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A REMARK ON A REMARK BY MACAULAY OR ENHANCING LAZARD STRUCTURAL THEOREM

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In [10] (also cf.[6]) Macaulay gave a construction which, to each monomial ideal $\mathsf{J} \subset k[X_1, \dots, X_n]$, associates a set of points $\mathsf{X} \subset k^n$ whose associated radical ideal I has, using modern lingo, J as the monomial ideal associated to its Gröbner basis — $\mathsf{J} = \mathbf{T}(\mathsf{I})$; moreover Macaulay explicitly stated a direct correspondence between the points X and the monomials $\tau \notin \mathsf{J}$ under the "Gröbner escalier" $\mathbf{N}(\mathsf{I})$.

Partial converse of the Macaulay's result appeared in the earlier research on Gröbner Technology:

- In 1981 Möller [1] introduced Duality in Computer Algebra proposing an algorithm which, for each finite set of points $\mathsf{X} \subset k^n$, computes the Gröbner basis and the "Gröbner escalier" of its associted radical ideal I.
- In 1985 Lazard [9] gave a characterization of the Gröbner basis of any ideal $\mathsf{I} \subset k[X_1, X_2]$ and such characterization is a refinement of Macaulay's result.
- In 1990 Cerlienco–Mureddu [2] gave an algorithm which, for each finite set of points $\mathsf{X} \subset k^n$, computes the "Gröbner escalier" $\mathbf{N}(\mathsf{I})$ of its associated radical ideal I and a direct correspondence between X and $\mathbf{N}(\mathsf{I})$.

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