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Mathematical Investigation of Soil Temperature Variation for Geothermal Applications

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ABSTRACT

This paper aims to predict the periodic variation of ground temperature with depth for time variant condition of ambient air temperature and solar radiation data for Jamshedpur, India. Fourier series and numerical techniques have been used to determine (hottest and coldest day) diurnal and annual temperature variation of the year 2015. The diurnal temperature variation is up to 0.2 m depth of soil whereas annual temperature variation is up to 3 m depth.

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NOMENCLATURE

α	Thermal diffusivity of soil (m^2/s)	k	Thermal conductivity of soil (W/mK)
α_0	Absorptivity of solar radiation at the surface	T	Soil temperature (^{0}C)
c	Specific heat of soil (J/kgK)	T_a	Air temperature (^{0}C)
ε	Emissivity of soil surface	T_e	Effective temperature (^{0}C)
h	Convective heat transfer coefficient on the soil surface (W/m^2K)	t	Time (s)
ΔR	Long-wave radiation (W/m^2)	ω	2Π/period (rad/s)
ρ	Density of ground (kg/m^3)	y	Vertical axis (m)
S	Intensity of solar radiation (W/m^2)		
m	Number of terms of Fourier series		

1. INTRODUCTION

Soil temperature has important applications in the field of heating and cooling of buildings and agricultural greenhouses. Design of geothermal system is necessary to know the ground temperature at different depths. The air temperature and solar radiation are the main meteorological parameters to change the regular periodic variation in thermal behaviour of the ground.

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Soni et al. [1] presented an excellent review of research in the area of earth-air heat exchangers. Khatry et al. [2] presented a technical note for ground temperature variation with depth, taking into account the periodicity of solar radiation and atmospheric temperature for Kuwait. Bharadwaj and Bansal [3] calculated daily and annual variations of the ground temperature for dry sunlit, wet sunlit, dry shaded and wet shaded surface conditions at New Delhi. Mihalakakouet et al. [4] and Mihalakakou [5] estimated ground surface temperature for bare and short-grass covered soil employing Fourier analysis and validated

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results by measurements in Athens and Dublin. Ozgener et al. [6] measured and predicted the temperature of the soil at various depths in Izmir, Turkey. Kurylyk and MacQuarrie [7] performed analytical solution for estimation of the ground temperature at different weather conditions. Hu et al. [8] estimates soil temperature, water properties and soil thermal properties by new Fourier series analytical based solution

In the present investigation, temperature variation of soil for the dry sunlit condition has been modeled employing Fourier as well as numerical techniques for time varying boundary condition for Jamshedpur, India.

2. MATHEMATICAL FORMULATION

The variation of ground temperature followed by one-dimensional, transient heat conduction equation is given by [9]:

$$\frac{\partial^2 T(y,t)}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T(y,t)}{\partial t} \tag{1}$$

Solution of above equation is subjected to the boundary condition at the ground surface given by [3]:

$$-k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T_a - T_{y=0}) - \varepsilon\Delta R + \alpha_0 S \tag{2}$$

The left side of above equation shows the conduction through ground surface. The first term of right side equation shows convective heat transfer between the ground surface and air. The second term of above equation is long wave radiation and the third term denotes solar radiation absorbed by the ground surface.

The above equation can be written in the form of general convective heat transfer boundary condition as:

$$-k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T_e - T_{y=0}) \tag{3}$$

The effective temperature T_e can be expressed as:

$$T_e = T_a + \alpha_0 S / h - \varepsilon\Delta R / h \tag{4}$$

2. 1. Fourier Analysis

Ambient air temperature T_a and solar radiation intensity S vary periodically which can be expressed as Fourier series. The effective temperature T_e will also be expressed as Fourier series:

$$T_e = a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega t - \psi_m) \tag{5}$$

The solution of one-dimensional heat conduction Equation (1) for T to be finite when $y \to \infty$ becomes [9]:

$$T(y,t) = A_0 + \sum_{m=1}^{\infty} A_m \exp[\left[i(m\omega t + \alpha_m y))\right] \tag{6}$$

where:

$$\alpha_m = -(1-i)(\omega\rho cm / 2k)^{1/2} \tag{7}$$

Substituting for T_e and $T(y,t)$ from Equations (5) and (6) respectively, into Equation (3), one obtains [3]:

$$T = a_0 + \sum_{m=1}^{\infty} B_m \exp(-m^{1/2}\alpha y)\cos(m\omega t - m^{1/2}\alpha y - \psi_m - \beta_m) \tag{8}$$

where:

$$B_m = a_m[(1 + m^{1/2}\mu)^2 + m\mu^2]^{-1/2} \tag{9}$$

$$\mu = (\frac{k\omega\rho c}{2})^{\frac{1}{2}} / h \tag{10}$$

$$\beta_m = \tan^{-1}\left[\frac{m^{1/2}\mu}{(1 + m^{1/2}\mu)}\right] \tag{11}$$

2. 2. Numerical Analysis

The computational domain is to be discretized by finite difference method shown in Figure 1. The one dimensional conduction equation can be discretised by forward differencing for the time derivative and central differencing is used for space derivative of the temperature as shown in Equation (12).

Discretized one dimensional conduction equation is:

$$T_{i,j+1} = r * T_{i+1,j} + (1 - 2 * r)T_{i,j} + r * T_{i-1,j} \tag{12}$$

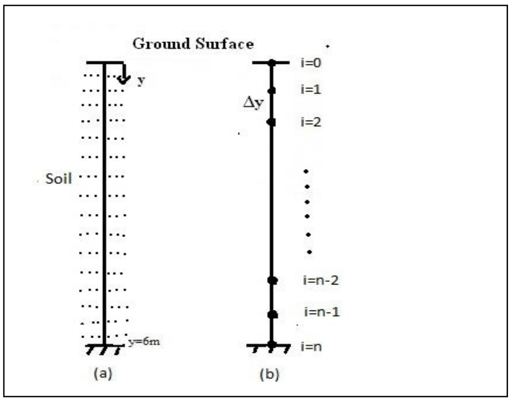

Figure 1. (a) Physical model and (b) computational domain

where the stability criteria is given by:

$$r = \frac{\Delta t * \alpha}{\Delta y^2} \leq 0.5 \quad (13)$$

Initial condition for computational domain is:

$$T[i][0] = 26.99 \quad (14)$$

2. 3. Boundary Condition

Temperature variation of upper surface of soil is the 1st boundary condition with respect to the effective temperature *(T_e)* value which is calculated with the help of ambient temperature and solar radiation data using Equation (4):

$$T_{0,j} = \frac{T_{i,j} + (\frac{\Delta y * h_s}{k}) T_e}{1 + \frac{h_s * \Delta T}{k}} \quad (15)$$

The 2nd boundary condition at the end point as constant temperature is the annual mean temperature:

$$T_{n,j} = T_{n-1.j} \quad (16)$$

3. RESULTS AND DISCUSSION

For Fourier and numerical analysis of ground temperature variation, the soil is taken homogeneous and its physical properties are assumed constant as given in the literature [1]:
$k = 0.51$ W/mK, $\rho = 2050$ Kg/m^3, $c = 1842.3$ J/KgKα_0=0.9, ε=1, ΔR=63.1 W/m^2 and h=22.7 W/m^{2}K (at 3 m/s wind speed).

The temperature distribution in the soil depends on the air temperature and solar radiation. Diurnal variation of hourly ambient temperature and solar radiation for hottest (10th June) and coldest (28th December) days of the year 2015 are shown in Figures 2 and 3, respectively. The temperature increases from 7 am to 15 pm due to increase in solar radiation. Between 15 pm to 18 pm air temperature decreases with decrease in solar radiation and temperature becomes constant and lowers during night at 32 ^{0}C and 12 ^{0}C during hottest and coldest days, respectively.

Figures 4 and 5 show the annual variation of monthly average of ambient air temperature and solar radiation, respectively. This graph shows the variation of temperature at three sessions of India. During the months 2 to 5, the temperature increases with increase in solar intensity of summer session. After that the variations become sinusoidal with decrease in temperature which indicates the rani session for next 4 months. After the 9th month, the temperature and solar radiation decrease that shows the winter session.

These variations can be expressed as Fourier series with six harmonics which are sufficient for matching all given data with approximately R^2= 99%. Fourier series coefficients for ambient air temperature, solar radiation, and effective temperature are given in Tables 1 and 2.

Figures 6 and 7 show a variation of soil temperature for the hottest and coldest days, respectively. At soil surface and in the depth of 0.1 m variation is like sinusoidal. The maximum soil temperature is at 15 pm and the minimum is in the mooring and night time. Results obtained by Fourier series and numerical methods are close.

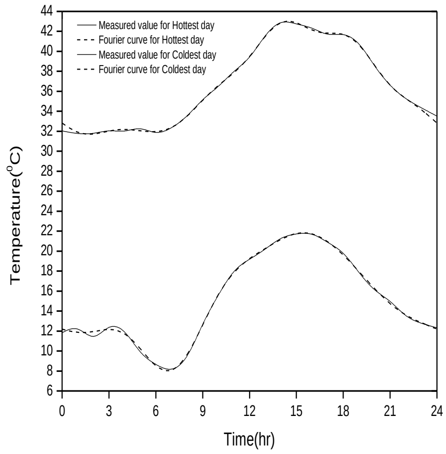

Figure 2. Hourly ambient air temperature for the hottest day (10th June) and the coldest day (28th Dec) in Jamshedpur, India for year 2015

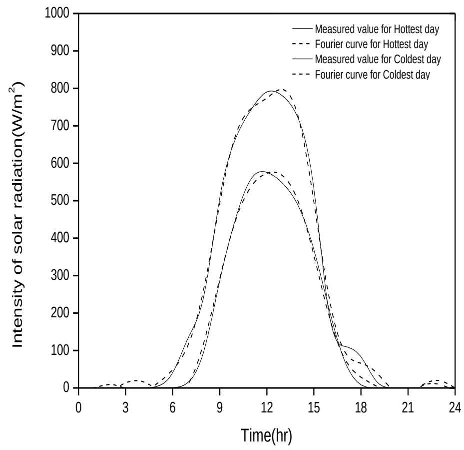

Figure 3. Hourly solar radiation intensity for the hottest day (10th June) and the coldest day (28th Dec) in Jamshedpur, India for year 2015

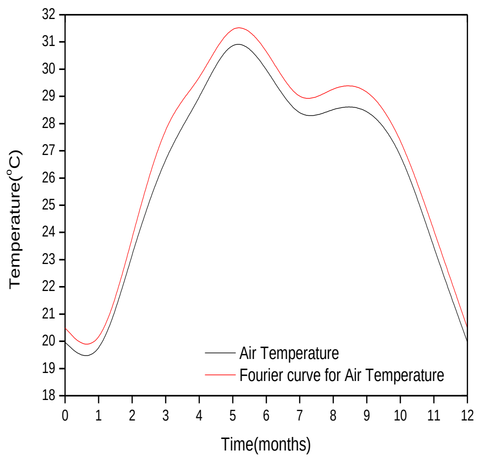

Figure 4. Monthly average ambient air temperature in Jamshedpur, India for year 2015

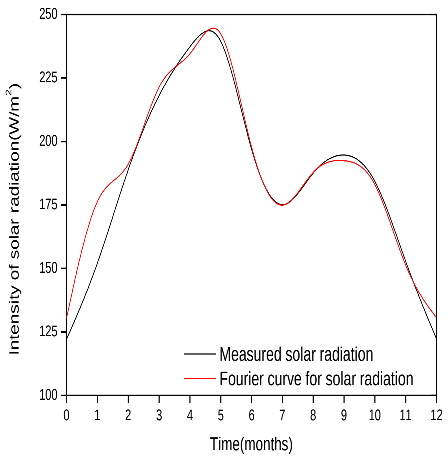

Figure 5. Monthly average solar radiation intensity in Jamshedpur, India for the year 2015

TABLE 1. Coefficients of Fourier series for ambient air temperature (T_a) in Jamshedpur, India

Fourier terms		0	1	2	3	4	5	6
Annual	a_m	26.99	4.910976	2.573121	0.916304	0.942657	0.479378	0.517238
	ψ_m		-0.08008	0.339793	-0.35989	1.013456	-0.37594	0.104728
Hottest day	a_m	36.58	5.74124	0.772853	0.73958	0.121835	0.512296	0.138914
	ψ_m		0.949696	-1.27728	-0.02204	-0.7817	-0.7484	0.6251
Coldest day	a_m	15.11	5.801712	1.907854	1.857241	0.685227	0.234354	0.038269
	ψ_m		1.005455	1.113712	0.398105	1.202187	0.2662	0.039154

TABLE 2. Coefficients of Fourier series for solar radiation intensity (S) in Jamshedpur, India

Fourier terms		0	1	2	3	4	5	6
Annual	a_m	191.7	34.99706	25.98472	14.50651	5.384759	16.90552	9.964501
	ψ_m		-0.93507	0.094001	-1.40186	0.14912	0.428086	-1.35907
Hottest day	a_m	232.6	376.6289	195.0089	6.141652	21.08999	38.40026	33.88953
	ψ_m		0.012397	0.009574	-1.26192	-0.86124	-0.39115	0.093973
Coldest day	a_m	153.2	261.5779	155.2413	19.99031	15.2605	24.03611	9.634712
	ψ_m		0.07201	0.126848	-0.18749	0.868574	0.596888	0.578877

As the depth of soil increases, the amplitude of temperature decreases. After a depth of 0.2 m, there is no diurnal variation in soil temperature.
Figure 8 shows an annual variation of soil temperature with depth. The maximum soil temperature is 36 ^{0}C during summer and the minimum is around 25 ^{0}C in winter. After a depth of 3 m, the soil temperature becomes constant. Results obtained by both Fourier series and numerical methods are close. The minor variation is due to input methods. In Fourier series, data for six harmonics are input whereas numerical simulation is run for monthly data points.

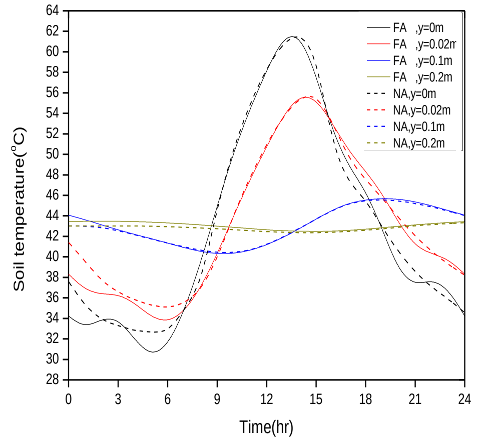

Figure.6. Variation of soil temperature for the hottest day at various depths

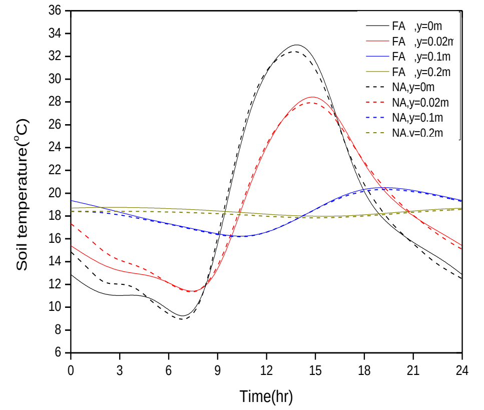

Figure 7. Variation of soil temperature for the coldest day at various depths

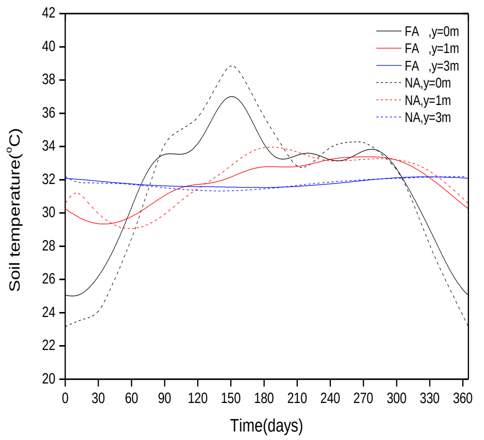

Figure 8. Annual variation of soil temperature with depth

4. CONCLUSION

The present investigation reports the variation of soil temperature with depth in Jamshedpur, India by the method of Fourier series and numerical methods. Both Fourier series and numerical simulation have been done for the hottest day (10th June) and the coldest day (28th December) in year 2015. Diurnal variation of soil temperature is up to the depth of 0.2 m whereas annual variation is up to the depth of 3 m.

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چکیده

هدف از این مقاله پیش بینی تغییرات دوره ای دمای زمین با عمق برای شرایط مختلف دمای هوا و داده های تابش خورشیدی برای جمشیدپور، هند است. سری فوریه و تکنیک های عددی برای تعیین دمای روزانه و سالانه داغترین و سردترین روز سال ۲۰۱۵ استفاده شده است. تغییرات دما روزانه تا عمق خاک ۰٫۲ متر است در حالیکه تغییرات دما سالانه تا عمق ۳ متر است.

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