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2-RECOGNIZABILITY OF THE SIMPLE GROUPS $B_n(3)$ AND $C_n(3)$ BY PRIME GRAPH

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Abstract. Let G be a finite group and let $GK(G)$ be the prime graph of G. We assume that $n \geqslant 5$, is an odd number. In the paper, we show that the simple groups $B_n(3)$ and $C_n(3)$ are 2-recognizable by their prime graphs. As the result, the characterizability of the groups $B_n(3)$ and $C_n(3)$ by their spectra and by the set of orders of maximal abelian subgroups are obtained. Also, we conclude that the AAM's conjecture is true for these groups.

1. Introduction

For a finite group G we denote by $\pi(G)$ the set of all prime divisors of $|G|$ and the *spectrum* $\omega(G)$ of G is the set of element orders of G, i.e., a natural number n is in $\omega(G)$ if there is an element of order n in G. The *prime graph* (or *Gruenberg-Kegel graph*) $GK(G)$ of G is the graph with vertex set $\pi(G)$ where two distinct vertices p and q are adjacent by an edge (briefly, adjacent) if $pq \in \omega(G)$, in which case, we write $(p, q) \in GK(G)$. Given an arbitrary subset ω of the set of natural numbers, denote by $h(\omega)$ the number of pairwise nonisomorphic finite groups G such that $\omega(G) = \omega$. Given a natural number m, a group G is said to be *m-recognizable by spectrum* if $h(\omega(G)) = m$. In particular, G is said to be *characterizable by spectrum* if $h(\omega(G)) = 1$. The recognition

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problem is solved for a finite group G if we know the value of $h(\omega(G))$. Along side the above definitions, we denote by $k(GK(G))$ the number of isomorphism classes of the finite groups H satisfying $GK(G) = GK(H)$. Hence, if G is a finite group then $k(GK(G)) \geqslant 1$. Given a natural number m, a finite group G is called *m-recognizable by prime graph* if $k(GK(G)) = m$. Usually, a 1-recognizable group by prime graph is called a *characterizable group by prime graph.* The recognition problem by prime graph have been considered for most of finite nonabelian simple groups with disconnected prime graphs ([8, 9, 10, 15, 20]), but $L_{16}(2)$ is the only group with connected prime graph which its recognizability by prime graph has been solved completely so far ([11, 21]). Here, we prove that the simple groups $B_n(3)$ and $C_n(3)$, where $n > 3$ is an odd number, are 2-recognizable by prime graph and since these groups have connected prime graphs when n is non-prime, thus we obtain the first example of two infinite series of finite simple groups with connected prime graphs which their recognition problem by prime graph are solved thoroughly. In fact, we have the following main theorem:

Main theorem. Let $n > 3$ be an odd number. The simple groups $B_n(3)$ and $C_n(3)$ are 2-recognizable by prime graph. In fact if G is a finite group with $GK(G) = GK(B_n(3))$, then $G \cong B_n(3)$ or $G \cong C_n(3)$. Also, if G is a finite group such that $GK(G) = GK(B_3(3))$, then $G \cong B_3(3), C_3(3), D_4(3)$, or $G/O_2(G) = \mathrm{Aut}(^2B_2(8))$.

Note that for an odd prime n, the above groups have disconnected prime graphs and their 2-recognizability by prime graph have been considered in [15]. Since the method we used to prove the main theorem is based on our result in [6], we give a new proof for 2-recognizability of these groups in the disconnected case as well.

In [22, 23], it has been proved that if $n > 3$ is an odd prime and $B_n(3)$ and $C_n(3)$ have disconnected prime graphs, then these groups are characterizable by their spectra. Since the knowledge of $\omega(G)$ determines $GK(G)$, as the first result of the main theorem, we can conclude that these groups are characterizable by their spectra in the connected case as well.

The non-commuting graph of a nonabelian group G, denoted by Γ_G, is the graph with vertex set $G \setminus Z(G)$, where two distinct vertices x and y are connected by an edge if $xy \neq yx$. Problem 16.1 in the Kourovka notebook [12] is the AAM's conjecture, which asserts that simple groups are determined uniquely by the non-commuting graph. As prominent corollaries of the main theorem, the validity of the AAM's conjecture and characterizability by the set of orders of maximal abelian subgroups can be obtained for the groups $B_n(3)$ and $C_n(3)$, where $n \geqslant 3$ is odd.

2. Preliminaries

If q is a natural number, r is an odd prime and $gcd(r, q) = 1$, then by $e(r, q)$ we denote the smallest natural number m such that $q^m \equiv 1 \pmod r$. If $r = 2$, then we put $e(2, q) = 1$ if $q \equiv 1 \pmod 4$, and $e(2, q) = 2$ otherwise.

Lemma 2.1. [18, Corollary to Zsigmondy's theorem] *Let q be a natural number greater than 1. For every natural number m there exists a prime r with $e(r, q) = m$, except for the cases $q = 2$ and $m = 1$, $q = 3$ and $m = 1$, and $q = 2$ and $m = 6$.*

The prime r with $e(r, q) = m$ is called a *primitive prime divisor* of $q^m - 1$. It is obvious that $q^m - 1$ can have more than one primitive prime divisor. We denote by $r_m(q)$ some primitive prime divisor of $q^m - 1$. Also, $\eta(n)$ for an integer n, has been defined in [17] as follow:

$$\eta(n) = \begin{cases} n \text{ if } n \text{ is odd}; \\ \frac{n}{2} \text{ otherwise.} \end{cases}$$

Lemma 2.2. [18, Proposition 2.4], [17, Proposition 3.1] *Let G be one of the simple groups of Lie type $B_n(q)$ or $C_n(q)$ over a field of characteristic p and let r, s be odd primes with $r, s \in \pi(G) \setminus \{p\}$. Let $k = e(r, q), l = e(s, q)$ and suppose that $1 \leq \eta(k) \leq \eta(l)$. The following statements hold:*

(1) *r and s are nonadjacent if and only if $\eta(k) + \eta(l) > n$ and $\frac{l}{k}$ is not an odd natural number,*
(2) *r and p are nonadjacent if and only if $\eta(e(r, q)) > n - 1$.*

Lemma 2.3. [17, Proposition 4.3] *Let $G = B_n(q)$ or $G = C_n(q)$ be a finite simple group of Lie type over a field of order q with odd characteristic p. Let r be an odd prime divisor of $|G|$, $r \neq p$, and $k = e(r, q)$. Then r and 2 are nonadjacent if and only if $\eta(k) = n$ and one of the following holds:*

(1) *n is odd and $k = (3 - e(2, q))n$;*
(2) *n is even and $k = 2n$.*

Definition 2.4. [2] *Let S be a finite simple group of Lie type in characteristic p. Let A be any abelian p-group with an S-action. An element $s \in S$ is said to be unisingular on A if s has a (non-zero) fixed point on A. S is said to be unisingular if every element $s \in S$ acts unisingularly on every finite abelian p-group A with an S-action.*

Lemma 2.5. [7, Theorem 1.3] *The simple groups $B_n(q)$ and $C_n(q)$ of characteristic p is unisingular if and only if $q = p \neq 2$.*

Lemma 2.6. [14, Lemma 1] *Let G be a finite group, let N be a normal subgroup of G and let G/N be a Frobenius group with Frobenius kernel F and cyclic complement C. If $gcd(|F|, |N|) = 1$ and F not lying in $NC_G(N)/N$ then $s.|C| \in \omega(G)$ for some $s \in \pi(N)$.*

3. Main results

Lemma 3.1. *If $n > 3$ is an odd number and q is the power of an odd prime p, then the groups $B_n(q)$ and $C_n(q)$ include subgroups isomorphic to Frobenius groups of the form $U : Z_{r_n(q)}$ and $T : Z_{r_{n-2}(q)}$, where U and T are nontrivial p-groups.*

Proof. By [13, Proposition 4.1.20], $B_n(q)$ contains a subgroup isomorphic to $U : SL_n(q)$, where U is a nontrivial p-group. Since $r_n(q) \in \pi(SL_n(q))$, $SL_n(q)$ has a cyclic subgroup of order $r_n(q)$, namely Z. Now by Lemma 2.2(2), since n is odd, we can conclude that $U : Z$ is the first desired Frobenius subgroup. Also, similar to the previous argument $B_{n-2}(q)$ has a Frobenius subgroup of the form $T : Z_{r_{n-2}(q)}$, where T is a nontrivial p-group.

Let $\mathrm{bd}(C_1, C_2, ..., C_m)$ denote a block-diagonal matrix with square blocks $C_1, C_2, ..., C_m$. Put

$$W_n = \mathrm{bd}(\overbrace{W, W, ..., W}^{\text{n times}}, 1), \text{ where } W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

We have $SO_{2n+1}(q) = \{X \in SL_{2n+1}(q) | \ X^t W_n X = W_n\}$ and $B_n(q)$ is the derived subgroup of $SO_{2n+1}(q)$. Of course $M = \{\mathrm{bd}(I_4, X) | X \in SL_{2n-3}(q), \ X^t W_{n-2} X = W_{n-2}\}$ is a subgroup of $SO_{2n+1}(q)$ and it is isomorphic to $SO_{2n-3}(q)$. Thus the derived subgroup of M is a subgroup of $B_n(q)$ and it is isomorphic to $B_{n-2}(q)$, so we can assume that $B_{n-2}(q) \hookrightarrow B_n(q)$ and hence, $B_n(q)$ has the second desired Frobenius subgroup as well.

Now we consider $C_n(q)$. For $J_n = \begin{bmatrix} \mathbf{0} & I_n \\ -I_n & \mathbf{0} \end{bmatrix}$ we have $Sp_{2n}(q) = \{X \in GL_{2n}(q) |\ X^t J_n X = J_n\}$. Let H and K be the following sets:

$$H := \left\{ \begin{bmatrix} I_n & A \\ \mathbf{0} & I_n \end{bmatrix} \mid A \in M_n(q)\ and\ A^t = A \right\},$$

$$K := \left\{ \begin{bmatrix} B & \mathbf{0} \\ \mathbf{0} & (B^t)^{-1} \end{bmatrix} \mid B \in GL_n(q) \right\}.$$

Obviously, H, $K \leq Sp_{2n}(q)$, $K \cong GL_n(q)$ and $K \leq N_{Sp_{2n}(q)}(H)$. Also, it is easy to check that H is a nontrivial p-group and does not contain the center of $Sp_{2n}(q)$ which consists of the matrices $\pm I_{2n}$ and hence $C_n(q)$ has a subgroup of the form $U : \bar{K}$, where $U = H\{\pm I_{2n}\}/\{\pm I_{2n}\}$ is a nontrivial p-group and $\bar{K} = K/\{\pm I_{2n}\}$. Since $r_n(q) \in \pi(\bar{K})$, $\bar{K}$ has a cyclic subgroup of order $r_n(q)$, namely Z. By Lemma 2.2(2) since n is odd, we conclude that $U : Z$ is one of the desired Frobenius subgroup of $C_n(q)$. Also $N = \{\mathrm{bd}(I_2, X, I_2) | X \in GL_{2(n-2)}(q),\ X^t J_{n-2} X = J_{n-2}\}$ is a subgroup of $Sp_{2n}(q)$ and it is isomorphic to $Sp_{2(n-2)}(q)$. Thus $N\{\pm I_{2n}\}/\{\pm I_{2n}\}$ is a subgroup of $C_n(q)$ which is isomorphic to $Sp_{2(n-2)}(q)$. But as it was mentioned, $Sp_{2(n-2)}(q)$ contains a subgroup of the form $U_0 : Z_0$, where U_0 a is nontrivial p-group and Z_0 is a cyclic subgroup of order $r_{n-2}(q)$. We claim that $U_0 : Z_0$ is a Frobenius subgroup of $Sp_{2(n-2)}(q)$. If not, then $U_0 : Z_0$ contains an element of order $pr_{n-2}(q)$, so $(p, r_{n-2}(q)) \in GK(Sp_{2(n-2)}(q))$. Thus since $p \neq 2$ and $C_{n-2}(q) = Sp_{2(n-2)}(q)/\{\pm I\}$, we deduce that $(p, r_{n-2}(q)) \in GK(C_{n-2}(q))$, contradicting Lemma 2.2(2). This leads us to find the second desired Frobenius subgroup of $C_n(q)$.

Proof of the main theorem. In [6] we have proved that if $n \geq 9$ and $GK(G) = GK(B_n(3))$, then there exists a finite nonabelian simple group S such that $S \leq G/K \leq \mathrm{Aut}(S)$ and $S \cong B_n(3)$ or $S \cong C_n(3)$, where K is the maximal normal solvable subgroup of G. Also, we know that $|\mathrm{Out}(B_n(3))| = |\mathrm{Out}(C_n(3))| = 2$, hence $G/K = S$ or $G/K = \mathrm{Aut}(S)$. Thus the last step to complete the proof is to show that $G \cong S$ and $K = 1$. We will do it in the following two steps:

Step I. In this step, we will prove that $G/K = S$. If not, then $G/K = \mathrm{Aut}(S)$ and we will consider the cases $S \cong B_n(3)$ and $S \cong C_n(3)$ separately:

Case 1. If $S \cong B_n(3)$, then since $[SO_{2n+1}(3) : B_n(3)] = 2$, we have $B_n(3)$ is a normal subgroup of $SO_{2n+1}(3)$ and hence,

$$N_{SO_{2n+1}(3)}(B_n(3)) = SO_{2n+1}(3),\ B_n(3) \cap C_{SO_{2n+1}(3)}(B_n(3)) \lhd B_n(3)$$
$$\text{and } B_n(3)C_{SO_{2n+1}(3)}(B_n(3)) \leq SO_{2n+1}(3).$$

Thus, since $B_n(3)$ is a nonabelian simple group, we deduce that the order of the group $C_{SO_{2n+1}(3)}(B_n(3))$ divides 2. If $|C_{SO_{2n+1}(3)}(B_n(3))| = 2$, then since $C_{SO_{2n+1}(3)}(B_n(3))$ is a normal subgroup of $SO_{2n+1}(3)$, we can easily conclude that $C_{SO_{2n+1}(3)}(B_n(3)) \subseteq Z(SO_{2n+1}(3))$ which leads us to a contradiction, because $Z(SO_{2n+1}(3)) = 1$. Thus $C_{SO_{2n+1}(3)}(B_n(3)) = 1$. Now we have $SO_{2n+1}(3) \cong N_{SO_{2n+1}(3)}(B_n(3))/C_{SO_{2n+1}(3)}(B_n(3))$. So $\mathrm{Aut}(S)$ contains a subgroup isomorphic to $SO_{2n+1}(3)$ and since $SO_{2n+1}(3)$ has a maximal torus of order $3^n - 1$, we conclude that

$$(2, r_n(3)) \in GK(G) = GK(B_n(3)),$$

which is impossible by Lemma 2.3.

Case 2. If $S \cong C_n(3)$, then $(G/K)/C_n(3) = \mathrm{Out}(C_n(3))$. As was stated in the proof of Lemma 3.1, for $J_n = \begin{bmatrix} \mathbf{0} & I_n \\ -I_n & \mathbf{0} \end{bmatrix}$, we have $Sp_{2n}(3) = \{X \in GL_{2n}(3) |\ X^t J_n X = J_n\}$ and $C_n(3) = Sp_{2n}(3)/\{\pm I\}$. According to [13, Relation 2.4.3 and Proposition 2.4.4], $\mathrm{Out}(C_n(3)) = \langle \ddot{\delta} \rangle$, where $\delta = \mathrm{bd}(-I_n, I_n)\{\pm I\}$ and $\ddot{\delta}$ denotes the image of δ in $\mathrm{Out}(C_n(3))$. Thus δ centralizes a subgroup $K = \{\mathrm{bd}(A, (A^t)^{-1})\{\pm I\} | A \in GL_n(3)\}$ of $C_n(3)$ which is isomorphic to $GL_n(3)/\{\pm I\}$. This shows that $\mathrm{Aut}(C_n(3))$ has an element of order $3^n - 1$ and we can get a contradiction similar to the previous case.

Step II. Here we want to show that $K = 1$. It suffices to show that $GK(G) \neq GK(S)$ whenever $K \neq 1$. Replacing K by K/K_1, where K_1 is a maximal subgroup of K which is normal in G allows us to assume that K is an elementary abelian p-group and S acts on K faithfully and irreducibly. If $p = 3$, then by Lemma 2.5 each element of S centralizes some nontrivial element of K, and so 3 and $r_n(3)$ are adjacent which is impossible by Lemma 2.2(2).

If $p \neq 3$, then we can assume that $e(p, 3) = u$. Of course by Lemma 3.1, there are Frobenius subgroups of the form $U : Z_{r_n(3)}$ and $T : Z_{r_{n-2}(3)}$ in $S = G/K$, where U and T are nontrivial 3-groups. According to Lemma 2.6, p is adjacent to $r_n(3)$ and $r_{n-2}(3)$ in $GK(S)$.

Since $(p, r_n(3)) \in GK(S)$, Lemma 2.3 implies that $p \neq 2$. Thus by Lemma 2.2(1), $n+\eta(u) \leq n$ or n/u is an odd number, because n is odd. This leads to $u \mid n$. Therefore u is an odd number, so $\eta(u) = u$. Also, $(p, r_{n-2}(3)) \in GK(S)$, so Lemma 2.2(1) implies that $n - 2 + \eta(u) = n - 2 + u \leq n$ or $(n-2)/u$ is an odd number. This shows that $u \leq 2$ or $u \mid (n, n-2) = 1$ and hence, $u = 1$, because n and u are odd numbers. This implies that $p = r_1(3)$ which is impossible by Lemma 2.1.

Thus, according to Steps I and II, the main theorem is obtained for $n \geqslant 9$. Also, if $n \leqslant 7$, then the Main theorem in [15] completes the proof.

As consequences of our main result, we have the following corollaries:

Corollary 3.2. *Let n be an odd number. The simple groups $B_n(3)$ and $C_n(3)$ are characterizable by their spectra except the case $B_3(3)$ which is 2-recognizable.*

Proof. Since $\omega(B_n(3)) \neq \omega(C_n(3))$ [16, Proposition] and $\omega(B_3(3)) = \omega(D_4(3))$ [4], thus the proof is straightforward by the main theorem. □

If G is a finite group, by $M(G)$ we denote the set of orders of maximal abelian subgroups of G and a group G is said to be characterizable by the set of orders of its maximal abelian subgroups, if G is uniquely determined by $M(G)$.

Lemma 3.3. [3, Lemma 2] *Let G and H be finite groups. If $M(G) = M(H)$, then $GK(G) = GK(H)$.*

Corollary 3.4. *If n is an odd number, then the simple groups $B_n(3)$ and $C_n(3)$ are characterizable by their sets of orders of maximal abelian subgroups.*

Proof. Table 2 in [19] shows that $M(C_n(3)) \neq M(B_n(3))$ and $M(B_3(3)) \neq M(D_4(3)) \neq M(C_3(3))$. Also, it implies that $\max(M(B_3(3))) = 3^5$, $\max(M(C_3(3))) = 3^6$ and $\max(M(^2B_2(8))) = 2^4$.
Also, since $[\mathrm{Aut}(^2B_2(8)) : {}^2B_2(8)] = 3$, $M(G/O_2(G)) \neq M(\mathrm{Aut}(^2B_2(8)))$, for every finite group G with $M(G) = M(B_3(3))$ or $M(G) = M(C_3(3))$ and hence, Lemma 3.3 and the main theorem complete the proof. □

The Non-commuting graph of a nonabelian group G , denoted by Γ_G is the graph with vertex set $G \setminus Z(G)$, where two distinct vertices x and y are adjacent by an edge if $xy \neq yx$. Problem 16.1 in the Kourovka notebook [12] is AAM's conjecture, which says simple groups are determined uniquely by the non-commuting graph. This conjecture is valid for all non-abelian finite simple groups with disconnected prime graph (for examples see paper by Darafsheh [5] and references quoted in that paper).

Lemma 3.5. [1, Theorem 2] *Let P be a finite nonabelian simple group and G is a group such that $\Gamma_G \cong \Gamma_P$, then $GK(G) = GK(P)$ and $M(G) = M(P)$.*

Corollary 3.6. *The AAM's Conjecture is true for the groups under study.*

Proof. By Lemma 3.5 and Corollary 3.4, the proof is straightforward. □

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