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n-CYCLICIZER GROUPS

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ABSTRACT. The cyclicizer of an element x of a group G is defined as $Cyc_G(x) = \{y \in G | \langle x, y \rangle \text{ is cyclic} \}$. Here, we introduce an ncyclicizer group and show that there is no finite n-cyclicizer group for n = 2, 3. We prove that for any positive integer $n \neq 2, 3$, there exists a finite n-cyclicizer group and determine the structure of finite 4 and 6-cyclicizer groups. Also, we characterize finite 5, 7 and 8-cyclicizer groups.

1. Introduction

Let G be a group. We know that the centralizer of an element $x \in G$ is defined as follows:

 $C_G(x) = \{ y \in G | \langle x, y \rangle \text{ is abelian} \}.$

If, in this definition, we replace the word abelian by the word cyclic, we get a subset of the centralizer of x. This subset is called the cyclicizer of x in G and it is denoted by $Cyc_G(x)$ [9, 10]. Thus,

 $Cyc_G(x) = \{y \in G | \langle x, y \rangle \text{ is cyclic} \}.$

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Also, Cyc(G), the cyclicizer of G, is defined as follows:

$$Cyc(G) = \{ y \in G | \langle x, y \rangle \text{ is cyclic for all } x \in G \}$$
$$= \bigcap_{x \in G} Cyc_G(x).$$

In general, for an element x of a group G, $Cyc_G(x)$ is not a subgroup of G. For example, in the group $G = \mathbb{Z}_2 \oplus \mathbb{Z}_4$, we have

 $Cyc_G((0,2)) = \{(0,0), (0,1), (0,2), (0,3), (1,1), (1,3)\},\$

which is not a subgroup of G.

In [1] and [2], the cyclicizers of a group are studied via a graph which is called the non-cyclic graph of the group.

For any non-cyclic group G, Cyc(G) is a subgroup, central, cyclic, normal and contained in all maximal cyclic subgroups of G. It is clear that for a nontrivial element x of G, $|Cyc_G(x)| \ge 2$ and $G = \bigcup_{1 \ne x \in G} Cyc_G(x)$. Also, for any group G and $x \in G$, if $\overline{G} = G/Cyc(G)$, then $Cyc_{\overline{G}}(xCyc(G))$ $= Cyc_G(x)/Cyc(G)$ [2] and it easily follows that $Cyc(\overline{G}) = 1$ (see also [10]).

For a finite group G, let $\operatorname{Cent}(G)$ denote the set of the centralizers of single elements of G. G is called an n-centralizer group if $|\operatorname{Cent}(G)| = n$. We know that there is no n-centralizer group for n = 2, 3. Let Z(G) denote the center of a group G. Then, $|\operatorname{Cent}(G)| = 4$ if and only if $G/Z(G) \cong C_2 \times C_2$ and $|\operatorname{Cent}(G)| = 5$ if and only if $G/Z(G) \cong C_3 \times C_3$ or S_3 [7], where C_2 is a cyclic group of size two and S_3 is a symmetric group on three letters.

Moreover, if |Cent(G)| = 6, then G/Z(G) is isomorphic to one of the groups $(C_2)^3$, $(C_2)^4$, A_4 or D_8 [6], where A_4 is an alternating group on four letters and D_8 is a dihedral group of size eight.

Also, |Cent(G)| = 7 if and only if G/Z(G) is isomorphic to one of the groups $C_5 \times C_5$, D_{10} or $\langle x, y | x^5 = y^4 = 1, y^{-1}xy = x^3 \rangle$ and if |Cent(G)| = 8, then G/Z(G) is isomorphic to one of the groups D_{12} , $(C_2)^3$ or A_4 [5].

Similarly, we can define an n-cyclicizer group, where n is a positive integer.

Definition 1.1. For a positive integer n, we say that G is an n-cyclicizer group if $|\{Cyc_G(x)|x \in G\}| = n$ and in this case, we write Cycl(G) = n.

It is obvious that G is a 1-cyclicizer group if and only if G is cyclic. Here, we show that there is no finite n-cyclicizer group for n = 2, 3 and prove that for any positive integer $n \neq 2, 3$, there exists a finite group

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G such that Cycl(G) = n. We also study finite *n*-cyclicizer groups for n = 4, 5, 6, 7 and 8.

2. *n*-Cyclicizer Groups for n = 4, 5, 6, 7 and 8

The following theorem is proved in [2].

Theorem 2.1. Let G be a finite non-cyclic group. Then, $|G/Cyc(G)| \leq max\{(s-1)^2(s-3)!, (s-2)^3(s-3)!\}, \text{ where s is the number of maximal cyclic subgroups of G.}$

It is clear that if G has n maximal cyclic subgroups, then $Cycl(G) \ge n$.

Lemma 2.2. Let G be a finite non-cyclic group such that Cycl(G) = n. Then, G has at most n - 1 maximal cyclic subgroups.

Proof. Assume that $\langle x \rangle$ is a maximal cyclic subgroup of G. Then, $Cyc_G(x) = \langle x \rangle$. Let Cycl(G) = n, and $\langle x_1 \rangle, \langle x_2 \rangle, \ldots, \langle x_r \rangle$ be distinct maximal cyclic subgroups of G. Since for any $i, 1 \leq i \leq r$, $Cyc_G(x_i) = \langle x_i \rangle$, then $r \leq n$. It is clear that $r \neq n$, since $Cyc_G(1) = G$. This completes the proof.

Lemma 2.3. Let G be a finite group. Then, $Cycl(\overline{G}) = n$ if and only if Cycl(G) = n.

Proof. Let $Cycl(\bar{G}) = n$ and C = Cyc(G). The key point of our proof is that $Cyc_G(x) \to Cyc_{\bar{G}}(\bar{x})$ is a one-to-one correspondence between the set of cyclicizers of G and those of \bar{G} (induced by the natural homomorphism $G \to \bar{G} = G/C$). For an element x of G, $\bar{X} = Cyc_G(x)/C$ and $\bar{x} = xC$. We know that $Cyc_{\bar{G}}(\bar{x}) = \bar{X}$. Assume that $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n$ be distinct cyclicizers of $\bar{x}_1 = C, \bar{x}_2, \ldots, \bar{x}_n$, respectively. It is clear that $Cycl(G) \ge n$. Without loss of generality, we can assume that X_1, X_2, \ldots, X_n are distinct cyclicizers of x_1, x_2, \ldots, x_n , respectively. Suppose that $Y = Cyc_G(g)$ is different from X_i , for any $i, 1 \le i \le n$. Then, $\bar{Y} = \bar{X}_i$, for some $i, 1 \le i \le n$. Thus, $Cyc_G(g)C = Cyc_G(x_i)C$. Therefore, for any $h_i \in Cyc_G(g)$, there exist c_i and $z_i \in C$ such that $h_ic_i = k_i z_i$, where $k_i \in Cyc_G(x_i)$, and so $h_i = k_i c_t$, for some $c_t \in C$. Since $k_i \in Cyc_G(x_i)$, it is not hard to see that $\langle h_i, x_i \rangle$ is a cyclic group. Hence, $Cyc_G(g) \subseteq Cyc_G(x_i)$. Similarly, $Cyc_G(x_i) \subseteq Cyc_G(g)$. Thus,

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 $Cyc_G(g) = Cyc_G(x_i)$. This contradiction indicates Cycl(G) = n. The converse is clear.

Lemma 2.4. Let $n \ge 2$ be an integer, and $Q_{4n} = \langle x, y | x^{2n} = 1, x^n = y^2, y^{-1}xy = x^{-1} \rangle$. Then, $Cycl(Q_{4n}) = n + 2$.

Proof. The set of all members of Q_{4n} is $\{1, x^j, x^i y, y | 1 \le j, i \le 2n - 1\}$. It is straightforward to check that

(i) for any $i, 0 \le i \le n-1, Cyc_{Q_{4n}}(x^iy) = \{1, y^2, x^iy, x^{n+i}y\};$ (ii) $Cyc_{Q_{4n}}(x) = \langle x \rangle;$

and

(*iii*) for any $i, 0 \le i \le n-1, Cyc_{Q_{4n}}(x^i y) = Cyc_{Q_{4n}}(x^{n+i}y).$ Therefore, $Cycl(Q_{4n}) = n+2.$

Corollary 2.5. Let n > 1 be an integer. Then, $Cycl(D_{2n}) = n + 2$.

Proof. It is well known that $Z(Q_{4n}) = \langle y^2 \rangle$, and we can see that $Z(Q_{4n}) = Cyc(Q_{4n})$ and $Q_{4n}/Z(Q_{4n}) = Q_{4n}/Cyc(Q_{4n}) \cong D_{2n}$, and so the proof follows from Lemma 2.3.

Corollary 2.6. Let n > 3 be an integer. Then, there exists a group G with Cycl(G) = n.

Theorem 2.7. There is no finite n-cyclicizer group for n = 2, 3.

Proof. First, note that there is no cyclic *n*-cyclicizer group for n = 2, 3. Assume G is a finite group such that Cycl(G) = 2. Now since the only proper cyclicizer of G is cyclic and G is covered by its all proper cyclicizers, it follows that G is cyclic, which is a contradiction.

Now, suppose for a contradiction that Cycl(G) = 3. Assume that $G = Cyc_G(x) \cup Cyc_G(y)$, where $Cyc_G(x)$ and $Cyc_G(y)$ are two distinct cyclicizers of G. By Lemma 2.2, G has at most two maximal cyclic subgroups. If G has exactly two maximal cyclic subgroups, then, without loss of generality, $G = \langle x \rangle \cup \langle y \rangle$, which is a contradiction. Thus, G has only one maximal cyclic subgroup. This means that G is a cyclic group. This contradiction completes the proof.

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Remark 2.8. Let p be a prime number and $m \in \mathbb{N}$. Then, $\frac{p^m-1}{p-1}$ is the number of subgroups of order p in $(C_p)^m$.

Theorem 2.9. Let p be a prime number and let G be a finite group such that $G/Cyc(G) \cong C_p \times C_p$. Then, Cycl(G) = p + 2.

Proof. Let $Cycl(C_p \times C_p) = r$. By Remark 2.8, $C_p \times C_p$ has p + 1maximal cyclic subgroups, and so $r \leq p + 1$. Let $\langle x_1 \rangle, \langle x_2 \rangle, ..., \langle x_{p+1} \rangle$ be maximal cyclic subgroups of $H = C_p \times C_p$. If $Y = Cyc_H(y) \neq H$ is different from $\langle x_i \rangle$, for any $i, 1 \leq i \leq p + 1$, then there exists $j, 1 \leq j \leq p + 1$, such that $y \in \langle x_j \rangle$. Therefore, $\langle x_j \rangle = \langle y \rangle \subseteq Y$. Let gbe an arbitrary element in Y. Then, for some integer $k, 1 \leq k \leq p + 1$, $\langle g, y \rangle = \langle x_k \rangle$. Thus, $y \in \langle x_j \rangle \cap \langle x_k \rangle$. If $j \neq k$, then y = 1, and so Y = H. This is a contradiction. Therefore, j = k. This implies that $Y = \langle x_j \rangle$. Now, Lemma 2.3 completes the proof.

Corollary 2.10. Let p be a prime number. Then, $Cyc(C_p \times C_p) = 1$.

Proof. By Lemma 2.9, we have that $C_p \times C_p$ has p+1 proper cyclicizers. Let $Cyc_{C_p \times C_p}(x) = \langle x \rangle$ and $Cyc_{C_p \times C_p}(y) = \langle y \rangle$ be two distinct proper cyclicizers of $C_p \times C_p$. If $\langle x \rangle \cap \langle y \rangle \neq 1$, then $|\langle x \rangle \cap \langle y \rangle| = |\langle x \rangle| = p$. Since $\langle x \rangle \cap \langle y \rangle \leq \langle x \rangle$, then $\langle x \rangle \cap \langle y \rangle = \langle x \rangle$. Therefore, $\langle x \rangle = \langle y \rangle$. This contradiction shows that $\langle x \rangle \cap \langle y \rangle = 1$ and the proof is complete.

Lemma 2.11. Let G be a finite p-group, for some prime number p. Then, $Cyc(G) \neq 1$ if and only if G is either a cyclic group or a generalized quaternion group. In this case, Cyc(G) = Z(G).

Proof. It follows from Proposition 2.2 of [2].

Lemma 2.12. Let G and H be finite groups such that (|G|, |H|) = 1. Then, $Cyc(G \times H) = Cyc(G) \times Cyc(H)$.

Proof. Let $(a,b) \in Cyc(G \times H)$. Then, for any $(g,h) \in G \times H$, there exists $(x,y) \in G \times H$ such that $\langle (g,h), (a,b) \rangle = \langle (x,y) \rangle$. Therefore, $\langle (g,a) \rangle \leq \langle x \rangle$. So $a \in Cyc(G)$. Similarly, $b \in Cyc(H)$. Thus, $Cyc(G \times H) \subseteq Cyc(G) \times Cyc(H)$.

Now, let $(a,b) \in Cyc(G) \times Cyc(H)$. Then, for any $g \in G$, $\langle g, a \rangle$ is

a cyclic group. Also, for any $h \in H$, $\langle b, h \rangle$ is a cyclic group. Since $\langle (g,h), (a,b) \rangle \leq \langle g, a \rangle \times \langle h, b \rangle$ and (|H|, |G|) = 1, then $(a,b) \in Cyc(G \times H)$. Thus, $Cyc(G) \times Cyc(H) \subseteq Cyc(G \times H)$.

Lemma 2.13. (i) Let p be a prime number and n be an integer such that (n,p) = 1. If $G = C_{pn} \times C_p$, then $G/Cyc(G) \cong C_p \times C_p$.

(ii) Let n be an odd positive integer. If $G = C_n \times Q_8$, then $G/Cyc(G) \cong C_2 \times C_2$.

Proof. (i) Let $H = C_p \times C_p$ and $K = C_n$. Since (|H|, |K|) = 1, then |Cyc(G)| = n. Thus, $|G/Cyc(G)| = p^2$. If G/Cyc(G) is a cyclic group, then G is also a cyclic group, which is a contradiction. Thus, $G/Cyc(G) \cong C_p \times C_p$.

(*ii*) Any Sylow subgroup of G is either a cyclic group or a generalized quaternion group, and so by Lemma 2.11, Cyc(G) = Z(G). We have |G/Cyc(G)| = |G/Z(G)| = 4 and G/Cyc(G) is not a cyclic group, since G is not a cyclic group. Therefore, $G/Cyc(G) \cong C_2 \times C_2$.

Lemma 2.14. Let p be a prime number and let G be a finite group such that $G/Cyc(G) \cong C_p \times C_p$. Then, G is not a cyclic group and

(i) if p = 2, then G is isomorphic to either $C_{2n} \times C_2$ or $C_n \times Q_8$, where n is an odd positive integer;

(ii) if $p \neq 2$, then $G \cong C_{pn} \times C_p$, where n is an integer such that (p,n) = 1.

Proof. If G is a cyclic group, then |G/Cyc(G)| = 1, which is a contradiction. If $G/Cyc(G) \cong C_p \times C_p$, then G/Cyc(G) is an abelian group. Since $G/Z(G) \cong \frac{G/Cyc(G)}{Z(G)/Cyc(G)}$, then G is a nilpotent group. Thus, $G = Syl_2 \times Syl_3 \times \cdots \times Syl_p \times \cdots$.

Since $|G/Cyc(G)| = p^2$, then Cyc(G) contains $C = \widehat{Syl}_p(\widehat{Syl}_p)$ is the product of all Sylow subgroups of G, except Syl_p). So, C is a cyclic group of size n such that (p, n) = 1. Thus, $|Cyc(G)| = p^m \times n$.

If $Cyc(G) \cap Syl_p = \langle 1 \rangle$, then $|Syl_p| = |G/Cyc(G)| = p^2$. If Syl_p is a cyclic group, then G is a cyclic group, which is a contradiction. Thus, $Syl_p \cong C_p \times C_p$. So $G \cong C_{pn} \times C_p$.

If $Cyc(G) \cap Syl_p \neq \langle 1 \rangle$, since $Cyc(G) \cap Syl_p \leq Cyc(Syl_p)$, then Syl_p

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is a *p*-group whose cyclicizer is nontrivial. Thus, Syl_p is a generalized quaternion group.

If $p \neq 2$, then G is not a generalized quaternion group.

If p = 2, then $|Cyc(Syl_p)| = 2$. Since $1 \neq |Cyc(G) \cap Syl_2| \leq |Cyc(Syl_2)| = 2$, then $|Syl_2| = 8$. Thus, $G \cong C_n \times Q_8$, and the proof is complete.

Lemma 2.15. Let G be a finite group. Then, Cycl(G) = 4 if and only if $G/Cyc(G) \cong C_2 \times C_2$.

Proof. Suppose that $G/Cyc(G) \cong C_2 \times C_2$. Since $Cycl(C_2 \times C_2) = 4$, then, by Lemma 2.3, Cycl(G) = 4.

If Cycl(G) = 4, then, by Lemma 2.2, G has at most three maximal cyclic subgroups. Now, Theorem 2.1 completes the proof.

Theorem 2.16. Let n be an odd positive integer, and G be a finite group. Then, Cycl(G) = 4 if and only if G is isomorphic to one of the following groups:

$$C_n \times Q_8, C_{2n} \times C_2.$$

Proof. It follows from Lemmas 2.14 and 2.15.

Theorem 2.17. Let n be an odd positive integer, and G be a finite group. Then, Cycl(G) = 6 if and only if G is isomorphic to one of the following groups:

$$C_n \times D_8, C_{4n} \times C_2, C_n \times Q_{16}.$$

Proof. Let Cycl(G) = 6. Then, $Cycl(\overline{G}) = 6$. Since G has at most five maximal cyclic subgroups, then, by Theorem 2.1, $|G/Cyc(G)| \leq 54$. It is easy to see (by the following programs in GAP [11]) that 6-cyclicizer groups whose orders are less than 54 are the followings:

$$C_4 \times C_2, D_8, Q_{16}, C_{12} \times C_2, C_3 \times D_8, C_{20} \times C_2, C_5 \times D_8, C_3 \times Q_{16}$$

a:=function(n)

local a; a:=AllSmallGroups(n); return a;

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end;
cycelement:=function(G,x)
   local c, e, i;
   e:=Elements(G);
   c:=[];
      for i in[1..Size(e)] do
          if IsCyclic(Group(x,e[i]))=true then Add(c,e[i]);
          fi;
      od;
   return c;
   end;
for n in[4..54] do
   G:=a(n);
      for i in [1..Size(G)] do
          h:=G[i];
          e:=Elements(h);
          l:=List(e,i->[cycelement(h,i)]);
             if Size(Set(1)) = 6 then
             Print(StructureDescription(h),"\n"); fi;
      od;
od;
```

But |Cyc(G/Cyc(G))| = 1, therefore, G/Cyc(G) is isomorphic to either $C_4 \times C_2$ or D_8 . We compute |Cyc(G)| by the following program:

```
CycG := function(G)
local c, e, i;
c:=G;
e:=Elements(G);
for i in[1..Size(G)] do
c:=Intersection(c,cycelement(G,e[i]));
od;
return c;
end;
```

Similar to the proof of Lemma 2.14, we can conclude that Cycl(G) = 6 if and only if G is isomorphic to either $C_n \times D_8$ or $C_{4n} \times C_2$ or $C_n \times Q_{16}$. \Box

Theorem 2.18. Let G be a finite group. Then, Cycl(G) = 5 if and only if G/Cyc(G) is isomorphic to either S_3 or $C_3 \times C_3$.

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Proof. Let Cycl(G) = 5. By Lemma 2.2, G has at most four maximal cyclic subgroups. Since $Cycl(\bar{G}) = 5$, then, by Theorem 2.1, we have $5 \leq |G/Cyc(G)| \leq 9$. On the other hand, |G/Cyc(G)| is not a prime number, and so |G/Cyc(G)| is either 6 or 8 or 9. If |G/Cyc(G)| = 8, then (by GAP) $Cycl(\bar{G}) \neq 5$, which is a contradiction. Thus, |G/Cyc(G)| = 6 or 9. Therefore, G/Cyc(G) is isomorphic to either S_3 or $C_3 \times C_3$. The converse is clear.

A covering for a group G is a collection of subgroups of G whose union is G. An *n*-cover for a group G is a cover with *n* members. A cover is irredundant if no proper subcollection is also a cover.

We write f(n) for the largest index |G:D| over all groups G having an irredundant *n*-cover with intersection D. Bryce et al. obtained f(5) = 16 [8]. Also, Abdollahi et al. obtained f(6) = 36, and f(7) = 81 [3, 4]. We use these results to prove the following theorems.

Theorem 2.19. Let G be a finite group. Then, Cycl(G) = 7 if and only if G/Cyc(G) is isomorphic to one of the following groups:

$$D_{10}, A = \langle x, y | x^5 = y^4 = 1, x^y = x^3 \rangle, C_5 \times C_5.$$

Proof. Let Cycl(G) = 7. By Lemma 2.2, G has at most six maximal cyclic subgroups. Since f(6) = 36, then $8 \le |G/Cyc(G)| \le 36$. Now, it is easy to see (by GAP) that G is isomorphic to one of the following groups:

$$D_{10}, C_5 \times C_5, A, Q_{20}, C_3 \times D_{10}.$$

On the other hand, |Cyc(G/Cyc(G))| = 1, and so G is isomorphic to either D_{10} or $C_5 \times C_5$ or A. The converse is clear.

Theorem 2.20. Let G be a finite group. Then, Cycl(G) = 8 if and only if G/Cyc(G) is isomorphic to one of the following groups:

$$(C_2)^3, A_4, D_{12}, C_8 \times C_2, C_8 : C_2, C_3 \times S_3, C_9 \times C_3, C_9 : C_3.$$

Proof. Let Cycl(G) = 8. By Lemma 2.2, G has at most seven maximal cyclic subgroups. As f(7) = 81, with an argument similar to the proof of Theorem 2.19, we can prove our claim. The converse is clear.

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