Studying the Behavior of Active Mass Drivers during an Earthquake Using Discrete Instantaneous Optimal Control Method

O. Bahar[1], M. R. Banan[2], and M. Mahzoon[3]

1. Structural Engineering Research Center, International Institute of Earthquake Engineering and Seismology (IIEES), Iran, email: omidbahar@iiees.ac.ir
2. Department of Civil Engineering, School of Engineering, Shiraz University, Shiraz, Iran
3. Department of Mech. Engineering, School of Engineering, Shiraz University, Shiraz, Iran

ABSTRACT: *In order to control the responses of a building, different control systems may be employed. To recognize and select a proper control system, a designer has to analyze many cases. This paper investigates the behavior of some control systems with respect to changes in different parameters of an AMD, and various combinations of masses and control forces of two or three AMDs, and also different locations of an AMD along the height of a building. In this study we used a recently proposed control algorithm, named discrete instantaneous optimal control method. A new discrete stable weighting matrix strengthens this method.*

Keywords: Active control; Discrete instantaneous optimal control; Digital state-space equation; Discrete stable weighting matrix; Active Mass Driver (AMD)

1. Introduction

In order to control a building, different control systems may be employed. In a building, there are many parameters that may be controlled (*e.g.* responses of the floors, maximum and average required control force, and responses of Active Mass Drivers, *AMDs*). Therefore, a designer has to recognize the main features of an efficient control system and contemplate the effect of any change in the main parameters of a control system on its final performance.

When an *AMD* is used for controlling the excessive responses of a building, the values of mass, damping, frequency, and its installation location have to be specified based on the specifications of the building and the characteristics of a design earthquake ground motion that may occur in the future. If the control system consists of two or more *AMDs*, selection of the main parameters will be more difficult, because the mass ratio and the control force ratio of the two or three *AMDs* will also be added to the parameters.

Many researchers have been working on the subject of the active structural control and have already presented many recommendations in this field [1-6]. They investigated different control systems and compared their performances. In general, in their works, the total mass of a control system and/or the total required control force was not fixed. These parameters strongly affect the behavior and efficiency of a control system and their changes cause the primary control system and the changed one to behave completely different.

In this paper in order to compare efficiencies of different cases, the total mass and the average required control force (as a criterion for the value of the required external energy) are given prescribed values. During this research, the frequencies of the *AMDs* are assumed to be close to the fundamental frequency of the model building, and they are not changed. In this paper by using a recently proposed algorithm, named *Discrete Instantaneous Optimal Control* (*DIOC*) method [7, 8], it is tried to investigate the behavior of a controlled building equipped by Active Mass Drivers (*AMDs)* during an earthquake ground motion.

In the following sections, a brief description of the Discrete Instantaneous Optimal Control and the procedure based on which a designer can find a stable weighting matrix, is presented. Using this algorithm, the behavior of an *AMD* with different mass and damping values and also its various locations along the height of a model building are investigated. The behavior of two or three *AMDs* with different mass ratios and/or control force ratios is also examined and discussed in detail.

2. Discrete Instantaneous Optimal Control Method

By employing the digital state-space equation and a new definition of the time-dependant performance index, *DIOC* method presents a powerful closed-open loop control rule similar to that of the classical optimal control method. A new Discrete Stable Weighting Matrix (*DSWM*) warrants the stability of the *DIOC* method. A brief description of this method is presented hereafter [7, 8].

The matrix equations of motion of a structure subjected to a ground acceleration $\ddot{x}_0(t)$ that is controlled by *AMDs* can be idealized by an *n*-degree of freedom linear system as follows:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Du(t) + Me\ddot{x}_0(t) \qquad (1)$$

in which *M*, *C*, and *K* are $n \times n$ mass, damping, and stiffness matrices, respectively, $x(t), \dot{x}(t)$ and $\ddot{x}(t)$ are *n*-dimensional displacement, velocity, and acceleration vectors, respectively, *D* is an $n \times r$ matrix that specifies the locations of active controllers, $u(t)$ is an *r*-dimensional control force vector, $e = [-1\ -1\ \ldots\ -1]^T$ is an *n*-dimensional vector which defines the ground acceleration influence on masses of the whole building. The first order digital state-space equation of motion of such structural system is defined as follows:

$$z_{k+1} = A_d z_k + B_d u_k + w_{1d}\ddot{x}_{0k}\ , \quad z(t_0) = z_0 \qquad (2)$$

where t_0 is the initial time and subscript *k* refers to time instant *t*, such that $t = k\Delta t$, and Δt is the time increment. Vector z_k is the state vector at time instant *t*, $[x_k\ \ \dot{x}_k]^T$, matrices A_d, B_d, and w_{1d} are the transition matrices corresponding to *A*, *B*, and w_1, respectively and are defined as follows:

$$A_d = exp(ADt),\ F = \int_0^{Dt} exp(A\eta)\,d\eta, \quad B_d = FB,\ w_{1d} = Fw_1 \qquad (3)$$

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, B = \begin{bmatrix} 0_{n\times r} & M^{-1}D \end{bmatrix}^T, \quad w_1 = \begin{bmatrix} 0_{n\times 1} & e \end{bmatrix}^T \qquad (4)$$

A new definition for quadratic time-dependent performance index $J(t)$ in discrete form is presented by authors [7, 8] as follows:

$$J(t) = \frac{1}{2}(z_{k+1}{}^T Q z_{k+1} + u_k{}^T R u_k) \qquad (5)$$

in which $2n \times 2n$ positive semi-definite *Q* matrix and $r \times r$ positive definite *R* matrix are weighting matrices related to the state variables and the control force, respectively. Minimizing $J(t)$ subject to the constraint of Eq. (2) at each time instant, the closed-open loop control force vector is obtained as follows:

$$u_k = -[R + B_d{}^T Q B_d]^{-1} B_d{}^T Q(A_d z_k + w_{1d}\ddot{x}_{0k}) \qquad (6)$$

This equation is equivalent to the first two terms of the control force vector in the closed-open loop method of the classical optimal control [9]. The only difference is the appearance of the weighting matrix *Q* in Eq. (6) instead of the Riccati matrix in closed-open loop classical optimal method. The third term in classical method, which is related to the external load and obtained from a backward solution of a matrix differential equation in time, is not appeared in Eq. (6).

3. Stable Weighting Matrix

A procedure based on the Lyapunov direct method [7-9] in discrete form is proposed. Consider a positive semi-definite matrix *Q*, such that

$$V(z_k) = z_k{}^T Q z_k \geq 0 \qquad (7)$$

which is a possible Lyapunov function. If the first difference of Eq. (7) with respect to the state-space vector results in a negative semi-definite matrix, the *Q* matrix is a discrete stable weighting matrix (*DSWM*). The above-mentioned first difference of Eq. (7) is as follows:

$$DV(z) = z_k{}^T \Big[A_d{}^T Q A_d - A_d{}^T Q B_d (R + B_d{}^T Q B_d)^{-1} \times B_d{}^T Q A_d - Q - (A_d - B_d G)^T Q B_d G \Big] z_k \qquad (8)$$

As a sufficient condition, it can be assumed that the sum of the first three terms of the bracket in Eq. (8) is equal to a negative semi-definite matrix, $-I_0$ in which I_0 is an arbitrary positive semi-definite matrix. By this definition we get

$$A_d^T Q A_d - A_d^T Q B_d (R + B_d^T Q B_d)^{-1} \times B_d^T Q A_d - Q + I_0 = 0 \quad (9)$$

This is the discrete Riccati matrix equation. By selecting a positive semi-definite matrix I_0, Eq. (9) is solved and the weighting matrix Q is obtained. Now, if as a necessary condition, the considered values of Eq. (7) in all time steps are non-negative, and as a sufficient condition, the bracket in Eq. (8) or its simpler form ($i.e.$,$[-I_0-(A_d-B_dG)^T QB_dG]$) is a negative semi-definite matrix, the computed weighting matrix Q is a *DSWM* for the differential equation of motion in Eq. (1).

4. Specifications of the Building and the Earthquake Record

4.1. Model Building

An eight-story planar shear-type building frame with similar story properties is selected as the model building. The structural properties of each story are as follows: floor mass is 345.6*tons*, elastic stiffness is 3.404*e*5*kN/m*, and internal damping coefficient is 2937*tons/sec* that corresponds to a 2% viscous damping of the first mode of the building without control system [1].

4.2. Control System

Active Mass Drivers (*AMDs*) are used as an active control system. The properties of *AMDs* are as follows: the frequency of each driver mass is 98% of the fundamental frequency of the building without control [1, 7, 10], the damping of each driver mass is 25*tons/sec*, such that their damping ratio are approximately 7.3%. In order to compare performances of different control systems, their average required control forces and the total mass of the *AMDs* are fixed to constant values, 72.68*kN*, and 29.63*tons*, respectively. The average required control force is determined as follows:

$$ACF = \frac{1}{t_f}\int_0^{t_f} \left| u(t) \right| \mathrm{d}t \quad (10)$$

where t_f is the terminating time.

4.3. Discrete Stable Weighting Matrix

First, the designer has to assign a proper I_0 matrix. Then by solving Eq. (9) the Q matrix will be found. If this matrix satisfies the necessary and sufficient conditions of the Lyapunov stability method [7, 8], presented in the previous section, the Q matrix will be a *DSWM*. After extensive analysis, the following matrix is selected for the I_0 matrix:

$$I_0 = \begin{bmatrix} \alpha K_\gamma & O \\ O & \beta M \end{bmatrix}, \text{ where } K_\gamma = \begin{bmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & \gamma \end{bmatrix} \quad (11)$$

in which M is the mass matrix of the whole building (building with control system), sub-matrix $K_{i,j}$ in matrix K_γ is a partition of the stiffness matrix of the controlled building, and factors α, β, and γ are three arbitrary scalar factors. These factors are assigned such that the Lyapunov stability conditions are satisfied and the average required control force remains equal to the prescribed value.

4.4. Control Force Related Weighting Matrix

This matrix, R, is a diagonal matrix with a dimension equal to the number of the *AMDs*, multiplied by a constant factor equal to 0.001. For cases with one driver mass this matrix is reduced to 1, and for more *AMDs* with force ratios equal to 1, this matrix is a unit matrix.

4.5. Earthquake Record

The *N-S* component of the 1940 El Centro earthquake record is used as the input excitation. The time increment is 0.02*sec*.

5. Behavior of the Building with Respect to Changes in Different Parameters of One *AMD*

Achieving high efficiency of a control system is mainly related to the employed control method, selected weighting matrices and the parameters of the applied control mechanism. The considered parameters of a control system with one *AMD* are the frequency, damping, mass, and the installation location of the driver mass. In this section, the behavior of the sample building controlled by an *AMD* with respect to changes in its different parameters is investigated. The investigated parameters include the value of mass and damping of the driver mass, and its location along the height of the building.

5.1. Effects of Different Values of the Mass of One AMD

In order to investigate the performance of the control system with respect to the different values of the mass of an *AMD*, seven cases are considered. The values of the masses of these cases are as follows: 1% of the effective modal mass related to the fundamental mode, (*i.e.*, 85.63%), 0.5%, 1%, 1.5%, 2%, 12.5%, and finally 25% of the total mass of the building. The last two cases refer to the weight of one and two floors of the building, which are used as a driver mass, respectively. After extensive analysis, the proper values for the three factors in I_0 matrix, Eq. (11), are specified as follows. The values of α for these cases are equal to 2.31, 7, 7, 7, 7, 8, and 4.19, respectively. The values of β are equal to 0.1, 0.21, 0.35, 0.54, 0.68, 1.0, and 0.5, respectively and finally the values of γ are equal to 21, 21, 21, 41, 71, 5001, and 10001. As mentioned earlier, these factors are specified such that the conditions of the Lyapunov stability method are satisfied as well as the average required control forces are fixed to 72.68*KN*.

In Table (1), comparing the first row with the other rows shows that, using *AMD* remarkably decreases the responses of the building, but increasing the mass value of the *AMD* does not considerably make more decrease in the responses of the floors. The heavier driver masses produce more reduction in the responses of the *AMD* and the maximum required control force. The decrease in the acceleration responses of the floors in the last two rows may be referred to the use of a larger value for the factor γ [7].

Therefore in order to control tall buildings, using heavy equipments of the building as *AMDs* is strongly recommended. For instance for the model building, using 12.5% of the total mass of

Table 1. The resulting responses for different values of the mass of AMD.

Mass Value (%)	8th floor Displ. (cm)	8th floor Accel. (m/sec²)	Driver Mass Responses Displ.* (m)	Driver Mass Responses Velocity* (m/sec)	Driver Mass Responses Accel. (m/sec²)	Max. Control Force (KN)	Max. Base Shear Reduction (%)
0.00	19.3	7.9	--	--	--	--	--
0.50	8.7	3.9	2.59	14.81	107.3	761.3	50.9
0.85	8.4	4.0	1.71	9.77	62.6	646.0	52.9
1.00	8.4	4.0	1.46	8.34	53.5	636.0	52.9
1.50	8.3	4.0	0.98	5.61	36.1	624.2	53.5
2.00	8.2	4.0	0.75	4.25	27.4	618.8	54.2
12.5	8.2	3.5	0.15	0.83	4.7	579.6	56.1
25.0	8.6	3.2	0.12	0.61	3.8	622.5	49.7

* Relative to the 8th floor responses.

the building as a driver mass means that the 9th floor acts as a driver mass for the eight floors below. In such a case, the inter-story deformation of the 8th floor with respect to the 7th floor without and with control system are 0.80 and 0.57*cm*, respectively, while the relative displacement of the driver mass (*i.e.*, the 9th floor), with respect to the 8th floor is about 15*cm*.

The last row of Table (1) is only presented to clarify the following point. Many researchers believe that when the mass value of a control system increases to a value about *5%* of the total mass of the building, the control system may fail and the controlled building may be unstable. In general, this idea may be correct. But, the results of our investigations show that this problem is only raised from the selected weighting matrices. In other words, a designer can always find a proper stable weighting matrix for a heavy driver mass such that it makes the control system very efficient without inducing instability in the controlled building.

5.2. Effects of Different Values of the Damping of One AMD

In order to investigate performance of the control system with respect to the different values of the damping ratio of an *AMD*, five cases are considered. The selected values of the damping ratios include zero and the values of the 1st to the 4th damping ratios of the considered building without control, *i.e.*, 2.5%, 7.41%, 12.07%, and 16.32%. For these analyses, other parameters are fixed to the before-mentioned parameters of one *AMD*. The coefficients of the I_0 matrix in Eq. (11) are specified as follows: the values of α and γ for all cases are equal to *7*, and *21*, respectively, and the values of β for the considered cases are equal to 0.478, 0.417, 0.429, 0.496, and 0.657.

The resulted responses of these considered cases are summarized in Figures (1) and (2). The values of the maximum required control forces of these cases are 592.6, 608.8, 631.5, 661.8, and 669.9*kN*, respectively. The results show that, by increasing the value of the damping of an *AMD*, the efficiency of the control system decreases, *i.e.*, the responses of the floors smoothly increase, and a larger maximum control force is needed.

Therefore, choosing damping ratio of the *AMD* between the 1st and the 2nd mode damping ratio of the building causes the most significant reduction in the responses of the building while the responses of the driver mass are maximized.

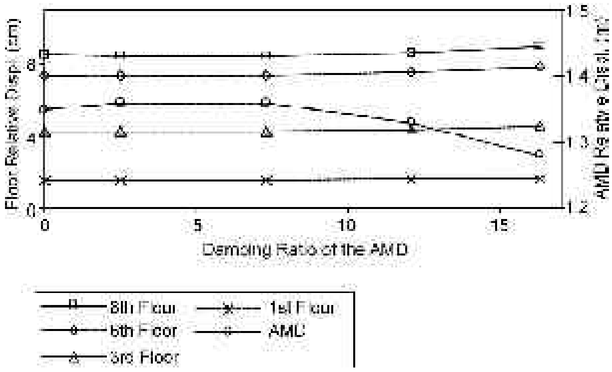

Figure 1. The displacements of the floors and driver mass for different damping of the AMD.

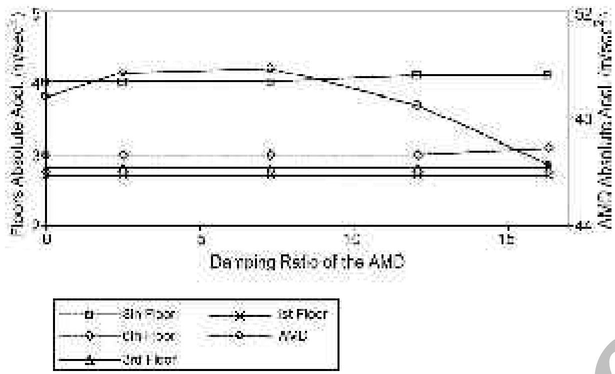

Figure 2. The accelerations of the floors and driver mass for different damping of the AMD.

5.3. Proper Location of One AMD

Analyses show that to achieve a better performance for a control system, when there is not any limitation for installation of *AMD* along the height of a building the best location is the top of the building. Four cases are presented such that in each case the *AMD* is installed on one floor along 8th, 7th, 6th, and 5th floors of the building, respectively. The coefficients of the I_0 matrix in Eq. (11) are specified as follows. The values of α and γ for all these cases are equal to *7*, and 21, respectively, and the values of β are equal to 0.417, 0.389, 0.337, and 0.296, respectively. The resulted responses of these cases are compared in Table (2) and Figure (3).

Based on the results set forth in Table (2), by moving the *AMD* to the lower floors, the responses of the control system and the maximum required control force remarkably increases, and maximum reduction of the base shear of the building effectively decreases. On the other hand, the results in Figure (3) show that, installing the *AMD* at the lower floors may slightly affect the displacement and acceleration responses of the floors. The velocity responses of the floors are not changed. Therefore, moving the location of an *AMD* system toward the higher floors causes the control system to be more efficient.

Table 2. Comparison of the resulting responses due to varying location of an AMD along the height of the controlled building.

Location of Driver Mass	Driver Mass Displ.* (m)	Velocity* (m/sec)	Accel. (m/sec^2)	Max. Control Force (KN)	Max. Base Shear Reduction (%)
8th floor	1.36	7.78	49.9	631.59	52.9
7th floor	1.40	8.00	50.0	643.45	51.5
6th floor	1.47	8.41	49.6	672.27	48.7
5th floor	1.59	9.00	52.1	705.04	44.2

* Relative to the responses of the floor that is equipped by AMD.

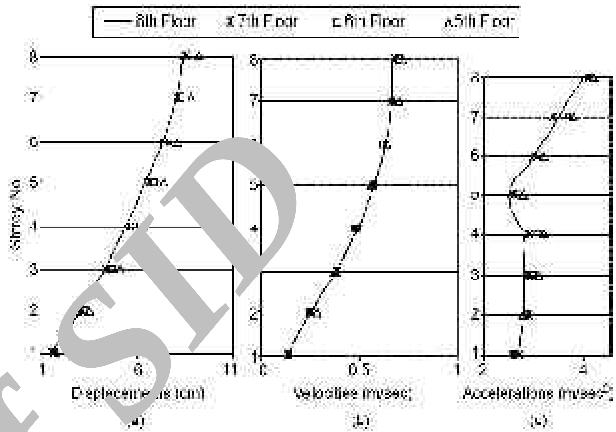

Figure 3. Comparison of the responses due to different locations of one AMD.

6. Behavior of the Building with respect to Changes in Different Parameters of Few *AMDs*

In this section, the behavior of the model building controlled by two or three *AMDs* with respect to changes in different parameters of *AMD*s is investigated. The investigated parameters include; various mass ratio and/or force ratio of two *AMDs* installed at the top floor. It is noted that, the average required control force is fixed to *72.68 KN*, and the total mass of the two or three *AMDs* are also fixed to 29.63 *ton*.

By using two *AMDs* the dimensions of I_0, Q and R matrices are equal to 20, 20, and 2, respectively. So, the factor γ in Eq. (11) and also, the force related weighting matrix, *i.e.*, R matrix, are changed as follows:

$$\gamma(\text{in Eq. (11)}) \rightarrow \gamma\begin{bmatrix}1 & 0\\ 0 & p\end{bmatrix} \text{ and } R = 0.001\begin{bmatrix}1 & 0\\ 0 & m\end{bmatrix} \quad (12)$$

where *p* and *m* are arbitrary values. For three *AMDs* the matrices given in Eq. (12) are defined as unit matrices with a dimension equal to 3.

6.1. Different Combinations of Mass Values of the AM Drivers

Sometimes, due to some practical limitations, the designer is forced to use few *AMDs* instead of one *AMD*. Here, some possible choices are investigated that include two *AMDs* with three different mass ratios equal to 1, 3 and 9, respectively, and three *AMDs* with equal masses and three *AMDs* with masses equal to 1/2, 1/3 and 1/6 of the total mass. It is noted that, the total mass values of all cases are fixed to 29.63*tons* and the total average required control forces are identical. In order to compare the efficiency of the control systems computations were done and the results of these cases and also one *AMD* are summarized in Table (3). All *AMDs* are installed on the top of the building considered.

Table 3. The responses of the entire building using few AMDs installed on the top floor.

Case No.	Mass Ratio	Displacement			Acceleration			Max. Total Force
		8^{th} floor (cm)	2^{nd} floor (cm)	DM # (m)	8^{th} floor (m/sec^2)	2^{nd} floor (m/sec^2)	DM # (m/sec^2)	(kN)
1	0	8.4	3.0	1.36*	4.0	2.8	49.9	[illegible]
2	1	8.7	3.1	1.31*	4.0	2.7	51.0	675
3	3	8.7	3.1	1.40*	3.9	2.7	52.[illegible]	667
4	9	8.7	3.1	1.40*	4.0	2.7	50.0	629
5	1**	9.0	3.2	1.26*	3.8	2.[illegible]	52.6	761
6	3**	9.0	3.2	1.40*	3.9	2.5	52.7	744

\# Driver Mass
* Relative to the 8^{th} floor response
** Ratio of the largest value to the smallest value

Based on these results, dividing a total mass into two or three parts does not significantly affect the responses of the building. In some cases the maximum responses of the control system may slightly increase. But, it is clear that, more *AMDs* need a greater peak in the total control force, and greater mass ratio needs smaller maximum of the total control force. This is because when more than one *AMD* is used the control force for each *AMD* is smaller than the total value. This certainly affects the performance of the control system.

The frequency response of the acceleration of the top floor of the building controlled by one *AMD* and two *AMDs* with different mass ratios are shown in Figure (4). One can observe that the frequency responses for different mass ratios are completely identical. By using only one *AMD* the first mode of the building can be well controlled. But, this control system has small effects on the other modes. Since, the accelerations of the floors highly depend on the higher modes, controlling them with one *AMD* is very difficult. On the other hand, a control system with two or more *AMDs* may affect more than one mode of the building.

Therefore, in order to control the longitudinal responses of buildings like the model building, by using a control system with a specified total mass and by tuning all frequencies of the *AMDs* to values close to the fundamental frequency of the building, some practical aspects are recommended which are using a fewer number of *Active Mass drivers* if possible, either using a greater mass ratio for two *AMDs*, or using equal mass values for three *AM* drivers.

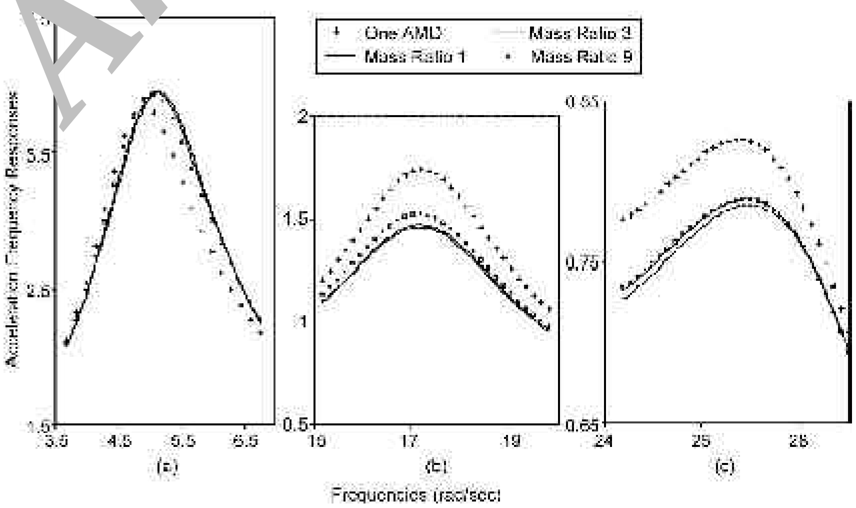

Figure 4. The acceleration frequency response of the top floor using one AMD and two AMDs; (a) the 1^{st} dominant frequency, (b) the 2^{nd} dominant frequency, and (c) the 3^{rd} dominant frequency of the controlled building.

6.2. Different Combinations of the Control Force Ratios of the Two AMDs

The values of the coefficients *p* and *m* defined in Eq. (12) were selected to be equal to *1* before. But, selecting larger values for these coefficients causes the required control force of the greater mass to be larger than the required control force of the smaller one. Extensive analysis shows that, the behavior of a control system using two *AMDs* with the same frequencies tuned to values close to the fundamental frequency of the building follows almost a regular trend with respect to changes in control force ratio of the two mass drivers.

In order to confirm this regularity, three combinations for the mass values of the two *AMDs* are evaluated. The factors γ and β are fixed to 21 and 0.5, respectively. The two coefficients *p* and *m* that will be noted hereafter by a coupled value (*p*, *m*) are changed. The coefficient α is selected such that the total average required control forces of all cases be equal to the specified value. Three coupled values, which are (1,1), (1,9) and (9,9), for each mass ratio are examined. The results of analysis are presented in Table (4).

The results given in Table (4) show that:

(a) using different control force ratios does not considerably affect the responses of the floors,

(b) using smaller control force ratios for a fixed mass ratio causes the maximum responses of the control system to decrease, but maximum of the total required control force to increase, and

(c) using greater mass ratios for an almost fixed control force ratio causes the maximum responses of the control system and the maximum of the required total control force to decrease (*e.g.*, the 3rd, 5th, and 7th rows in Table (4)).

7. Conclusions

Behavior of different Active Mass Driver Systems (*AMDs*) with respect to various changes in their parameters was investigated and their efficiencies were compared, in detail. All of analyses are carried out by using a recently proposed control algorithm named "*DIOC*". A new discrete stable weighting matrix formed based on the Lyapunov direct method, strengthens this method. Different parameters of one *AMD* including mass and damping values and its location along the height of a sample building are investigated. It is shown that, although an *AMD* with mass value more than 0.85% of the total mass of the building, cannot make more reduction in the responses of the floors, but a heavy *AMD*, even 25% of the total mass of the building, can effectively reduce the responses of the control system. It can also lessen the maximum required control force without inducing instability in the controlled building. This is because of using a proper stable weighting matrix. Installing an *AMD* above the mid height of the building can properly reduce the responses of the floors. But, installation in the higher floors produces smaller responses of the *AMD* and the maximum required control force.

An extensive analysis shows that, dividing a total mass into two or three portions does not produce considerable effects on the responses of the building. But, more *AMDs* need greater maximum of the total control force, and greater mass ratio needs smaller maximum of the total control force. The results of the frequency response of the acceleration of the top floor show that, by using only one *AMD* the first mode of the building can be well controlled. But, this control system has small effects on the other

Table 4. The responses of the building using two AMDs with the frequencies tuned close to the fundamental frequency of the building.

Coupled Value	Mass Ratio	Displacement 8th floor (cm)	Displacement 2nd floor (cm)	Displacement DM # (m)	Acceleration 8th floor (m/sec^2)	Acceleration 2nd floor (m/sec^2)	Acceleration DM # (m/sec^2)	Control Force Ratio	Max. Total Control Force (kN)
(1,1)	1	8.7	3.1	1.31*	4.0	2.7	51.0	1.0	675
(1,9)	1	8.7	3.1	1.66*	3.9	2.6	73.9	2.5	670
(9,9)	1	8.9	3.2	2.20*	3.9	2.5	93.5	7.7	762
(1,1)	3	8.7	3.1	1.40*	3.9	2.7	52.1	3.0	667
(1,9)	3	8.5	3.0	1.53*	4.0	2.7	58.6	6.9	633
(9,9)	3	8.7	3.1	1.71*	4.0	2.7	64.2	20.9	646
(1,1)	9	8.7	3.1	1.40*	4.0	2.7	50.0	8.5	629
(1,9)	9	8.5	3.0	1.44*	4.0	2.8	53.1	21.0	628
(9,9)	9	8.6	3.1	1.48*	4.0	2.8	54.4	43.2	624

* Relative to the 8th floor responses   # Driver Mass

modes, while a control system with two or more *AMDs* can affect more than one mode of the building. Therefore, controlling the longitudinal responses of buildings like the model building using a control system with a specified total mass can be undertaken by: 1) using Smaller number of *AMDs* if possible, 2) tunning their frequencies close to the fundamental frequency of the building, 3) using a greater mass ratio for two *AMDs*, 4) using equal mass values for three *AM* drivers.

Finally, various control force ratios for two *AMDs* are investigated. The results show that, using different control force ratios does not considerably affect the responses of the floors, and using greater mass ratio for an almost fixed control force ratio causes the maximum responses of the control system and the maximum of the total required control force to decrease.

Acknowledgments

The first author is grateful to emeritus Prof. T. Kobori (Kobori research complex, Kajima Corp., Japan) and Prof. Kitagawa (the first author's supervisor in Hiroshima Univ., Japan) for their support and cooperation for examining some actively controlled buildings in Tokyo, and attaining actual information for checking the computer program.

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