

**A REMARK ON A REMARK BY MACAULAY
OR
ENHANCING LAZARD STRUCTURAL THEOREM**

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In [10] (also cf.[6]) Macaulay gave a construction which, to each monomial ideal $J \subset k[X_1, \dots, X_n]$, associates a set of points $X \subset k^n$ whose associated radical ideal I has, using modern lingo, J as the monomial ideal associated to its Gröbner basis — $J = T(I)$; moreover Macaulay explicitly stated a direct correspondence between the points X and the monomials $\tau \notin J$ under the “Gröbner escalier” $N(I)$.

Partial converse of the Macaulay’s result appeared in the earlier research on Gröbner Technology:

- In 1981 Möller [1] introduced Duality in Computer Algebra proposing an algorithm which, for each finite set of points $X \subset k^n$, computes the Gröbner basis and the “Gröbner escalier” of its associated radical ideal I .
- In 1985 Lazard [9] gave a characterization of the Gröbner basis of any ideal $I \subset k[X_1, X_2]$ and such characterization is a refinement of Macaulay’s result.
- In 1990 Cerlienco–Mureddu [2] gave an algorithm which, for each finite set of points $X \subset k^n$, computes the “Gröbner escalier” $N(I)$ of its associated radical ideal I and a direct correspondence between X and $N(I)$.

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