is the average time to signal when an assignable cause occurs, *L* is expected cost per hour, *U* is a possible combination of design parameters, P_s is the minimum value of power. In addition, α_s, ATS_s and L_s are the maximum values of α, ATS and L, respectively. They are determined by decision maker (DM) to achieve desired solutions.

To solve *np-MOESD* problem, there are lots of methods which can be used. The *DEA* approach is one of the most suitable methods among them. Already, the *DEA* approach has been used for $\bar{X}$ control charts. However, to the best of our knowledge, it is not applied for the *np* (*p*) control chart up to now. In this paper, we use this approach for the *np-MOESD* model.

3. DATA ENVELOPMENT ANALYSIS

As discussed in the introduction section, the *DEA* is a mathematical programming model used to measure the relative efficiency of DMUs. The relative efficiency for *j*th *DMU* is computed as:

$$E_j = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}}.$$

Thus, if a *DMU* wants to have upper efficiency, the denominator of above ratio namely, input data should be minimized and output data should be maximized. If we assume that there are *n DMU*s, each with *m* inputs and *s* outputs, the efficiency of *j*th *DMU* is computed using Equation (10).

$$E_j(U) = \frac{\sum_{r=1}^{s} u_r Y_{rj}(U)}{\sum_{i=1}^{m} v_i X_{ij}(U)}, \qquad j = 1,\ldots,n, \qquad (10)$$

where, u_r is the weight of output *r*, v_i is the weight of input *i*, Y_{rj} is the value of output *r* for *j*th *DMU*, X_{ij} is the value of input *i* for *j*th *DMU*.

In our proposed *np-MOESD* model, the statistical objectives including ARL_0 and *P* are considered as outputs because of their maximizing nature, and *L* and *ATS* are investigated as inputs. Therefore, *DEA* is designed to have two inputs and two outputs for determining the efficiency of *DMUs*.

3. 1. *CCR* Model

To obtain the input and output weights, we used the *CCR* model which proposed by Charnes et al. [25]:

$$\begin{aligned}
&\text{Maximize } E_0(U) = \frac{\sum_{r=1}^{s} u_r Y_{r0}(U)}{\sum_{i=1}^{m} v_i X_{i0}(U)} \\
&\text{s.t.} \\
&\frac{\sum_{r=1}^{s} u_r Y_{rj}(U)}{\sum_{i=1}^{m} v_i X_{ij}(U)} \le 1, && j = 1,\ldots n \\
&u_r \ge 0, && r = 1,\ldots s \\
&v_i \ge 0. && i = 1,\ldots,m.
\end{aligned} \qquad (11)$$

where, DMU_0 is the *DMU* under evaluation.

The *CCR* model is equivalently transformed into the linear programming model using transformation method in Charnes and Cooper's [26]. The corresponding *LP* model of Equation (11) is:

$$\begin{aligned}
&\text{Maximize} E_0(U) = \sum_{r=1}^{s} u_r Y_{r0}(U) \\
&\text{s.t.} \\
&\sum_{i=1}^{m} v_i X_{i0}(U) = 1, \\
&\sum_{r=1}^{s} u_r Y_{rj}(U) - \sum_{i=1}^{m} v_i X_{ij}(U) \le 0, && j = 1,\ldots,n \\
&u_r \ge 0, && r = 1,\ldots s \\
&v_i \ge 0. && i = 1,\ldots,m
\end{aligned} \qquad (12)$$

The above model is solved for each of *DMUs* and relative efficiency for each of them are obtained. As a result, at least one of the *DMUs* will be efficient. However, usually more than one *DMU* have efficiency equal to one. Therefore, the *DEA* efficient units should be ranked. In this paper, we use maximin weight model proposed by Wang et al. [20] for ranking the *DMU*s.

3. 2. Maximin Weight Model

Consider the DMU_0 as the efficient unit that specified by *CCR* model. Therefore, the following results for this DMU_0 are obtained.

$$\begin{aligned}
&\sum_{r=1}^{s} u_r y_{r0} - \sum_{i=1}^{m} v_i x_{i0} = 0, \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, && j = 1,\ldots,n \\
& && r = 1,\ldots s \\
&u_r \ge 0, && i = 1,\ldots,m \\
&v_i \ge 0.
\end{aligned} \qquad (13)$$

Wang et al. [20] proposed a maximin weight model for ranking the *DEA* efficient units. This model is solved for all *DMUs* with efficiency equal to one. This model is introduced in Equation (14).

$$\begin{aligned}
&\text{Maximize } \varepsilon \\
&\text{s.t.} \\
&\sum_{i=1}^{m} v_i = 1, \\
&\sum_{r=1}^{s} u_r \hat{y}_{r0} - \sum_{i=1}^{m} v_i \hat{x}_{i0} = 0, \\
&\sum_{r=1}^{s} u_r \hat{y}_{rj} - \sum_{i=1}^{m} v_i \hat{x}_{ij} \le 0, && j = 1,\ldots,n \\
& && r = 1,\ldots s \\
&u_r \ge \varepsilon, && i = 1,\ldots,m \\
&v_i \ge \varepsilon.
\end{aligned} \qquad (14)$$

where, ε is the maximin weight which can keep DMU_0 as an efficient unit, $\hat{x}_{ij}(i = 1,\ldots,m; j = 1,\ldots,n)$ is normalized input data and $\hat{y}_{rj}(r = 1,\ldots,s; j = 1,\ldots,n)$ is normalized output data by Equation (15).

$$\hat{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}}, \qquad i = 1,\ldots,m;\ j = 1,\ldots,n$$
$$\hat{y}_{rj} = \frac{y_{rj}}{\sum_{j=1}^{n} y_{rj}}, \qquad r = 1,\ldots,s;\ j = 1,\ldots,n \qquad (15)$$

If there are k *DEA* efficient units, by solving maximin weight model in Equation (14) for each efficient unit, we can obtain a group of maximin weights, $\varepsilon_{i1}^*, \varepsilon_{i2}^*, \ldots, \varepsilon_{ik}^*$. The unit with largest ε^* is the most efficient unit. Then other efficient units are ranked based on their calculated ε^* in descending order.

Genrally, the *CCR* and maximin weight models are applied to solve the *np-MOESD* model for determining the most efficient of combination of design parameters.

4. METHODOLOGY

In this paper, we apply the proposed algorithm by Chen and Liao [14]. There are many differences between our procedure and Chen and Liao's method. First, our *np-MOESD* model is developed for the *np* (*p*) control chart instead of $\bar{X}$ control chart. Second, in the *np-MOESD* model, average time to signal is considered as an objective function and two constraints on *ATS* and cost are considered in addition to their model. Third, our procedure includes one step more than Chen and Liao's. This step is used to rank the *DEA* efficient units and to determine the most efficient one. The proposed methodology to solve the *np-MOESD* model is as follows:

Step 0. *Determining the potential combinations of design parameters.*
First, by considering the bounds of design parameters, the potential combinations of the design parameters are specified and then the value of each objective function is computed. In this paper, the bounds of each parameter are assumed as follows:

- The bounds of sample size *n* are 1 and 45, increases by 1.
- The sampling interval *h* is confined between 0.5 and 6, increases by 0.5.
- The sample number of nonconforming items *d* is between 0 and *n*, increases by 1. For example if *n* equals to 5, *d* can be between 0 and 5.

Note that to solve the optimization model based on discrete optimization approach, the analyzer can limit the solution space. This is the method used by many authors (see for example Chen and Liao [14], Asadzadeh and Khoshalhan [17]). Our simulation studies showed that the values of *n* and *h* larger than 45 and 6, respectively usually lead to obtaining dominated solutions. Hence, to avoid additional computations, we limited the solution space with the above bounds on the design parameters *n* and *h*.

Step 1. *Discrimination of feasible combinations.*
Determination of the feasible combinations by the following assumption constraints:

$\alpha \le \alpha_s \qquad P \ge P_s \qquad ATS \le ATS_s \qquad L \le L_s.$

Then, the feasible combinations with the same sample size *n* are separately gathered into a set Q_n.

Step 2. *Partial solution selection.*
Determine the non-dominated solution (*NDS*) points for each set of Q_n. The *NDS* solution (combination) in the set Q_n means that there is no other solution in the same set that is dominated in terms of statistical properties and cost.

Step 3. *Global Pareto solution selection.*
Combine all determined partial Pareto solution from step 2 into a set *W*, and then select the global Pareto solutions. In this step the efficiency score of each combination can be calculated using *DEA*.

Step 4. *The most efficient unit determination.*
Compute ε for each *DEA* efficient unit by solving Equation (14). The unit with the largest maximin weight ε will be the most efficient unit. In addition, the other *DEA* efficient unit can be ranked from the largest to the smallest based on the maximin weight ε.

5. NUMERICAL EXAMPLE AND COMPARISON

In this section, the numerical example of Duncan [1] is employed to investigate the performance of *np-MOESD* model. Then, the proposed approach is compared with pure economic design (Duncan's model, [1]). In addition, we applied Chen and Liao [14]'s model and solved it with the proposed method. Then we compared the results of the proposed model to the results of Chen and Liao's model. Finally, sensitivity analysis is performed to evaluate performance of the proposed model.

5. 1. Duncan's Numerical Example

In this subsection, the numerical example of Duncan [1] with minor changes is considered to present the *np-MOESD* model.

The time between occurring successive assignable causes is exponentially distributed with average of 10. The average time in hours to looking for the assignable cause is equal to 2. The time to take and test the sample size *n* is equal to 0.05. Furthermore, the distribution of the number of nonconforming items in a sample is binomial with mean np_0 (p_0= 0.01). In addition, the value of δ is 1.5 and hence p_1 is equal to 0.16. The values of the other parameters are summarized in Table 2. The constraints of *np-MOESD* model are also assumed to be as follows:

$\alpha \leq 0.05 \qquad P \geq 0.9 \qquad ATS \leq 4 \qquad L \leq 3.5$

Based on the proposed methodology in section 4, the following steps are done to solve the model:

First, the feasible solutions are separated based on the constraints mentioned above. Then, the sets Q_n are determined and the *NDS* of each set are selected. Next, the relative efficiently of each *NDS* combination is computed by *DEA* through *DEAP* software. Finally, the maximin weight model is applied to rank the *DEA* efficient units and to determine the most efficient combination of design parameters. The determined efficient units using *DEA* are summarized in Table 3.

Furthermore, results of maximin weight model for all of efficient units are presented in Table 4. Based on results of maximin weighted model, combinations (40,3,3) and (40,3.5,3) are the most efficient units which are specified by * in Table 4.

5. 2. Comparisons With the Pure Economic Model

In this subsection, a comparison between the pure economic model presented by Duncan [1] and the proposed *np-MOESD* model is performed and the results are summarized in upper half of Table 5. Note that the two most efficient design parameters determined in Table 4 are compared with the results reported by Duncan [1] for *np* control chart.

TABLE 2. The input value of cost parameters for Duncan (1978) example

Parameter	Value ($)
T	25
W	12.5
M	20
b	1
c	0.1

TABLE 4. Ranking of DEA efficient combinations by maximin weight model for the modified Duncan (1978) example

n	*H*	*d*	ε^*	Rank
23	1.5	1	0.05491	4
24	4	1	0.03064	5
40	2.5	3	0.15809	3
40	3	3	0.16003	1*
40	3.5	3	0.16003	1*
40	4	3	0.15895	2
43	2.5	2	Not feasible	6

TABLE 3. Efficient combination of the determined design parameters obtained by *DEA* method

n	*h*	*d*	*ATS*	*L*	ARL_0	*P*	Efficiency
23	1.5	1	1.3536	3.4889	45.4545	0.9024	1
24	4	1	3.6604	2.2245	41.841	0.9151	1
40	2.5	3	2.254	3.2251	1458.56	0.9016	1
40	3	3	2.7048	2.9514	1458.56	0.9016	1
40	3.5	3	3.1556	2.7734	1458.56	0.9016	1
40	4	3	3.6064	2.6551	1458.56	0.9016	1
43	2.5	2	2.4417	3.4159	108.6957	0.9767	1

TABLE 5. Comparison between the pure economic model and Chen and Liao's model with *np-MOESD* model for numerical example

Design	*n*	*h*	*d*	Efficiency	*ATS*	Improvement rate (%)	ARL_0	Improvement rate (%)	*P*	Improvement rate (%)	*L*	Improvement rate (%)
pure economic	17	4.5	1	0.91	3.5158		81.30081		0.7813		2.0527	
np-MOESD	40	3	3	1	2.7048	23.1	1458.555	1694	0.9016	15.4	2.9514	-43.8
	40	3.5	3	1	3.1556	10.2	1458.555	1694	0.9016	15.4	2.7734	-35.1

Design	*n*	*h*	*d*	Efficiency	*ATS*	Improvement rate (%)	ARL_0	Improvement rate (%)	*P*	Improvement rate (%)	*L*	Improvement rate (%)
Chen and liao [14]	40	6	3	1	5.4096		1458.555		0.9016		2.4816	
np-MOESD	40	3	3	1	2.7048	50	1458.555	0	0.9016	0	2.9514	-18.9
	40	3.5	3	1	3.1556	41.7	1458.555	0	0.9016	0	2.7734	-11.8

In addition, a comparison between the proposed *np-MOESD* model and the model by Chen and Liao [14] is done and the results are summarized in lower half of Table 5.

The results of upper half of Table 5 show that the relative efficiency of the best combination determined by the pure economic model is less than 1. Selected two most efficient units by the proposed *np-MOESD* model have improved *ATS* about 23.1% [(3.5158-2.7048)/3.5158] and 10.2%, respectively. In addition, the sampling interval is decreased in *np-MOESD* design. Furthermore, the hourly expected cost (*L*) is increased about 43.8% [(2.0527-2.9514)/2.0527] and 35%, respectively by the proposed combinations. The values of ARL_0 for *np*-MOESD design are very suitable and the power of control chart is improved about 15.4% [(0.9016-0.7813)/0.7813]. Although, the cost of the *np* control charts is increased but statistical performance is improved using *np-MOESD* model. Finally, it can be concluded that the combination determined by pure economic model cannot be stated in the Pareto optimal solution in the multi-objective space. Its efficiency score confirm this conclusion.

The results of lower half of Table 5 show that the relative efficiency of the best combination obtained by the Chen and Liao [14]'s model is equal to 1. However, this combination is not feasible in the proposed model because *ATS* is greater than 4. Although the values of ARL_0 and *P* are the same based on two models, the most efficient units of the proposed *np-MOESD* model have improved *ATS* about 50% and 41.7% . The cost (*L*) is also increased in the proposed *np-MOESD* model. It is true that the cost of the proposed model is increased but the values of *ATS* have been significantly improved using *np-MOESD* model.

5. 3. Sensitivity Analysis In this subsection, sensitivity analysis of the input cost parameters and resources of constraints is done which exhibit the robustness of the proposed *np-MOESD*.

5. 3. 1. Sensitivity Analysis on Input Parameters of Cost Function Sensitivity analysis of the input cost parameters is done to investigate the effect of input parameters on the selection of most efficient units. The parameters include constant and variable sampling costs (*b* and *c*), loss per hour due to producing nonconforming items in out-of-control state (*M*), the average cost of verifying a false alarm (*T*), the average cost of detecting the assignable cause (*W*) and also the time for sampling and analyzing (*g*). The results are summarized in Table 6 and illustrated in Figures 1 and 2.

TABLE 6. Sensitivity analysis of input cost parameters

Changed Parameter	Value	*n*	*h*	*d*	*ATS*	*L*	ARL_0	*P*
-		40	3	3	2.7048	2.9514	1458.555	0.9016
		40	3.5	3	3.1556	2.7734	1458.555	0.9016
c	0.05	40	2.5	3	2.254	2.4252	1458.555	0.9016
		40	3	3	2.7048	2.2848	1458.555	0.9016
	0.5	32	3.5	2	3.16855	3.3997	250	0.9053
		32	4	2	3.6212	3.1924	250	0.9053
b	0.5	40	3	3	2.7048	2.7912	1458.555	0.9016
		40	3.5	3	3.1556	2.6372	1458.555	0.9016
	5	32	4	2	3.6212	3.3413	250	0.9053
g	0.025	40	3	3	2.7048	2.7534	1458.555	0.9016
		40	3.5	3	3.1556	2.5754	1458.555	0.9016
	0.25	24	4	1	3.6604	3.1747	41.841	0.9151
T	12.5	40	3	3	2.7048	2.9487	1458.555	0.9016
		40	3.5	3	3.1556	2.771	1458.555	0.9016
	50	40	3	3	2.7048	2.957	1458.555	0.9016
		40	3.5	3	3.1556	2.7782	1458.555	0.9016
M	10	40	3	3	2.7048	2.3737	1458.555	0.9016
		40	3.5	3	3.1556	2.1653	1458.555	0.9016
	40	24	3	1	2.7453	3.4295	41.841	0.9151
W	6.25	40	3	3	2.7048	2.8896	1458.555	0.9016
		40	3.5	3	3.1556	2.7115	1458.555	0.9016
	25	40	3	3	2.7048	3.0752	1458.555	0.9016
		40	3.5	3	3.1556	2.8972	1458.555	0.9016

TABLE 7. Sensitivity analysis on resources of constraints

Changed Parameter	Value	*n*	*h*	*d*	*ATS*	*L*	*ARL₀*	*P*
-		40	3	3	2.7048	2.9514	1458.555	0.9016
		40	3.5	3	3.1556	2.7734	1458.555	0.9016
α_s	0.025	40	3	3	2.7048	2.9514	1458.555	0.9016
		40	3.5	3	3.1556	2.7734	1458.555	0.9016
	0.1	40	3	3	2.7048	2.9514	1458.555	0.9016
		40	3.5	3	3.1556	2.7734	1458.555	0.9016
P_s	0.875	38	3	3	2.6337	2.8817	1779.359	0.8779
		38	3.5	3	3.0726	2.7163	1779.359	0.8779
	0.95	43	2.5	2	2.44175	3.4159	108.6957	0.9767
ATS_s	2.5	40	2.5	3	2.254	3.2251	1458.555	0.9016
	5	40	3	3	2.7048	2.7534	1458.555	0.9016
		40	3.5	3	3.1556	2.5754	1458.555	0.9016
L_s	3	40	3	3	2.7048	2.9487	1458.555	0.9016
		40	3.5	3	3.1556	2.771	1458.555	0.9016
	4	40	2	3	1.8032	3.6661	1458.555	0.9016

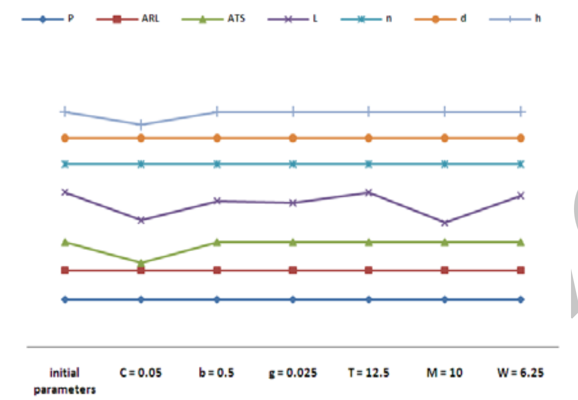

Figure 1. Effect of decreasing input cost parameters on objective functions and design parameters

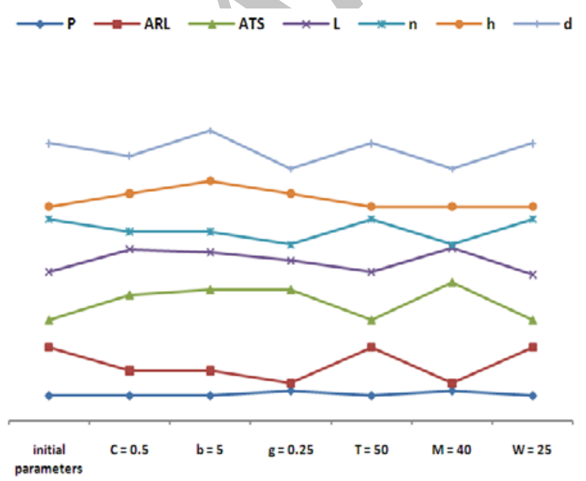

Figure 2. Effect of increasing input cost parameters on objective functions and design parameters

The *np-MOESD* model is robust to decreasing in the input cost parameters. In other words, when the input cost parameters decrease, design parameters including the sample size (*n*), the sampling interval (*h*) and the sample number of nonconforming items (*d*) are constant except the sampling interval which changes with the change of parameter *c*.

Furthermore, the *np-MOESD* model is robust to increasing in the average cost of verifying a false alarm (*T*) and the average cost of detecting an assignable cause (*W*). When, the input parameters increases, cost function also increases and some of previous design parameters combinations would be infeasible. As a result, number of feasible solutions decreases and finally optimal design parameters changes.

Note that the first two rows of Table 6 are the results of the numerical example based on the initial cost parameters discussed in subsection 5.1.

5. 3. 2. Sensitivity Analysis on the Right Hand Sides of Constraints In this segment, sensitivity analysis on the right hand side of constraints including ($\alpha_s, P_s, ATS_s, L_s$) is done. To do that, each of these resources is altered and the effect of these changes on the selection of the most efficient units is investigated.

When the bound of constraint on probability of Type I error (α) increases or decreases, the most efficient combination of design parameters does not change. On the other hand, *np-MOESD* is not dependent to decreasing in L_s and increasing in ATS_s. Moreover, when the bounds of constraints on *L* and *ATS* increases or decreases respectively, the sampling frequency (*h*) and *ATS* of the most efficient combination of design parameters decreases and the expected cost value (*L*) increases. The most sensitive right hand side in the *np-*

MOESD model is P_s because When this parameter changes, the most efficient unit changes as well.

6 CONCLUSIONS

In this paper, we proposed a multi-objective economic-statistical model for *np* control chart (*np-MOESD* model). This model contains four objective functions including economic cost function, in-control average run length (ARL_0), detection power (*P*) and also average time to signal (*ATS*). *DEA* approach was applied for measuring relative efficiency of different combinations of the design parameters (*n, h and d*). Then, the maximin weight model (Wang et al., [20]) was used to determine the most efficient unit. The numerical example from Duncan [1] was applied to investigate the proposed model. Comparison between our *np-MOESD* model with two models (pure economic model in Duncan [1] and Chen and Liao [14]'s model) showed the better statistical properties of the proposed model. In addition, the sensitivity analysis showed the robustness of *np-MOESD* model to decreasing in all of the input cost parameters except the sampling interval which changes with the change of parameter *c*. Moreover, the proposed *np-MOESD* model is robust to *T* and *W* parameters as well. Furthermore, the most sensitive right hand side of the constraints in *np-MOESD* is P_s because when this parameter changes, the most efficient unit changes as well.

For the future research, one can use other *MCDM* methods to determine design parameters of *np* control chart. The proposed approach of this paper can be also applied in other control charts such as cumulative sum (*CUSUM*) or exponentially weighted moving average (*EWMA*) control charts. Moreover, when several assignable causes occur in the process, the proposed approach can be developed with some changes in the economic cost function.

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Multi-objective Efficient Design of *np* Control Chart Using Data Envelopment Analysis

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P A P E R I N F O

Paper history:
Received 20 September 2012
Received in revised form 18 November 2012
Accepted 24 January 2013

Keywords:
Economic-statistical Design
Data Envelopment Analysis (*DEA*)
MOESD
np Control Chart
Maxmin Weight Model

چکیده

نمودارهای کنترلی از ابزارهای پرکاربرد کنترل فرآیند آماری می باشند که به تفکیک انحرافات بادلیل از انحرافات تصادفی پرداخته و موجب بهبود کیفیت فرآیند می شوند. به منظور طراحی نمودارهای کنترلی، سه پارامتر مورد نیاز است که عبارتنداز: اندازه نمونه، فاصله بین دو بار نمونه گیری و ضریب حدود کنترل. رویکردهای مختلفی توسط محققان از جمله طراحی آماری، طراحی اقتصادی، طراحی اقتصادی- آماری برای طراحی نمودارهای کنترلی مد نظر قرار گرفته است. اخیراً طراحی چند هدفه نمودارهای کنترل توسط محققان مورد بررسی قرار گرفته است. در این مقاله، طراحی اقتصادی- آماری چندهدفه نمودار کنترل *np* پیشنهاد شده است. اهداف در نظر گرفته شده عبارتند از متوسط هزینه در ساعت، متوسط طول دنباله تحت کنترل، توان نمودار کنترل و متوسط مدت زمان تا هشدار. برای حل مدل چندهدفه پیشنهادی از روشی جهت جستجوی حل های بهینه پارتو استفاده شده و با ترکیب آن با روش تحلیل پوششی داده ها کاراترین پارامترهای نمودار کنترل تعیین شده است. یک مثال عددی از مقاله دانکن [۱] استخراج شده است و از آن برای بررسی و ارزیابی روش پیشنهادی در مقایسه با مدل اقتصادی دانکن [۱] و یک مدل دیگر در ادبیات موضوع استفاده شده است. در ادامه نیز به تحلیل حساسیت روش پیشنهادی پرداخته شده است. نتایج نشان دهنده بهبود خواص آماری نمودار کنترل *np* توسط مدل پیشنهادی می‌باشد.

doi: *10.5829/idosi.ije.2013.26.06c.07*