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ON THE FISCHER-CLIFFORD MATRICES OF A MAXIMAL SUBGROUP OF THE LYONS GROUP Ly

J. MOORI AND T. SERETLO*

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ABSTRACT. The non-split extension group $\overline{G} = 5^{3\cdot}L(3,5)$ is a subgroup of order 46500000 and of index 1113229656 in Ly. The group $\overline{G}$ in turn has $L(3,5)$ and $5^2{:}2.A_5$ as inertia factors. The group $5^2{:}2.A_5$ is of order 3000 and is of index 124 in $L(3,5)$. The aim of this paper is to compute the Fischer-Clifford matrices of $\overline{G}$, which together with associated partial character tables of the inertia factor groups, are used to compute a full character table of $\overline{G}$. A partial projective character table corresponding to $5^2{:}2A_5$ is required, hence we have to compute the Schur multiplier and projective character table of $5^2{:}2A_5$.

1. Introduction

The Lyons group Ly, is a sporadic simple group of order $2^8.3^7.5^6.7.11.31.37.67 = 51765179004000000$. It was discovered in 1970 by Richard Lyons [21], using the concept of classifying simple groups with an involution centralizer $2^{\cdot}A_n$. The smallest value of n for which $2^{\cdot}A_n$ has non-central involutions is $n = 8$, for which the McLaughlin group M^cL, has an involution centralizer $2^{\cdot}A_8$. The only other case that arises is $n = 11$ which is in the Lyons group Ly, that is the Lyons group has an involution centralizer $2^{\cdot}A_{11}$. Moreover, a 3-cycle in $2^{\cdot}A_{11}$ centralizes $2^{\cdot}A_8$ and the full centralizer of this 3-cycle in Ly is the triple

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∗Corresponding author

cover $3{\cdot}M^cL$ of the McLaughlin group. The normalizer of the group generated by this 3-cyle is $3{\cdot}M^cL{:}2$.

The existence of this group and its uniqueness up to isomorphism was shown by Sims [32, 33], using *coset enumeration* and it is often referred to as the "Lyons-Sims" group. The group Ly has elements of order 37 and 67 which cannot be found in the monster and is one of the six sporadic simple groups called the "pariahs" which are not subgroups of the monster. The other five "pariahs" being $J_1, J_3, J_4, O'N$ and Ru, the last to be determined in [36] being J_1. The group Ly has nine conjugacy classes of maximal subgroups. One of the maximal subgroups of the form, $\overline{G} = N{\cdot}G$ is a group of order $46500000 = 2^6.3.5^6.31$, where $N \cong 5^3$ and $G \cong L(3,5)$. The group $5^3{\cdot}L(3,5)$ is also maximal in the Baby Monster B. The aim of this paper is to compute the Fischer-Clifford matrices which together with partial character tables of inertia factor groups will be used to compute a character table for $\overline{G}$. This work is taken from [31], the notation used is consistent with that of the $\mathbb{ATLAS}$ [9] and $\mathbb{ATLAS}$ of group representations V3 [35].

The method used is based on Ficher-Clifford Theory. Let $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a group extension. The character table of $\overline{G}$ can be constructed once we have

- the character tables (ordinary or projective) of the inertia factor groups,
- the fusions of classes of the inertia factors into classes of G,
- the Fischer-Clifford matrices of $\overline{G} = N{\cdot}G$.

Let $\bar{g} \in \bar{G}$ be a lifting of $g \in G$ under the natural homomorphism $\bar{G} \longrightarrow G$ and let $[g]$ be a conjugacy class of elements of G with representative g. Let $\{\theta_1, \theta_2, \ldots, \theta_t\}$ be a set of representatives of the orbits of $\bar{G}$ on $Irr(N)$ such that for $1 \leq i \leq t$, we have inertia groups $\bar{H}_i = I_{\bar{G}}(\theta_i)$ with the corresponding inertia factors H_i and let ψ_i be a projective character of $\bar{H}_i$ with factor set $\bar{\alpha}_i$ such that $(\psi_i)_N = \theta_i$. For each $[g]$ we obtain the matrix $M(g)$ given by

$$M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix} \quad ,$$

where $M_i(g)$ is the submatrix corresponding to the inertia group $\bar{H}i$ and its inertia factor H_i. If $H_i \cap [g] = \emptyset$, then $M_i(g)$ will not exist and $M(g)$ does not contain $M_i(g)$. The size of the matrix $M(g)$ is $l \times c(g)$ where l is the number of α_i^{-1}-regular conjugacy classes of elements of the inertia factors H_i's for $1 \leq i \leq t$ which fuse into $[g]$ in G and $c(g)$ is the number of conjugacy classes of elements of $\bar{G}$ which correspond to the coset $\bar{g}N$. Then $M(g)$ is the *Fischer-Clifford matrix* of $\bar{G}$ corresponding to the coset $\bar{g}N$. The partial character table of $\bar{G}$ on the classes $\{x_1, x_2, \ldots, x_{c(g)}\}$ is given by

$$\begin{bmatrix} C_1(g)M_1(g) \\ C_2(g)M_2(g) \\ \vdots \\ C_t(g)M_t(g) \end{bmatrix}$$

where the Fischer-Clifford matrix $M(g)$ is divided into blocks with each block corresponding to an inertia group $\bar{H}_i$ and $C_i(g)$ is the partial projective character table of H_i with factor set α_i^{-1} consisting of the columns corresponding to the α_i^{-1}-regular classes that fuse into $[g]$ in G. We obtain the characters of $\bar{G}$ by multiplying the relevant columns of the projective characters of H_i with factor set α_i^{-1} by the rows of $M(g)$.

The theory of Fischer-Clifford matrices, which is based on Clifford Theory (see Clifford [8]), was developed by B. Fischer ([12], [13] and [14]). This technique has also been discussed and applied to both split and non-split extension in several publications, for example in [1–6, 23, 25]. One can read more on Fischer-Clifford theory and projective characters from [11, 22, 24, 34] and [10, 17, 18, 20, 26–28] respectively. For the theory of characters one can also read Character Theory of Finite Groups by Isaacs [19].

2. Construction of $\overline{G} \cong 5^{3\cdot}L(3,5)$ and $G \cong L(3,5)$

From the $\mathbb{ATLAS}$ of group representation [35] we get two 111×111 matrices a, b over $GF(5)$, with $o(a) = 2$, $o(b) = 5$, $o(ab) = 14$ and $Ly =< a, b >$. From [35] we get Programme I, which computes the generators of the 3rd maximal subgroup of Ly as used in [31]. Here we use, $a = input[1]$ and $b = input[2]$ and we obtain $\bar{x} = output[1]$ and $\bar{y} = output[2]$, where $o(\bar{x}) = 2$, $o(\bar{y}) = 3$, $o(\bar{x}\bar{y}) = 31$ and $\overline{G} =< \bar{x}, \bar{y} >$. From [35] we see that $o(\bar{x}\bar{y}\bar{x}\bar{y}^2) = 25$ and if we let $gen[1] = (\bar{x}\bar{y}\bar{x}\bar{y}^2)^5$, then $o(gen[1]) = 5$, we also get that gen[2] $= \bar{y}gen[1]\bar{y}^{-1}$, gen[3] $= \bar{x}gen[2]\bar{x}^{-1}$ and $N = 5^3 =< gen[1], gen[2], gen[3] >$. Let $\lambda_i = gen[i],\ i = 1, 2, 3$. We use GAP to compute the conjugacy classes of $\overline{G}$ and also the fusion of its classes into Ly. These are given in Table 1. The conjugacy classes of $\overline{G}$ are represented in the well-known format of *coset analysis technique* applied to both spilt and non-split group extensions. This technique has been used by various authors and several MSc and PhD students of the first author, such as Mpono [22, 23], Rodrigues [29] and Whitely [34].

Table 1: Conjugacy Classes of $5^{3\cdot}L(3,5)$

$[g]_{L(3,5)}$	$[x]_{5^3\cdot L(3,5)}$	$C_{5^3\cdot L(3,5)}(x)$	$\longrightarrow$ Ly
1A	1A	46500000	1A
	5A	375000	5A
2A	2A	2400	2A
	10A	600	10A
3A	3A	120	3A
	15A	30	15B
4A	4A	480	4A
4B	4B	480	4A
4C	4C	80	4A
	20A	20	20A
	5B	2500	5A
5A	5C	1250	5B
	5D	1250	5B
5B	25A	25	25A
6A	6A	120	6B
	30A	30	30B
8A	8A	24	8B
8B	8B	24	8B
	10B	100	10A
10A	10C	50	10B
	10D	50	10B
12A	12A	24	12B
12B	12B	24	12B
20A	20B	20	20A
20B	20C	20	20A
24A	24A	24	24C
24B	24B	24	24B
24C	24C	24	24B
24D	24D	24	24C
31A	31A	1	31B
31B	31B	31	31A
31C	31C	31	31E
31D	31D	31	31D
31E	31E	31	31C
31F	31F	31	31B
31G	31G	31	31A
31H	31H	31	31E
31I	31I	31	31D
31J	31J	31	31C

Since for the application of Fischer-Clifford theory we are required to act $\overline{G}$ and G on N and on $Irr(N)$, we should represent these groups in terms of 3×3 matrices over $GF(5)$. We used the technique which was developed for determining these actions (see for example [31]). For this purpose we regard N as a vector space V of dimension 3 over $GF(5)$. For us to be able to act on a three dimensional vector space V it becomes necessary to rewrite generators of $\overline{G}$ from 111×111 matrices to 3×3 matrices. To do this we have to act $\overline{G}$ on N by letting the two generators of $\overline{G}$, $\bar{x}$ and $\bar{y}$, to act on the generators of N, $\lambda_i,\ i = 1, 2, 3$ by conjugation, using GAP [15]. Writing these as maps we get :

$$\bar{x} : \lambda_1 \to \lambda_1^4,\ \lambda_2 \to \lambda_3,\ \lambda_3 \to \lambda_2;$$

$$\bar{y} : \lambda_1 \to \lambda_2,\ \lambda_2 \to \lambda_1\lambda_2\lambda_3^4,\ \lambda_3 \to \lambda_2^2\lambda_3^4;$$

and in 3×3 matrix form over $GF(5)$ we obtain matrices

$$x = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, y = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 4 \\ 0 & 2 & 4 \end{pmatrix}.$$

We let $G = < x, y >$. Then $G \cong L(3, 5)$ which means that the action of $\overline{G}$ on N is isomorphic to $L(3, 5)$.

3. Inertia factors of $\bar{G}$

We use GAP [15] to compute the permutation character of Ly acting on $5^{3}{\cdot}L(3, 5)$. That is

$\chi(Ly|5^{3}{\cdot}L(3, 5)) = 1a + 45694a + 381766a + 1534500aa + 3028266a + 4226695aa + 11834746a + 18395586abc + 19212250a + 21312500ab + 22609664abc + 27252720aabbcd + 28787220aa + 29586865a + 33813560aa + 38734375a + 43110144abcde + 45648306b + 45694000ab + 56022921a + 64906250a + 71008476a$.

We then use Programme C from [31] to compute the orbit lengths of the actions on N and on $Irr(N)$. We let G act on a full row vector space V of dimension 3 over $GF(5)$. We get two orbits on N of lengths 1 and 124. By Brauer's Theorem [7] when G acts on $Irr(N)$, we also get two inertia groups H_1 and H_2 of index 1 and 124 in $\overline{G}$, respectively.

Since the affine subgroup of $GL(3,5)$ is of the form $5^2{:}GL(2,5)$, which also sits maximally inside $L(3,5)$ (note that $GL(3,5) = 4 \times L(3,5)$), we can easily see that the affine subgroup of $L(3,5)$ is of the form $5^2{:}SL(2,5) \cong 5^2{:}(2.A_5)$. Thus the full inertia groups are $\bar{H}_i = 5^3.H_i,\ i = 1,2$, where $H_1 = L(3,5)$ and $H_2 = 5^2{:}(2.A_5)$. We used GAP [15] to calculate the character table of H_2. We give the fusion of H_2 into $L(3,5)$ in Table 2.

Table 2. The fusion of $5^2{:}2.A_5$ into $L(3,5)$

$[x]_{5^2:2.A_5}$	$\longrightarrow$	$[g_1]_{L(3,5)}$
$1a$		$1A$
$2a$		$2A$
$3a$		$3A$
$4a$		$4C$
$5a$		$5A$
$5b$		$5B$
$5c$		$5B$
$5d$		$5A$
$5e$		$5B$
$5f$		$5B$
$5g$		$5A$
$6a$		$6A$
$10a$		$10A$
$10b$		$10A$

4. Projective character Table of $5^2{:}2.A_5$

From the fusions and orbit lengths and centralizer orders, we compute the Fischer-Clifford matrix $M(1A)$ of $\bar{G}$, that is

$$M(1A) = \begin{bmatrix} 1 & 1 \\ 124 & -1 \end{bmatrix}.$$

Having computed $M(1A)$ we want to determine the type of partial character tables we are going to use for our computations. We will show that the partial projective character table of H_2 is required. We follow the methods used in [1,4] and we use the character table of $Ly = < a, b >$. Let $Irr(Ly) = \{\Psi_i : 1 \leq i \leq 53\}$, where the notation is the same as the one used in the $\mathbb{ATLAS}$ [9]. From the list we take the values of $\Psi_2,\ \Psi_3,\ \Psi_4$ on $1A$ and $5A$.

$\|C_{\overline{G}}(x)\|$	46500000	375000
$[x]_{Ly}$	1A	5A
Ψ_2	2480	-20
Ψ_3	2480	-20
Ψ_4	45694	69

Let γ_1, γ_2 be the rows of the Fischer-Clifford matrix $M(1A)$. Then

$$< (\Psi_2)_N, 1_N >= \frac{1}{125}(2480 - 20.124) = 0.$$

Since $< (\Psi_2)_N, 1_N >= 0$, we get that $2480 = 0{+}20.124$, so that $(\Psi_2)_N = 0.\gamma_1 + 20.\gamma_2$. Let $[x_1, \cdots, x_t]$ be the transpose of the partial entries for the ordinary characters of $H_2 = 5^2{:}2.A_5$ on $1A \in L(3,5)$. Then $C_2(1A)M(1A)$ is a $t \times 2$ matrix with entries on the first column $124x_1 = 2480$. Hence $x_1 = 20$. But from the ordinary character table of $H_2 = 5^2{:}2.A_5$ one can see that there is no character of degree 20. Similarly

$$< (\Psi_4)_N, 1_N >= \frac{1}{125}(45694 - 69.124) = 434,$$

which gives us $x_1 = 365$ and this is a very large character degree that is not possible for $H_2 = 5^2{:}2.A_5$, and this holds for the remaining characters. Hence we have to use the projective character table of H_2. There are three primes dividing the order of H_2 namely $2, 3$ and 5. Using the fact that $H_2 = 5^2{:}2.A_5$ is a perfect group, we use GAP to determine its Schur multiplier (one can also use MAGMA which has a programme that computes the Schur multiplier in general). These are also given as Programmes J and J' in [31]. The p - Sylow subgroups corresponding to $p = 2$ and 3 are cyclic , using methods from [1,4] the Schur multipliers of both p-Sylow subgroups are trivial. Hence the Schur multiplier of H_2 is the cyclic group of order 5. The projective characters of H_2 with factor set α^{-1} where $\alpha^5 \sim 1$ is given in Table 3. Note that from this table we can see that $5a, 5b, 5c, 5e, 5f$ are all not α regular classes and we have a total of nine α regular classes.

Let $\omega = -E(5) - E(5)^4$, and $\omega^* = 1 - \omega = -E(5)^2 - E(5)^3$. Then $\omega + \omega^* = 1,\ \omega^*\omega = \omega\omega^* = -1,\ \omega^2 + (\omega^*)^2 = 3,\ \omega^3 + (\omega^*)^3 = 4$. In fact we get a Fibonacci sequence, with $f_{i+1} = f_i + f_{i-1},\ i \geq 2$, where $f_i = \omega^i + (\omega^*)^i$. This helps us to compute the Fischer-Clifford matrices and character table of $\overline{G} = 5^3{\cdot}L(3,5)$.

TABLE 3. The projective character table of $5^2{:}2.A_5$ with factor set α^{-1}

	1a	5a	2a	4a	3a	6a	5b	5c	5d	10a	5e	5f	5g	10b
χ_1	5	0	1	1	1	1	0	0	1	1	0	0	1	1
χ_2	15	0	3	-1	0	0	0	0	ω	ω	0	0	ω^*	ω^*
χ_3	15	0	3	-1	0	0	0	0	ω^*	ω^*	0	0	ω	ω
χ_4	20	0	4	0	1	1	0	0	-1	-1	0	0	-1	-1
χ_5	20	0	4	0	1	-1	0	0	-1	1	0	0	-1	1
χ_6	25	0	5	1	-1	-1	0	0	0	0	0	0	0	0
χ_7	10	0	2	0	-1	1	0	0	$-\omega$	ω	0	0	$-\omega^*$	ω^*
χ_8	10	0	2	0	-1	1	0	0	$-\omega^*$	ω^*	0	0	$-\omega$	ω
χ_9	30	0	6	0	0	0	0	0	1	-1	0	0	1	-1

TABLE 4. The Fischer-Clifford matrices of $5^{3\cdot}L(3,5)$

$M(1A) = \begin{bmatrix} 1 & 1 \\ 124 & -1 \end{bmatrix}$	$M(2A) = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$
$M(3A) = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$	$M(4C) = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$
$M(5A) = \begin{bmatrix} 1 & 1 & 1 \\ 10 & -5\omega^* & -5\omega \\ 10 & -5\omega & -5\omega^* \end{bmatrix}$	$M(10A) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -\omega & -\omega^* \\ 2 & -\omega^* & -\omega \end{bmatrix}$
$M(6A) = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$	All Others $= \begin{bmatrix} 1 \end{bmatrix}$

5. Fischer-Clifford matrices of $\overline{G}$

Having computed the projective character table of H_2 (see Table 3), we get the α-regular conjugacy classes. These together with the fusions of $5^2{:}2.A_5$ into $L(3,5)$ given in Table 2 help us to compute the sizes of the Fischer-Clifford matrices of $\overline{G}$. We use the projective characters, the fusions, the centralizer orders of $\overline{G}$ and properties of Fisher-Clifford matrices, to compute the Fischer - Clifford matrices, which are given in Table 4.

To compute the character table of $5^{3\cdot}L(3,5)$, as an example consider the following. Let $C_1(5A)$ and $C_2(5A)$ be the partial character tables of the inertia factors for the classes that fuse to $5A \in L(3,5)$. The portions of the character table of $\overline{G} = 5^{3\cdot}L(3,5)$ corresponding to the coset $5A$ are (note that $5d$ of H_2 fuses to $5A$ of $L(3,5)$):

$$C_1(5A)M_1(5A) = \begin{bmatrix} 1\\5\\6\\6\\6\\-4\\-4\\-4\\-4\\-4\\-4\\-4\\-4\\-4\\-4\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\-1\\0\\5\\5\\5\\11 \end{bmatrix} [1\ 1\ 1] = \begin{bmatrix} 1&1&1\\5&5&5\\6&6&6\\6&6&6\\6&6&6\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-4&-4&-4\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\-1&-1&-1\\0&0&0\\5&5&5\\5&5&5\\5&5&5\\11&11&11 \end{bmatrix},$$

$$C_2(5A)M_2(5A) = \begin{bmatrix} 1&1\\A&A^*\\A^*&A\\-1&-1\\0&0\\-A&-A^*\\-A^*&-A\\-1&-1\\1&1 \end{bmatrix} \begin{bmatrix} 10&-5A^*&-5A\\10&-5A&-5A^* \end{bmatrix} = \begin{bmatrix} 20&-5&-5\\10&10&-15\\10&-15&10\\-20&5&5\\0&0&0\\-10&-10&15\\-10&15&-10\\-20&5&5\\20&-5&-5 \end{bmatrix}.$$

The fusion of $\overline{G}$ to Ly together with the restriction of characters of Ly to $\overline{G}$ forces the signs of the Fischer-Clifford matrices and the orders of the elements of the conjugacy classes of $\overline{G}$.

6. Power maps and character Table of $\overline{G}$

We used a programme written in GAP (see Programe E in [31])together with the fusion map from $\overline{G}$ to Ly and computed the power maps of elements of $\overline{G}$. These are given in Table 5. The character table of $5^{3\cdot}L(3,5)$ is given in Table 6.

TABLE 5. The Power Maps of elements of $5^{3\cdot}L(3,5)$

$[g]_{L(3,5)}$	$[x]_{5^{3}\cdot L(3,5)}$	2	3	5	31	$[g]_{L(3,5)}$	$[x]_{5^{3}\cdot L(3,5)}$	2	3	5	31
1A	1A	1A	1A	1A	1A	2A	2A	1A	2A	2A	2A
	5A	5A	5A	1A	5A		10A	5A	10A	2A	10A
3A	3A	3A	1A	3A	3A	4A	4A	2A	4A	4A	4A
	15A	15A	5A	1A	15A	4B	4B	2A	4A	4A	4A
						4C	4C	2A	4C	4C	4C
5A	5B	5B	5B	1A	5B	5B	25A	25A	25A	5A	25A
	5C	5C	5C	1A	5C						
	5D	5D	5D	1A	5D						
6A	6A	3A	2A	6A	6A	10A	10B	5A	10B	2A	10B
	30A	15A	5A	6A	30A		10C	5A	10C	2A	10C
							10D	5A	10D	2A	10D
8A	8A	4A	8A	8A	8A	8B	8B	4B	8B	8B	8B
12A	12A	6A	4A	12A	12A	12B	12B	6A	4B	12B	12B
20A	20A	10A	20A	4A	20A	20B	20B	10A	20B	4B	20B
24A	24A	12A	8A	24A	24A	24B	24B	12B	8B	24B	24B
24C	24C	12A	8A	24C	24C	24D	24D	12B	8B	24D	24D
31A	31A	31A	31A	31A	1A	31B	31B	31B	31B	31B	1A
31C	31C	31C	31C	31C	1A	31D	31D	31D	31D	31D	1A
31E	31E	31E	31E	31EA	1A	31F	31B	31F	31F	31F	1A
31G	31G	31G	31G	31G	1A	31H	31H	31H	31H	31H	1A
31I	31I	31I	31I	31I	1A	31J	31J	31J	31J	31J	1A

TABLE 6. The character table of $5^{3\cdot}L(3,5)$

	$1A$		$2A$		$3A$		$4A$	$4B$	$4C$			$5A$	$5B$	
	1a	5a	2a	10a	3a	15a	4a	4b	4c	20a	5b	5c	5d	25a
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	30	30	6	6	0	0	6	6	2	2	5	5	5	0
χ_3	31	31	7	7	1	1	-5	-5	-1	-1	6	6	6	1
χ_4	31	31	-5	-5	1	1	A	/A	1	1	6	6	6	1
χ_5	31	31	-5	-5	1	1	/A	A	1	1	6	6	6	1
χ_6	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_7	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_8	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_9	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{10}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{11}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{12}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{13}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{14}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{15}	96	96	0	0	0	0	0	0	0	0	-4	-4	-4	1
χ_{16}	124	124	4	4	1	1	4	4	0	0	-1	-1	-1	-1
χ_{17}	124	124	4	4	1	1	4	4	0	0	-1	-1	-1	-1
χ_{18}	124	124	4	4	1	1	-4	-4	0	0	-1	-1	-1	-1
χ_{19}	124	124	4	4	1	1	-4	-4	0	0	-1	-1	-1	-1
χ_{20}	124	124	-4	-4	-2	-2	B	-B	0	0	-1	-1	-1	-1
χ_{21}	124	124	-4	-4	-2	-2	-B	B	0	0	-1	-1	-1	-1
χ_{22}	124	124	-4	-4	1	1	-B	B	0	0	-1	-1	-1	-1
χ_{23}	124	124	-4	-4	1	1	B	-B	0	0	-1	-1	-1	-1
χ_{24}	124	124	-4	-4	1	1	-B	B	0	0	-1	-1	-1	-1
χ_{25}	124	124	-4	-4	1	1	B	-B	0	0	-1	-1	-1	-1
χ_{26}	125	125	5	5	-1	-1	5	5	1	1	0	0	0	0
χ_{27}	155	155	11	11	-1	-1	-1	-1	-1	-1	5	5	5	0
χ_{28}	155	155	-1	-1	-1	-1	C	/C	1	1	5	5	5	0
χ_{29}	155	155	-1	-1	-1	-1	/C	C	1	1	5	5	5	0
χ_{30}	186	186	-6	-6	0	0	6	6	-2	-2	11	11	11	1
χ_{31}	620	-5	4	-1	-4	1	0	0	-4	1	20	-5	-5	0
χ_{32}	1860	-15	12	-3	0	0	0	0	4	-1	10	10	-15	0
χ_{33}	1860	-15	12	-3	0	0	0	0	4	-1	10	-15	10	0
χ_{34}	2480	-20	16	-4	-4	1	0	0	0	0	-20	5	5	0
χ_{35}	3100	-25	20	-5	4	-1	0	0	-4	1	0	0	0	0
χ_{36}	1240	-10	-8	2	4	-1	0	0	0	0	-10	-10	15	0
χ_{37}	1240	-10	-8	2	4	-1	0	0	0	0	-10	15	-10	0
χ_{38}	2480	-20	-16	4	-4	1	0	0	0	0	-20	5	5	0
χ_{39}	3720	-30	-24	6	0	0	0	0	0	0	20	-5	-5	0

The character table of $5^3L(3,5)$(continued)

	$6A$		$8A$	$8B$	$10A$			$12A$	$12B$	$20B$	20C	$24A$	$24B$	$24C$	$24D$
	6a	30a	8a	8b	10b	10c	10d	12a	12b	20b	20c	24a	24b	24c	24d
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	0	0	0	0	1	1	1	0	0	1	1	0	0	0	0
χ_3	1	1	-1	-1	2	2	2	1	1	0	0	-1	-1	-1	-1
χ_4	1	1	D	-D	0	0	0	-1	-1	F	/F	D	-D	D	-D
χ_5	1	1	-D	D	0	0	0	-1	-1	/F	F	-D	D	-D	D
χ_6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	1	1	2	2	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1
χ_{17}	1	1	-2	-2	-1	-1	-1	1	1	-1	-1	1	1	1	1
χ_{18}	1	1	E	-E	-1	-1	-1	-1	-1	1	1	-D	D	-D	D
χ_{19}	1	1	-E	E	-1	-1	-1	-1	-1	1	1	D	-D	D	-D
χ_{20}	2	2	0	0	1	1	1	-E	E	-D	D	0	0	0	0
χ_{21}	2	2	0	0	1	1	1	E	-E	D	-D	0	0	0	0
χ_{22}	-1	-1	0	0	1	1	1	-D	D	D	-D	G	/G	-G	-/G
χ_{23}	-1	-1	0	0	1	1	1	D	-D	-D	D	/G	G	-/G	-G
χ_{24}	-1	-1	0	0	1	1	1	-D	D	D	-D	-G	-/G	G	/G
χ_{25}	-1	-1	0	0	1	1	1	D	-D	-D	D	-/G	-G	/G	G
χ_{26}	-1	-1	-1	-1	0	0	0	-1	-1	0	0	-1	-1	-1	-1
χ_{27}	-1	-1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1
χ_{28}	-1	-1	-D	D	-1	-1	-1	1	1	D	-D	-D	D	-D	D
χ_{29}	-1	-1	D	-D	-1	-1	-1	1	1	-D	D	D	-D	D	-D
χ_{30}	0	0	0	0	-1	-1	-1	0	0	1	1	0	0	0	0
χ_{31}	4	-1	0	0	4	-1	-1	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	2	-3	2	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	2	2	-3	0	0	0	0	0	0	0	0
χ_{34}	4	-1	0	0	-4	1	1	0	0	0	0	0	0	0	0
χ_{35}	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	4	-1	0	0	2	-3	2	0	0	0	0	0	0	0	0
χ_{37}	4	-1	0	0	2	2	-3	0	0	0	0	0	0	0	0
χ_{38}	-4	1	0	0	4	-1	-1	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	-4	1	1	0	0	0	0	0	0	0	0

The character table of $5^{3\cdot}L(3,5)$(continued)

	$31A$	$31B$	$31C$	$31D$	$31E$	$31F$	$31G$	$31H$	$31I$	$31J$
	$31a$	$31b$	$31c$	$31d$	$31e$	$31f$	$31g$	$31h$	$31i$	$31j$
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_3	0	0	0	0	0	0	0	0	0	0
χ_4	0	0	0	0	0	0	0	0	0	0
χ_5	0	0	0	0	0	0	0	0	0	0
χ_6	H	/H	L	/L	K	/K	J	/J	I	/I
χ_7	/H	H	/L	L	K	/L	/K	/K	/I	I
χ_8	I	/I	H	/H	L	/L	K	/K	J	/J
χ_9	/I	I	/H	H	/L	L	/K	K	/J	J
χ_{10}	J	/J	I	/I	H	/H	L	/L	K	/K
χ_{11}	/J	J	/I	I	/H	H	/L	L	/K	K
χ_{12}	K	/K	J	/J	I	/I	H	/H	/L	L
χ_{13}	/K	K	/J	J	/I	I	/H	H	/L	L
χ_{14}	L	/L	K	/K	J	/J	I	/I	H	/H
χ_{15}	/L	L	/K	K	/J	J	/I	I	/H	H
χ_{16}	0	0	0	0	0	0	0	0	0	0
χ_{17}	0	0	0	0	0	0	0	0	0	0
χ_{18}	0	0	0	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0	0
χ_{20}	0	0	0	0	0	0	0	0	0	0
χ_{21}	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	0	0	0	0	0	0	0	0
χ_{24}	0	0	0	0	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	0	0	0
χ_{26}	1	1	1	1	1	1	1	1	1	1
χ_{27}	0	0	0	0	0	0	0	0	0	0
χ_{28}	0	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0

A = -1+6*E(4) = -1+6*ER(-1) = -1+6i
B = 4*E(4) = 4*ER(-1) = 4i
C = -5+6*E(4) = -5+6*ER(-1) = -5+6i
D = E(4) = ER(-1) = i
E = 2*E(4) = 2*ER(-1) = 2i
F = -1+E(4) = -1+ER(-1) = -1+i
G = $-E(24)^{11} + E(24)^{19}$

H = $E(31) + E(31)^5 + E(31)^{25}$
I = $E(31)^3 + E(31)^{13} + E(31)^{15}$
J = $E(31)^8 + E(31)^9 + E(31)^{14}$
K = $E(31)^{11} + E(31)^{24} + E(31)^{27}$
L = $E(31)^2 + E(31)^{10} + E(31)^{19}$

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J. Moori
Department of Mathematical Sciences, University of North West(Mafeking), Private Bag X 2046, Mmabatho 2735, South Africa
Email: Jamshid.Moori@nwu.ac.za

T. Seretlo
Department of Mathematical Sciences, University of North West(Mafeking), Private Bag X 2046, Mmabatho 2735, South Africa
Email: Thekiso.Seretlo@uniwest.ac.za