TRANSACTIONS B: Applications Vol. 29, No. 2, (February 2016) 137-144

International Journal of Engineering

Journal Homepage: www.ije.ir

Mathematical Model and Vibration Analysis of Aircraft with Active Landing Gear System using Linear Quadratic Regulator Technique

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PAPER INFO

Paper history:
Received 26 November 2015
Received in revised form 22 December 2015
Accepted 26 January 2016

Keywords:
Linear Quadratic Regulator Controller
Full Aircraft Model
Active Landing Gear System
Suspension System

ABSTRACT

This paper deals with the study and comparison of passive and active landing gear system of the aircraft and dynamic responses due to runway irregularities while the aircraft is taxying. The dynamic load and vibration caused by the unevenness of runway will result in airframe fatigue, discomfort of passengers and the reduction of the pilot's ability to control the aircraft. One of the objectives of this paper is to obtain a mathematical model for the passive and active landing gears for full aircraft model. The main purpose of current paper is to design linear quadratic regulator (LQR) for active landing gear system that chooses damping and stiffness performance of suspension system as control object. Sometimes conventional feedback controller may not perform well because of the variation in process dynamics due to nonlinear actuator in active control system, change in environmental conditions and variation in the character of the disturbances. To overcome the above problem, we have designed a controller for a second order system based on Linear Quadratic Regulator. The performance of active system is compared with the passive landing gear system by numerical simulation. The results of current paper in compared with the previous work mentioned in reference, demonstrates 37.04% improvement in body acceleration, 20% in fuselage displacement and 13.8% in the shock strut travel. The active landing gear system is able to increase the ride comfort and good track holding by reducing the fuselage acceleration and displacement and load induced to airframe caused by runway excitation.

doi: *10.5829/idosi.ije.2016.29.02b.01*

NOMENCLATURE

P_{accum}	High pressure in accumulator	M	Sprung mass (kg)
P_{res}	Low pressure in reservoir	m	Landing gear unsprung mass (kg)
Ks	gear sprung mass stiffness rate (N/m)	F_Q	Active Control force
Kt	Gear sprung mass stiffness rate (N/m)	$\mathbf{Q}_{flow}$	Fluid flow quantity from servo valve
Cs	Gear unsprung mass damper rate (N.s/m)	ρ	Density of hydraulic fluid
Ct	Tire damper rate coefficient (N.s/m)	C_d	Coefficient of discharge

1. INTRODUCTION

Developing improved methods for achieving better quality ride control from rough and unexpected runway conditions is one of the major challenges currently faced by the aerospace industry [1]. To improve ride comfort and landing gear maneuverability, a good suspension system needs to reduce sprung mass acceleration and provide adequate suspension deflection to maintain tire-terrain contact. An aircraft landing gear system must absorb the kinetic energy produced by a landing impact and excitations caused by the aircraft travelling over an uneven runway surface and provide

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Please cite this article as: A. Toloei, E. Aghamirbaha, M. Zarchi, Mathematical Model and Vibration Analysis of Aircraft with Active Landing Gear System using LQR Technique, International Journal of Engineering (IJE), TRANSACTIONS B: Applications Vol. 29, No. 2, (February 2016) 137-144

ride comfort for passengers and make the aircraft easier to control on the ground before take-off and after landing.

This is the necessary requirement of a successfully designed landing gear system. It is impossible to adjust the control forces that are naturally generated in real-time landing and runway environments. The parameters are generally fixed and a passive suspension has the ability to store energy via a spring and to dissipate it via a damper. An active suspension system has the ability to store, dissipate and to introduce energy to the system. It may vary its parameters depending upon operating conditions. However, in active controlled landing gear system, the hydraulic fluid flow to the shock strut is controlled depending on ground induced aircraft vibration loads, thereby changing the hydraulic damping. So, the focus on active landing gear system is essential to overcome the difficulties in passive landing gear system. From 1970s, the active control and semi-active controls began to be popular and widely used in vibration control of constructions and vehicle suspensions. Compared with the passive control, the active and semi-active controls have excellent tunabilities due to their flexible structure. Previous analytical studies by McGhee [2] and Wignot et al. [3], indicated the feasibility and potential benefits of applying active load control to the landing gear to limit the ground loads applied to the airframe. An active landing gear system was first demonstrated by Ross and Edson to reduce landing loads and vibrations under various runway profiles [3]. Active control technology has become popular in recent years and has been applied to many systems such as an automobile suspension system, precision machine platform and building structures [4].

The study explained in Freymann [5] proved analytically and experimentally the benefits of actively controlled landing gears in reducing landing loads and vibrations under various runway profiles [5]. Active control schemes have been investigated on landing gears [2]. The dynamic performances of active control of damping have been measured for a range of aircraft speeds and for random and discrete bump models of the runway surface [6]. The reductions of peak and root mean square (RMS) accelerations at various fuselage stations are addressed. A mathematical model and the nonlinear equations for a telescopic main gear modified with an external hydraulic system have been carried out [7].The analysis and test result for an A-6 intruder landing gear system has been studied. Investigation results from an F-106B fighter interceptor aircraft involving both passive and active control modes show that the active landing gear system significantly reduces the loads to the airframe during landing and ground operations [8]. Investigations of development of a mathematical model of a single active landing gear system with a proportional integral derivative (PID) controller has shown the improvement in performances of a passive landing gear system [7]. Zarchi designed the Proportional Integral Derivative and Fuzzy controller for linear and nonlinear model of semi-active and active landing gear system based on Bees Intelligent Algorithm as the optimization technique that chooses damping performance of suspension system at touchdown as optimization object [9]. Sivakumar [10] studied on a mathematical vibration model of an aircraft with active landing gear system has been developed and its performance simulated using MATLAB/Simulink. In this system a PID controller has been used and the gains of the PID controllers have been tuned using the Ziegler-Nichols method. The outputs of the independent controllers are used to operate the servo control system which applies the control forces in the respective active landing gear [10].

In this paper we shall give a short view of the so-called Linear Quadratic Regulator theory which can be consulted for more details in reference [4]. In order to realize the full potential of active suspensions, the controller should have the capability of adapting to changing road environments. Two important suspension performance metrics considered in the literature are passenger comfort and suspension deflection, i.e., the relative displacement between the fuselage and landing gear assembly. It is widely accepted that lower vertical acceleration levels correspond to increased comfort.

Optimal controllers, however, minimize a defined performance index. Current hardware technology and knowledge of optimal control theory allows us to employ a sophisticated electronically controlled active suspension system in a vehicle at a reasonable cost.

2. MODEL OF ACTIVE LANDING GEAR SYSTEM

Figure 1 illustrates the schematic diagram of active landing gear system; the active landing gear system consists of a low pressure reservoir, a hydraulic pump, a high pressure accumulator, a servo actuator and an electronic controller. The passive system does not consist of a servo actuator, transducers and electronic controllers. The transducers fitted in the landing gear send a signal to the electronic controllers depending on the impact conditions to actuate the servo system to supply hydraulic oil into the landing gears. The generation of active control energy is to extenuate the vibrations to improve the ride comfort [10].

3. AIRCRAFT AND LANDING GEAR SYSTEM MATHEMATICAL VIBRATION MODEL

A mathematical vibration model of six Degree of Freedom for the airplane and landing gears is shown in Figure 2.

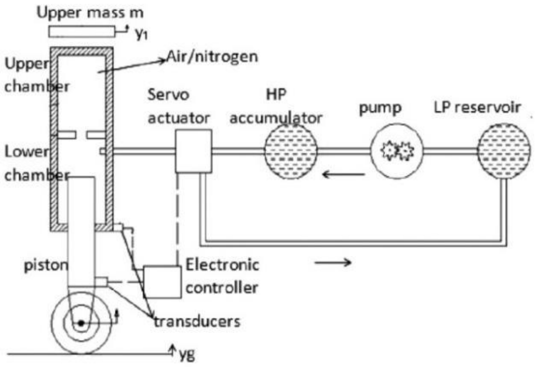

Figure 1. Schematic diagram of active landing gear system. HP=high-pressure, LP=low-pressure [10]

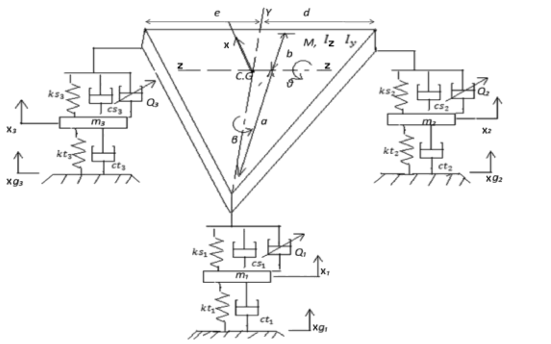

Figure 2. Schematic sketch of the dynamic model of the active landing gear system [10]

In the full aircraft model, the fuselage or sprung mass is free to roll and pitch. The sprung mass is connected to the three unsprung masses which are front, rear left and rear right landing gears. They are free to bounce vertically with respect to the sprung mass. The full aircraft model contains three degrees of freedom (d.o.f) for the sprung mass (bounce, roll, pitch) and three for the vertical motions of the nose and the rear main landing gear's unsprung masses.

3. 1. Six DOF Dynamic Equilibrium Equation of Motion

The six d.o.f. vibration model of the full aircraft is illustrated in Figure 2. In this model u,ϴ, and β explain the bounce, pitch and roll motion of the aircraft, respectively, while u_1, u_2, and u_3 demonstrate the displacement of the nose, left and right main landing gears, respectively. In this figure *a* is distance between center of gravity (CG) and the nose landing gear, *b* is distance between CG to the main landing gears, *d* is distance between CG to left main landing gear, *e* is distance between CG to right main landing gear. Using Newton's second law of motion, the second order differential equations of motion explaining dynamics of the active landing gear system can be written as For bounce motion of the sprung mass:

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{x}}\}+[\boldsymbol{C}]\{\dot{\boldsymbol{x}}\}+[\boldsymbol{K}]\{\boldsymbol{x}\}=\{\boldsymbol{F}\} \tag{1}$$

$$[M]=\begin{bmatrix} M & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{yy} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{xx} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 \end{bmatrix} \tag{2}$$

$$[C]=\begin{bmatrix} Z_1 & Z_2 & Z_3 & -cs_1 & -cs_2 & -cs_3 \\ Z_2 & Z_4 & Z_5 & acs_1 & -bcs_2 & -bcs_3 \\ Z_3 & Z_5 & Z_6 & hcs_1 & dcs_2 & -ecs_3 \\ -cs_1 & acs_1 & hcs_1 & cs_1+ct_1 & 0 & 0 \\ -cs_2 & -bcs_2 & dcs_2 & 0 & cs_2+ct_2 & 0 \\ -cs_3 & -bcs_3 & -ecs_3 & 0 & 0 & cs_3+ct_3 \end{bmatrix} \tag{3}$$

$$Z_1 = cs_1 + cs_2 + cs_3$$
$$Z_2 = -acs_1 + bcs_2 + bcs_3$$
$$Z_3 = -hcs_1 - dcs_2 + ecs_3$$
$$Z_4 = a^2cs_1 + b^2cs_2 + b^2cs_3$$
$$Z_5 = hacs_1 - dbcs_2 + ebcs_3$$
$$Z_6 = h^2cs_1 + d^2cs_2 + e^2cs_3$$

$$[K]=\begin{bmatrix} R_1 & R_2 & R_3 & -ks_1 & -ks_2 & -ks_3 \\ R_2 & R_4 & R_5 & aks_1 & -bks_2 & -bks_3 \\ R_3 & R_5 & R_6 & hks_1 & dks_2 & -eks_3 \\ -ks_1 & aks_1 & hks_1 & ks_1+kt_1 & 0 & 0 \\ -ks_2 & -bks_2 & dks_2 & 0 & ks_2+kt_2 & 0 \\ -ks_3 & -bks_3 & -eks_3 & 0 & 0 & ks_3+kt_3 \end{bmatrix} \tag{4}$$

$$R_1 = ks_1 + ks_2 + ks_3$$
$$R_2 = -aks_1 + bks_2 + bks_3$$
$$R_3 = -hks_1 - dks_2 + eks_3$$
$$R_4 = a^2ks_1 + b^2ks_2 + b^2ks_3$$
$$R_5 = haks_1 - dbks_2 + ebks_3$$
$$R_6 = h^2ks_1 + d^2ks_2 + e^2ks_3$$

$$\{X\}=\begin{Bmatrix} x \\ \theta \\ \beta \\ x_1 \\ x_2 \\ x_3 \end{Bmatrix} \tag{5}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}=\begin{bmatrix} O_{6\times6} & I_{6\times6} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix}+\begin{bmatrix} O_{6\times6} \\ M^{-1} \end{bmatrix}F \tag{6}$$

$$\{F\}=\begin{Bmatrix} -Q_1 \\ -Q_2 \\ -Q_3 \\ kt_1ug_1 + ct_1\dot{u}g_1 + Q_1 \\ kt_2ug_2 + ct_2\dot{u}g_2 + Q_2 \\ kt_3ug_3 + ct_3\dot{u}g_3 + Q_3 \end{Bmatrix} \tag{7}$$

$$\{\ddot{\boldsymbol{x}}\}=[\boldsymbol{M}]^{-1}\{\boldsymbol{F}\}-[\boldsymbol{M}]^{-1}[\boldsymbol{C}]\{\dot{\boldsymbol{x}}\}-[\boldsymbol{M}]^{-1}[\boldsymbol{K}]\{\boldsymbol{x}\} \tag{8}$$

4. CONTROLLER DESIGN

The simple form of loop-shaping in scalar systems does not extend directly to multivariable (MIMO) plants, which are characterized by transfer matrices instead of transfer functions. The notion of optimality is closely tied to MIMO control system design. Optimal controllers, i.e., controllers that are the best possible, according to some figure of merit, turn out to generate only stabilizing controllers for MIMO plants. In this sense, optimal control solutions provide an automated design procedure, we have only to decide what figure of merit to use. The linear quadratic regulator (LQR) is a well-known design technique that provides practical feedback gains.

4. 1. Introduction of LQR Controller

The most important and most commonly used linear controller can be linear quadratic regulator (LQR) noted. Linear optimal control problem and dual Kalman filter, the estimator is optimized for a given observer random noise and disturbances that affect the behavior of the system, LQR state feedback to be optimized for cost. Also, it has good performance for linear systems. On the other hand, LQR control will use all of state variables to form a linear controller. The purpose of LQR design applying a control (u) to the system with the following equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$

That the performance index, or the cost function (J) be optimized:

$$\min J = \frac{1}{2} x(t_2)^T Sx(t_2) + \int_0^{t_f} (x^T Qx + u^T Ru + x^T Nu)dt \quad (10)$$

As the product of x in u (Cross term) is usually not considered. In Equation (2), Q is a semi positive definite matrix and R a real symmetric matrix. By choosing the elements of the matrices R and Q can control system state variables to be weighted together. R and Q are also called weight matrices.

$$\begin{cases} ifQ : P.S.D \leftrightarrow \forall x \neq 0 : x^T Qx \geq 0 \\ ifR : P.D \leftrightarrow \forall u \neq 0 : u^T Ru \geq 0 \end{cases} \quad (11)$$

A soluon to a complex problem and solve it exactly the feedback state.

$$u(t) = -k(t)x(t) \quad (12)$$

Matrix k (t) achieve optimal control vector (12) into equation in solving the LQR controller.

$$k(t) = R^{-1}B^T P(t) \quad (13)$$

Also, Riccati differential equation into the algebraic equation is calculated as follow:

RDE:

$$Q + A^T P + PA - PBR^{-1}B^T P = -\dot{P}(t) \quad (14)$$

Answer Riccati differential equation P(t) varies with time, which proves that the amount of performance index (J) is optimized. For calculation, Riccati equation solution with the command LQR had run by MATLAB.

When R → 0 (minimum R = 0) control is cheap, it means that a lot of energy is applied into the system, and a lot of energy is wasted. When R → ∞, LQR consume less energy and the system is slower and less force is applied to the system and vice versa, as well as the matrix Q. In LQR controller we can change weight matrices R and Q, to prevent growing too large state variables and control signals.

Due to the popularity of LQR controllers, in addition to ease of implementation, is robustness and high performance properties.

Computer simulation work has been performed based on Equations (1) and (2). Comparison between passive and active suspension for SIX degrees of freedom aircraft model is observed. For the LQR controller, the best parameters Q and R by trial and error method is set to be:

$$Q = 10e9 \times \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}_{12\times12}$$

$$R = 10e-1 \times \begin{bmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{bmatrix}_{6\times6}$$

4. 2. Active Control Force

The active control force F_Q is a function of the flow output of the servo valve. The servo valve displacement l(t) is controlled by the LQR controller. There is no exact relationship between the active control force F_Q and the flow quantity Q_{flow} from the servo valve. It is often determined through experiments or by empirical formula. It is assumed that the active control force is described by:

$$F_Q = k_a Q_{flow} |Q_{flow}| \quad (15)$$

The flow quantity Q_{flow} is calculated by:

$$Q_{flow} = C_d \sqrt{\frac{|P_{accum} - P_{res}|}{\rho}} \quad (16)$$

When the displacement *l(t)*> 0, the hydraulic oil would have positive flow from the accumulator into the landing gear system and a positive control force F_Q>0. When *l(t)*< 0, oil is drawn from the landing gear into the *LP* reservoir, so that F_Q<0, where *l(t)* is the displacement determined by the LQR controller.

When the displacement *l(t)*> 0, the hydraulic oil would have positive flow from the accumulator into the landing gear system and a positive control force F_Q>0. When *l(t)*< 0, oil is drawn from the landing gear into the *LP* reservoir so that F_Q<0, where *l(t)* is the displacement determined by the LQR controller.

5. DYNAMIC RESPONSE OF THE AIRCRAFT FOR A RUNWAY WITH HALF SINE WAVE BUMPS

Figure 3 as displayed the runway ramp input impulse. It is designed with half sine wave height of 40mm for nose landing gear, duration of impulse 0.8 s and frequency 7.85 rad/s. It can be mentioned that the airplane travels, and main landing gears the runway bumps are designed with half sine wave of height 60mm for the right main landing gear and 100mm for the left main landing gear. Duration of impulse is 0.8 s and frequency is 7.85 rad/s over which the airplane travels, and is described by Equations (17), (18) and (19).

$$x_{g1} = 40(1-\cos\omega t) \quad 0.2 \le t \le 1.0 \qquad 0 \quad \textit{Otherwise} \quad (17)$$

$$x_{g2} = 60(1-\cos\omega t) \quad 2.6 \le t \le 3.4 \qquad 0 \quad \textit{Otherwise} \quad (18)$$

$$x_{g3} = 100(1-\cos\omega t) \quad 5.0 \le t \le 5.8 \qquad 0 \quad \textit{Otherwise} \quad (19)$$

6. NUMERICAL SIMULATION AND CONTROL ANALYSIS

Based on the analysis described in Sections 2 and 3, and using MATLAB (Simulink, 2014) numerical simulations of the active landing gear system, responses are derived using LQR. To illustrate the approaches, we investigate a Fokker airplane according to [10] with body and landing gear masses of 22,000 and 650 kg, taxying at a speed of 55.5 m/s on a runway. For representation purposes, on six DOF model, an assumed half sine type runway bump of height 40mm for concentrated landing gears is used for the airplane travels. The transient response of the aircraft with the passive and active landing gear system with LQR techniques is simulated for the runway with half sine wave bumps and for the random runway.

7. RESULTS AND DISCUTIONS

A mathematical vibration model of an aircraft with active landing gear system has been developed and its performance simulated using MATLAB/Simulink. In this paper, LQR technique is applied to a second order system. Figure 4 illustrate the schematic of simulink model for LQR technique.

Two kinds of control methods including passive, LQR active control is used in the computer simulation. By comparing the performance of the passive and active suspension system using LQR technique, it is clear that active suspension can give lower amplitude and faster settling time. In the process of simulation, the comparison is dynamic response of fuselage acceleration, vertical displacement of aircraft, suspension travel, and force generated for landing gear/shock absorber actuators for a runway with half sine wave are as given in Figures 5-12, respectively.

Figures 5-12 illustrate that both peak values and settling time have been reduced by the active landing gear system. The vertical displacement of the aircraft is an important parameter in designing an aircraft landing gear system. It is expected that an aircraft rapidly returns to its original equilibrium state with runway disturbance. Through numerical simulation, and according to the parameters defining the stability conditions, we found that Figures 8 and 5 show that there is 21.6, 59.5% decrease of the aircraft's displacement and fuselage acceleration response, respectively. This makes taxying smoother, and therefore the crew/ passenger comfort improved. The passive system requires approximately 3sec for the aircraft to return to its static equilibrium position. This time is reduced to approximately 1sec using active system with LQR that demonstrates a significant improvement over the performance of the passive system. The amplitude of the spring force transmitted to the airframe and landing gear affects the structural strength and their fatigue life. Figures 9, 10 and 11 show that this force is reduced using LQR active system and indicate that there are 22.9, 53.33 and 28.1% decrease of transmitted force in the passive landing gear, respectively.

Shock strut travel in Figure 12 can reduce the amplitude and settling time compare to passive landing gear. In Table 3, results of this work and research done by [10] is compared. As seen, significant improvement is achieved by means of LQR method for active landing gear system.

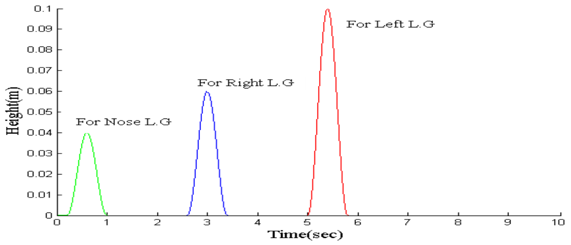

Figure 3. Runway Input Excitation

TABLE 1. The parameters obtained from the linear part of nonlinear curves used in the numerical simulations [10]

Description	Symbol	Value	Units
Sprung mass	M	22000	Kg
Nose landing gear unsprung mass	m1	130	Kg
Rear left gear unsprung mass	m2	260	Kg
Rear right gear unsprung mass	m3	260	Kg
Nose gear sprung mass stiffness rate	ks1	6.73e5	N/m
Rear left gear sprung mass stiffness rate	ks2	4.08e5	N/m
Rear right gear sprung mass stiffness rate	ks3	4.08e5	N/m
Nose gear sprung mass damper rate	cs1	1.43e5	N.s/m
Rear left gear sprung mass damper rate	cs2	6.25e5	N.s/m
Rear right gear sprung mass damper rate	cs3	6.25e5	N.s/m
Nose gear unsprung mass stiffness rate	kt1	1.59e6	N /m
Rear left gear unsprung mass stiffness rate	kt2	1.59e6	N /m
Rear right gear unsprung mass stiffness rate	kt3	1.59e6	N /m
Nose gear unsprung mass damper rate	ct1	4066	N.s/m
Rear left gear unsprung mass damper rate	ct2	4066	N.s/m
Rear right gear unsprung mass damper rate	ct3	4066	N.s/m
Mass moment of inertia about XX axis	Ixx	65e3	$Kg.m^2$
Mass moment of inertia about YY axis	Iyy	100e3	$Kg.m^2$
Longitudinal distance from CG to nose landing gear	a	7.76	m
Distance from CG to left main landing gear	d	3.8425	m
Longitudinal distance from CG to horizontal axis of main landing gear	b	3.8425	m
Distance from CG to right main landing gear	e	3.8425	m

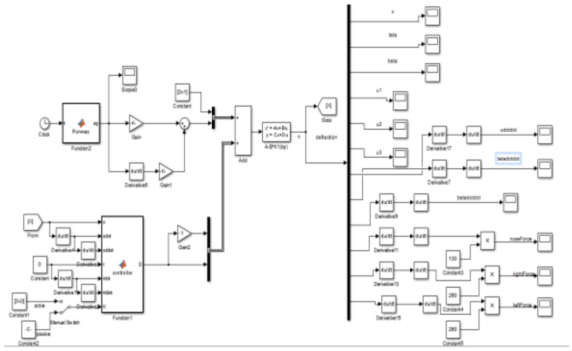

Figure 4. The Schematic of simulink model

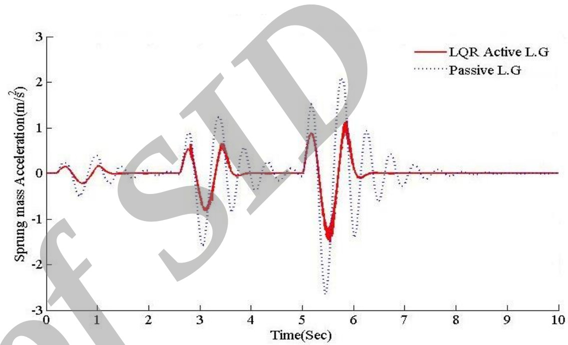

Figure 5. The Fuselage acceleration of the aircraft with passive and active landing gear

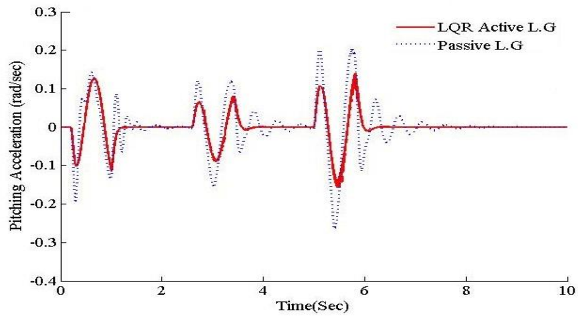

Figure 6. The sprung mass pitch acceleration of the aircraft with passive and active landing gear

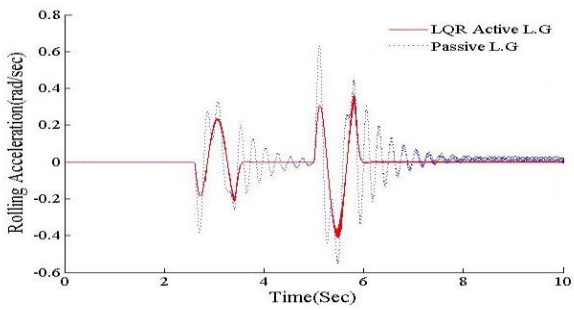

Figure 7. The sprung mass roll acceleration of the aircraft with passive and active landing gear

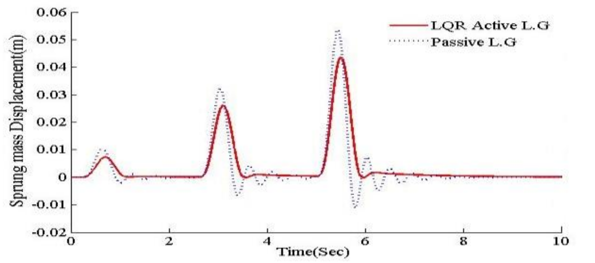

Figure 8. The sprung mass displacement of the aircraft with passive and active landing gear

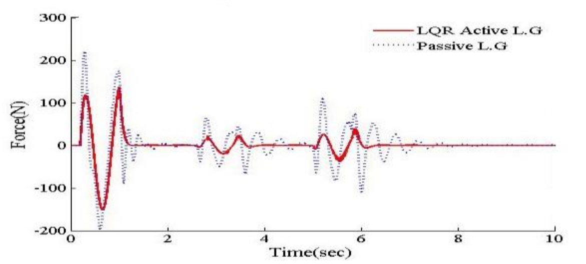

Figure 9. Force generated for Nose Landing Gear Actuator

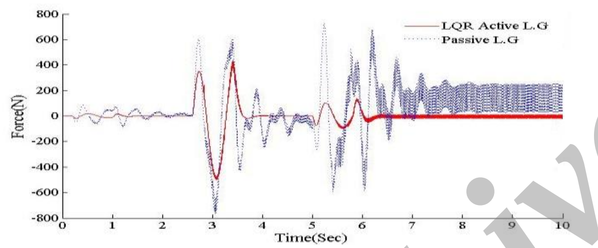

Figure 10. Force generated for Right Landing Gear Actuator

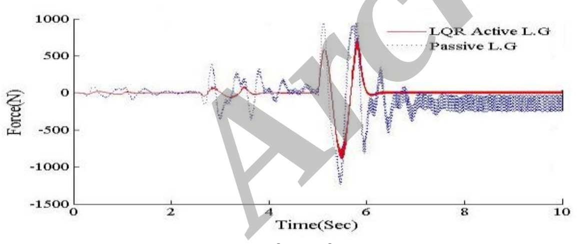

Figure 11. Force generated for left Landing Gear Actuator

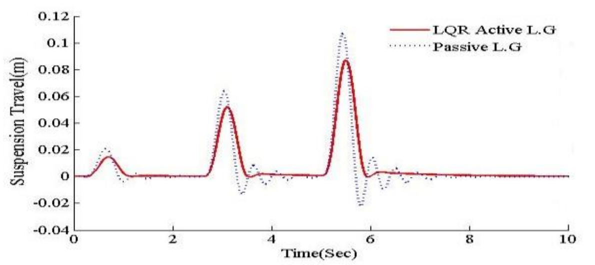

Figure 12. The shock strut travel of passive and active landing gear

TABLE 2. Comparison of passive and LQR active landing gears

	Passive L.G			Active L.G		
Parameter	**Nose**	**Right**	**Left**	**Nose**	**Right**	**Left**
Fuselage Acc (m/s^2)	0.427	1.529	2.580	0.173	0.751	1.423
Fuselage Disp (m)	0.010	0.032	0.053	0.008	0.027	0.044
Shock Strut Travel (m)	0.021	0.065	0.107	0.015	0.053	0.087
Force (N)	195.665	760.331	1227.56	150.867	495.868	883.013

TABLE 3. Comparison of PID active landing gear and LQR active landing gears

	PID Active L.G			LQR Active L.G		
Parameter	**Nose**	**Right**	**Left**	**Nose**	**Right**	**Left**
Fuselage Acc (m/s^2)	0.52	1.28	2.26	0.173	0.751	1.423
Fuselage Disp (m)	0.01	0.0288	0.050	0.008	0.027	0.044
Shock Strut Travel (m)	0.0174	0.0517	0.101	0.015	0.053	0.087

From the figures, the peak to peak values are taken for comparison of passive and active landing gears and are listed in Table 2.

The overall average peak to peak value of the aircraft's displacement response decreased 16.98% by the active landing gear system. The overall average peak to peak value of the aircraft's acceleration response and the value of the force reduced 48.35 and 28.58% with the active landing gear system, respectively. The settling time is also reduced to 25% with the active landing gear system. The sprung mass roll and pitch acceleration of the aircraft with active landing gear reduced 23 and 40.7%, respectively. Thereby, the aircraft taxies more smoothly, crew/passenger comfort is improved and a better runway holding is achieved by using the active landing gear system.

9. CONCLUSION

Comparison of passive control and LQR with half sine wave height runway input impulse is identify the effectiveness of the second and third through significant reduction in the magnitude of the displacement of the center of gravity of the aircraft and the load transmitted to the airframe by the landing gear during aircraft taxying. In this research, design of LQR technique is considered and it is further demonstrated that by using this method in active landing gear system, a reduction in the time length of responses to return to their static equilibrium positions is achieved. However, improving the performance of the landing gear, the fatigue life of the airframe and landing system, crew and passenger comfort, the pilot's ability to control the airplane during ground operations and a reduction of the influence of runway unevenness is attained in compared to passive performance.

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Mathematical Model and Vibration Analysis of Aircraft with Active Landing Gear System using Linear Quadratic Regulator Technique

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P A P E R   I N F O

Paper history:
Received 26 November 2015
Received in revised form 22 December 2015
Accepted 26 January 2016

Keywords:
Linear Quadratic Regulator Controller
Full Aircraft Model
Active Landing Gear System
Suspension System

چکیده

این مقاله به بررسی و مقایسه پاسخ دینامیکی هواپیما با سیستم ارابه فرود فعال و غیر فعال به سبب بی نظمی باند در حالت تاکسی هواپیما می‌پردازد. بار دینامیکی و ارتعاشی بر اثر ضربه فرود و ناهمواری باند به خستگی بدنه، ناراحتی مسافران و کاهش توانایی خلبان برای کنترل هواپیما منجر خواهد شد. یکی از اهداف این مقاله به دست آوردن یک مدل ریاضی برای ارابه فرود فعال و غیر فعال برای مدل هواپیمای کامل است. هدف اصلی مقاله حاضر طراحی تکنیک‌های تنظیم کننده خطی درجه دوم برای سیستم ارابه فرود فعال است که عملکرد میرایی و سختی سیستم تعلیق را به عنوان هدف کنترلی انتخاب می‌کند. گاهی اوقات کنترلر فیدبک معمولی به دلیل تغییر در دینامیک فرایند با توجه به محرک غیرخطی در سیستم کنترل فعال، تغییر شرایط محیطی و تغییر در خصوصیات اغتشاشات نمی‌تواند عملکرد خوبی داشته باشد. برای غلبه بر مشکل فوق، این تحقیق یک کنترل کننده برای یک سیستم مرتبه دوم را طراحی می‌کند. عملکرد سیستم فعال با سیستم ارابه فرود غیرفعال از طریق شبیه سازی عددی مورد مقایسه قرار گرفته است. نتایج مقاله حاضر در مقایسه با کار قبلی اشاره شده در مرجع نشان می‌دهد ٪۳۷/۰٤ بهبود در شتاب بدنه، ٪۲۰ در جابه‌جایی بدنه و٪۱۳/۸ در کورس ضربه‌گیر دارد. با سیستم ارابه فرود فعال به طور قابل توجهی راحتی مسافر و کیفیت هدایت خلبان با کاهش شتاب بدنه، کاهش جابجایی عمودی بدنه و نیروی وارد بر بدنه هواپیما ناشی از اغتشاش باند افزایش یافته است.

doi*: 10.5829/idosi.ije.2016.29.02b.01*