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ON THE NON-SPLIT EXTENSION GROUP $2^{6\cdot}Sp(6,2)$

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Abstract. In this paper we first construct the non-split extension $\overline{G} = 2^{6\cdot}Sp(6,2)$ as a permutation group acting on 128 points. We then determine the conjugacy classes using the coset analysis technique [J. Moori, On the Groups G^+ and $\overline{G}$ of the form $2^{10}{:}M_{22}$ and $2^{10}{:}\overline{M}_{22}$, PhD Thesis, University of Birmingham, 1975] and [J. Moori, On certain groups associated with the smallest Fischer group, *J. London Math. Soc.(2)* 23 (1981), no. 1, 61–67.], inertia factor groups and Fischer matrices, which are required for the computations of the character table of $\overline{G}$ by means of Clifford-Fischer Theory. There are two inertia factor groups namely $H_1 = Sp(6,2)$ and $H_2 = 2^5{:}S_6$, the Schur multiplier and hence the character table of the corresponding covering group of H_2 were calculated. Using information on conjugacy classes, Fischer matrices and ordinary and projective tables of H_2, we concluded that we only need to use the ordinary character table of H_2 to construct the character table of $\overline{G}$. The Fischer matrices of $\overline{G}$ are all listed in this paper. The character table of $\overline{G}$ is a 67×67 integral matrix, it has been supplied in the PhD Thesis [A. B. M. Basheer, Clifford-Fischer Theory Applied to Certain Groups Associated with Symplectic, Unitary and Thompson Groups, University of KwaZulu-Natal, Pietermaitzburg, 2012] of the first author, which could be accessed online.

1. Introduction

Let $G = Sp(2n,q)$ be the symplectic group consisting of $2n \times 2n$ matrices over $\mathbb{F}_q$ that preserve a non-degenerate alternating bilinear form

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and let $V = q^{2n}$ be a $2n-$dimensional vector space over $\mathbb{F}_q$. In [10] Dempwolff proved that a non-split extension of the form $\overline{G}_n = 2^{2n\cdot}Sp(2n,2)$ does exist for all $n \geq 2$, where $\overline{G}_n/2^{2n} \cong Sp(2n,2)$ acts faithfully on 2^{2n}. Moreover such an extension is unique, up to isomorphism, since $\dim_{\mathbb{F}_2} H^2(Sp(2n,2),2^{2n}) = 1$ for all $n \geq 2$, where $H^2(K,M)$ is the second cohomology group of a group K with coefficients in M. In the case $n = 2$, the non-split extension $\overline{G}_2 = 2^{4\cdot}Sp(4,2) \cong 2^{4\cdot}S_6$ is a maximal subgroup of the sporadic simple group Higman-Sims HS (see the $\mathbb{ATLAS}$ [8]). This group has been fully investigated by T. Seretlo [26], its Fischer matrices and the character table were determined. The character table of $\overline{G}_2$ is also available in a GAP library (see [14]). The split extension $2^6{:}Sp(6,2)$ as a maximal subgroup of F_{22} is fully studied by Mpono and Mpono and Moori [21, 19], its Fischer matrices and character table were calculated. The Fischer matrices of our group $\overline{G} = 2^{6\cdot}Sp(6,2)$ and its character table are not known. In this paper our main aims are to fully study this group, to determine its inertia factor groups (and their respective ordinary and projective character tables) and to compute the Fischer matrices. It will turn out that the character table of $\overline{G}$ is a 67×67 integral matrix and coincides with the character table of the split extension $2^6{:}Sp(6,2)$, constructed in Mpono [21] and which is also available in GAP. If one only interested in the calculation of the character table, then it could be computed by using GAP or Magma and the generators $\overline{g}_1$ and $\overline{g}_2$ of $\overline{G}$. But Clifford-Fischer Theory provides many other interesting information on the group and on the character table, in particular the character table produced by Clifford-Fischer Theory is in a special format that could not be achieved by direct computations using GAP or Magma. Also providing various examples for the applications of Clifford-Fischer Theory to both split and non-split extensions is making sense, since each group requires individual approach. The readers (particulary young researchers) will highly benefit from the theoretical background required for these computations. GAP and Magma are computational tools and would not replace good powerful and theoretical arguments.

We have built a small subroutine (see Section 9.2 of [4]) in GAP to construct the group $\overline{G}$ in terms of permutations of a set of cardinality 128. The following two elements $\overline{g}_1$ and $\overline{g}_2$ generate the group $\overline{G}$.

$\overline{g}_1$ = (1 12)(2 111 86 50)(3 57 94 69)(4 95 23 121)(5 70 48 58)(6 71 21 59)(8 72 73 117)(9 108 33 74)(10 118 61 18)(11 119 31 19)(13 102 97 87)(14 100 27 101)(15 85 55 66)(16 115 88 67)(17 79 52 78)(20 93 120 104)(22 28 24 105)(25 106 47 29)(30 44 107 90)(32 45 34 91)(35 92 60 46)(36 65 64 37)(38 126 122 53)(39 42 116 89)(40 43 84 54)(41 63 56 125)(51 83 127 68)(62 96)(76 98)(77 99)(80 81 112 113)(82 114 128 103)(109 123)(110 124),

$\overline{g}_2$ = (1 104 103 118 93 113 55 52 83 15 98 8 18 17 107)(2 73 12 69 68 91 57 81 89 86 114 42 123 30 45)(3 48 49 32 72 115 34 36 125 94 90 63 77 24 67)(4 53 35 127 102 99 106 47 60 59 112 14 82 6 27)(5 40 20 22 75 61 44 84 10 64 101 120 117 100 110)(9 50 88 43 79)(11 128 126 124 71 21 31 29 80 41 51 25 56 23 87)(13 105 96 28 97)(16 111 33 78 54)(19 116 74 66 92)(37 76 121)(38 70 62 58 122)(39 119 46 85 108)(65 109 95).

with $o(\overline{g}_1) = 4,\ o(\overline{g}_2) = 15$ and $o(\overline{g}_1\overline{g}_2) = 9$.

Note 1.1. *Note that the generators $\overline{g}_1$ and $\overline{g}_2$ fix the points 7 and 26. Thus $\overline{G}$ acts transitively on a set Ω of 126 points. Hence we have a permutation character $\chi(\overline{G}|\Omega) = \chi$ of degree 126. In Table 11.1 of* [4] *we listed the values of χ on $\overline{G}-$classes and using the character table of $\overline{G}$ (Table 11.12 of* [4]*) we can see that $\chi = \chi_1 + \chi_6 + \chi_7 + \chi_{31}$.*

Now having the group $\overline{G}$ constructed in GAP, it is easy to obtain its normal subgroups. In fact the only non-trivial proper normal subgroup that $\overline{G}$ contains is a group of order 64 and thus must be isomorphic to the elementary abelian group $N = 2^6$. The following 6 permutations $n_1, n_2, \cdots, n_6$ generate the normal subgroup N.

n_1 = (6 25)(11 35)(16 43)(19 46)(22 48)(29 59)(32 61)(37 65)(40 67)(49 75)(53 87)(55 89)(62 96)(68 103)(69 104)(71 106)(73 107)(74 108)(77 110)(79 111)(81 113)(83 114)(85 116)(90 117)(92 119)(94 120)(95 121)(97 122)(99 124)(101 125)(102 126)(127 128),

n_2 = (5 24)(6 25)(10 34)(11 35)(15 42)(16 43)(18 45)(19 46)(21 47)(22 48)(28 58)(29 59)(31 60)(32 61)(36 64)(37 65)(39 66)(40 67)(54 88)(55 89)(70 105)(71 106)(76 109)(77 110)(80 112)(81 113)(84 115)(85 116)(91 118)(92 119)(98 123),

n_3 = (99 124) (4 23)(5 24)(6 25)(9 33)(10 34)(11 35)(14 41)(15 42)(16 43)(18 45)(19 46)(28 58)(29 59)(36 64)(37 65)(44 72)(50 78)(52 86)(54 88)(55 89)(57 93)(63 100)(69 104)(74 108)(80 112)(81 113)(83 114)(90 117)(95 121)(97 122)(102 126)(127 128),

n_4 = (3 20)(5 24)(6 25)(8 30)(10 34)(11 35)(13 38)(15 42)(16 43)(21 47)(22 48)(31 60)(32 61)(39 66)(40 67)(44 72)(50 78)(51 82)(54 88)(55 89)(57 93)(63 100)(69 104)(73 107)(79 111)(84 115)(85 116)(90 117)(94 120)(97 122)(101 125)(127 128),

n_5 = (2 17)(3 20)(4 23)(5 24)(8 30)(9 33)(10 34)(16 43)(19 46)(22 48)(27 56)(29 59)(32 61)(36 64)(39 66)(49 75)(50 78)(55 89)(62 96)(63 100)(69 104)(70 105)(77 110)(79 111)(80 112)(83 114)(84 115)(90 117)(91 118)(99 124)(101 125)(102 126),

n_6 = (1 12)(5 24)(6 25)(8 30)(9 33)(15 42)(16 43)(18 45)(19 46)(21 47)(22 48)(27 56)(36 64)(37 65)(39 66)(40 67)(50 78)(51 82)(52 86)(53 87)(57 93)(62 96)(70 105)(71 106)(79 111)(83 114)(90 117)(94 120)(95 121)(98 123)(99 124)(127 128).

In Magma or GAP one can easily check for the complements of the normal subgroup $N = \langle n_1, n_2, \cdots, n_6\rangle$ in $\overline{G} = \langle \overline{g}_1, \overline{g}_2\rangle$, which in our case will return an empty list of complements. This shows that the group $\overline{G}$ constructed using the generators $\overline{g}_1$ and $\overline{g}_2$ is indeed a non-split extension of the elementary abelian group $N = 2^6$ by the symplectic group $Sp(6,2)$.

Section 2 is devoted to review the conjugacy classes of group extensions in general and the conjugacy classes of $\overline{G}$. In Section 3 we review the fundamentals of Clifford-Fischer Theory. In Section 4 we determine the inertia factors $H_1 = Sp(6,2)$ and $H_2 = 2^5{:}S_6$ and the fusion of H_2 in $Sp(6,2)$ as well as the ordinary and projective character tables of H_2. In Section 5 we calculate the Fischer matrices of $\overline{G}$ and we see that these matrices are integral valued matrices and their sizes range between one and four. Finally in Section 6 we show how to construct the table of $\overline{G}$. The character table of $\overline{G}$ is given as Table 11.12 of Basheer [4]. It could be accessed online via
"http://researchspace.ukzn.ac.za/xmlui/handle/10413/6674?show=full".

Throughout this paper,

- all the mentioned groups are finite,
- an ordinary or projective character will always mean a character over $\mathbb{C}$,
- 1_G means the neutral element of a finite group G, while $\mathbf{1}_G$ (bolded) means the trivial character of G,
- if $H \leq G$, then $\chi\uparrow_H^G$ and $\chi\downarrow_H^G$ will denote the induction and restriction of a character χ from H to G and from G to H respectively.

- $\inf(\chi)$ denotes the inflation (lift) of a character χ of a quotient group G/K to G, where K is any normal subgroup of G.

We would like to remark that most of the results mentioned in this paper follow [4] and [5].

2. Conjugacy classes of group extensions and of $\overline{G} = 2^{6\cdot}Sp(6,2)$

In this section we use the method of the *coset analysis* to calculate the conjugacy classes of $\overline{G} = 2^{6\cdot}Sp(6,2)$. This technique has been developed by the second author of this paper in his PhD Thesis [17] and also in [18]. The coset analysis can be used for any extension (spilt or non-split) $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and also whether the kernel N of the extension is abelian or not. It has been used by various authors such as Barraclough [3] and in particular by several MSc and PhD students, such as Mpono [19], Rodrigues [24], Whitely [27] and in [5] and [6] by the authors of this paper. In the following we give a shortened description on how the coset analysis can be used to determine the conjugacy classes of any group extension.

For each $g \in G$ let $\overline{g} \in \overline{G}$ map to g under the natural epimorphism $\pi : \overline{G} \longrightarrow G$ and let $g_1 = N\overline{g}_1, g_2 = N\overline{g}_2, \cdots, g_r = N\overline{g}_r$ be representatives for the conjugacy classes of $G \cong \overline{G}/N$. Therefore $\overline{g}_i \in \overline{G}$, $\forall i$, and by convention we take $\overline{g}_1 = 1_{\overline{G}}$. The method of the coset analysis constructs for each conjugacy class $[g_i]_G$, $1 \leq i \leq r$, a number of conjugacy classes of $\overline{G}$. That is each conjugacy class of $\overline{G}$ corresponds uniquely to a conjugacy class of G. This method can be described briefly in the following steps:

- For fixed $i \in \{1, 2, \cdots, r\}$, act N (by conjugation) on the coset $N\overline{g}_i$ and let the resulting orbits be $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$. If N is abelian (regardless to whether the extension is split or not), then $|Q_{i1}| = |Q_{i2}| = \cdots = |Q_{ik_i}| = \frac{|N|}{k_i}$.
- Act $\overline{G}$ on $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ and suppose f_{ij} orbits fuse together to form a new orbit Δ_{ij} and let the total number of the new resulting orbits in this action be $c(g_i)$ (that is $1 \leq j \leq c(g_i)$). Then $\overline{G}$ has a conjugacy class $[g_{ij}]_{\overline{G}}$ that contains Δ_{ij} and $|[g_{ij}]_{\overline{G}}| = |[g_i]_G| \times |\Delta_{ij}|$.
- Repeat the above two steps, for all $i \in \{1, 2, \cdots, r\}$.

Lemma 2.1. *$\forall\, i \in \{1, 2, \cdots, r\}$, write $g_i = N\overline{g}_i = \bigcup_{j=1}^{c(g_i)} \left(N\overline{g}_i \bigcap [g_{ij}]_{\overline{G}}\right) = \bigcup_{j=1}^{c(g_i)} \Delta_{ij}$. Then $\{\overline{g}_{i1}, \overline{g}_{i2}, \cdots, \overline{g}_{ic(g_i)}\}$ is a complete set of representatives for the conjugacy classes of $\overline{G}$ that correspond (under the natural epimorphism) to $[g_i]_G$.*

Proof. One can refer to Barraclough [3] with slight difference in notations. □

Thus each $[g_i]_G$ affords $c(g_i)$ conjugacy classes in $\overline{G}$.

Remark 2.2. *For fixed $i \in \{1, 2, \cdots, r\}$, the conjugacy class $[g_{ij}]_{\overline{G}}$ is partitioned into $|[g_i]_G|$ equal size subsets $\Delta_{ij1}, \Delta_{ij2}, \cdots, \Delta_{ij|[g_i]_G|}$, where $|\Delta_{iju}| = |\Delta_{ij}|$, for each $1 \leq u \leq |[g_i]_G|$ (we can take $\Delta_{ij1} = \Delta_{ij}$). Moreover, for fixed i and $s \in \{1, 2, \cdots, |[g_i]_G|\}$, the relation $\sum_{j=1}^{c(g_i)} |\Delta_{ijs}| = |N|$ holds. If the extension splits, then Δ_{i1s} is the intersection of $[g_{ij}]_{\overline{G}}$ with an element of $[g_i]_G$, for all $1 \leq s \leq |[g_i]_G|$.*

Therefore information about every conjugacy class of $\overline{G}$ can be obtained by examining one coset $N\overline{g}_i = g_i \in G$ for each conjugacy class of G. The following two propositions relate the orders of the elements of $\overline{G}$ with those of G.

Proposition 2.3. *Let $\overline{G} = N{:}G$, where N is an abelian group. Also let $\overline{G} \ni \overline{g} = ng$, for some $n \in N$ and $g \in G$. Then $o(g)|o(\overline{g})$.*

Proof. Let $o(\overline{g})$ and $o(g)$ be k and m respectively. We have $1_{\overline{G}} = \overline{g}^k = (ng)^k = nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}} g^k$. Since G acts on N, we have $n, n^g, n^{g^2}, n^{g^3}, \cdots, n^{g^{k-1}} \in N$ and therefore $nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}} \in N$. Now since $N \bigcap G = \{1_{\overline{G}}\}$ and $nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}} g^k = 1_{\overline{G}}$, we must have $nn^g n^{g^2} n^{g^3} \cdots n^{g^{k-1}}$ and g^k equal to 1_N and 1_G respectively. Hence $m|k$. □

Proposition 2.4. *With the settings of* Proposition 2.3 *and its proof, assume further that N is an elementary abelian $p-$group. Then $k \in \{m, pm\}$.*

Proof. See Mpono [21]. □

Further results on the conjugacy classes of $\overline{G} = N{\cdot}G$, when N is abelian or the extension splits, can be found in many sources such as Ali [1], Barraclough [3], Moori [17], [18], Mpono [21], Rodrigues [24] or Whitely [27].

In Table 1 we list the conjugacy classes of $\overline{G} = 2^{6\cdot}Sp(6,2)$, where in this table, the values of

- k_i's represent the number of fixed points of g_i on its action on $N = \langle n_1, n_2, \cdots, n_6 \rangle$,
- f_{ij}'s represent the number of orbits (of the action of N on $N\overline{g}_i$) fused together under the action of $\overline{G} = \langle \overline{g}_1, \overline{g}_2 \rangle$,
- m_{ij}'s are weights (attached to each class of $\overline{G}$) that will be used later in computing the Fischer matrices of $\overline{G}$. These weights are computed through the formula

$$m_{ij} = [N_{\overline{G}}(N\overline{g}_i) : C_{\overline{G}}(g_{ij})] = |N| \frac{|C_G(g_i)|}{|C_{\overline{G}}(g_{ij})|}. \tag{2.1}$$

Table 1. The conjugacy classes of $\overline{G} = 2^{6\cdot}Sp(6,2)$

$[g_i]_G$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{\overline{G}})\rvert$	$\lvert C_{\overline{G}}(g_{ij})\rvert$
$g_1 = 1A$	$k_1 = 64$	$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	1	92897280
		$f_{12} = 63$	$m_{12} = 63$	g_{12}	2	63	1474560
$g_2 = 2A$	$k_2 = 32$	$f_{21} = 13$	$m_{21} = 2$	g_{21}	4	126	737280
		$f_{22} = 15$	$m_{22} = 30$	g_{22}	4	1890	49152
		$f_{23} = 16$	$m_{23} = 32$	g_{23}	2	2016	46080
$g_3 = 2B$	$k_3 = 16$	$f_{31} = 1$	$m_{31} = 4$	g_{31}	2	1260	73728
		$f_{32} = 3$	$m_{32} = 12$	g_{32}	2	3780	24576
		$f_{33} = 12$	$m_{33} = 48$	g_{33}	4	15120	6144
$g_4 = 2C$	$k_4 = 16$	$f_{41} = 1$	$m_{41} = 4$	g_{41}	4	3780	24576
		$f_{42} = 3$	$m_{42} = 12$	g_{42}	4	11340	8192
		$f_{43} = 4$	$m_{43} = 16$	g_{43}	2	15120	6144
		$f_{44} = 8$	$m_{44} = 32$	g_{44}	4	30240	3072
$g_5 = 2D$	$k_5 = 8$	$f_{51} = 1$	$m_{51} = 8$	g_{51}	4	30240	3072
		$f_{52} = 1$	$m_{52} = 8$	g_{52}	2	30240	3072
		$f_{53} = 3$	$m_{53} = 24$	g_{53}	4	90720	1024
		$f_{54} = 3$	$m_{54} = 24$	g_{54}	4	90720	1024
$g_6 = 3A$	$k_6 = 16$	$f_{61} = 1$	$m_{61} = 4$	g_{61}	3	2688	34560
		$f_{62} = 15$	$m_{62} = 60$	g_{62}	6	40320	2304
$g_7 = 3B$	$k_7 = 1$	$f_{71} = 1$	$m_{71} = 64$	g_{71}	3	143360	648

Continued on next page

$[g_i]_G$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{\overline{G}}\rvert$	$\lvert C_{\overline{G}}(g_{ij})\rvert$
$g_8 = 3C$	$k_8 = 4$	$f_{81} = 1$	$m_{81} = 16$	g_{81}	3	215040	432
		$f_{82} = 3$	$m_{82} = 48$	g_{82}	6	645120	144
$g_9 = 4A$	$k_9 = 4$	$f_{91} = 1$	$m_{91} = 16$	g_{91}	4	60480	1536
		$f_{92} = 3$	$m_{92} = 48$	g_{92}	4	181440	512
$g_{10} = 4B$	$k_{10} = 8$	$f_{10,1} = 1$	$m_{10,1} = 8$	$g_{10,1}$	8	60480	1536
		$f_{10,2} = 3$	$m_{10,2} = 24$	$g_{10,2}$	8	181440	512
		$f_{10,3} = 4$	$m_{10,3} = 32$	$g_{10,3}$	4	241920	384
$g_{11} = 4C$	$k_{11} = 8$	$f_{11,1} = 1$	$m_{11,1} = 8$	$g_{11,1}$	8	60480	1536
		$f_{11,2} = 3$	$m_{11,2} = 24$	$g_{11,2}$	8	181440	512
		$f_{11,3} = 4$	$m_{11,3} = 32$	$g_{11,3}$	4	241920	384
$g_{12} = 4D$	$k_{12} = 4$	$f_{12,1} = 1$	$m_{12,1} = 16$	$g_{12,1}$	4	181440	512
		$f_{12,2} = 1$	$m_{12,2} = 16$	$g_{12,2}$	4	181440	512
		$f_{12,3} = 2$	$m_{12,3} = 32$	$g_{12,3}$	4	362880	256
$g_{13} = 4E$	$k_{13} = 4$	$f_{13,1} = 1$	$m_{13,1} = 16$	$g_{13,1}$	8	725760	128
		$f_{13,2} = 1$	$m_{13,2} = 16$	$g_{13,2}$	4	725760	128
		$f_{13,3} = 1$	$m_{13,3} = 16$	$g_{13,3}$	8	725760	128
		$f_{13,4} = 1$	$m_{13,4} = 16$	$g_{13,4}$	4	725760	128
$g_{14} = 5A$	$k_{14} = 4$	$f_{14,1} = 3$	$m_{14,1} = 48$	$g_{14,1}$	10	2322432	40
		$f_{14,2} = 1$	$m_{14,2} = 16$	$g_{14,2}$	5	774144	120
$g_{15} = 6A$	$k_{15} = 8$	$f_{15,1} = 1$	$m_{15,1} = 8$	$g_{15,1}$	12	80640	1152
		$f_{15,2} = 3$	$m_{15,2} = 24$	$g_{15,2}$	12	241920	384
		$f_{15,3} = 4$	$m_{15,3} = 32$	$g_{15,3}$	6	322560	288
$g_{16} = 6B$	$k_{16} = 4$	$f_{16,1} = 1$	$m_{16,1} = 16$	$g_{16,1}$	6	161280	576
		$f_{16,2} = 3$	$m_{16,2} = 48$	$g_{16,2}$	12	483840	192
$g_{17} = 6C$	$k_{17} = 1$	$f_{17,1} = 1$	$m_{17,1} = 64$	$g_{17,1}$	6	1290240	72
$g_{18} = 6D$	$k_{18} = 4$	$f_{18,1} = 1$	$m_{18,1} = 16$	$g_{18,1}$	12	483840	192
		$f_{18,2} = 1$	$m_{18,2} = 16$	$g_{18,2}$	6	483840	192
		$f_{18,3} = 2$	$m_{18,3} = 32$	$g_{18,3}$	12	967680	96
$g_{19} = 6E$	$k_{19} = 2$	$f_{19,1} = 1$	$m_{19,1} = 32$	$g_{19,1}$	12	1290240	72
		$f_{19,2} = 1$	$m_{19,2} = 32$	$g_{19,2}$	6	1290240	72
$g_{20} = 6F$	$k_{20} = 4$	$f_{20,1} = 1$	$m_{20,1} = 16$	$g_{20,1}$	6	645120	144
		$f_{20,2} = 3$	$m_{20,2} = 48$	$g_{20,2}$	6	1935360	48
$g_{21} = 6G$	$k_{21} = 2$	$f_{21,1} = 1$	$m_{21,1} = 32$	$g_{21,1}$	12	3870720	24
		$f_{21,2} = 1$	$m_{21,2} = 32$	$g_{21,2}$	6	3870720	24
$g_{22} = 7A$	$k_{22} = 1$	$f_{22,1} = 1$	$m_{22,1} = 64$	$g_{22,1}$	7	13271040	7
$g_{23} = 8A$	$k_{23} = 2$	$f_{23,1} = 1$	$m_{23,1} = 32$	$g_{23,1}$	8	2903040	32
		$f_{23,2} = 1$	$m_{23,2} = 32$	$g_{23,2}$	8	2903040	32
$g_{24} = 8B$	$k_{24} = 2$	$f_{24,1} = 1$	$m_{24,1} = 32$	$g_{24,1}$	8	2903040	32
		$f_{24,2} = 1$	$m_{24,2} = 32$	$g_{24,2}$	8	2903040	32
$g_{25} = 9A$	$k_{25} = 1$	$f_{25,1} = 1$	$m_{25,1} = 64$	$g_{25,1}$	9	10321920	9
$g_{26} = 10A$	$k_{26} = 2$	$f_{26,1} = 1$	$m_{26,1} = 32$	$g_{26,1}$	20	4644864	20
		$f_{26,2} = 1$	$m_{26,2} = 32$	$g_{26,2}$	10	4644864	20
$g_{27} = 12A$	$k_{27} = 2$	$f_{27,1} = 1$	$m_{27,1} = 32$	$g_{27,1}$	24	1935360	48
		$f_{27,2} = 1$	$m_{27,2} = 32$	$g_{27,2}$	12	1935360	48
$g_{28} = 12B$	$k_{28} = 2$	$f_{28,1} = 1$	$m_{28,1} = 32$	$g_{28,1}$	24	1935360	48
		$f_{28,2} = 1$	$m_{28,2} = 32$	$g_{28,2}$	12	1935360	48
$g_{29} = 12C$	$k_{29} = 1$	$f_{29,1} = 1$	$m_{29,1} = 64$	$g_{29,1}$	12	7741440	12
$g_{30} = 15A$	$k_{30} = 1$	$f_{30,1} = 1$	$m_{30,1} = 64$	$g_{30,1}$	15	6193152	15

Remark 2.5. *Note that from* Table 1*, the group* $\overline{G} = 2^{6\cdot}Sp(6,2)$ *contains 6 conjugacy classes of involutions, while from* Table 6.1 *of Mpono* [21]*, the split extension* $T = 2^6{:}Sp(6,2)$ *contains 8 conjugacy classes of involutions. This confirms that the group* $\overline{G}$ *constructed using the generators* $\overline{g}_1$ *and* $\overline{g}_2$ *is different from the group* T *but later it will be shown that the character tables of the two groups will be the same.*

3. The theory of Clifford-Fischer matrices

Let $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a group extension. To construct the character table of $\overline{G}$ we need to have

- the character tables (ordinary or projective) of the inertia factor groups,
- the fusions of classes of the inertia factors into classes of G,
- the Fischer matrices of the extension $\overline{G} = N{\cdot}G$.

The theory of Clifford-Fischer matrices, which is based on Clifford Theory (see Clifford [9]), was developed by B. Fischer ([11], [12] and [13]). This technique has also been discussed and applied to both split and non-split extension in several publications, for example see Ali and Moori [2], Barraclough [3], Fischer [13], Moori [17], Moori and Basheer [5] and [6], Moori and Mpono [19], Moori and Zimba [20], Pahlings [22], Rodrigues [24], Whitely [27], Zimba [29] and in a recent book by K. Lux and H. Pahlings [23].

Let $\overline{H} \trianglelefteq \overline{G}$ and let $\phi \in \mathrm{Irr}(\overline{H})$. For $\overline{g} \in \overline{G}$, define $\phi^{\overline{g}}$ by $\phi^{\overline{g}}(h) = \phi(\overline{g}h\overline{g}^{-1})$, $\forall\, h \in \overline{H}$. It follows that $\overline{G}$ acts on $\mathrm{Irr}(\overline{H})$ by conjugation and we define the *inertia group* of ϕ in $\overline{G}$ by $\overline{H}_\phi = \{\overline{g} \in \overline{G} |\ \phi^{\overline{g}} = \phi\}$. Also for a finite group $\mathcal{K}$, we let $\mathrm{IrrProj}(\mathcal{K}, \alpha^{-1})$ denotes the set of irreducible projective characters of $\mathcal{K}$ with factor set α^{-1}.

Theorem 3.1 (**Clifford Theorem**). *Let* $\chi \in \mathrm{Irr}(\overline{G})$ *and let* $\theta_1, \theta_2, \cdots, \theta_t$ *be representatives of orbits of* $\overline{G}$ *on* $\mathrm{Irr}(N)$*. For* $k \in \{1, 2, \cdots, t\}$*, let* $\theta_k^{\overline{G}} = \{\theta_k = \theta_{k1}, \theta_{k2}, \cdots, \theta_{ks_k}\}$ *and let* $\overline{H}_k$ *be the inertia group in* $\overline{G}$ *of* θ_k*. Then*

$$\chi\downarrow_N^{\overline{G}} = \sum_{k=1}^{t} e_k \sum_{u=1}^{s_k} \theta_{ku}, \quad \textit{where} \quad e_k = \left\langle \chi\downarrow_N^{\overline{G}}, \theta_k \right\rangle.$$

Moreover, for fixed k

$$\begin{aligned}\mathrm{Irr}(\overline{H}_k,\theta_k) &:= \left\{\psi_k\in \mathrm{Irr}(\overline{H}_k)\,|\, \left\langle \psi_k\downarrow_N^{\overline{H}_k},\theta_k\right\rangle\neq 0\right\}\\ &\longleftrightarrow \left\{\chi\in \mathrm{Irr}(\overline{G})\,|\, \left\langle \chi\downarrow_N^{\overline{G}},\theta_k\right\rangle\neq 0\right\} := \mathrm{Irr}(\overline{G},\theta_k)\end{aligned}$$

under the map $\psi_k \longmapsto \psi_k\uparrow_{\overline{H}_k}^{\overline{G}}$.

Proof. See Theorems 4.1.5 and 4.1.7 of Ali [1] with the difference in notations. □

Theorem 3.2. *Further to the settings of* Theorem 3.1, *assume that for* $k\in\{1,2,\cdots,t\}$, *there exists* $\psi_k\in \mathrm{Irr}(\overline{H}_k,\theta_k)$. *Then*

$$\mathrm{Irr}(\overline{G}) = \bigcup_{k=1}^{t}\left\{(\psi_k \inf(\zeta))\uparrow_{\overline{H}_k}^{\overline{G}}\,|\ \zeta\in \mathrm{Irr}(\overline{H}_k/N)\right\}. \tag{3.1}$$

Proof. See Ali [1] or Whitley [27]. □

Remark 3.3. *It is by no means necessarily the case that there exists an extension* ψ_k *of* θ_k *to the inertia group (that is the case* $\mathrm{Irr}(\overline{H}_k,\theta_k)=\emptyset$, *the empty set, is feasible). However, there is always a projective extension* $\widetilde{\psi}_k\in \mathrm{IrrProj}(\overline{H}_k,\overline{\alpha}_k^{-1})$ *for some factor set* $\overline{\alpha}_k$ *of the Schur multiplier of* $\overline{H}_k$. *Thus the more proper formula for* Equation (3.1) *is (see* Remark 4.2.7 *of Ali* [1]*)*

$$\begin{aligned}\mathrm{Irr}(\overline{G}) &= \bigcup_{k=1}^{t}\left\{(\widetilde{\psi}_k \inf(\zeta))\uparrow_{\overline{H}_k}^{\overline{G}}\,|\ \widetilde{\psi}_k\in \mathrm{IrrProj}(\overline{H}_k,\overline{\alpha}_k^{-1}),\right.\\ &\qquad\left.\zeta\in \mathrm{IrrProj}(\overline{H}_k/N,\alpha_k^{-1})\right\},\end{aligned}\tag{3.2}$$

where the factor set α_k *is obtained from* $\overline{\alpha}_k$ *as described in* Corollary 7.3.3 *of Whitely* [27]. *Hence the character table of* $\overline{G}$ *is partitioned into* t *blocks* $\mathcal{K}_1,\mathcal{K}_2,\cdots,\mathcal{K}_t$, *where each block* $\mathcal{K}_k$ *of characters (ordinary or projective) is produced from the inertia subgroup* $\overline{H}_k$.

Note 3.4. *Observe that if* $\alpha_k\sim[1]$ *in* Equation (3.2)*, then we get* Equation (3.1). *That is* $\mathrm{IrrProj}(\overline{H}_k,\overline{1})=\mathrm{Irr}(\overline{H}_k)$ *and* $\mathrm{IrrProj}(H_k,1)=\mathrm{Irr}(H_k)$.

By convention we take $\theta_1=\mathbf{1}_N$, the trivial character of N. Thus $\overline{H}_{\theta_1}=\overline{H}_1=\overline{G}$ and thus $\overline{H}_1/N\cong G$. Since $\{\mathbf{1}_{\overline{G}}\}\subseteq \mathrm{Irr}(\overline{G},\mathbf{1}_N)$ and such that $\mathbf{1}_{\overline{G}}\downarrow_N^{\overline{G}}=\mathbf{1}_N$, the block $\mathcal{K}_1$ will consists only of the ordinary irreducible characters of G.

We now fix some notations for the conjugacy classes.

- With π being the natural epimorphism from $\overline{G}$ onto G, we use the notation $U = \pi(\overline{U})$ for any subset $\overline{U} \subseteq \overline{G}$. We have seen from Section 2 that $\pi^{-1}([g_i]_G) = \bigcup_{j=1}^{c(g_i)} [g_{ij}]_{\overline{G}}$ for any $1 \leq i \leq r$. Let us assume that $\pi(g_{ij}) = g_i$ and by convention we may take $g_{11} = 1_{\overline{G}}$. Note that $c(g_1)$ is the number of $\overline{G}-$conjugacy classes obtained from N.
- $[g_{ij}]_{\overline{G}} \bigcap \overline{H}_k = \bigcup_{n=1}^{c(g_{ijk})} [g_{ijkn}]_{\overline{H}_k}$, where $g_{ijkn} \in \overline{H}_k$ and by $c(g_{ijk})$ we mean the number of $\overline{H}_k-$conjugacy classes that form a partition for $[g_{ij}]_{\overline{G}}$. Since $g_{11} = 1_{\overline{G}}$, we have $g_{11k1} = 1_{\overline{G}}$ and thus $c(g_{11k1}) = 1$ for all $1 \leq k \leq t$.
- $[g_i]_G \bigcap H_k = \bigcup_{m=1}^{c(g_{ik})} [g_{ikm}]_{H_k}$, where $g_{ikm} \in H_k$ and by $c(g_{ik})$ we mean the number of H_k-conjugacy classes that form a partition for $[g_i]_G$. Since $g_1 = 1_G$, we have $g_{1k1} = 1_G$ and thus $c(g_{1k1}) = 1$ for all $1 \leq k \leq t$. Also $\pi(g_{ijkn}) = g_{ikm}$ for some $m = f(j,n)$.

Proposition 3.5. *With the notations of* Theorem 3.2 *and the above settings, we have*

$$(\widetilde{\psi}_k \inf(\zeta))\uparrow_{\overline{H}_k}^{\overline{G}}(g_{ij}) = \sum_{m=1}^{c(g_{ik})} \zeta(g_{ikm}) \sum_{n=1}^{c(g_{ijk})} \frac{|C_{\overline{G}}(g_{ij})|}{|C_{\overline{H}_k}(g_{ijkn})|} \widetilde{\psi}_k(g_{ijkn}).$$

Proof. See Ali [1] or Barraclough [3]. □

We proceed to define the Fischer matrix $\mathcal{F}_i$ corresponds to the conjugacy class $[g_i]_G$. We label the columns of $\mathcal{F}_i$ by the representatives of $[g_{ij}]_{\overline{G}}$, $1 \leq j \leq c(g_i)$ obtained by the coset analysis and below each g_{ij} we put $|C_{\overline{G}}(g_{ij})|$. Thus there are $c(g_i)$ columns. To label the rows of $\mathcal{F}_i$ we define the set $\overline{J}_i$ to be (this equivalent to the notation $R(g)$ used by Ali [1] (page 49), where g is a representative for a conjugacy class of G)

$$\overline{J}_i = \{(k, g_{ikm})|\ 1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ g_{ikm} \text{ is } \alpha_k^{-1} - \text{regular class}\},$$

or for more brevity we let

$$\begin{aligned} J_i \;=\; & \{(k,m)|\ 1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}), \\ & g_{ikm} \text{ is } \alpha_k^{-1} - \text{regular class}\}. \end{aligned} \tag{3.3}$$

Then each row of $\mathcal{F}_i$ is indexed by a pair $(k, g_{ikm}) \in \overline{J}_i$ or $(k, m) \in J_i$. For fixed $1 \leq k \leq t$, we let $\mathcal{F}_{ik}$ be a sub-matrix of $\mathcal{F}_i$ with rows correspond to the pairs $(k, g_{ik1}), (k, g_{ik2}), \cdots, (k, g_{ikr_{ik}})$ or for brevity $(k,1), (k,2), \cdots, (k, r_k)$. Now let

$$a_{ij}^{(k,m)} := \sum_{n=1}^{c(g_{ijk})} \frac{|C_{\overline{G}}(g_{ij})|}{|C_{\overline{H}_k}(g_{ijkn})|} \widetilde{\psi}_k(g_{ijkn}) \tag{3.4}$$

(for which $\pi(g_{ijkn}) = g_{ikm}$). For each i, corresponding to the conjugacy class $[g_i]_G$, we define the Fischer matrix $\mathcal{F}_i = \left(a_{ij}^{(k,m)}\right)$, where $1 \leq k \leq t,\ 1 \leq m \leq c(g_{ik}),\ 1 \leq j \leq c(g_i)$. The Fischer matrix $\mathcal{F}_i$,

$$\mathcal{F}_i = \left(a_{ij}^{(k,m)}\right) = \left(\begin{array}{c} \mathcal{F}_{i1} \\ \hline \mathcal{F}_{i2} \\ \hline \vdots \\ \hline \mathcal{F}_{it} \end{array}\right)$$

together with additional information required for their definition are presented as follows:

$\mathcal{F}_i$

g_i		g_{i1}	g_{i2}	$\cdots$	$g_{ic(g_i)}$
$\lvert C_{\overline{G}}(g_{ij})\rvert$		$\lvert C_{\overline{G}}(g_{i1})\rvert$	$\lvert C_{\overline{G}}(g_{i2})\rvert$	$\cdots$	$\lvert C_{\overline{G}}(g_{ic(g_i)})\rvert$
(k,m)	$\lvert C_{H_k}(g_{ikm})\rvert$				
$(1,1)$	$\lvert C_G(g_i)\rvert$	$a_{i1}^{(1,1)}$	$a_{i2}^{(1,1)}$	$\cdots$	$a_{ic(g_i)}^{(1,1)}$
$(2,1)$	$\lvert C_{H_2}(g_{i21})\rvert$	$a_{i1}^{(2,1)}$	$a_{i2}^{(2,1)}$	$\cdots$	$a_{ic(g_i)}^{(2,1)}$
$(2,2)$	$\lvert C_{H_2}(g_{i22})\rvert$	$a_{i1}^{(2,2)}$	$a_{i2}^{(2,2)}$	$\cdots$	$a_{ic(g_i)}^{(2,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$(2,r_2)$	$\lvert C_{H_2}(g_{i2r_{i2}})\rvert$	$a_{i1}^{(2,r_2)}$	$a_{i2}^{(2,r_2)}$	$\cdots$	$a_{ic(g_i)}^{(2,r_2)}$
$(u,1)$	$\lvert C_{H_u}(g_{iu1})\rvert$	$a_{i1}^{(u,1)}$	$a_{i2}^{(u,1)}$	$\cdots$	$a_{ic(g_i)}^{(u,1)}$
$(u,2)$	$\lvert C_{H_u}(g_{iu2})\rvert$	$a_{i1}^{(u,2)}$	$a_{i2}^{(u,2)}$	$\cdots$	$a_{ic(g_i)}^{(u,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(u,r_u)	$\lvert C_{H_u}(g_{iur_{iu}})\rvert$	$a_{i1}^{(u,r_u)}$	$a_{i2}^{(u,r_u)}$	$\cdots$	$a_{ic(g_i)}^{(u,r_u)}$
$(t,1)$	$\lvert C_{H_t}(g_{it1})\rvert$	$a_{i1}^{(t,1)}$	$a_{i2}^{(t,1)}$	$\cdots$	$a_{ic(g_i)}^{(t,1)}$
$(t,2)$	$\lvert C_{H_t}(g_{it2})\rvert$	$a_{i1}^{(t,2)}$	$a_{i2}^{(t,2)}$	$\cdots$	$a_{ic(g_i)}^{(t,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(t,r_t)	$\lvert C_{H_t}(g_{itr_{it}})\rvert$	$a_{i1}^{(t,r_t)}$	$a_{i2}^{(t,r_t)}$	$\cdots$	$a_{ic(g_i)}^{(t,r_t)}$
m_{ij}		m_{i1}	m_{i2}	$\cdots$	$m_{ic(g_i)}$

In the above the last entries give the weights m_{ij} as defined by Equation (2.1). These weights are required for computing the entries of $\mathcal{F}_i$ (see Proposition 3.6).

Fischer matrices satisfy some interesting properties, which help in computations of their entries. We gather these properties in the following Proposition.

Proposition 3.6. (i) $\sum_{k=1}^{t} c(g_{ik}) = c(g_i)$,

(ii) *$\mathcal{F}_i$ is non-singular for each i,*

(iii) $a_{ij}^{(1,1)} = 1, \ \forall\ 1 \leq j \leq c(g_i)$,

(iv) *If $N\overline{g}_i$ is a split coset, then* $a_{i1}^{(k,m)} = \frac{|C_G(g_i)|}{|C_{H_k}(g_{ikm})|}$, $\forall i \in \{1, 2, \cdots, r\}$. *In particular for the identity coset we have* $a_{11}^{(k,m)} = [G : H_k]\theta_k(1_N)$, $\forall\ (k,m) \in J_1$,

(v) *If $N\overline{g}_i$ is a split coset, then $|a_{ij}^{(k,m)}| \leq |a_{i1}^{(k,m)}|$ for all $1 \leq j \leq c(g_i)$. Moreover if $|N| = p^{\alpha}$, for some prime p, then $a_{ij}^{(k,m)} \equiv a_{i1}^{(k,m)} (\mathrm{mod}\ p)$,*

(vi) *For each $1 \leq i \leq r$, the weights m_{ij} satisfy the relation* $\sum_{j=1}^{c(g_i)} m_{ij} = |N|$,

(vii) ***Column Orthogonality Relation:***

$$\sum_{(k,m) \in J_i} |C_{H_k}(g_{ikm})| a_{ij}^{(k,m)} \overline{a_{ij'}^{(k,m)}} = \delta_{jj'} |C_{\overline{G}}(g_{ij})|,$$

(viii) ***Row Orthogonality Relation:***

$$\sum_{j=1}^{c(g_i)} m_{ij} a_{ij}^{(k,m)} \overline{a_{ij}^{(k',m')}} = \delta_{(k,m)(k',m')} a_{i1}^{(k,m)} |N|.$$

Proof. Proofs for many assertions of Proposition 3.6 can be founded in Moori's students theses, for example see Ali [1] or Mpono [21] and some other assertions are provided in Schiffer [25]. □

3.1. **Character Table of $\overline{G}$.** For fixed $1 \leq k \leq t$ and $1 \leq i \leq r$, let $\mathcal{K}_{ik}$ be the fragment of the projective character table of H_k, with factor set α_k^{-1}, consisting of columns correspond to the conjugacy classes $g_{ik1}, g_{ik2}, \cdots, g_{ikr_{ik}}$ of H_k (those are the α_k^{-1}−regular classes of H_k that fuse to $[g_i]_G$ and thus $r_{ik} = c(g_{ik})$). Then the characters of $\overline{G}$ on the classes $[g_{ij}]_{\overline{G}}$, $1 \leq j \leq c(g_i)$, is given by the matrix $\mathcal{K}_{ik}\mathcal{F}_{ik}$, where $\mathcal{F}_{ik}$ is the sub-matrix of $\mathcal{F}_i$ defined previously with rows correspond to the pairs $(k, g_{ik1}), (k, g_{ik2}), \cdots, (k, g_{ikr_{ik}})$. Note that the size of $\mathcal{K}_{ik}$ is $|\text{IrrProj}(H_k, \alpha_k^{-1})| \times r_{ik}$ and the size of $\mathcal{F}_{ik}$ is $r_{ik} \times c(g_i)$. Therefore the character table of $\overline{G}$ will have the form

	g_1			g_2			$\cdots$	g_r		
	g_{11}	$\cdots$	$g_{1c(g_1)}$	g_{21}	$\cdots$	$g_{2c(g_2)}$	$\cdots$	g_{r1}	$\cdots$	$g_{rc(g_r)}$
$\mathcal{K}_1$	$\mathcal{K}_{11}\mathcal{F}_{11}$			$\mathcal{K}_{12}\mathcal{F}_{12}$			$\cdots$	$\mathcal{K}_{1r}\mathcal{F}_{1r}$		
$\mathcal{K}_2$	$\mathcal{K}_{21}\mathcal{F}_{21}$			$\mathcal{K}_{22}\mathcal{F}_{22}$			$\cdots$	$\mathcal{K}_{2r}\mathcal{F}_{2r}$		
$\vdots$	$\vdots$			$\vdots$			$\ddots$	$\vdots$		
$\mathcal{K}_t$	$\mathcal{K}_{t1}\mathcal{F}_{t1}$			$\mathcal{K}_{t2}\mathcal{F}_{t2}$			$\cdots$	$\mathcal{K}_{tr}\mathcal{F}_{tr}$		

Note 3.7. *Observe that characters of $\overline{G}$ consisted in $\mathcal{K}_1$ are just* $\text{Irr}(G)$ *and therefore the size of $\mathcal{K}_{1i}\mathcal{F}_{1i}$, for each $1 \le i \le r$, is* $|\text{Irr}(G)| \times c(g_i)$. *In particular, columns of $\mathcal{K}_{11}\mathcal{F}_{11}$ are the degrees of irreducible characters of G repeated themselves $c(g_1)$ times, where we know that $c(g_1)$ is number of $\overline{G}-$conjugacy classes obtained from the normal subgroup N.*

4. Inertia factor groups of $\overline{G} = 2^{6\cdot}Sp(6,2)$

We have seen in Section 2 that the action of $\overline{G}$ on N yielded two orbits of lengths 1 and 63, where the first orbit consists of the identity element 1_N while the other orbit consists of all the involutions of N. By Brauer Theorem (for example see Lemma 4.5.2 of Gorenstein [15] or Theorem 5.1.5 of Mpono [21]), it follows that the action of $\overline{G}$ on $\text{Irr}(N)$ will also produce two orbits. These two orbits must necessarily have lengths 1 and 63 and the first orbit consists of the identity character $\mathbf{1}_N$ while the other orbit consists of the other non-trivial linear characters of N. Thus the corresponding inertia factor groups H_1 and H_2 have indices 1 and 63 respectively in $Sp(6,2)$. By looking at the maximal subgroups of $Sp(6,2)$ (see $\mathbb{ATLAS}$), it is readily verified that $H_1 = Sp(6,2)$ and $H_2 = 2^5{:}S_6$.

The character table of $H_1 = Sp(6,2)$ is available in the $\mathbb{ATLAS}$, the electronic ATLAS of Wilson or can be obtained from Magma or GAP. The fusion of the conjugacy classes of $H_2 = 2^5{:}S_6$ into classes of $Sp(6,2)$ and the character table of H_2 can be found in many sources such as Ali [1], Magma, GAP or Mpono [21]. For the convenience of the reader, we supply the fusion of H_2 into G in Table 2. Also the character table of H_2 appears as Table 11.11 of Basheer [4].

Note that from Table 1, the number of the conjugacy classes of $\overline{G}$ is 67. Thus $|\text{Irr}(\overline{G})| = 67$. Since $|\text{Irr}(H_1)| = |\text{Irr}(Sp(6,2))| = 30$, the other

TABLE 2. The fusion of classes of $H_2 = 2^5{:}S_6$ into classes of $Sp(6,2)$

Inertia Factor Group H_2	Class of H_2 $\hookrightarrow$	Class of $Sp(6,2)$	Class of H_2 $\hookrightarrow$	Class of $Sp(6,2)$
$H_2 = 2^5{:}S_6$	$1a = g_{121}$	$1A$	$4g = g_{13,22}$	$4E$
	$2a = g_{221}$	$2A$	$4h = g_{11,22}$	$4C$
	$2b = g_{421}$	$2C$	$4i = g_{10,22}$	$4B$
	$2c = g_{321}$	$2B$	$4j = g_{13,23}$	$4E$
	$2d = g_{222}$	$2A$	$5a = g_{14,21}$	$5A$
	$2e = g_{422}$	$2C$	$6a = g_{16,21}$	$6B$
	$2f = g_{521}$	$2D$	$6b = g_{18,21}$	$6D$
	$2g = g_{522}$	$2D$	$6c = g_{15,21}$	$6A$
	$2h = g_{423}$	$2C$	$6d = g_{15,22}$	$6A$
	$2i = g_{523}$	$2D$	$6e = g_{18,22}$	$6D$
	$2j = g_{322}$	$2B$	$6f = g_{19,21}$	$6E$
	$3a = g_{621}$	$3A$	$6g = g_{21,21}$	$6G$
	$3b = g_{821}$	$3C$	$6h = g_{20,21}$	$6F$
	$4a = g_{11,21}$	$4C$	$8a = g_{23,21}$	$8A$
	$4b = g_{10,21}$	$4B$	$8b = g_{24,21}$	$8B$
	$4c = g_{12,21}$	$4D$	$10a = g_{26,21}$	$10A$
	$4d = g_{921}$	$4A$	$12a = g_{27,21}$	$12A$
	$4e = g_{13,21}$	$4E$	$12b = g_{28,21}$	$12B$
	$4f = g_{12,22}$	$4D$		

inertia factor group $H_2 = 2^5{:}S_6$ must contribute with 37 characters in order to construct the ordinary character table of $\overline{G}$. Although we know that $|\mathrm{Irr}(H_2)| = |\mathrm{Irr}(2^5{:}S_6)| = 37$, which suggests to consider the set $\mathrm{Irr}(2^5{:}S_6)$, there is no reason, at this stage, allowing us to use the set $\mathrm{Irr}(2^5{:}S_6)$ as the group $\overline{G}$ does not split and consequently we do not know whether we will use the ordinary or a projective character table of H_2. To determine the type of the character table (ordinary or a projective) we have to calculate the Schur multiplier of H_2, consider all the projective character tables and we may need to test all possible choices, and one is successful if all but one lead to a contradiction. The following sequence of Magma commands reveals the Schur multiplier of H_2 and also the ordinary character table of the full covering group of H_2.

```
> H2:= PermutationGroup< 62 | (1,2)(3,5)(4,7)(6,10)(12,16)(13,18)(14,20)(15,21)
                            (19,23)(24,28)(29,31)(30,32),(1,3,6,11)(2,4,8,13,
                             19,10,15,22,18,25,29,32)(5,9,14,21,16,23,27,20,
                             26,30,31,28)(7,12,17,24)>;
> Order(H2);
23040
> pMultiplicator(H2,2);
[ 2, 2 ]
> pMultiplicator(H2,3);
[ 1 ]
> pMultiplicator(H2,5);
[ 1 ]
> F := FPGroup(H2);
```

```
> F2 := pCover(H2, F, 2);
> Order(F2);
92160
> p, K:= CosetAction(F2, sub<F2|>);
> s:= SylowSubgroup(K, 3);
> p2, K1:= CosetAction(K, s);
> Order(K1);
92160
> ct:= CharacterTable(K1);
> ct;
```

From the above we can see that the Schur multiplier $M(H_2)$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_1 \times \mathbb{Z}_1 \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \cong 2^2$. Also the *covering group* $M(H_2){\cdot}H_2$[1] (central extension of $M(H_2)$ by H_2) is isomorphic to $2^2{\cdot}(2^5{:}S_6)$. The command "ct:= CharacterTable(K1);" will display the character table of $2^2{\cdot}(2^5{:}S_6)$, where we get 70 irreducible characters for this group, in which 37 of these are the ordinary characters of H_2 (see Table 11.11 of Basheer [4]) for the ordinary characters of H_2). Thus we deduce that $|\mathrm{IrrProj}(H_2, \alpha^{-1})| \leq 33$, for any non-trivial factor set α of $M(H_2) = 2^2$. Therefore if we use a projective character table of H_2, with non-trivial factor set α, to construct the character table of $\overline{G}$, we obtain

$$|\mathrm{Irr}(H_1)| + |\mathrm{IrrProj}(H_2, \alpha^{-1})| \leq 30 + 33 = 63 < 67 = |\mathrm{Irr}(\overline{G})|.$$

This shows that we should use the set $\mathrm{Irr}(2^5{:}S_6)$ to construct the ordinary character table of $\overline{G}$. This leads to the following corollary.

Corollary 4.1. *The second inertia group $\overline{H}_2$ has a character ψ of degree 63 such that $\psi\downarrow_N^{\overline{H}_2} = \sum_{i=2}^{64} \theta_i$, where $\theta_2, \theta_3, \cdots, \theta_{64}$ are the non-trivial linear characters of N.*

5. Fischer matrices of $\overline{G} = 2^{6\cdot} Sp(6, 2)$

We recall that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of g_{ij}, $1 \leq j \leq c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 1 we supplied $|C_{\overline{G}}(g_{ij})|$ and m_{ij}, $1 \leq i \leq 30$, $1 \leq j \leq c(g_i)$. Also having obtained the fusions of the inertia factor group H_2 into $Sp(6, 2)$, we are able to label the rows of the Fischer matrices as described in Section 3. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 1 that the sizes of the Fischer matrices of $\overline{G} = 2^{6\cdot} Sp(6, 2)$ range between 1 and

[1]some authors refer to this group as the *representation group*.

4 for every $i \in \{1, 2, \cdots, 30\}$. We have used the arithmetical properties of Fischer matrices, given in Proposition 3.6, to calculate some of the entries of the Fischer matrices and also to build an algebraic system of equations. With the help of the symbolic mathematical package Maxima [16], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}
$o(g_{1j})$		1	2
$\lvert C_{\overline{G}}(g_{1j})\rvert$		92897280	1474560
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$		
$(1,1)$	1451520	1	1
$(2,1)$	23040	63	-1
m_{1j}		1	63

$\mathcal{F}_2$

g_2		g_{21}	g_{22}	g_{23}
$o(g_{2j})$		4	4	2
$\lvert C_{\overline{G}}(g_{2j})\rvert$		737280	49152	46080
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$			
$(1,1)$	23040	1	1	1
$(2,1)$	23040	1	1	-1
$(2,2)$	768	30	-2	0
m_{2j}		2	30	32

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}
$o(g_{3j})$		2	2	4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		73728	24576	6144
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$			
$(1,1)$	4608	1	1	1
$(2,1)$	1536	3	3	-1
$(2,2)$	384	12	-4	0
m_{3j}		4	12	48

$\mathcal{F}_4$

g_4		g_{41}	g_{42}	g_{43}	g_{44}
$o(g_{4j})$		4	4	2	4
$\lvert C_{\overline{G}}(g_{4j})\rvert$		24576	8192	6144	3072
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$				
$(1,1)$	1536	1	1	1	1
$(2,1)$	1536	1	1	1	-1
$(2,2)$	768	2	2	-2	0
$(2,3)$	128	12	-4	0	0
m_{4j}		4	12	16	32

$\mathcal{F}_5$

g_5		g_{51}	g_{52}	g_{53}	g_{54}
$o(g_{5j})$		4	2	4	4
$\lvert C_{\overline{G}}(g_{5j})\rvert$		3072	3072	1024	1024
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$				
$(1,1)$	384	1	1	1	1
$(2,1)$	128	3	3	-1	-1
$(2,2)$	128	3	-3	-1	1
$(2,3)$	384	1	-1	1	-1
m_{5j}		8	8	24	24

$\mathcal{F}_6$

g_6		g_{61}	g_{62}
$o(g_{6j})$		3	6
$\lvert C_{\overline{G}}(g_{6j})\rvert$		34560	2304
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$		
$(1,1)$	2160	1	1
$(2,1)$	144	15	-1
m_{6j}		4	60

$\mathcal{F}_7$

g_7		g_{71}
$o(g_{7j})$		3
$\lvert C_{\overline{G}}(g_{7j})\rvert$		648
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$	
$(1,1)$	648	1
m_{7j}		64

$\mathcal{F}_8$

g_8		g_{81}	g_{82}
$o(g_{8j})$		3	6
$\lvert C_{\overline{G}}(g_{8j})\rvert$		432	144
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$		
$(1,1)$	108	1	1
$(2,1)$	36	3	-1
m_{8j}		16	48

$\mathcal{F}_9$

g_9		g_{91}	g_{92}
$o(g_{9j})$		4	4
$\lvert C_{\overline{G}}(g_{9j})\rvert$		1536	512
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$		
$(1,1)$	384	1	1
$(2,1)$	128	3	-1
m_{9j}		16	48

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$
$o(g_{10j})$		8	8	4
$\lvert C_{\overline{G}}(g_{10j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{10j}		8	24	32

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$	$g_{11,3}$
$o(g_{11j})$		8	8	4
$\lvert C_{\overline{G}}(g_{11j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{11j}		8	24	32

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$	$g_{12,3}$
$o(g_{12j})$		4	4	4
$\lvert C_{\overline{G}}(g_{12j})\rvert$		512	512	256
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$			
$(1,1)$	128	1	1	1
$(2,1)$	128	1	1	-1
$(2,2)$	64	2	-2	0
m_{12j}		16	16	32

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$	$g_{13,3}$	$g_{13,4}$
$o(g_{13j})$		8	4	8	4
$\lvert C_{\overline{G}}(g_{13j})\rvert$		128	128	128	128
(k,m)	$\lvert C_{H_k}(g_{13km})\rvert$				
$(1,1)$	32	1	1	1	1
$(2,1)$	32	1	1	-1	-1
$(2,2)$	32	1	-1	-1	1
$(2,3)$	32	1	-1	1	-1
m_{13j}		16	16	16	16

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$	$g_{14,2}$
$o(g_{14j})$		10	5
$\lvert C_{\overline{G}}(g_{14j})\rvert$		40	120
(k,m)	$\lvert C_{H_k}(g_{14km})\rvert$		
$(1,1)$	30	1	1
$(2,1)$	10	3	-1
m_{14j}		16	48

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$	$g_{15,3}$
$o(g_{15j})$		12	12	6
$\lvert C_{\overline{G}}(g_{15j})\rvert$		1152	384	288
(k,m)	$\lvert C_{H_k}(g_{15km})\rvert$			
$(1,1)$	144	1	1	1
$(2,1)$	144	1	1	-1
$(2,2)$	24	6	-2	0
m_{15j}		8	24	32

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$
$o(g_{16j})$		6	12
$\lvert C_{\overline{G}}(g_{16j})\rvert$		576	192
(k,m)	$\lvert C_{H_k}(g_{16km})\rvert$		
$(1,1)$	144	1	1
$(2,1)$	48	3	-1
m_{16j}		16	48

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$
$o(g_{17j})$		6
$\lvert C_{\overline{G}}(g_{17j})\rvert$		72
(k,m)	$\lvert C_{H_k}(g_{17km})\rvert$	
$(1,1)$	72	1
m_{17j}		64

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$	$g_{18,2}$	$g_{18,3}$
$o(g_{18j})$		12	6	12
$\lvert C_{\overline{G}}(g_{18j})\rvert$		192	192	96
(k,m)	$\lvert C_{H_k}(g_{18km})\rvert$			
$(1,1)$	48	1	1	1
$(2,1)$	48	1	1	-1
$(2,2)$	24	2	-2	0
m_{18j}		16	16	32

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$
$o(g_{19j})$		12	6
$\lvert C_{\overline{G}}(g_{19j})\rvert$		72	72
(k,m)	$\lvert C_{H_k}(g_{19km})\rvert$		
$(1,1)$	36	1	1
$(2,1)$	36	1	-1
m_{19j}		32	32

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$	$g_{20,2}$
$o(g_{20j})$		6	6
$\lvert C_{\overline{G}}(g_{20j})\rvert$		144	48
(k,m)	$\lvert C_{H_k}(g_{20km})\rvert$		
$(1,1)$	36	1	1
$(2,1)$	12	3	-1
m_{20j}		16	48

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$	$g_{21,2}$
$o(g_{21j})$		12	6
$\lvert C_{\overline{G}}(g_{21j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{21km})\rvert$		
$(1,1)$	12	1	1
$(2,1)$	12	1	-1
m_{21j}		32	32

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$
$o(g_{22j})$		7
$\lvert C_{\overline{G}}(g_{22j})\rvert$		7
(k,m)	$\lvert C_{H_k}(g_{22km})\rvert$	
$(1,1)$	7	1
m_{22j}		64

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$	$g_{23,2}$
$o(g_{23j})$		8	8
$\lvert C_{\overline{G}}(g_{23j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{23km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{23j}		32	32

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$	$g_{24,2}$
$o(g_{24j})$		8	8
$\lvert C_{\overline{G}}(g_{24j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{24j}		32	32

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$
$o(g_{25j})$		9
$\lvert C_{\overline{G}}(g_{25j})\rvert$		9
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$	
$(1,1)$	9	1
m_{25j}		64

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$	$g_{26,2}$
$o(g_{26j})$		20	10
$\lvert C_{\overline{G}}(g_{26j})\rvert$		20	20
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$		
$(1,1)$	10	1	1
$(2,1)$	10	1	-1
m_{26j}		32	32

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$	$g_{27,2}$
$o(g_{27j})$		24	12
$\lvert C_{\overline{G}}(g_{27j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{27j}		32	32

$\mathcal{F}_{28}$

g_{28}		$g_{28,1}$	$g_{28,2}$
$o(g_{28j})$		24	12
$\lvert C_{\overline{G}}(g_{28j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{28km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{28j}		32	32

$\mathcal{F}_{29}$

g_{29}		$g_{29,1}$
$o(g_{29j})$		12
$\lvert C_{\overline{G}}(g_{29j})\rvert$		12
(k,m)	$\lvert C_{H_k}(g_{29km})\rvert$	
$(1,1)$	12	1
m_{29j}		64

$\mathcal{F}_{30}$

g_{30}		$g_{30,1}$
$o(g_{30j})$		15
$\lvert C_{\overline{G}}(g_{30j})\rvert$		15
(k,m)	$\lvert C_{H_k}(g_{30km})\rvert$	
$(1,1)$	15	1
m_{30j}		64

Remark 5.1. *We note that the Fischer matrix $\mathcal{F}_i$, $1 \leq i \leq 30$, corresponds to class $[g_i]_{Sp(6,2)}$, of $\overline{G} = 2^{6\cdot}Sp(6,2)$ coincides with Fischer matrix corresponds to class $[g_i]_{Sp(6,2)}$ of $2^6{:}Sp(6,2)$ (see Mpono [21]), except for the classes $6A$ and $6B$ of $Sp(6,2)$, where the Fischer matrices correspond to these two classes are interchanged.*

6. Character table of $\overline{G} = 2^{6\cdot}Sp(6,2)$

From Sections 2, 4 and 5 we have

- the conjugacy classes of $\overline{G} = 2^{6\cdot}Sp(6,2)$ (Table 1),
- the fusion of H_2 into $Sp(6,2)$ (Table 2),
- the character table of H_2 (Table 11.11 of Basheer [4]),
- the Fischer matrices of $\overline{G}$ (see Section 5).

By Section 3, it follows that the full character table of $\overline{G}$ can be constructed easily. We give an example on how to construct the character table of $\overline{G}$, which is partitioned into 60 parts corresponding to the 30 cosets and the two inertia factor groups. As an example we construct the parts $\mathcal{K}_{21}\mathcal{F}_{21}$ and $\mathcal{K}_{22}\mathcal{F}_{22}$ of the character table of $\overline{G}$ (this means that we are listing the values of all the irreducible characters of $\overline{G}$ on the classes g_{21}, g_{22}, and g_{23} of $\overline{G}$, which correspond to the conjugacy class $2A$ of $Sp(6,2)$). The two parts $\mathcal{K}_{21}\mathcal{F}_{21}$ and $\mathcal{K}_{22}\mathcal{F}_{22}$ can be derived as follows: From Table 2 we can see that there are two classes, namely $2a = g_{221}$ and $2d = g_{222}$ of H_2 that fuse into the class $g_2 = [2A]_{Sp(6,2)}$. To construct the part $\mathcal{K}_{21}\mathcal{F}_{21}$, we multiply the column of the character table of $H_1 = Sp(6,2)$ corresponds to the class $2A$ of $Sp(6,2)$ (see the $\mathbb{ATLAS}$), by the first row of $\mathcal{F}_2$, namely $(1 \quad 1 \quad 1)$ and thus the part $\mathcal{K}_{21}\mathcal{F}_{21}$ of size 30×3, consists of the column of the character table of $Sp(6,2)$ corresponds to the class $2A$ repeated 3 times. To construct the part $\mathcal{K}_{22}\mathcal{F}_{22}$, select the two columns of the character table of $H_2 = 2^5{:}S_6$, correspond to the classes $2a$ and $2d$ of H_2 (see Table 11.11 of Basheer [4]) and multiply these two columns by the two rows of $\mathcal{F}_2$ correspond to the pair $(2,1)$ and $(2,2)$. Thus we get a part in the character table of $\overline{G}$ of size 37×3. The above two parts have the following form:

$$\mathcal{K}_{21}\mathcal{F}_{21} = \begin{pmatrix} 1 \\ -5 \\ -5 \\ 9 \\ -11 \\ 15 \\ -5 \\ 15 \\ -24 \\ -10 \\ 4 \\ -35 \\ 25 \\ 5 \\ 40 \\ 40 \\ 21 \\ -51 \\ -39 \\ 50 \\ 10 \\ -24 \\ -40 \\ 40 \\ -45 \\ -16 \\ -30 \\ 45 \\ 20 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{array}{c} \begin{matrix} g_{21} & g_{22} & g_{23} \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 \\ -5 & -5 & -5 \\ -5 & -5 & -5 \\ 9 & 9 & 9 \\ -11 & -11 & -11 \\ 15 & 15 & 15 \\ -5 & -5 & -5 \\ 15 & 15 & 15 \\ -24 & -24 & -24 \\ -10 & -10 & -10 \\ 4 & 4 & 4 \\ -35 & -35 & -35 \\ 25 & 25 & 25 \\ 5 & 5 & 5 \\ 40 & 40 & 40 \\ 40 & 40 & 40 \\ 21 & 21 & 21 \\ -51 & -51 & -51 \\ -39 & -39 & -39 \\ 50 & 50 & 50 \\ 10 & 10 & 10 \\ -24 & -24 & -24 \\ -40 & -40 & -40 \\ 40 & 40 & 40 \\ -45 & -45 & -45 \\ -16 & -16 & -16 \\ -30 & -30 & -30 \\ 45 & 45 & 45 \\ 20 & 20 & 20 \\ 0 & 0 & 0 \end{pmatrix} \end{array},$$

$$
\mathcal{K}_{22}\mathcal{F}_{22} = \begin{pmatrix}
1 & 1 \\
1 & -1 \\
5 & -3 \\
5 & 3 \\
5 & -1 \\
5 & 1 \\
-6 & -4 \\
-6 & 4 \\
9 & -3 \\
9 & 3 \\
-10 & -4 \\
-10 & -4 \\
10 & -2 \\
-10 & 4 \\
-10 & 4 \\
10 & 2 \\
15 & 7 \\
15 & -5 \\
15 & -7 \\
15 & 5 \\
16 & 0 \\
-20 & 0 \\
-24 & 8 \\
-24 & -8 \\
-30 & -4 \\
30 & -2 \\
30 & 2 \\
-30 & 4 \\
-36 & 0 \\
-40 & 8 \\
-40 & -8 \\
-40 & 0 \\
-40 & 0 \\
45 & -9 \\
45 & 9 \\
45 & 3 \\
45 & -3
\end{pmatrix}
\begin{pmatrix}
1 & 1 & -1 \\
30 & -2 & 0
\end{pmatrix}
= \begin{pmatrix}
g_{21} & g_{22} & g_{23} \\
31 & -1 & -1 \\
-29 & 3 & -1 \\
-85 & 11 & -5 \\
95 & -1 & -5 \\
-25 & 7 & -5 \\
35 & 3 & -5 \\
-126 & 2 & 6 \\
114 & -14 & 6 \\
-81 & 15 & -9 \\
99 & 3 & -9 \\
-130 & -2 & 10 \\
-130 & -2 & 10 \\
-50 & 14 & -10 \\
110 & -18 & 10 \\
110 & -18 & 10 \\
70 & 6 & -10 \\
225 & 1 & -15 \\
-135 & 25 & -15 \\
-195 & 29 & -15 \\
165 & 5 & -15 \\
16 & 16 & -16 \\
-20 & -20 & 20 \\
216 & -40 & 24 \\
-264 & -8 & 24 \\
-150 & -22 & 30 \\
-30 & 34 & -30 \\
90 & -38 & 30 \\
90 & 26 & -30 \\
-36 & -36 & 36 \\
200 & -56 & 40 \\
-280 & -24 & 40 \\
-40 & -40 & 40 \\
-40 & -40 & 40 \\
-225 & 63 & -45 \\
315 & 27 & -45 \\
135 & 39 & -45 \\
-45 & 51 & -45
\end{pmatrix}.
$$

Similarly one can obtain all the other 58 parts $\mathcal{K}_{ik}\mathcal{F}_{ik}$, $1 \leq i \leq 30,\ i \neq 2,\ 1 \leq k \leq 2$ and hence the full character table of $\overline{G}$, which is a 67×67 $\mathbb{Z}-$valued matrix. The full character table of $\overline{G}$ appears in Basheer [4] as Table 11.12. By referring to Table 6.15 of Mpono [21], one can see that the entries of the character table of $2^6{:}Sp(6,2)$ coincide with the entries of the character table of $2^{6\cdot}Sp(6,2)$ constructed in this paper.

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