Robustness of Weighted Median Filters Based on Estimation Approaches

Rastislav Lukac and Alexander Varga

Abstract—**Weighted median (WM) filters, a nonlinear filter class based on a median operator and a weight vector associated with samples inside the filter window, take their popularity from the robust order-statistic theory, the noise attenuation capability and the degree of the freedom related to filter design. In order to adapt a filter behavior for a variety of statistics describing the desired signal and the noise distribution, there were developed some optimization algorithms based on estimation and structural approaches.**

In this paper, we focus on optimal weighted median filters based on the estimation approach. Besides well-known WM optimization algorithms that utilize linear and sigmoidal approximation of a sign function, we test and analyze a genetic approach that outperforms others optimal WM algorithms especially in terms of the signal-detail preservation.

Index Terms—**Nonlinear image filtering, weighted median filters, estimation optimization approaches, genetic algorithms, impulsive noise.**

I. Introduction

In general, the success of denoising approaches depends on three factors such as the original signal, the nature of corruption and finally, the measure of the solution accuracy [1] as the compromise between the original and the nature of corruption. If the noise corruption is modeled as non-Gaussian or impulsive, in order to suppress the noise effectively, the nonlinear filter classes based on the robust order-statistic theory are preferred [1]-[3]. Many order-statistic filters pass to a filter output the sample from the input set and thus, it minimizes the local distortion of processed images. The measure of the noise suppression depends on the choice of order-statistics as the filter output. Many nonlinear filtering classes [1], [2], [4] such as weighted median filters [5], lower-upper-middle (LUM) smoothers [6], [7], L-filters [3], Ll-filters [3] incorporate the weight coefficients into the filter structure to express the influence of input samples, where relationship between the pixel under consideration and each sample in the filter window should be reflected in the decision for the weight coefficients. In the adaptive design, the weights provide the degree to which the input vector contributes to the filter output. This way can improve [8] the estimation capability, extend the design possibilities and scale the filter behavior for a variety of the signal and noise statistics.

In the case of weighted median filters, there were developed some algorithms for their optimization. These methods outcome from a stack filter framework [9]-[11], where the optimization approaches can be differentiated into two classes such as structural [9], [12] and estimation approaches [13]. The structural approach is based on the filter optimization under structural constraints reflecting position and orientation of small edges and image details. As the estimation approaches [13] are used non-adaptive and adaptive least mean absolute (LMA) or least mean square (LMS) algorithms. This paper focuses on the genetic optimization of weighted median filters. It will be demonstrated, that the genetic optimization improves the signal detail preservation capability of optimal weighted median filters.

The rest of the paper is organized as follows. In the next Section, a class of weighted median filters is described. Section III focuses on standard estimation approaches used in the weighted median filter design. In Section IV, the proposed method taking the advantages of global solution provided by genetic algorithms is presented. Section V is devoted to the analysis of the presented filtering approaches in the dependence on the intensity of impulsive noise corruption. This section also contains a number of simulations, tests and filtering results, together with tables and graphs depicting the objective image quality measures. Finally, the main ideas, achieved results and future plans are summarized in conclusion section.

II. Weighted Median Filters

The principle of weighted median (WM) filtering [1], [3], [4], [14] lies in the median filter generalization, where each input sample spawned by a filter window is associated with a weight coefficient.

Let us consider discrete-time continuous-valued input set $\mathbf{x}(n)=\{x_1(n),x_2(n),...,x_N(n)\}$ associated with the nonnegative integer weight vector $\mathbf{w}=\{w_1,w_2,...,w_N\}$. The output of WM filter [4] is given by

$$y(n)=med\{w_1\Diamond x_1(n),w_2\Diamond x_2(n),...,w_N\Diamond x_N(n)\} \tag{1}$$

where N is the window size, n represents the position of a running filter window, med characterizes the median operator and $\Diamond$ is the duplication operator defined by

$$w_i\Diamond x_i(n)=\overbrace{x_i(n),x_i(n),...,x_i(n)}^{w_i-times}. \tag{2}$$

The filtering procedure defined by (1) requires following steps such as the ordering of samples determined by the filter window, the duplication of ordered samples to the number of the corresponding weight w_i and choice of the median value from the new sequence.

Let w_i, for $i=1,2,...,N$, be positive real weights. The WM filter of the input set $\mathbf{x}(n)=\{x_1(n),x_2(n),...,x_N(n)\}$ is the value $y(n)$ minimizing the expression

Manuscript received May 1, 2002; revised December 4, 2002.

R. Lukac is with the Slovak Image Processing Center, Jarkova 343, 049 25 Dobsina, Slovak Republic (e-mail: lukacr@ ieee.org).

A. Varga is with the Technical University of Kosice, Letna 9, 041 20 Kosice, Slovak Republic.

Publisher Item Identifier S 1682-0053(03)0156

$$L(y(n)) = \sum_{i=1}^{N} w_i \left| x_i(n) - y(n) \right| \tag{3}$$

If $w_i \geq 0$, for $i = 1, 2, ..., N$ and the function $L(y(n))$ is piecewise linear and convex, then $y(n)$ is the sample from the input set [13]. In the case of positive real weights, the computation of WM filter output requires the ordering of the input samples and the successive summing up the weights corresponding with ordered samples until the sum exceeds half of the total sum of weights. The WM filter output is the sample corresponding with the last added weight.

WM filters include the standard median filter and the center-weighted median filters as special cases. If $w_i = 1$, for $i = 1, 2, ..., N$, the definition of WM filters (1) is identical with the definition of the standard median filter given by

$$y = med\{x_1(n), x_2(n), ..., x_N(n)\} \tag{4}$$

where med is the median operator, $x_1(n), x_2(n), ..., x_N(n)$ represent the input set and N is the window size.

If the set of weights is defined by

$$w_i = \begin{cases} w^* & \text{for } i = (N+1)/2 \\ 1 & \text{otherwise} \end{cases} \tag{5}$$

where only the central weight w^* that is forced to be an odd positive integer whereas other weights are equal to 1, then definition of WM filters is identical with the definition of center-weighted median (CWM) filters [15]. Note that the CWM filters are equivalent to the LUM smoothers [7], [16], that can be designed to provide the best balance between the noise suppression and the signal-detail preservation.

III. Standard Estimation Approaches

It is clear that a number of various weight vectors, i.e. various WM filters grows rapidly with the window size N. Using the most popular square window shape of nine samples results in 172 958 integer weight vectors. For that reason, it is very difficult to determine the weight vector so that the WM filter would be robust with respect to the introduced corruption and also preserve the useful information.

Let us consider a running filter window of the size N and the WM filtering operation (i.e., (1) or (3)) resulting in the filter output $y(n)$. The aim of the optimal WM filtering is the searching for the WM filter with the window size N so that the error criteria such as the mean absolute error or the mean square error between the filter output $y(n)$ and desired output $o(n)$ were minimized.

A. Non-Adaptive LMA WM Approach

The non-adaptive WM optimization algorithm utilizes the global signal statistics based on the correlation between signal and noise. In practice, autocorrelation matrixes are estimated as follows [13]:

$$\mathbf{R}_{i,j} = \frac{1}{L}\sum_{n=0}^{L-1}\left(x_{\max}(n) - x_{\min}(n) - 2\left|x_i(n) - x_j(n)\right|\right) \tag{6}$$

$$\mathbf{R}_j^s = \frac{1}{L}\sum_{n=0}^{L-1}\left(x_{\max}(n) - x_{\min}(n) - 2\left|o(n) - x_j(n)\right|\right) \tag{7}$$

where $i = 1, 2, ..., N$, $j = 1, 2, ..., N$, $x_{\max}(n)$ is the maximum and $x_{\min}(n)$ the minimum of the input set $\mathbf{x}(n)$, $o(n)$ is the original (desired) sample in the time position n and L represents the signal length.

The non-adaptive algorithm includes some steps such as the initialization, iteration and convergence. The initialization requires to set the start weight vector $\mathbf{w}(0)$ to any positive values and the iteration constant to a small positive value.

The iteration performs an update of the weight vector according to

$$\mathbf{w}^T(k+1) = P[(\mathbf{I} - \mu\mathbf{R})\mathbf{w}^T(k) + \mu\mathbf{R}^{sT}] \tag{8}$$

for $0 < \mu < 2/(\lambda_1 + \lambda_N)$. Projection operation $P(.)$ remains positive values unchanged, whereas negative values sets to zero. $\mathbf{I}$ is the identity matrix, λ_1 and λ_N are the smallest and the largest eigenvalues of $\mathbf{R}$, μ is the fixed step-size and k describes the iteration index.

When the condition

$$\sum_{i=1}^{N}(w_i(k+1) - w_i(k))^2 \leq \alpha_t \tag{9}$$

is satisfied, then the convergence occurs and the weight vector $\mathbf{w}(k+1)$ is accepted as a solution. If the condition (9) is not satisfied, then the algorithm performs the next iteration.

If original and corrupted signals are jointly stationary, the non-adaptive algorithms provide good results. In general, the use of non-adaptive WM algorithms is disadvantageous for missing global statistics of corrupted and original signals.

B. Adaptive LMA WM with Sigmoidal Approximation

Adaptive LMA WM filtering is preferred when the global statistics of corrupted and original signals are not available. In general, if original and corrupted signal are jointly stationary, the non-adaptive algorithms provide good results. On the other hand, the adaptive WM algorithms follow the time-varying statistics, save the memory space and are easy to implement.

Now, we describe simple adaptive LMA WM algorithms [13] based on linear and sigmoidal approximation. In the case of sigmoidal approximation of the sign function, the convergence to global optimal solution cannot be guaranteed, since the optimization problem based on sigmoidal approximation has no global minimum.

The adaptive LMA WM filtering with the linear approximation updates the weight coefficients as follows:

$$w_i(n+1) = w_i(n) + 2\mu(n)[x_{\max}(n) - x_{\min}(n) - 2\left|o(n) - x_i(n)\right| - \\ -\sum_{j=1}^{N}\left(w_j(n)(x_{\max}(n) - x_{\min}(n) - 2\left|x_i(n) - x_j(n)\right|\right)] \tag{10}$$

where $i = 1, 2, ..., N$, $j = 1, 2, ..., N$, $\mu(n)$ is the adaptive step-size, e.g. a positive constant or a time-varying sequence. The algorithm is restricted by

$$w_i(n+1) = 0 \quad \text{if } w_i(n) < 0 \tag{11}$$

It can be easily seen that the adaptive LMA algorithm is significantly simpler than that of the non-adaptive WM filtering. Note that LMS algorithms can be derived from LMA structures by a simple change of an absolute norm to

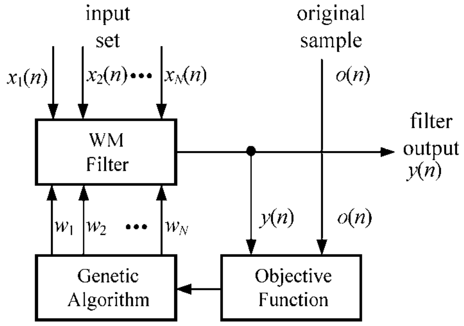

Fig. 1. Principle of genetically optimized WM filter.

a square [13].

C. Adaptive LMA WM with Linear Approximation

Consider the input set written as $\mathbf{x}(n) = \{x_1(n), x_2(n), ..., x_N(n)\}$ and the original (desired) sample $o(n)$, all for the time position *n*. In the case of sigmoidal approximation [13], the adaptive LMA WM algorithm can be simplified to the following expression

$$w_i(n+1) = P\left[w_i(n) + 2\mu(o(n) - y(n))\text{sgn}(x_i(n) - y(n))\right] \quad (12)$$

where $i = 1, 2, ..., N$, $P(.)$ characterizes the projection operation and $\text{sgn}_s(.)$ is the sign function approximated by sigmoidal function defined as

$$\text{sgn}_S(a) = \frac{2}{1+e^a} - 1 \quad (13)$$

If the actual WM output is smaller than the original value, the weights corresponding with the samples which are larger than the actual output are incremented. Note that the sigmoidal function is a very popular choice in the field of neural networks.

IV. Proposed Method

Now, we provide the genetic optimization (Fig. 1) of the weight coefficients. Genetic algorithms (GA) [17], [18] belongs to the field of biologically-oriented computational techniques. The GA is useful for searching the optimal solution in situations, where other optimization techniques may fail dramatically. The reason lies in a complex specification of initial conditions related to mathematically oriented methods.

In general, solved problems can be specified through a wide space of possible solutions, where it is very difficult to determine the optimal one. In the GA optimization problem [18], an individual represented by a set of parameters (genes in chromosomes) expresses the potential solution of the problem. The optimization requires the quality evaluating of generated solutions. The measure of the individual quality is known as a fitness value and it reflects the degree of negotiability enforcing during the evolution. Note that $o(n)$ represents the desired signal used to determine the quality of solutions. Since, the GA utilizes the mechanism of natural selection through the evolution method, an optimal solution can be represented by the individual with the largest fitness in the last iteration.

The genetic optimization [17], [18] starts with randomly generated solutions of the problem. The number of solutions is represented by the size of population. The

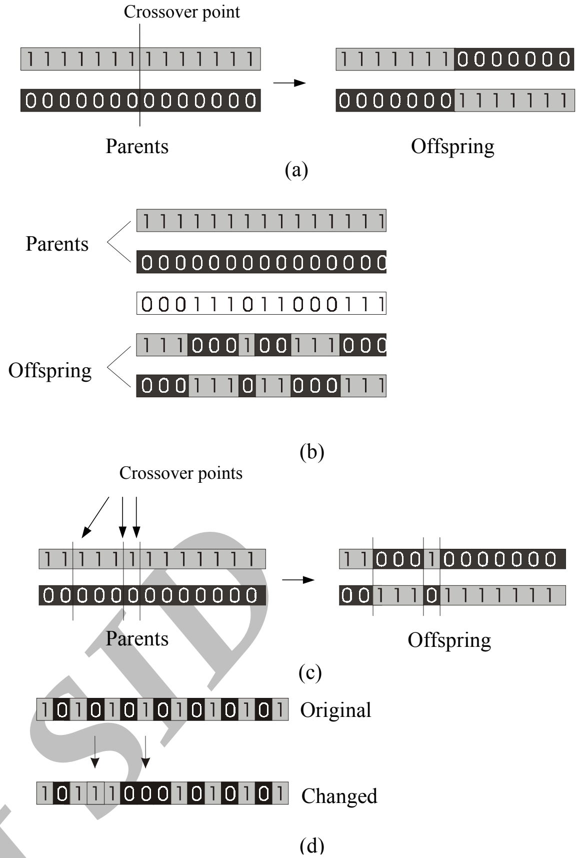

Fig. 2. Crossover and mutation: (a) one-point crossover, (b) uniform crossover, (c) three-point crossover, (d) mutation.

following generation is created from the chromosomes in the current population. However, because of the genetic evolution, the individual (chromosome) with larger fitness tends to yield a superior offspring which means better solution of the problem. For that reason, the significant emphasis is imposed on the set of parent chromosomes that is selected by a roulette wheel selection. After the selection, the genes of the parent chromosomes are mixed and recombined for the production of the offspring in the next generation.

Besides the selection procedure, the genetic optimization requires the applying of two operators (Fig. 2) such as the crossover and the mutation operators [17], [18]. After a randomly selected crossover point, the right-positioned parts of two parent chromosomes are exchanged to form the offspring. A crossover operator generates the offspring from two individuals of a current population. After the crossover operation, the random information is introduced to the offspring by a mutation operator.

In this paper, the parameters of the GA were set as follows: the population size consisted of 120 chromosomes and the number of training cycles was 1000. Fig. 3 shows the dependence of the number of iterations necessary for searching the minimum error on crossover probability p_c and mutation probability p_m related to the genetically optimized WM filter. It can be seen that in many cases, the GA results in a good solution after 50th iteration. Note that in the rest of the paper, the considered optimal weight coefficients related to the genetically optimized WM filter were achieved with the crossover probability $p_c = 0.95$ and the mutation probability $p_m = 0.05$.

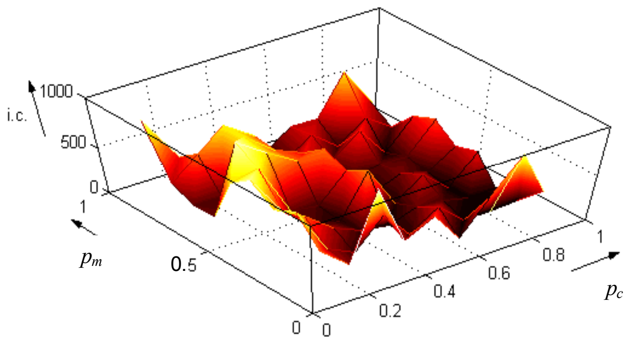

Fig. 3. Dependence of the number of iterations necessary for searching the minimum error on the crossover probability p_c and the mutation probability p_m related to the genetically optimized WM filter.

V. Experimental Results

In this Section, the performance of presented approaches is tested for two well-known images such (Figs. 4(a) and 5(a)) as the images Lena and Bridge with a various complexity. We will observe the impulsive noise (bit errors) attenuation capability, the measure of the signal-detail preservation and also the filter robustness.

A. Noise Model

In order to simulate the noise corruption (Figs. 4(b) and 5(b)), we use the model of the random valued impulsive noise [1], [16] expressed as

$$x_{i,j} = \begin{cases} o_{i,j} & \text{with probability } 1-p_\upsilon \\ \upsilon & \text{with probability } p_\upsilon \end{cases} \tag{14}$$

where i,j characterize the sample position, $o_{i,j}$ is the sample from the original image, $x_{i,j}$ represents the sample from the noisy image, p_υ is a corruption probability and υ is a random image intensity value.

Although the occasions for the generation of impulsive noise are various, it is often introduced through bit errors [1], especially in modern communication technologies and multimedia applications. Mathematically, the bit errors are defined by following expression

$${}^*k_{i,j}^m = \begin{cases} k_{i,j}^m & 1-p_\upsilon \\ 1-k_{i,j}^m & p_\upsilon \end{cases} \tag{15}$$

where i,j characterize the sample position, m is a bit level forced to be between 1 and B (a number of bits per sample), p_υ is a bit error probability and finally $\{k\}$ and $\{{}^*k\}$ characterize original and corrupted bit levels. Note that the original sample is expressed as

$$o_{i,j} = k_{i,j}^1 2^{B-1} + k_{i,j}^2 2^{B-2} + \ldots + k_{i,j}^{B-1} 2 + k_{i,j}^B \tag{16}$$

whereas a sample from the noisy image is defined by

$$x_{i,j} = {}^*k_{i,j}^1 2^{B-1} + {}^*k_{i,j}^2 2^{B-2} + \ldots + {}^*k_{i,j}^{B-1} 2 + {}^*k_{i,j}^B \tag{17}$$

It is clear that the degree of the impulsive noise corruption depends on the impulse probability p_υ and the affected bit level m.

B. Objective Criteria

The quality of the filtered images or in other words, the difference between filtered and original image is evaluated by two objective criteria [1], [16] such as the mean absolute error (MAE) and mean square error (MSE). The first criterion expresses the signal-detail preservation well, whereas the second one represents the measure of the noise suppression. Two-dimensional definitions of MAE and MSE are expressed as

$$MAE = \frac{1}{KL}\sum_{i=1}^{K}\sum_{j=1}^{L}\left|o_{i,j} - x_{i,j}\right| \tag{18}$$

$$MSE = \frac{1}{KL}\sum_{i=1}^{K}\sum_{j=1}^{L}\left(o_{i,j} - x_{i,j}\right)^2 \tag{19}$$

where K, L represent image dimensions, i, j determine the time position, $o_{i,j}$ and $x_{i,j}$ are samples from original and noisy (filtered) images, respectively.

Note that two objective criteria are necessary, since the noise filtering problem is a multicriterion task, where the balance between the noise suppression (expressed through MSE) and the signal detail preservation (expressed through MAE) should be achieved.

C. Achieved Results

Now, we analyze the genetic optimization of WM filters. Let us consider the training set given by the test image Lena corrupted by 10% impulsive noise ($p_\upsilon = 0.10$). Fig. 6 shows the error criteria of genetic optimization in the dependence on the crossover probability p_c and mutation probability p_m. Note that Fig. 6 is related to the population size and subpopulation size equal to 120 and 80, respectively. It can be seen that the GA error criteria increases with the increased mutation probability, because the new population includes a number of new individuals out of the relevant solutions.

Another results of the genetic optimization are shown in Fig. 7 and Tables I and II. Note that Table I is related to $p_c = 0.9$ and $p_m = 0.1$, whereas Table II is related to the population and subpopulation size 120 and 80, respectively. These results confirm that in many situations the GA optimization is time consuming, because the GA reaches the optimal solution after some iterations. Fig. 7 corresponds to MAE and MSE error criteria related to four various initial settings of the GA. Thus, GA1, GA2, GA3 and GA4 characterize the settings described in Table III. The results presented in Fig. 7 show that the convergence to a good solution depends on initial conditions. To illustrate the used processing time necessary to perform 1000 iterations of the GA optimization, the optimization procedure written in C language required 8 hours and 20 minutes using the personal computer (PC) with processor AMD K6-2 500 MHz; and 4 hours and 25 minutes using the PC with processor AMD Duron 900 MHz, respectively.

In general, the computational time increases with the increased population and subpopulation size and also with the increased crossover and mutation probabilities.

In order to observe the performance of all relevant filtering approaches, the presented filtering algorithms were tested for the impulsive noise corruption p_υ ranged from no corruption to 0.30. Note that all the pictures from no corruption to 0.30. Note that all the pictures presented in Figs 4(c) to 4(f) and Figs. 5(c) to 5(f) are related to the filtering of Fig. 4(b) ($p_\upsilon = 0.1$) and Fig. 5(b) ($p_\upsilon = 0.05$), respectively. The evaluating of results is presented in Tables IV to VII. Fig. 4(c) corresponds to the output of the well-known median filter that provides excellent noise

Fig. 4. Achieved results using the test image Lena: (a) original image, (b) noisy image with $p_\upsilon = 0.1$, (c) output of the median filter, (d) output of the non-adaptive LMA WM filter, (e) output of the adaptive sigmoidal WM filter, (f) output of the genetically optimized WM filter.

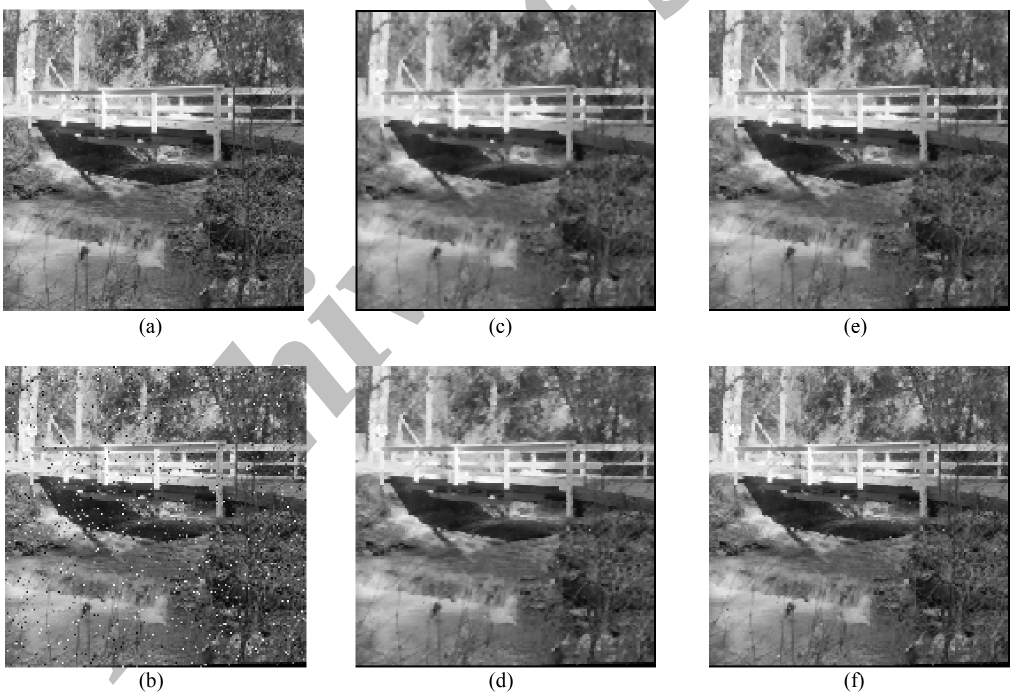

Fig. 5. Achieved results using the test image Bridge (a) original image, (b) noisy image with $p_\upsilon = 0.05$, (c) output of the median filter, (d) output of WM1 filter with $\mathbf{w} = [1,1,2,3,5,3,2,1,1]$, (e) output of the adaptive linear WM filter, (f) output of the genetically optimized WM filters.

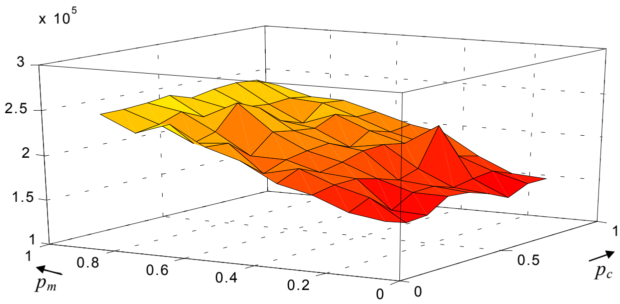

Fig. 6. Dependence of the GA error function on the crossover probability p_c and the mutation probability p_m related to the genetically optimized WM filter.

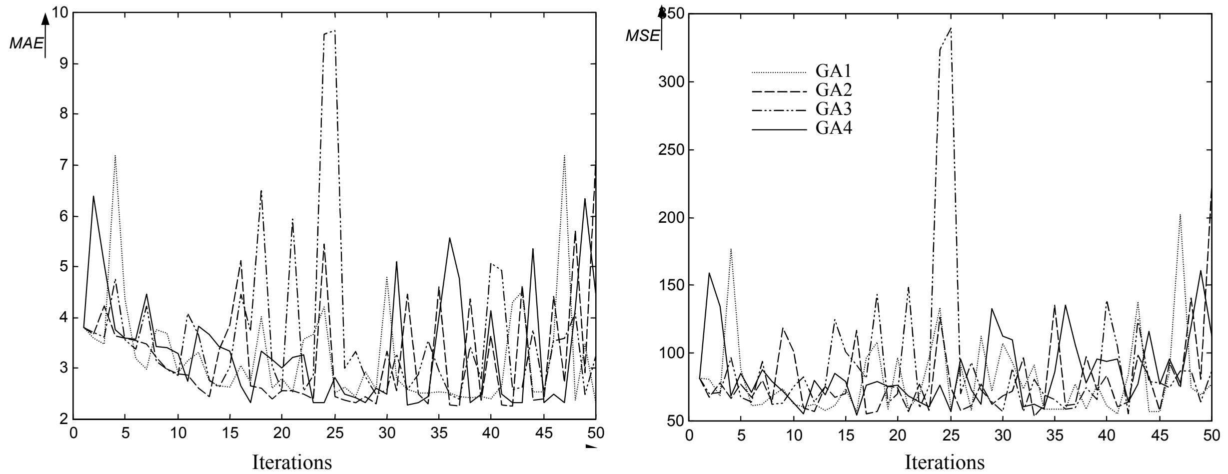

Fig. 7. Performance of the genetically optimized WM filter in the dependence on the number of iterations. We used the test image Lena corrupted by 10% impulsive noise: (a) MAE, (b) MSE.

TABLE I
COMPUTATIONAL TIME [SEC.] OF GWM OPTIMIZATION USING 10 ITERATIONS

Population / Subpopulation	50	40	30	20
120	275	236	183	145
100	266	218	174	139
80	261	192	170	121
60	254	181	165	114

TABLE II
COMPUTATIONAL TIME [SEC.] OF GWM OPTIMIZATION USING 10 ITERATIONS

Crossover / Mutation	0.1	0.2	0.3	0.4
0.9	399	410	419	426
0.8	366	374	386	399
0.7	325	337	352	375
0.6	295	314	331	356

TABLE III
DESCRIPTION OF THE GA PARAMETERS

Criteria / Method	GA1	GA2	GA3	GA4
Population size	120	100	80	80
Subpopulation size	80	80	60	40
Crossover probability	0.9	0.9	0.95	0.95
Mutation probability	0.1	0.1	0.05	0.05

TABLE IV
ACHIEVED RESULTS USING 2% IMPULSIVE NOISE

Image	Lena		Bridge	
Method	MAE	MSE	MAE	MSE
Noisy	1.456	157.4	1.583	182.9
Median	4.373	79.8	7.481	151.6
WM1	2.903	55.7	4.340	77.6
WM2	3.020	47.0	4.598	76.5
NWM	2.622	45.0	4.552	87.6
ALWM	2.751	46.8	4.524	84.7
ASWM	1.807	26.9	3.357	63.3
GWM	1.331	21.8	2.218	40.4

TABLE V
ACHIEVED RESULTS USING 5% IMPULSIVE NOISE

Image	Lena		Bridge	
Method	MAE	MSE	MAE	MSE
Noisy	3.540	374.3	3.568	398.9
Median	4.563	85.4	7.644	157.5
WM1	3.119	63.1	4.604	86.1
WM2	3.204	53.0	4.814	83.3
NWM	2.801	50.0	4.786	94.6
ALWM	2.918	51.2	4.747	91.5
ASWM	2.033	34.3	3.667	75.7
GWM	1.639	35.4	2.631	59.4

attenuation capability, however, too much smoothing introduced to the output image results in a blurring of thin image details and edges. WM1 and WM2 defined by weight vectors $\mathbf{w}=[1,1,2,3,5,3,2,1,1]$ and $\mathbf{w}=[1,4,1,4,7,4,1,4,1]$, respectively, achieve better detail the preservation in comparison with the median filter, however, in some situations the above-mentioned setting of the filter weights can lead to worse filter performance. Optimized WM filters can achieve the balance between the noise suppression and the signal-detail preservation. The results presented in Tables IV to VII, Figs. 4(d) to 4(f) and optimized on the image Lena degraded by 10% impulsive Figs. 5(e) and 5(f) are related to optimal WM filters noise ($p_\upsilon = 0.1$). In general, the performance of non-adaptive LMA WM filter (NWM) shown in Fig. 4(d) and adaptive WM filter with the linear approximation (ALWM) is robust for the whole range of the noise corruption. In case of adaptive WM filter with the sigmoidal approximation (ASWM) and genetically optimized WM filter (GWM), these filters (Figs. 4(e) and 4(f)) achieve excellent improvement in comparison with other presented methods, especially in terms of the MAE.

Their preservation capability decreases with the increased degree of the noise corruption. It is possible to observe some impulses presented in the output images that result in larger value of MSE (Fig. 7(b)). Concerning the MAE criterion, this dependence does not change so dramatically (Fig. 7(a)). The same behavior can be observed for the second test image Bridge. The filter outputs shown in Figs. 5(c) to 5(e) are related to the filtering of the image corrupted by impulse noise with $p_\upsilon = 0.05$ (Fig. 5(b)). In Fig. 8, we also provided error criteria in dependence on the degree of impulsive noise using the test image Bridge. We also provide Figs. 10 and 11 that correspond to estimation errors of relevant filters related to the images different from the training set and

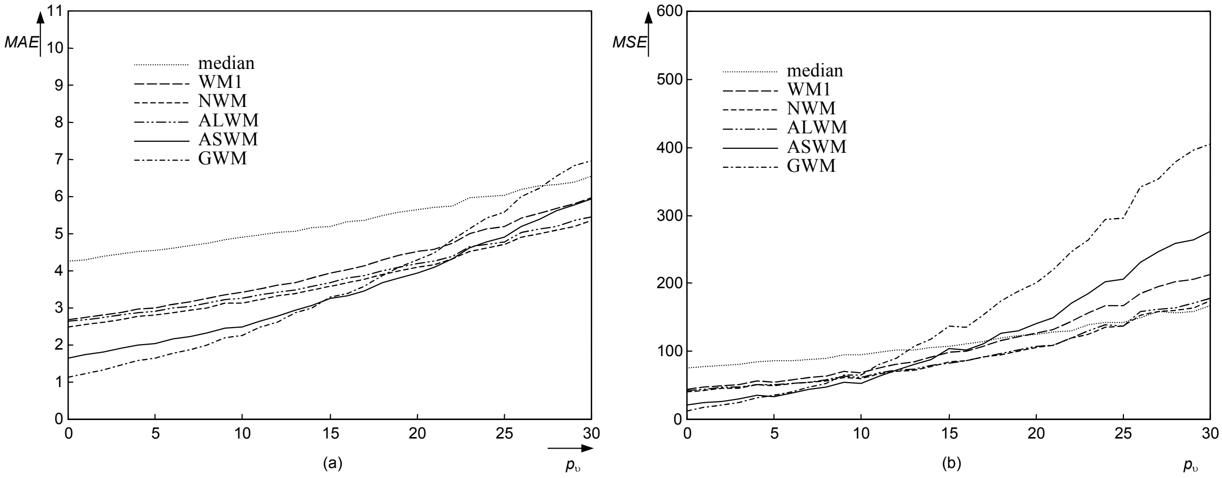

Fig. 8. Dependence of error criteria on degree of the impulse noise corruption related to the test image Lena: (a) MAE, (b) MSE.

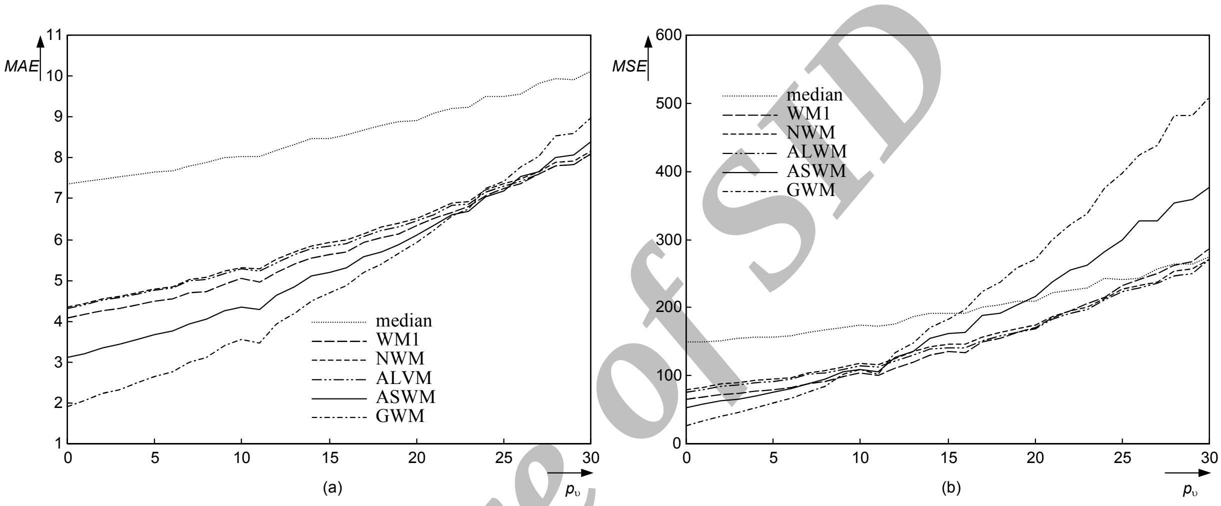

Fig. 9. Dependence of error criteria on degree of the impulse noise corruption related to the test image Bridge: (a) MAE, (b) MSE.

TABLE VI
ACHIEVED RESULTS USING 10% IMPULSIVE NOISE

Image	Lena		Bridge	
Method	MAE	MSE	MAE	MSE
Noisy	7.018	759.1	7.221	807.6
Median	4.888	94.3	8.042	173.7
WM1	3.536	76.9	5.197	113.4
WM2	3.584	65.0	5.392	107.9
NWM	3.138	60.3	5.318	118.0
ALWM	3.261	62.2	5.283	115.2
ASWM	2.488	53.2	4.350	108.9
GWM	2.254	64.3	3.561	108.4

TABLE VII
ACHIEVED RESULTS USING 20% IMPULSIVE NOISE

Image	Lena		Bridge	
Method	MAE	MSE	MAE	MSE
Noisy	13.168	1414.1	13.616	1522.3
Median	5.640	125.4	8.909	208.6
WM1	4.705	139.6	6.512	179.0
WM2	4.615	119.2	6.675	174.0
NWM	4.096	106.2	6.525	174.9
ALWM	4.194	108.1	6.444	169.9
ASWM	3.951	141.1	6.091	216.0
GWM	4.276	201.2	5.919	270.8

Fig. 12 that shows enlarged fragments of output images achieved using the training set. It can be seen again that the traditional filtering schemes fail on the image details and edges, because their excessive smoothing capability distorts the edges and thin details in the image. In case of the ASWM and the proposed GWM filter, their estimation errors are the result of insufficient noise attenuation capability. The same behavior is observed in enlarged fragment of output images shown in Fig. 12. The standard filtering schemes (Figs. 12(c) and 12(d)) blur the image details, whereas AWVM and GWM approaches provide excellent estimation capabilities.

Apart from the numerical behavior of the presented optimization approaches, their computational complexity and overall evaluation are realistic measures (Table VIII)of their usefulness. Excellent detail preservation capability and adaptability to training condition are the best advantages of the proposed method. Its drawback is a high computational complexity so that searching for global optimum is time consuming. The adaptive ALWM and ASWM need a few seconds to achieve optimal weights, whereas the proposed WM optimization can take some computational hours using the same processor. On the other hand, the proposed method is able to achieve the global solution of the filtering problem. However, too high specialization on the training set can lead to the decreased robustness against noise.

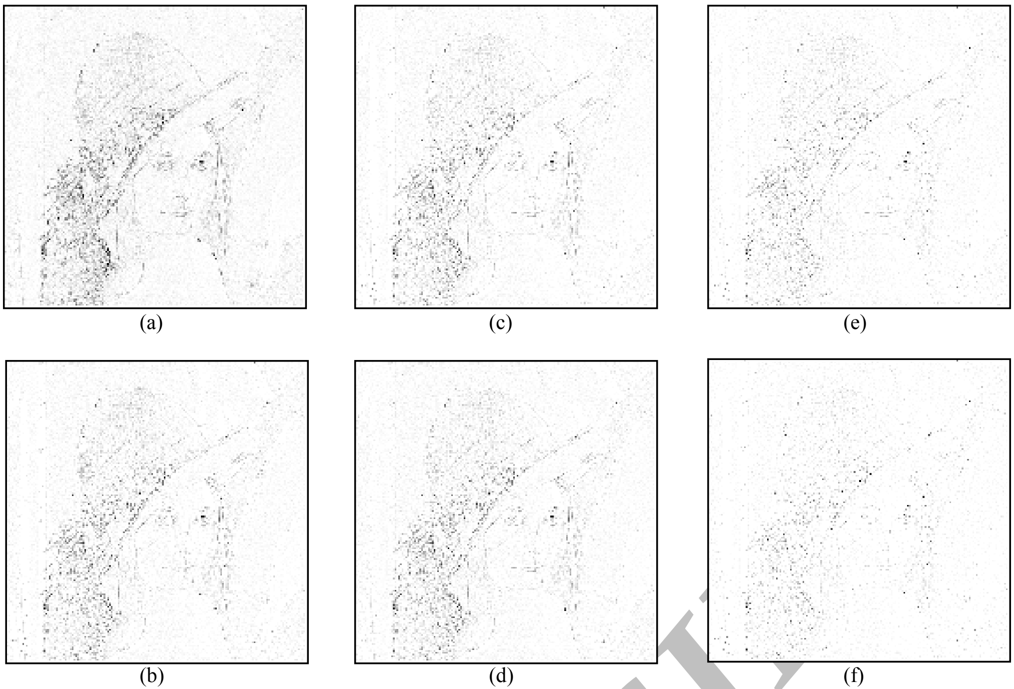

Fig. 10. Estimation errors emphasized by factor 3 related to the test image Lena degraded by 2% impulsive noise (optimal filters were optimized using the image Lena and 10% impulsive noise): (a) median, (b) WM1, (c) NWM, (d) ALWM, (e) ASWM, (f) GWM.

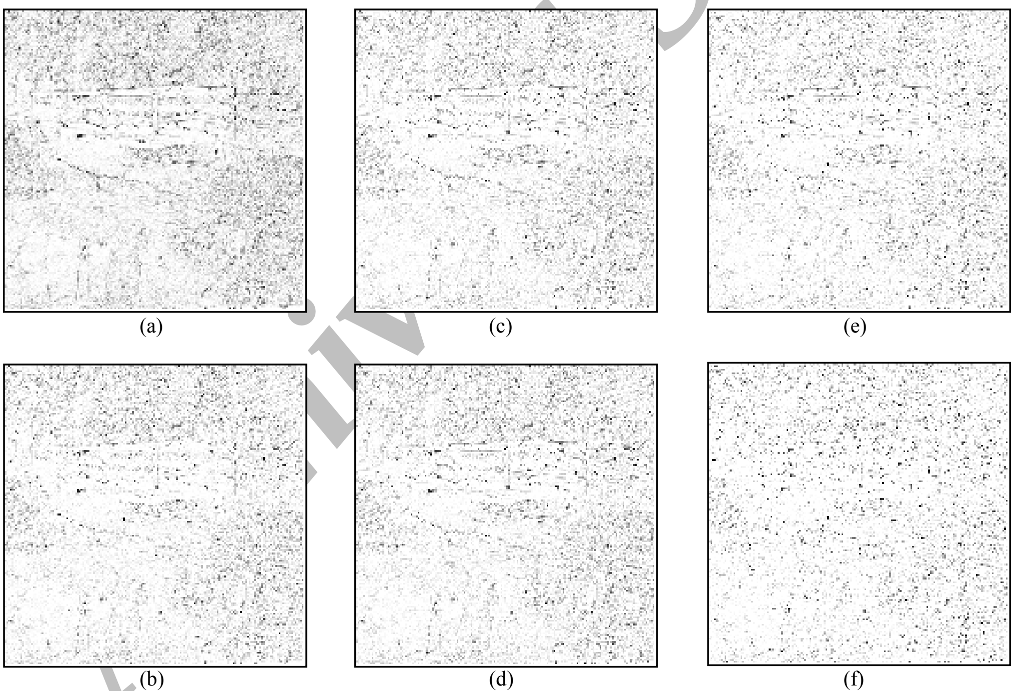

Fig. 11. Estimation errors emphasized by factor 3 related to the test image Bridge degraded by 15% impulsive noise (optimal filters were optimized using the image Lena and 10% impulsive noise): (a) median, (b) WM1, (c) NWM, (d) ALWM, (e) ASWM, (f) GWM.

VI. Conclusion

In this paper, the standard estimation approaches and the proposed genetic optimization of WM filters were analyzed in terms of impulsive noise attenuation, signal detail preservation and robustness. In order to adapt the WM filters to removal of impulsive noise, WM filters were optimized using four optimization approaches. The achieved weight vectors have been successfully used for a variety of test images and degrees of the impulse noise corruption. Concerning filter performance, the proposed genetically optimized WM filter can outperform the well-

TABLE VIII
Overall comparison of the WM optimization approaches

Criteria / Method	NWM	ALWM	ASWM	GWM
Robustness against noise	Excellent	Excellent	Good	Good
Preservation capability	Worse	Worse	Good	Excellent
Complexity	High	Low	Low	High
Adaptability	Good	Good	Excellent	Excellent
Globally optimal	Yes	No	No	Yes

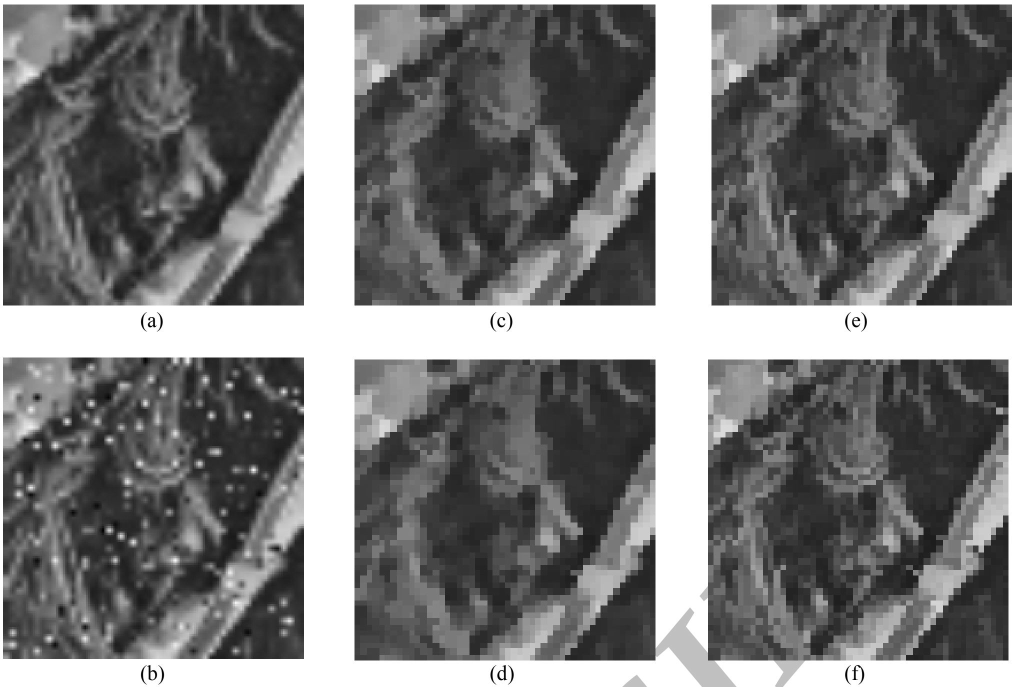

Fig. 12. Zoomed results obtained using the Lena test image: (a) original image, (b) training set (10% impulsive noise), (c) NWM output, (d) ALWM output, (e) ASWM output, (f) GWM output.

known estimation approaches especially in terms of the signal detail preservation, however, its drawbacks are high computational complexity and decreased robustness for highly corrupted images. Another advantage of the proposed method lies in the globally optimal weight vector for given training set.

REFERENCES

[1] J. Astola and P. Kuosmanen, *Fundamentals of Nonlinear Digital Filtering*, CRC Press LLC, 1997.

[2] S. J. Mitra and G. L. Sicuranza, *Nonlinear Image Processing*, Academic Press, 2001.

[3] I. Pitas and A. N. Venetsanopoulos, *Nonlinear Digital Filters, Principles and Applications*, Kluwer Academic Publishers, 1990.

[4] I. Pitas and A. N. Venetsanopoulos, "Order statistics in digital image processing," *Proceedings of the IEEE*, vol. 80, no. 12, pp. 1892-1919, Dec. 1992.

[5] P. T. Yu and W. H. Liao, "Weighted order statistics filters – their classification, some properties, and conversion algorithm," *IEEE Trans. on Signal Processing*, vol. 42, no. 10, pp. 2678-2691, Oct. 1994.

[6] R. Lukac and S. Marchevsky, "Boolean expression of LUM smoothers," *IEEE Signal Processing Letters*, vol. 8, no. 11, pp. 292-294, Nov. 2001.

[7] R. Lukac, "Binary LUM smoothing," *IEEE Signal Processing Letters*, vol. 9, no. 12, pp. 400-403, Dec. 2002.

[8] S. Peltonen, M. Gabbouj, and J. Astola, "Nonlinear filter design: methodologies and challenges," in *Proc. 2nd IEEE EURASIP Symp. ISPA'01*, pp. 102-107, Pula, Croatia, Jun. 2001.

[9] M. Gabbouj and E. J. Coye, "Minimum mean absolute error stack filtering with structural constraints and goals," *IEEE Trans. on Acoustics Speech and Signal Processing*, vol. 38, no. 6, pp. 955-968, Jun. 1990.

[10] J. L. Paredes and G. R. Arce, "Optimization of stack filters based on mirrored threshold decomposition," *IEEE Trans. on Signal Processing*, vol. 49, no. 6, pp. 1179-1188, Jun. 2001.

[11] M. K. Prasad and Y. H. Lee, "Stack filters and selection probabilities," *IEEE Trans. on Signal Processing*, vol. 42, no. 10, pp. 2628-2643, Oct. 1994.

[12] R. Yang, L. Yin, M. Gabbouj, J. Astola, and Y. Neuvo, "Optimal weighted median filtering under structural constraints," *IEEE Trans. on Signal Processing*, vol. 43, no. 3, pp. 591-604, Mar. 1995.

[13] L. Yin, R. Yang, M. Gabbouj, and Y. Neuvo, "Weighted median filters: a tutorial," *IEEE Trans. on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 43, no. 3, pp. 157-192, Mar. 1996.

[14] R. Yang, M. Gabbouj, and Y. Neuvo, "Fast algorithms for analyzing and designing weighted median filters," *Signal Processing*, vol. 41, no. 2, pp. 135-192, Jan. 1995.

[15] R. Lukac and S. Marchevsky, "Conversion between center-weighted medians and positive Boolean functions," in *Proc. of 2nd IEEE EURASIP Symp. on Image and Signal Processing and Analysis ISPA'01*, pp. 396-398, Pula, Croatia, Jun. 2001.

[16] R. Lukac and S. Marchevsky, "LUM smoother with smooth control for noisy image sequences," *EURASIP Journal on Applied Signal Processing*, vol. 2001, no. 2, pp. 110-120, Jun. 2001.

[17] D. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading, Massachusetts, 1989.

[18] K. F. Man, K. S. Tang, and S. Kwong, *Genetic Algorithms (Concept and Design)*, Springer Verlag, London, 1999.

Rastislav Lukac received the Diploma in Telecommunications degree with honors in 1998 and the Ph.D. degree in 2001, both at the Technical University of Kosice, Slovak Republic. From February 2001 till August 2002 he was an Assistant Professor at the Department of Electronics and Multimedia Communications at the Technical University of Kosice. Since August 2002 he is a researcher in Slovak Image Processing Center in Dobsina, Slovak Republic and since February 2003 also a post-doc at Artificial Intelligence and Information Analysis Lab at the Aristotle University of Thessaloniki, Greece. In 2003, he was awarded the NATO Science Fellowship for a one-year stay at Multimedia Laboratory of the University of Toronto, Canada.

Dr. Lukac is a member of the IEEE Signal Processing Society. He is an active member of Review and Program Committees at various European conferences and a reviewer for some scientific journals. Recently, his research interests include nonlinear digital filters, impulse detection, color image processing, image sequence processing and the use of Boolean functions and permutation theory in filter design.

Alexander Varga received the M.Sc. degree at the Technical University of Kosice, the Slovak Republic, at the Department of Electronics and Multimedia Communications in 2001. The title of his M.Sc. work is Weighted Median Filter Optimization by Genetic Algorithm.